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Vindication of entanglement-based witnesses of non-classicality in hybrid systems

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Abstract

Recently, Vedral and one of us proposed an entanglement-based witness of non-classicality in systems that need not obey quantum theory, based on information-theoretic ideas [1, 3], which offers a robust foundation for newly proposed table-top tests of non-classicality in gravity, [2, 3]. The witness asserts that if a mediator can entangle locally two quantum systems, then it has to be non-classical. Hall and Reginatto [4, 7] claimed that there are classical systems that can entangle two quantum systems, thus violating our proposed witness. Here we refute that claim, explaining that the counterexample proposed by Hall and Reginatto in fact validates the witness, vindicating the witness of non-classicality in its full generality.

Keywords: hybrids systems, witnesses of non-classicality, entanglement

1. Witnesses of non-classicality

Quantum theory is one of the best existing explanations of the physical world and it has so far proven unchallenged by experimental evidence. There are however several arguments [8] supporting the idea that quantum theory ceases to apply at a certain scale, for instance due to

the collapse of macroscopic quantum superpositions. It is often suggested that gravity may be responsible for this phenomenon. One of the reasons for this claim is that there is currently no unique viable proposal for a theory of quantum gravity; in addition, the predictions of existing models appear to be beyond current experimental capabilities, [9]. The best explanation of gravity is, still, general relativity—which is a classical theory, in the sense that it does not admit quantum superpositions and related effects.

Recently, an experimental scheme has been proposed in order to refute classical theories of gravity in the presence of quantum masses, [2, 3]. This scheme is based on probing gravity with two masses, each in a superposition. If gravity can entangle them, then gravity must be non-classical. As perceptively noted by Hall and Reginatto, [7], it is improper to talk about such tests as proving that gravity obeys a particular quantum theory model. What they actually do is to show that, if gravitational entanglement is observed between the masses, gravity must be non-classical—which is a lesser property than having the full set of quantum features. By non-classical we mean that gravity must have at least two variables that are necessary to describe its features, and yet cannot be measured to arbitrarily high accuracy simultaneously (i.e. by the same measuring system). This notion generalises what in quantum theory is called complementarity and it can be defined in general information-theoretic terms, [1].

There are two notable features in this proposed witness, which make it interesting. One is that it probes a regime where quantum-gravity effects are relevant and measurable, but general relativity's effects are not, because gravity can be described in the Newtonian limit. The other is that the witness is based on a general theorem that can be proven with minimal assumptions about the dynamics that gravity obeys. This, in short, is the theorem: if a system $\mathbf{M}$ (e.g. gravity) can entangle two quantum systems $\mathbf{Q_A}$ and $\mathbf{Q_B}$ (e.g. two masses) by local interactions (by $\mathbf{Q_A}$ coupling to $\mathbf{M}$ and $\mathbf{M}$ to $\mathbf{Q_B}$, but without $\mathbf{Q_A}$ and $\mathbf{Q_B}$ interacting directly) then $\mathbf{M}$ must be non-classical.

The theorem, in this general form, rests on two main principles, explicitly expressed in [1]:

- One is the principle of locality, or no action at a distance, that has to be satisfied by the dynamical theories describing the mediator and the probes—this principle then allows one to assume pairwise interactions of each quantum probe with the mediator, but not with each other.
- The other is the principle of interoperability of information, [11]. Informally, this principle requires that information can be copied from one system that is capable of containing information, to any other such system with at least the same capacity. Note that 'capable of containing information' has a precise operational definition, in terms of possible and impossible tasks. Specifically, given two systems that can each be prepared in any of a number of states that can be permuted in all possible ways, and copied, the interoperability principle requires that their composite system must, too, have a set of attributes that can all be permuted and copied. This principle is a pillar of both quantum and classical information theory, as proven in [11].

These two principles are obeyed by our best physical theories proposed so far, both classical and quantum: locality is necessary for the existence of independent subsystems, and both principles are necessary for testability. By relying on these two principles, one need not assume a specific dynamics for the mediator. This is essential in order to interpret the test of non-classicality in gravity as ruling out, in case of observing entanglement, a large class of classical theories for the mediator, not just a specific decohered version of linear quantum gravity (as instead is assumed in [2, 5]).

