2024 Spring VLSI DSP Homework Assignment #1

Due date: 2024/3/19

1. Least square optimization problem

   For an over constrained linear system $\mathbf{Ax}=\mathbf{b}$, find the least square solution of $\mathbf{x}$ using

   a) Pseudo inverse $\hat{\mathbf{x}} = \mathbf{A}^{+} \cdot \mathbf{b}$

   b) QR decomposition,

   c) Compare if a) and b) yield the same result?

   Note: the pseudo inverse function is pinv() in matlab

$$\mathbf{A}_{8\times 4} = \begin{bmatrix} 15 & -13 & 20 & -8 \\ -5 & -15 & -4 & -4 \\ -17 & 16 & -2 & 9 \\ 10 & -19 & -14 & -15 \\ -7 & 8 & -7 & 15 \\ 14 & 10 & -8 & -17 \\ -5 & -3 & 16 & -2 \\ 13 & -5 & -10 & -19 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 13 \\ 10 \\ -15 \\ 9 \\ 3 \\ 18 \\ 3 \\ 20 \end{bmatrix}$$

2. Eigen decomposition

   For a symmetric 8×8 matrix $\mathbf{M}$ shown below, find its Eigen value decomposition using a QR decomposition based iterative algorithm.

$$\mathrm{M} = \begin{bmatrix} -2 & 16 & -6 & -16 & 3 & 15 & -6 & -19 \\ 16 & -17 & 10 & -2 & 7 & 8 & 3 & 5 \\ -6 & 10 & 15 & -1 & -15 & -18 & 9 & -8 \\ -16 & -2 & -1 & 9 & 0 & 0 & 0 & 18 \\ 3 & 7 & -15 & 0 & 14 & 19 & -12 & 11 \\ 15 & 8 & -18 & 0 & 19 & 10 & -8 & -17 \\ -6 & 3 & 9 & 0 & -12 & -8 & 15 & 20 \\ -19 & 5 & -8 & 18 & 11 & -17 & 20 & 20 \end{bmatrix}$$

   Eigen decomposition means to find a matrix decomposition of the format shown below

$$\mathbf{M} = \mathbf{Q} \cdot \mathbf{\Lambda} \cdot \mathbf{Q}^{t} \quad \text{where} \quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & 0 & \\ & 0 & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$ is a diagonal matrix consisting of real

   Eigen values. $\mathbf{Q}$ is orthogonal with the property $\mathbf{Q} \cdot \mathbf{Q}^{t} = \mathbf{I}$ and all its column vectors as Eigen vectors.

Iterative Algorithm

Note 1:

Step 1: Perform tri-diagonalization first using the Givens rotations to obtain $\widetilde{\mathbf{M}} = \mathbf{Q} \cdot \mathbf{M} \cdot \mathbf{Q}^t$, where $\widetilde{\mathbf{M}}$ is tri-diagonal. The numbers in the equation of $\widetilde{\mathbf{M}}$ indicate the ordering of nullification using Givens rotations without causing any fill-ins. Since $\mathbf{M}$ is symmetric, only the nullifications of the lower left portion are performed. The upper right portion will be identical to the lower left potion after applying the corresponding $\mathbf{Q}^t$ to the right.

$$\widetilde{\mathbf{M}} = \begin{bmatrix} x & x & 6 & 5 & 4 & 3 & 2 & 1 \\ x & x & x & 11 & 10 & 9 & 8 & 7 \\ 6 & x & x & x & 15 & 14 & 13 & 12 \\ 5 & 11 & x & x & x & 18 & 17 & 16 \\ 4 & 10 & 15 & x & x & x & 20 & 19 \\ 3 & 9 & 14 & 18 & x & x & x & 21 \\ 2 & 8 & 13 & 17 & 20 & x & x & x \\ 1 & 7 & 12 & 16 & 19 & 21 & x & x \end{bmatrix}$$

Step 2: Apply QR decomposition to make $\mathbf{Q}\widetilde{\mathbf{M}}$ and upper triangular matrix. Then compute $\mathbf{Q}\widetilde{\mathbf{M}}\mathbf{Q}^t$. This is called one "sweep" in the iterative process. Repeat the process until all off diagonal elements become negligible. The condition of convergence is $\det(\mathbf{Q}\widetilde{\mathbf{M}}\mathbf{Q}^t - diagonal(\mathbf{Q}\widetilde{\mathbf{M}}\mathbf{Q}^t)) / \det(\mathbf{M}) < 10^{-4}$

Verify your results with the matlab result using the function `[V,D] = eig(M)` and indicate the number of sweeps needed to obtain the results.