Echoes of the regularized dilatonic black hole

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Abstract

In present work, the evolution of scalar field and electromagnetic field under the background of the regularized dilatonic black bounces spacetimes are investigated, we obtain an obvious echoes signal which appropriately reports the properties of regularized dilatonic black bounces spacetimes and disclose the physical reasons behind such phenomena. By studying the quasinormal ringdown of the three states of regularized dilatonic black bounces spacetimes, it shows that the echoes signal only appears when $\boldsymbol{b > 2k}$.

Keywords: echoes, quasinormal ringdown, regularized dilatonic black bounces spacetimes

1 Introduction

The paper is organized as follows. In Sec.2, we focus on the properties of the regularized dilatonic black bounces spacetimes. In Sec.3, the time domain integration method and WKB method are introduced. In Sec.4, we study time-domain profiles of the electromagnetic field and scalar field in the background of the black hole. In Sec.5, the QNM frequencies of regularized dilatonic black bounces spacetimes are presented. Finally, the work is summarized in Sec.6.

2 Studies on the properties of the regularized dilatonic black bounces spacetimes

We consider the action of matter S_m as a combination of a scalar field $\phi(x)$ and a nonlinear electromagnetic field minimally coupled to gravity

$$S_m = \int \sqrt{-g} d^4x \left[2h(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) - \mathcal{L}(\mathcal{F})\right], \tag{1}$$

where the function $h(\phi)$ realizes the scalar field parametrization freedom, $V(\phi)$ represents a certain potential and $\mathcal{L}(\mathcal{F})$ reads a nonlinear electromagnetic field with the Lagrangian density.

According to the Eq.(1), we can derive the metric of the regularized dilatonic black bounces spacetimes after regularizing[1]

$$ds^2 = \frac{1-2k/x}{1+p/x} dt^2 - \frac{1+p/x}{1-2k/x} du^2 - x(x+p) d\Omega^2, \quad x = \sqrt{u^2+b^2}, \tag{2}$$

where $k > 0$ and Q (the electric charge) are integration constants($p = \sqrt{k^2 + Q^2\left(1+\lambda^2\right)} - k > 0$), and m reads the Schwarzschild mass $m = k + \frac{p}{2} = \frac{Q^2}{p}$.

Evidently, the range of u is $u \in \mathbb{R}$, the metric Eq.(2) is asymptotically flat at $u \in \pm\infty$ and describes: (1) if $b < 2k$, a regular black hole with two horizons at $u = \pm\sqrt{4k^2 - b^2}$ and a black bounce at $u = 0$; (2) if $b = 2k$, a regular extremal black hole with a single extremal horizon at $u = 0$; (3) if $b > 2k$, a symmetric traversable wormhole with a throat at $u = 0$, the throat radius is $r_{th} = minr(u) = \sqrt{b(p+b)}$.

The general covariant Klein-Gordon equation of scalar field can be expressed as

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Psi) = 0, \tag{3}$$

in order to reduce the above equation, we assume the scalar field separated in the standard form [2]

$$\Psi(t, r, \theta, \phi) = \frac{1}{\sqrt{x(x+p)}} \Phi_0(r, t) Y_{l_0}^{\hat{m}}(\theta, \phi), \tag{4}$$

where $Y_l^{\hat{m}}$ is spherical harmonic function of degree l_0 related to the angular coordinates θ,ϕ, and according to the tortoise coordinate $r_* = \int \frac{dr}{A(r)}$, we have

$$[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + V_s(r)] \Phi_s(r, t) = 0, \tag{5}$$

where $v_s = \sqrt{\frac{A}{B}} (\sqrt{\frac{A}{B}})' h^{-1} h' + \frac{Al(l+1)}{h}^2$, $A = \frac{1-2k/x}{1+p/x}$, $B = \frac{1+p/x}{1-2k/x}$ and $h^2 = x(x+p)$.

We recall the wave equations for a test electromagnetic field

$$\frac{1}{\sqrt{-g}}\partial_v\left[\sqrt{-g}g^{\alpha\mu}g^{\sigma\nu}\left(A_{\sigma,\alpha}-A_{\alpha,\sigma}\right)\right]=0, \tag{6}$$

