### Applying the Klein-Gordon Theory to Gravitation

Modelling Newtonian gravitation as a classical scalar field theory obeying Klein-Gordon structure

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## Towards Classical Field Theory

### The Inverse Square Law

Gravitational force:

$$F_m = -G \frac{Mm}{r^2}$$

Electrostatic force:

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{Q_e q_e}{r^2}$$

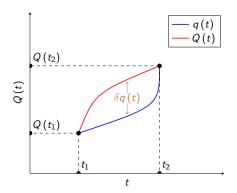
Magnetic force:

$$F_b = \frac{\mu_0}{4\pi} \frac{Q_b q_b}{r^2}$$

# Formal Analogies Between the Gravitational and Electrostatic Forces

	Gravitation	Static electricity
Newton's second law	$a^i = -\partial^i V$	$E^i = -\partial^i \phi$
	$-\overrightarrow{\nabla}V$	$-\vec{ abla}\phi$
Gauss' law	$\sum_{i=1}^{3} \nabla_{i} a^{i} = -4\pi G \rho_{m}$	$\sum_{i=1}^{3} \nabla_{i} E^{i} = \frac{1}{\epsilon_{0}} \rho_{e}$
	$ec{ abla} \cdot ec{a}$	$ec{ abla} \cdot ec{a}$
Poisson's equation	$\sum_{i=1}^{3} \nabla_{i} \partial^{i} V = 4\pi G \rho_{m}$	$\sum_{i=1}^{3} \nabla_{i} \partial^{i} \phi = -\frac{1}{\epsilon_{0}} \rho_{e}$
	$\underbrace{i=1}_{\nabla^2 V}$	$\underbrace{i=1}_{ abla^2\phi}$

### Lagrangian Mechanics



Nature 'selects' the unique on-shell trajectory q(t) given the boundary conditions  $(t_1, Q(t_1))$  and  $(t_2, Q(t_2))$  for a system.

$$\underbrace{Q(t)}_{Off-shell} = \underbrace{q(t)}_{On-shell} + \underbrace{\delta q(t)}_{Variation}$$

▶ Each trajectory Q(t) between the endpoints is associated with a corresponding number called the action.

$$S\left[Q\left(t\right)\right]\left(t_{1},t_{2}\right)=\int_{t_{1}}^{t_{2}}dt\,L\left(Q\left(t\right),\dot{Q}\left(t\right),t\right)$$

The integrand  $L\left(Q\left(t\right),\dot{Q}\left(t\right),t\right)$  is known as the Lagrangian of the system being modelled and encodes the dynamics of the system.

▶ In general, the action S maps Q(t) to a real number determined by the above integral. Therefore, it is a functional, i.e. a higher-order function which takes in infinite values of the form  $\{(t, Q(t)) : t \in \mathbb{R}\}$  and spits out a real.

$$S: \begin{cases} \mathbb{R}^{\mathbb{R}} & \to \mathbb{R} \\ \frac{Q(t)}{Q(t)} & \mapsto \int_{t_1}^{t_2} dt \ L\left(\frac{Q(t), \dot{Q}(t), t}{Q(t), t}\right) \end{cases}$$

### Principle of Stationary Action