

Problem 1.37

We already have:

$$H[x] = - \int p(x) \ln p(x) dx$$
$$H[y|x] = - \int \int p(x, y) \ln p(y|x) dx dy$$

It's straightforward by adding two above equations:

$$\begin{aligned} H[x] + H[y|x] &= - \left(\int p(x) \ln p(x) dx + \int \int p(x, y) \ln p(y|x) dx dy \right) \\ &= - \left(\int \int p(x, y) \ln p(x) dx dy + \int \int p(x, y) \ln p(y|x) dx dy \right) \\ &= - \left(\int \int p(x, y) \ln (p(x)p(y|x)) dx dy \right) \\ &= - \int \int p(x, y) \ln p(x, y) dx dy = H[x, y] \end{aligned} \quad (\text{Solved})$$