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HW #2

1. a. If the algorithm is constant, than for both input size 100 and 500, the running time is 1 ms.

b. $x \cdot 100 = 1$ $\quad .01n = .01 \cdot 500 = 5$ ms
$x = .01$

c. $x \cdot 100^2 = 1$ $\quad .0001 n^2 = .0001 \cdot 500^2 = 25$ ms
$x = .0001$

2. a. $x^{.99} > \frac{x}{\log x} > 2 = \log\log\log x = \frac{1}{x}$
b. The last three all have very similar growth rates

b. $x! > 2^x > x \cdot 2^{\frac{x}{2}} > \log x^x$

3. $2^n - n^2 \in \Theta(2^n + n^2)$

a. $0 \leq \frac{1}{8}(2^n + n^2) \leq 2^n - n^2 \leq 1(2^n + n^2)$
for all $n \geq 8$

b. $\lim_{n \to \infty} \frac{2^n - n^2}{2^n + n^2} = \frac{1 - \frac{n^2}{2^n}}{1 + \frac{n^2}{2^n}} = \frac{\lim_{n\to\infty} 1 - \frac{n^2}{2^n}}{\lim_{n\to\infty} 1 + \frac{n^2}{2^n}} = \frac{1}{1} = 1$

When computing the limit, if the solution is finite and non-zero, then $f(n) = \Theta(g(n))$