Permutation Matrix

- Matrix P is a *permutation matrix* if every element is 0 except for one 1 in each row and each column.
- The product PA is A with rows permuted.
- The product AP^T is A with columns permuted the same way.
- Vector p is a *permutation vector* if it contains every number in range(n) exactly once.
- P = np.eye(n)[p, :] is a permutation matrix such that:
  - row permutation: A[p, :] is equal to PA
  - column permutation: A[:, p] is equal to AP^T

Symmetric Positive Definite (SPD) Matrix

- An SPD matrix acts sort of like a positive number.
- A is *symmetric* if $a_{ij} = a_{ji}$, for all i and j.
- Several equivalent conditions for A to be *positive definite*:
  - All eigenvalues are > 0
  - LU factorization without pivoting succeeds, and all pivots are > 0
  - For every nonzero vector x , the number $x^T A x > 0$
- SPD matrices come up a lot in scientific computing & data analysis!
- The temperature matrix is SPD.

Orthogonal Matrix

- Matrix Q is *orthogonal* if the matrix $Q^T Q = I$ is the identity.
- An n-by-n orthogonal matrix represents a rotation or reflection of vectors in n-space.
- It acts sort of like a number whose absolute value is 1.
- Examples: I (identity matrix), any permutation matrix P.
- The inverse of Q is the transpose of Q.
- The columns of Q are mutually perpendicular (orthogonal).
- Every column of Q has length equal to 1.
- The same holds for the rows of Q.