In this sense, the theorem supporting these witnesses is like Bell's theorem: by violating Bell's inequalities, one can rule out a vast class of theories obeying general probabilistic assumptions; likewise, when observing entanglement between the masses, one can rule out all classical theories of gravity obeying the above-mentioned general principles: classical theories that are known (e.g. quantum field theory in curved spacetime, general relativity, some collapse theories, [1]) and those that have yet to be discovered.

As a pedagogical explanation of the theorem, consider a quantum example, where the mediator **M** between two quantum systems $\mathbf{Q}_A$ and $\mathbf{Q}_B$ is a channel operating by means of Local Operations (LO). Assuming that the mediator is classical implies, in this example, that the LO it can induce is a form of classical communication (CC) between the two quantum system. In other words, a classical mediator is an instance of a LOCC, as considered in [2, 5]. It is well-known that LOCC cannot increase the entanglement already present in the system [6]; hence, any classical mediator in this sense would be ruled out if one could use it to enhance the entanglement between $\mathbf{Q}_A$ and $\mathbf{Q}_B$. Specifically, let the initial state of $\mathbf{Q}_A$ and $\mathbf{Q}_B$ be a product:

$$\rho_{AB}(t_0) = \rho_A \otimes \rho_B,$$

where ρ_A and ρ_B are density operators. Assume the LOCC to be modelled by a set Krauss operators $\{\hat{A}_i \otimes \hat{B}_i\}$ acting on ρ_{AB} with probability $\{p_i\}$. The evolved state

$$\rho_{AB}(t) = \sum_i p_i(\hat{A}_i \otimes \hat{B}_i)\rho_{AB}(t_0)(\hat{A}_i^\dagger \otimes \hat{B}_i^\dagger) \tag{1}$$

is still separable, for any initial condition where the two systems $\mathbf{Q}_A$ and $\mathbf{Q}_B$ are in a separable state. Note that it is key to assume that the interactions between M and each of $\mathbf{Q}_A$ and $\mathbf{Q}_B$ are local, so that when M interacts with $\mathbf{Q}_A$ it cannot affect $\mathbf{Q}_B$; and likewise for $\mathbf{Q}_B$ and **M**. Note also that it is crucial to assume that **M** can be coupled to $\mathbf{Q}_A$ and $\mathbf{Q}_B$ in some classical basis, so that the interoperability of information is satisfied.

The theorem in [1] proves this result under more general axioms, that do not assume quantum theory, but are compatible with it. This provides the witness of non-classicality a much broader validity: the weaker the assumptions on the mediator M, the stronger the witness. It is this broader validity that has been questioned by Hall and Reginatto's counterexample, which we now proceed to refute. Our proposed vindication of the witness of non-classicality is important for understanding the implications of the recently proposed tests of quantum effects in gravity which are based on this witness. It also illustrates how the theorem in [1] could be regarded as an information-theoretic no-go theorem for hybrid models.

2. Hall and Reginatto's proposed hybrid model

There is a rich literature on hybrid quantum–classical systems, see e.g. [7, 10, 12–14, 17] and references therein. Broadly speaking, they consist of inequivalent dynamical models where a classical and quantum sector coexist and can interact with each other, under given constraints that are required for consistency. Some of these models are not expressible as the fully decohered version of a quantum dynamical model; hence, it is of the essence to have the general theorem in [1] in order to rule them out as viable models for the generation of entanglement by a classical theory of gravity.

Hall and Reginatto [7] asserted that there can be hybrid statistical quantum–classical models where the mediator **M**, despite being classical, can still create entanglement between two quantum probes. Happily, as we shall explain, the proposed model in fact validates the witness. In short, this is because the model conceals a hidden non-locality in the configuration space

dynamics, in contradiction with the locality assumption of our theorem; and when considering a local account, the model involves a non-classical mediator, with non-commuting variables that can be empirically accessed, as required by our witness.

Let us summarise here the model, following [7, 17]. Consider two quantum systems Q and Q' and a third one, C which is classical. The statistical hybrid used by [7] is defined as follows: any point in configuration space is described by the triplet of real numbers $z = (q, q', x)$, representing each the position coordinate of Q, Q' and C. The description based on the configuration space is based on the map of classical and quantum observables onto the so-called ensemble observables, defined as:

$$f(x, \nabla_x S) \to C_f[P, S] := \int \mathrm{d}z \, P f(x, \nabla_x S)$$
$$\hat{M} \to Q_{\hat{M}}[P, S] := \langle \psi | \hat{M} | \psi \rangle \tag{2}$$

for any classical observable f and quantum observable $\hat{M}$. Note that for both sectors, $C_f[P, S]$ and $Q_{\hat{M}}[P, S]$ are functionals of $(P(z), S(z))$. These are called ensemble observables of the hybrid ensemble model.

