

$V =$ set of all nodes

$$E = \left\{ (x, n) \mid x \in V, n \in N(x) \right\}$$

$$N(x) = \{n_1, n_2, \dots\}$$

$$G = (V, E)$$

$$N = \left\{ (x, N(x)) \mid x \in V \right\}$$

$$S = \{V, N\}$$

$$\bigwedge_i N_i \in N(x),$$

$$\bigcap_i N_i \in N(x), \quad \bigcup_i N_i \in N(x)$$

$$1. d(x, y) = d(y, x)$$

$$2. d(x, n_i) = d(x, n_j) \forall n_i, n_j \in N(x)$$

$$\text{Define } N_1(x) = \{n_1 \mid n_1 \in N_1(x)\} = N(x)$$

$$N_2(x) = \{n_2 \mid n_2 \in N(n_1) \forall n_1 \in N_1(x)\}$$

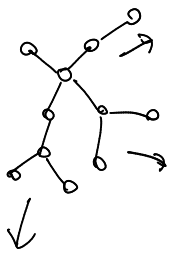
\vdots

$$N_{k+1}(x) = \{n_{k+1} \mid n_{k+1} \in N(n_k) \forall n_k \in N_k(x)\}$$

Then,

$$3. d(x, n_k) = d(x, n'_k) \forall n_k, n'_k \in N_k(x), x \in V$$

$$4. d(x, n_k) < d(x, n_{k+p}) \text{ if } p \in \mathbb{N} \forall n_k \in N_k(x), n_{k+p} \in N_{k+p}(x), x \in V$$



$$T = \{x_0, \mathcal{N}_1(x_0), \mathcal{N}_2(x_0), \dots\}$$

x_0 is 'origin'

$$T \equiv S \text{ iff } x \in \mathcal{N}_k(x_0) \mid k \in \mathbb{W}, x \in T$$



Affine space structure

Vector
bundle
structure

Thanks, Felix Hausdorff