CS 111: Sample Problems for Midterm 1

See Exam e01 on the course GitHub page for midterm rules, syllabus, and more sample problems.

1. Let x, p, and A be defined by the following numpy statements:

```
x = np.array([31, 41, 59])
p = np.array([2, 0, 1])
A = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
```

What is the value of y after each of the following?

1a. `y = x[p]`

1b. `y[p] = x`

1c. `y = A[[2, 0], 1:]`

1d. `y = A[:2, p]`

2. Suppose that:

```
A = np.array([[1, 1, 1], [1, 2, 3], [1, 3, 6]])
x = npla.solve(A, [3, 8, 15])
y = A @ [3, -1, 0]
L, U, p = cs111.LUfactor(A)
B = spla.cholesky(A, lower = True)
C = npla.inv(A)
D = L @ U
E = U @ L
```

Match each of x, y, L, U, p, B, C, D, E with one of the following:

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & .5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1.5 & 1 \\ 7 & 4.5 & 5 \\ -.5 & -.25 & -.5 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 6 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & -.5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$$(2, 1, 0)^T$$

$$(0, 1, 2)^T$$

$$(0, 2, 1)^T$$

3. Suppose A is an n by n symmetric, positive definite matrix and b is a column vector of dimension n. Recall that the (lower triangular) Choleksy factor L of A satisfies $LL^T = A$. Fill in the blanks below to compute the solution x to $Ax = b$. You may use L but not A.

```
L = spla.cholesky(A, lower=True)


y = ______________________________


x = ______________________________
```

4. True or false:

- LU factorization without pivoting works on every nonsingular square matrix.
- LU factorization without pivoting works on every symmetric positive definite matrix.
- LU factorization with partial pivoting works on every nonsingular square matrix.
- LU factorization with partial pivoting works on every symmetric positive definite matrix.
- Cholesky factorization works on every nonsingular square matrix.
- Cholesky factorization works on every nonsingular symmetric matrix.
- Cholesky factorization works on every symmetric positive definite matrix.
- Conjugate gradient works on every nonsingular square matrix.
- Conjugate gradient works on every nonsingular symmetric matrix.
- Conjugate gradient works on every symmetric positive definite matrix.
- QR factorization works on every nonsingular square matrix.
- QR factorization works on every nonsingular symmetric matrix.

- QR factorization works on every symmetric positive definite matrix.

5. Let A and B be n-by-n matrices, and let v be an n-vector. As a function of n in asymptotic terms (such as $O(n \log n)$, ignoring constants), how many floating-point arithemtic operations are performed in each of the following:

- Computing the matrix-matrix product $A^T B$.
- Computing the matrix-vector product Av.
- Computing the vector inner product $v^T v$.
- Using `cs111.LUsolve` to solve $Ax = v$ for x.

6. Consider the sparse matrix A returned by `A = cs111.make_A(20)`. How many rows does A have? How many of A's rows have exactly 1 nonzero? Exactly 2 nonzeros? Exactly 3 nonzeros? Exactly 4 nonzeros? Exactly 5 nonzeros? Exactly 6 nonzeros?

7. Which of the following matrices are orthogonal?

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}, \quad D = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$