We can introduce the hybrid Poisson brackets

$$\{A, B\}_H := \int \mathrm{d}z \left[\frac{\delta A}{\delta P} \frac{\delta B}{\delta S} - \frac{\delta A}{\delta S} \frac{\delta B}{\delta P} \right],$$

where $\frac{\delta A}{\delta f}$ denotes the variational derivative of the functional $A[f]$ with respect to f. This induces a map from the algebra of quantum and classical sectors to the Poisson algebra of variables C_f and $Q_{\hat{M}}$ in the configuration space, defined as follows:

$$\{C_f, C_g\}_H = C_{\{f,g\}_P}, \qquad \{Q_{\hat{M}}, Q_{\hat{N}}\}_H = Q_{[\hat{M}, \hat{N}]/i\hbar}$$

with f, g and $\hat{M}, \hat{N}$ being respectively some generic classical and quantum observables. Via this representation, the state of the composite system is specified by the probability density $P(z)$ and its conjugate density $S(z)$, while the time evolution driven by the Hamiltonian H reads

$$\frac{\partial P}{\partial t} = \frac{\delta H}{\delta S}, \quad \frac{\partial S}{\partial t} = -\frac{\delta H}{\delta P}.$$

Hall and Reginatto then analysed the interaction for the hybrid system Q, Q' and C given by:

$$H[P, S] = g_1 \int \mathrm{d}q \, \mathrm{d}q' \mathrm{d}x P(\partial_q S) x + g_2 \int \mathrm{d}q \, \mathrm{d}q' \mathrm{d}x P(\partial_x S) q' \tag{3}$$

which represents, according to the authors, a pairwise coupling between Q and C and then between C and Q', with g_1 and g_2 being the respective coupling constants.

By applying the statistical hybrid model dynamical laws, one obtains that, for this particular Hamiltonian, the global state of the hybrid ensemble evolves as:

$$\psi_t(q, q', x) = \sqrt{P(q, q', x)} \mathrm{e}^{iS(q,q',x)/\hbar} = \mathrm{e}^{-itH_{\mathrm{eff}}/\hbar} \psi_0(q, q', x), \tag{4}$$

where $H_{\mathrm{eff}} = (g_1 \hat{p}\hat{x} + g_2 \hat{q}\hat{k})$, $\hat{p} = -i\hbar\partial_q$ and $\hat{k} = -i\hbar\partial_x$ are the momenta operators of Q and C respectively. It is easy to check that by starting the three system in say a product state $\psi_0(q, q', x) = \psi_Q(q)\psi_{Q'}(q')\psi_C(x)$, it is possible to obtain a state $\psi_t(q, q', x)$ where Q and Q' are entangled, by evolving it as specified by H. How can this be, given the initial assumption about C being classical? Hall and Reginatto conclude that this is a counterexample to the proposed witness. We shall now see that in fact this model validates it.

3. What is really going on

There are two main explanations of what is going on in the example, each of which is compatible with the witness. The first is manifest when considering the form of H_{eff}.

3.1. The mediator is non-classical

Let us first clarify an important point. The state in (4) is entangled if, when describing Q and Q' in the formalism of quantum theory, one can violate Bell's inequalities by measuring quantum observables of Q and Q'. So let us then assume this is the case, and consider carefully what this implies for the mediator C. As correctly pointed out by [7], $\hat{k}$ and $\hat{x}$ are two variables of the sector C that do not commute. Hence, the system C is a non-classical system, in line with the witness' predictions. However there is a subtlety: Hall and Reginatto claim that C is still effectively classical, because C's non-classical variable $\hat{k}$ is not empirically accessible. For it is an assumption of the statistical hybrid model that 'classical observables' must take the form (for some well-behaved function f):

$$C_f[P, S] = \int \mathrm{d}x \, P f(x, \nabla_x S). \tag{5}$$

In other words, observables of C must be functions of x and $\nabla_x S$ only. For this reason, according to Hall and Reginatto, C is still a classical system, even when governed by a dynamical evolution that involves $\hat{k}$, because the latter is assumed to be unobservable in their hybrid model.