The four-potential A_μ can be expanded in terms of the 4-dimensional vector spherical harmonics as

$$\begin{aligned}A_\mu(t,r,\theta,\phi)=\sum_{l,m}&\left(\begin{bmatrix}0\\0\\\frac{a(t,r)}{\sin\theta}\partial_\phi Y_{lm}\\-a(t,r)\sin\theta\partial_\theta Y_{lm}\end{bmatrix}\right.\\&\left.+\begin{bmatrix}f(t,r)Y_{lm}\\h(t,r)Y_{lm}\\k(t,r)\partial_\theta Y_{lm}\\k(t,r)\partial_\phi Y_{lm}\end{bmatrix}\right),\end{aligned} \tag{7}$$

in which Y_{lm} represents the spherical harmonics. Thus we find the following secondorder differential equation for the radial part

$$\frac{d^2\Psi\left(r_*\right)}{dr_*^2}+\left[\omega^2-V_E\left(r_*\right)\right]\Psi\left(r_*\right)=0, \tag{8}$$

with the effective potential

$$V_E=\frac{Al(l+1)^2}{h}. \tag{9}$$

3 The time domain integration method and WKB method

The external perturbation field satisfies the equation under the regularized dilatonic black bounces spacetimes in terms of the tortoise coordinate is given by

$$\frac{d^2\Phi_b}{dt^2}-\frac{d^2\Phi_b}{dr_*^2}+V_b(r)\Phi_b=0, \tag{10}$$

since QNMs are complex frequencies related with purely outgoing waves at spatial infinity, from Eq.(10) we have $\Phi_b(r_*,t)\to e^{\pm i\omega r_*}e^{-i\omega t}$ as $r_*\to\pm\infty$, and for an asymptotically flat spacetime, V_b approaches 0. Based on the features of the effective potentials, we will deal with the scalar and vector QNMs in context of the charged black-bounce spacetimes, the existence of QNMs can be directly observed in the time evolution of the scalar field and vector field obtained by integrating the wave equation (Eq.(10)). In view of the obscure expression of the effective potentials, it is difficult to obtain the analytical solutions, here we intend to adopt the time domain integration method which does not depend on the form of the potential barrier, thus we rewritten Eq.(10) in terms of the light-cone variables $u = t - r_*$ and $v = t + r_*$ as

$$-4\frac{\partial^2}{\partial u \partial v}\Phi_b(u,v) = V(r(u,v))\Phi_b(u,v), \tag{11}$$

where u and v are integral constants.

Next, we move on towards the numerical integration of the Eq.(11), regarding the characteristic initial value problem, the initial data is specified on the two null surfaces $u = u_0$ and $v = v_0$. In view of the fact that the field decay is almost independent of the initial conditions, and the initial condition is the Gaussian wave packet [3–5], thus we start with a Gaussian pulse ($u = u_0$) and set the field to zero ($v = v_0$), i.e.

$$\Phi_b(u = u_0, v) = exp[-\frac{(v - v_c)^2}{2\sigma^2}], \Phi_b(u, v = v_0) = 0, \tag{12}$$

where the Gaussian pulse with a width is $\sigma = 3$ and the center of the Gaussian pulse is $v_c = 10$. In addition, the appropriate discretization scheme in terms of the Taylors theorem [6, 7] is

$$\begin{aligned}\Phi_b(N) =& \Phi_b(w) + \Phi_b(E) - \Phi_b(S) - \\ & \Delta^2 \frac{V(w)\Phi_b(w) + V(E)\Phi_b(E)}{8} + O(\Delta^4),\end{aligned} \tag{13}$$

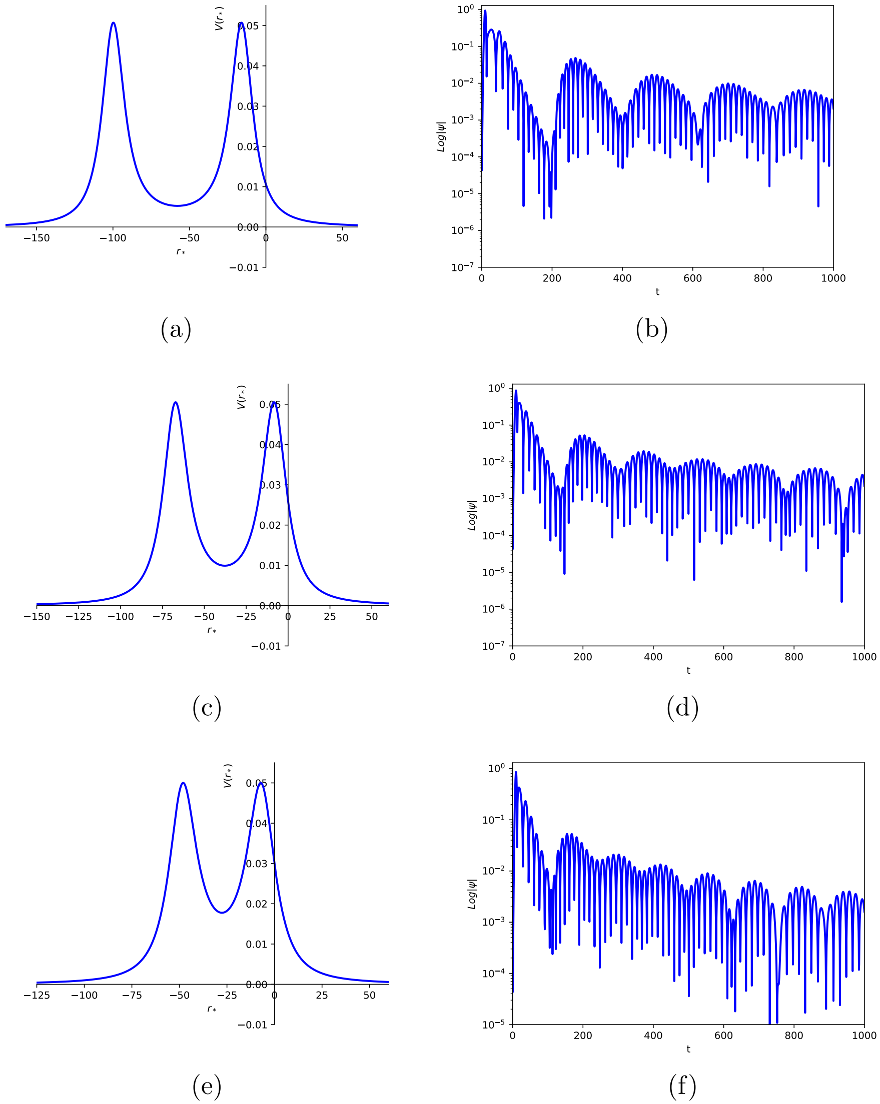
where the following designations for the points were used: $N = (u+\Delta, v+\Delta)$, $w = (u + \Delta, v)$, $E = (u, v + \Delta)$ and $S = (u, v)$, thus using the corresponding initial conditions along u and v lines we numerically integrate to derive the time-domain profiles. In addition, in order to facilitate the extraction of the quasinormal frequencies, we will employ the Prony method[8] which helps us to fit the signal by a sum of exponents with some excitation factors.

