CELESTIAL HOLOGRAPHY

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Abstract

Celestial holography proposes a duality between the gravitational &-matrix and correlators in a conformal field theory living on the celestial sphere. In this white paper, solicited for the 2022 Snowmass process, we review the motivation from asymptotic symmetries, fundamentals of the proposed holographic dictionary, potential applications to experiment and theory, and some important open questions.

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1 Motivation

The formulation of a complete theory of quantum gravity remains a major outstanding problem in modern physics. This fundamental, yet elusive, theory would provide the microscopic structure underlying systems whose long distance behavior is governed by Einstein's theory of general relativity. To date, remarkable progress has emerged from concrete realizations [1–4] of the holographic principle [5–7], underscoring the power of identifying a lower-dimensional nongravitational description of the same physics. Celestial holography provides a new approach to quantum gravity in asymptotically flat spacetimes by seeking to establish such a holographic correspondence.

More specifically, celestial holography proposes a duality between gravitational scattering in asymptotically flat spacetimes and a conformal field theory living on the celestial sphere [8–11]. One of the core features of this program is the reorganization of observables according to the principle of putting symmetries front and center. This framework makes manifest infinite dimensional enhancements that are typically hidden in soft theorems [8]. The central objects of study are celestial amplitudes, which recast the gravitational S-matrix into a basis of boost eigenstates. These transform like the correlation functions of a conformal field theory, providing a key entry in the holographic dictionary. The quest for a self-consistent intrinsically-defined dual theory is thus isomorphic to the S-matrix program [12–14].

In this white paper we review the asymptotic symmetry origins of this program and the basic building blocks of the celestial holographic dictionary, before presenting some of the prospects for merging tools from the relativity, amplitudes, and bootstrap communities in the form of potential applications and open questions. For concreteness we restrict to massless scattering in 3+1 dimensions throughout, though our story naturally generalizes to both higher dimensions [15–18] and massive external states [15, 19–22].

2 Symmetries from the Infrared

Matching the symmetries on both sides of the proposed duality is the natural first step towards constructing a holographic dictionary from the ground up. In the bulk the relevant question is: what are the symmetries of asymptotically flat spacetimes? This question was first tackled by Bondi, van der Burg, Metzner, and Sachs [23–25]. Asymptotically flat spacetimes are a class of solutions to Einstein's equations with vanishing cosmological constant and localized matter stress-energy [23–27], which capture the physics of isolated gravitational systems. Asymptotic symmetries are diffeomorphisms that preserve the causal structure of the boundary in a conformal compactification. They can be identified as the residual diffeomorphisms that remain after fixing appropriate gauge and boundary conditions. Namely

Asymptotic Symmetries =
$$\frac{\text{Allowed Symmetries}}{\text{Trivial Symmetries}}$$
, (2.1)

where we quotient by symmetries that act trivially on the phase space. The procedure for identifying the asymptotic symmetries involves a delicate interplay between the allowed boundary conditions and resulting asymptotic symmetry group. For example, BMS employed boundary conditions that led to an enhancement of the Poincaré group that includes angle-dependent translations of the generators of null infinity, known as supertranslations. However, it was not until rather recently that studies considered boundary conditions that were sufficiently relaxed to permit local enhancements of the Lorentz subgroup, called superrotations [28, 29].

This sensitivity to the choice of boundary conditions lends a degree of arbitrariness that one would like to mitigate. A directly observable consequence of asymptotic symmetries, known as the memory effect, informs this choice, singling out physically relevant solutions and symmetries. The celestial holography program is rooted in the interesting observation [30,31] that the perturbative gravitational &-matrix provides additional guidance in addressing this question. In particular, the Ward identities for asymptotic symmetries manifest as soft theorems of the &-matrix (see [8] and references therein)

$$\langle out|Q^{+}S - SQ^{-}|in\rangle = 0 \Leftrightarrow \langle out|a(\omega)S|in\rangle \underset{\omega \to 0}{\propto} \langle out|S|in\rangle.$$
 (2.2)

In fact there is an equivalence [32–34] between memory effects [35–43], soft theorems [44–50], and Ward identities [51–72] for asymptotic symmetries [73–82]. Moreover, the symmetry generators are naturally represented by currents in a two-dimensional theory [16, 30, 92–96]. In a particularly rewarding example of applying this equivalence, the aforementioned speculation about the physicality of superrotations led to the discovery of a subleading soft graviton theorem [47]. This in turn established the Ward identity for a Virasoro symmetry [52] and revealed a new memory

¹For their generalizations to higher dimensions and massive external states, see [83–88] and [89–91].

effect [33], as well as a 4D scattering mode that behaves like the stress tensor of a 2D conformal field theory [93].

Thus, a foundational lesson that emerges from symmetry is that a holographic dual to quantum gravity in four-dimensional asymptotically flat spacetimes must admit a two-dimensional conformal symmetry! Accordingly, the proposed dual theory built upon this 2D conformal symmetry is referred to as the *celestial conformal field theory* (CCFT).