But, remarkably, the assumption that $\hat{k}$ is unobservable is inconsistent with the claim that Q and Q' can be entangled by C. This is because, by confirming entanglement on Q and Q' via e.g. a protocol to violate Bell's inequalities, one can empirically access the degree of freedom $\hat{k}$ of the system C, effectively performing its tomographic reconstruction. For, after the action of H_{eff}, the time-evolved quantum observables of Q and of Q' become both functions of both variables $\hat{x}$ and $\hat{k}$. So, by measuring the *joint* observables of Q and Q' to confirm entanglement, e.g. an entanglement witness $\langle \mathcal{W} \rangle \approx \langle \psi | q\hat{p}' + q'\hat{p} | \psi \rangle$, one measures indirectly the non-commuting degrees of freedom of the mediator ($\hat{x}$ and $\hat{k}$). Therefore, the dynamical evolution generated by the Hamiltonian (3) is precisely a way to access to $\hat{k}$ empirically. This violates the assumption (5) that $\hat{k}$ does not qualify as an 'observable' for the classical sector. Hence, the proposed hybrid model precisely adheres to the theorem in [3]: in order for C to entangle Q and Q', it must be non-classical (i.e. its dynamics must allow for two non-commuting physical variables, $\hat{x}$ and $\hat{k}$).

Interestingly, a similar inconsistency is present in a different hybrid model, considered by [20]. There, a superselection rule is imposed on the classical system: a variable called $\hat{\pi}$ (which does not commute with the position variable of the classical system) is assumed not to be an observable; but at the same time it is allowed that the joint Hamiltonian of the classical system and of the quantum system coupled to it depend on $\hat{\pi}$. These two requirements are inconsistent, because it is possible to access $\hat{\pi}$ empirically via a protocol similar to that discussed in this note, where the classical system entangles two other quantum systems.

Finally, it is important to notice that there is an inherent ambiguity in the definition of entanglement in these hybrid models, arising from the definition of ensemble observables for the quantum sector (see (2)). The state in (4) can only be considered as describing two entangled systems Q and Q' if it is possible to define a protocol to confirm entanglement by measuring certain types of correlations between these two systems. However, in [7] it is never explained how to confirm entanglement within the hybrid ensemble model. To do so, one would have

to model the measurement of correlations between local ensemble observables of Q and Q' (defined via the map in (2)), as in standard protocols to violate e.g. Bell's inequalities. The problem with modelling this process in the hybrid ensemble is that a joint quantum observable of Q and Q' (such as an entanglement witness) cannot in general be written as a product of local *ensemble observables* of Q and Q', such as $E_{\hat{M}} = \langle \psi | \hat{M} | \psi \rangle$ or $E_{\hat{M}'}$. In general for any two quantum observable $\hat{M}$ and $\hat{M}'$, $E_{\hat{M}} E_{\hat{M}'} \neq E_{\hat{M}\hat{M}'}$. The equality fails, in particular, when Q and Q' are quantum-entangled. This is manifest, from the definition (2), when considering the marginals expectation values in Q and Q', so that:

$$\langle \psi | \hat{M} \otimes \mathrm{1\!l} | \psi \rangle \langle \psi | \mathrm{1\!l} \otimes \hat{N} | \psi \rangle \neq \langle \psi | \hat{M} \otimes \hat{N} | \psi \rangle \tag{6}$$

for every $|\psi\rangle$ describing two entangled quantum systems Q and Q'. Therefore it is questionable that the state in (4) can be interpreted as entangled in the hybrid ensemble model, if the observables of Q and Q' are interpreted as ensemble observables in dynamical hybrid phase space. Because, in this case, the operational procedure to confirm entanglement between subsystems of the hybrid ensemble is not properly defined. Under this interpretation, the two ensembles corresponding to Q and Q' would therefore be simply classically correlated when in state (4), and the theorem in [1] supporting the witness is trivially satisfied, as there would be no entanglement generated.

In summary if, like Hall and Reginatto do, one assumes that Q and Q' become quantum-entangled via C, one also implicitly assumes that there are available interactions jointly on Q and Q' that can measure their joint observables to violate Bell inequalities; and this corresponds to an indirect measurement of C's non-commuting degrees of freedom, which violates the central tenet of 'classicality' (equation (5)) of the mediator C, in line with the theorem's predictions.