However, it is tricky to analytically solve the time-independent, second-order differential equation with the potential for the regularized dilatonic black hole. The WKB method can be used for the effective potential, which configurates the form of a potential barrier and takes constant values at the event horizon and spatial infinity. Specifically speaking, this method is based on matching the asymptotic WKB solutions at spatial infinity and the event horizon with a Taylor expansion near the top of the potential barrier through two turning points. By using the considered potential functions, one can obtain the QNMs frequencies through a sixth-order WKB method, and as seen in Ref.[9], this method is the most accurate one for finding the quasinormal spectrum with lower overtones. The BH potential $V(r)$ in the present of the sixth order formula is

$$\frac{i(\omega^2 - V_0)}{\sqrt{-2V_0''}} - \sum_{i=2}^{6} \Lambda_i = n + \frac{1}{2}, n = 0, 1, 2, ..., \tag{14}$$

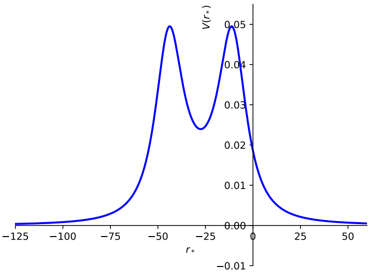
among them, V_0 is the maximum effective potential of $V(r)$ at the tortoise coordinate r_*, n is the overtone number (we study the case $n = 0$) and the correction term Λ_i can be obtained in Ref.[10]. And generally speaking, the quasinormal frequencies ω take the form $\omega = \omega_R - i\omega_I$, where the real part and the imaginary part of ω denote actual field oscillation and damping of the perturbation respectively.

4 The pictures of echoes and the effective potential for the regularized dilatonic black bounces spacetimes



(a) (b) (c) (d) (e) (f)

Fig. 1



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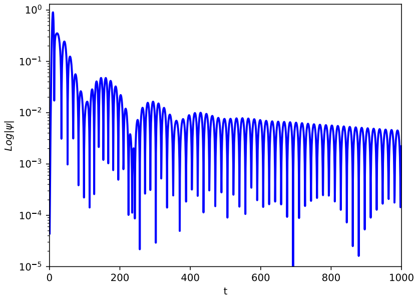

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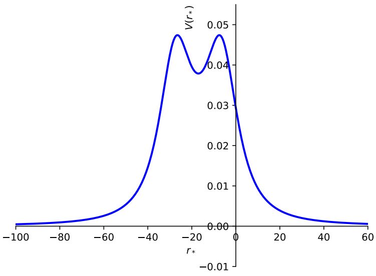

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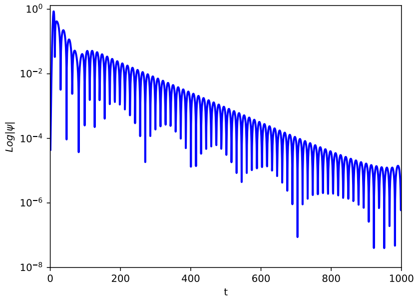