3 Building a Holographic Dictionary

The subleading soft graviton mode couples to an external particle of helicity ℓ and energy ω through the operators [47,54]

$$\hat{h} = \frac{1}{2}(\ell - \omega \partial_{\omega}), \quad \hat{\bar{h}} = \frac{1}{2}(-\ell - \omega \partial_{\omega}).$$
 (3.1)

Scattering states that diagonalize \hat{h} and $\hat{\bar{h}}$ are constructed by transforming from momentum to boost eigenstates [15, 19, 54, 93, 97–99]. For massless particles, this can be achieved via a Mellin transform in the energy

$$\langle \mathfrak{G}_{\Delta_1}^{\pm}(z_1, \bar{z}_1) ... \mathfrak{G}_{\Delta_n}^{\pm}(z_n, \bar{z}_n) \rangle = \prod_{i=1}^n \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} \langle out | \mathcal{S} | in \rangle, \tag{3.2}$$

where (z_i, \bar{z}_i) parametrize the direction of propagation of the *i*th particle, $\Delta_i = h_i + \bar{h}_i$ is its scaling dimension or Rindler energy, and the \pm superscripts distinguish between incoming and outgoing operators. Generalizations of this basis have been explored in [15,100–105]. We thus have a map between traditional \mathcal{S} -matrix elements and correlation functions on the celestial sphere.

The equivalence between Lorentz transformations in four dimensions and global conformal transformations on the celestial sphere guarantees that the operators constructed from Mellintransformed wavepackets transform as quasi-primaries with conformal weights (h, \bar{h}) . In a gravitational theory, these are enhanced to Virasoro primaries via their coupling to the subleading soft graviton [52, 93, 98, 106].

Given the central correspondence (3.2), a more complete holographic dictionary is readily built by directly transforming known amplitudes and features thereof. Universal properties including soft and collinear limits are of particular interest. We present a summary of the current status, organized in terms of standard CFT data, namely the spectrum and OPE coefficients.

The Spectrum Finite-energy scattering states, characterized by normalizable single-particle wavepackets in the bulk [15], are captured by celestial operators with a spectrum [97]

$$\Delta = 1 + i\lambda, \quad \lambda \in \mathbb{R}. \tag{3.3}$$

Nevertheless, it has proven judicious to analytically continue Δ over the complex plane [107] in order to incorporate both the action of translations which shift $\Delta \to \Delta + 1$ [108–110], as well as soft theorems which involve operators with (half)-integer dimensions [106, 108, 111–116]. The conformal dimensions in the latter admit shortened representations of the global conformal symmetry [17, 18, 116–120], explaining the observed organization of soft theorems into these multiplets.

Remarkably, factorization in the soft limit of scattering persists in the boost eigenstate basis [98, 106, 111, 113, 121–123]. In particular, soft theorems, which prescribe the Laurent coefficients of an expansion about $\omega \to 0$, become statements specifying the residues of simple poles in the corresponding external dimension Δ . More generally, IR and UV phenomena can each have a rather stark and distinct effect on the analyticity of celestial amplitudes in their conformal dimensions [124–126]. For example, as a function of the net conformal dimension (which is Mellin-conjugate to the center of mass energy), celestial amplitudes admit a series of simple poles. The locations of the poles distinguish UV completions of quantum field theories from expected UV completions of quantum gravity and the residues reproduce the coefficients in an EFT expansion.

OPE Coefficients The OPE data of the celestial CFT encode the behavior of scattering in collinear limits [106, 114, 121, 127–132]. This follows from the equivalence between the limit in which two operators approach one another on the celestial sphere and the limit in which two massless on-shell momenta become collinear. Celestial OPE coefficients obey symmetry constraints intrinsic to the CCFT. While superrotations establish a Virasoro symmetry naturally incorporated into the standard CFT technology, global translations, as well as the infinity of soft symmetries associated with (half)-integer dimension insertions, imply non-standard constraints on the CFT data. Curiously the Ward identities arising from soft theorems lead to recursion relations for the OPE data [114,131] and, given certain analyticity assumptions in Δ , are sufficient to completely bootstrap the leading and, in some cases, subleading collinear behavior of scattering [114, 128, 130, 131, 133–135]. This quite unique feature of CCFT highlights the power of our symmetry-based approach, which apparently allows 4D kinematics to determine the dynamics of the 2D dual. Conversely, there is even recent evidence that celestial symmetries can be used to determine certain bulk dynamics [136].

There are many more features of the celestial holographic dictionary that are under active investigation for which the following two approaches have proven fruitful:

- 1. Determine properties of CCFT from known behavior of scattering amplitudes.
- 2. Establish intrinsic CCFT structure by applying known CFT properties or techniques.