3.2. Locality is violated

The other explanation to refute the counterexample is based on the violation of the locality principle in the hybrid ensemble model. The Hamiltonian (3) is presented as providing a local coupling between Q and C; and separately between Q' and C. But, does it? As explained in [16], it is possible to show that ensemble Hamiltonians of the form (3) acting on a given subsystem have zero hybrid Poisson brackets with ensemble observables of other subsystems. However, this is a fortunate coincidence, as the hybrid Poisson brackets vanishes only for at most linear terms in $\nabla_x S$ for the classical observable. But in the general case, where the classical observables include terms that are not linear in $\nabla_x S$, one has

$$\{Q_{\hat{M}}, C_f\}_H \neq 0$$

for $\hat{M}$ and f observables in the quantum and classical sectors respectively, and for a generic wavefunction for Q, Q' and C. Hence the quantum hybrid model violates the principle of no-action at a distance, which is one of the two axioms of the theorem in [1].

As a possible way out, one could attempt to introduce a superselection rule that forces interactions to proceed always through configuration and momentum ensemble observables, as suggested in [16]. This move in itself is problematic, because this superselection rule does not have a corresponding rule in known quantum theory and standard classical mechanics, where all the interactions are allowed. Hence the correspondence between such superselected hybrid model and existing well-corroborated physical theories would be violated.

Furthermore, even assuming superselected interactions involving only configuration and linear momentum ensemble observables, the microscopic descriptors of the classical system C

constituting the hybrid ensembles violate the axiom of no-action at a distance of the theorem in [3]. Hence the proposed hybrid model is ruled out *a priori*, according to this explanation. As in Bohmian mechanics, [21], the problem is of course that the functional forms of $P(q, x, q')$ and $S(q, x, q')$ (when describing an entangled state) are responsible for the non-local behaviour; and the dynamical classical linear momenta are functions of S. The theorem in [3] requires no-action at a distance to be applied, at the level of the complete descriptors of each subsystems: but this is violated by the fact that the classical momentum $\nabla_x S(q, x, q')$, when Q and Q' are quantum entangled, can be modified by quantum operations that involve only Q or only Q', both space-like separated from C. The canonical commutator being zero for all physical variables of space-like separated system is necessary for a local theory of measurement. In the hybrid model scenario, for this condition of *strong separability* to hold it is necessary to restrict attention to configurations where the systems are not quantum-entangled, as already remarked [17–19]. But this condition is precisely violated in Hall and Reginatto's model, given that Q and Q' must become entangled at the end of the interaction.

This violation of no-action at a distance at the level of fundamental descriptors also disqualifies the hybrid model as a faithful description of quantum and classical systems constituting the ensembles. For the principle of no-action at a distance is satisfied by quantum theory (as it is manifest in the Heisenberg picture) and classical general relativity: thus the hybrid ensemble model, given that it violates the principle of locality, cannot be a faithful representation of either quantum theory or general relativity.

We remark that the non-locality is strictly related to the non-linearity of Hall and Reginatto's model: in fact other hybrid, but linear, constructions like [14], do not share the same problems. The case of a classical oscillator mediating the interaction in a bipartite system of spins was already analysed in [15]. In line with the theorem in [1], any increment of the entanglement during the evolution has been excluded in such semiclassical models. The same authors highlight that when considering the protocol but adopting Hall and Reginatto's model, the conclusion would be plausibly different, and the intrinsic non-locality of their model would affect the entanglement.

4. Discussion

To summarise: the proposed model is not a counterexample, but is in fact an example that adheres to the theorem supporting the witness. In configuration space, the hybrid model violates locality; but locality is necessary to apply the witness, hence the model is not a counterexample. Indeed, according to the theorem supporting the non-classicality witness there may be non-local hybrid quantum–classical systems that can create entanglement, but such models are not physically plausible, given that locality is a pillar of both general relativity, non-relativistic quantum theory and quantum field theory.

On the other hand, assuming that Q and Q' are entangled, if we consider the local account of the proposed model which involves the Hamiltonian H_{eff}, we see that it includes two non-commuting variables for the mediator, $\hat{k}$ and x. Hence the mediator is non-classical and it provides an example of a non-classical system mediating entanglement. In addition, the variable $\hat{k}$ is, pace Hall and Reginatto, empirically accessible—through measurements that involve both Q and Q' and their mutual entanglement at the end of the entangling protocol. Thus, the hybrid model in this case validates the witness proposed in [3], because it described a local generation of entanglement via a non-classical mediator.

We have therefore provided the promised vindication of the general witness of non-classicality and of the recently proposed schemes to test non-classical effects in gravity.

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Data availability statement

No new data were created or analysed in this study.

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