In the first category, we have a wide variety of efforts to understand loop effects [125, 137–139], double copy relations [140–143], supersymmetry [21, 22, 115, 131, 143–149], changes in signature [102, 150, 151], twistor methods [104, 111, 152, 153], and more general amplitudes techniques [146, 147, 154–156] and features [125, 126, 157]. For the second, there have been extensive investigations of constraints from celestial symmetries [17, 18, 81, 109, 110, 114, 116–120, 128, 131, 133–135, 148, 152, 158–164], the appropriate representations [20, 165–168], 2D EFT models for conformally soft sectors [94, 96, 98, 169–178], the celestial state operator correspondence [103], and conformal block decomposition [101, 102, 132, 179, 180]. These highlights are only a snapshot of an active venture. While there is a rich variety of open questions regarding technical aspects of the CCFT dictionary, let us turn now to the big picture goals for this program that we hope to accomplish in the near future.

4 Applications and Open Questions

Measuring Memory Effects The theoretical framework of asymptotic flatness is expected to describe a wide range of experimentally relevant systems from collider to astrophysical (but subcosmological) scales. Memory effects are low-frequency observable signatures [35–38] of symmetry enhancements [32–34, 41] and thus provide an experimental litmus test for the relevance of the assumed boundary conditions to a given physical process. The soft physics program has identified new memory effects in both gravity (spin memory [33]) and gauge theory (color memory [41]), adding to the known examples of leading gravitational [32,35–37] and electromagnetic [34,38,39] memory. In fact, experimental investigations into the leading gravitational memory effect are already underway [181] and the choice of BMS frame plays an important role in extracting the correct gravitational waveform [182]. Moreover, color memory is a natural observable [42] in the physical regime that will be probed by the recently launched electron-ion collider (EIC) project [183,184]. It is tantalizing to ask: Can spin memory be measured by LISA? Will color memory be observed at the EIC? At the moment there are various proof-of-concept measurement proposals [33,34,39,41] and experimental feasibility studies [40] for these lesser known memories, but we expect there is much more to come.

Constraining Black Hole Evaporation The infinite-dimensional symmetries that motivated the celestial holography program have interesting applications in their own right. In the context of infrared divergences, the notion of asymptotic charge non-conservation provides a beautiful interpretation of the vanishing amplitude for emitting a finite number of soft particles [185]. Upon including an infinite number of these soft emissions, one obtains a finite result *precisely because* the contributions from the soft quanta render the asymptotic charges conserved. By identifying

symmetry as the underlying origin of infrared divergences, this perspective establishes a general principle for constructing infrared safe amplitudes [120, 125, 185–192].

The implications of asymptotic symmetry conservation laws are even more rich in the presence of horizons, which can carry soft hair [193–196]. For example, asymptotic symmetries constrain black hole evaporation and thereby provide insight into questions about information loss and firewalls [197–205]. Determining the precise phrasing of these statements within the CCFT framework is an active area of research [98, 206, 207] which will help bridge the gap between the celestial holography program and standard techniques from AdS holography for quantum information exchange. To this end, it would prove fruitful to understand how to recover celestial CFT from a flat limit of AdS by, for example, zooming into a bulk point [208, 209] or relaxing the standard boundary conditions to allow for radiation [210].

A Booststrap Program The celestial holography program is inherently isomorphic to the &-matrix program. Namely any intrinsic definition of the celestial CFT should include a set of rules for constructing consistent (gravitational) scattering matrices as a function of their on-shell data. Our discussion of the celestial holographic dictionary demonstrates the success with which we have exploited soft and collinear behavior of scattering amplitudes to constrain and in some cases determine properties of celestial amplitudes. While CCFT may be an exotic 2D CFT [99,154], we have already seen indications of a simple analytic structure in the boost weights that encodes information about bulk physics [125, 126, 157]. Recently, the CCFT framework facilitated the identification of a $w_{1+\infty}$ algebra underlying the soft symmetries of the &-matrix [116,131,148,152, 160,162]. In turn, this symmetry algebra supplies the CCFT with yet more powerful organizing principles. Codifying the rules for a consistent CCFT will be an important step for bootstrapping the &-matrix in the boost basis.

Connecting to String Theory A crowning achievement for the celestial holography program would be for it to determine concretely whether string theories are the only consistent theories of (asymptotically flat) quantum gravity. Thus far, it has been observed that even at tree-level, string theoretic amplitudes are sufficiently well-behaved in the UV to admit a celestial representation [124,125]. Yet another interesting line of inquiry is into the relation (and possible equivalence?) between CCFTs and worldsheet constructions of quantum gravity. Recent results matching celestial OPEs to worldsheet OPEs in both standard and twistor string contexts [153, 162], motivated by the discovery of symmetry algebras common to single helicity sectors of CCFT [116, 131, 148, 152, 160, 162] and stringy constructions, evoke even more anticipation!

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