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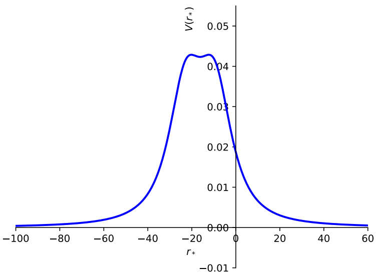

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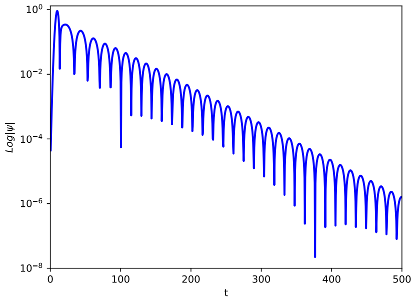

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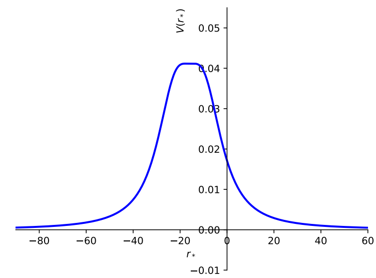

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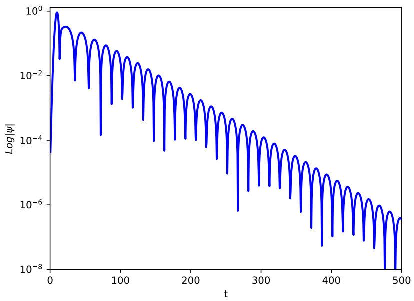

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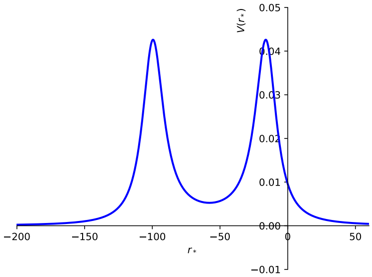
Fig. 2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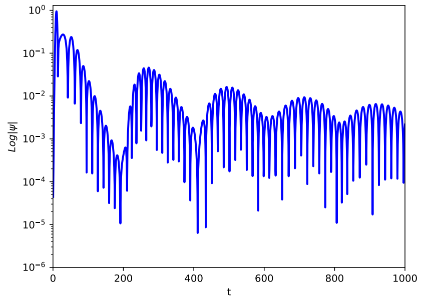
(a)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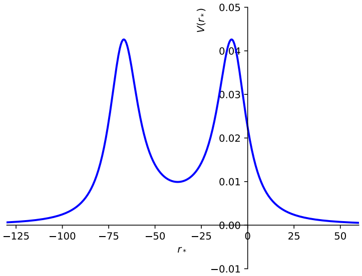
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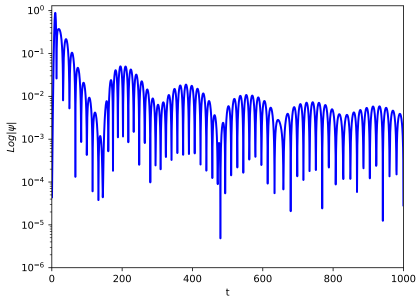
(c)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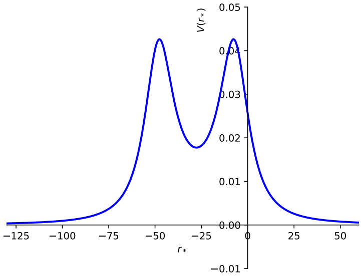
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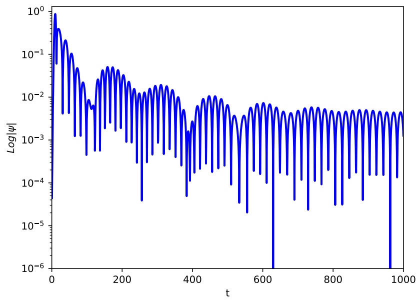
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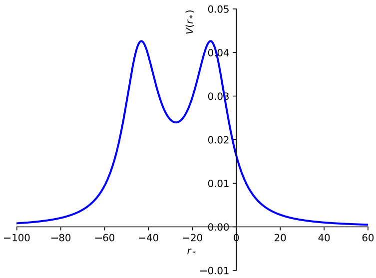
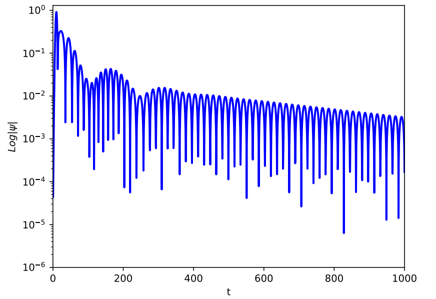
Fig. 3



(a)



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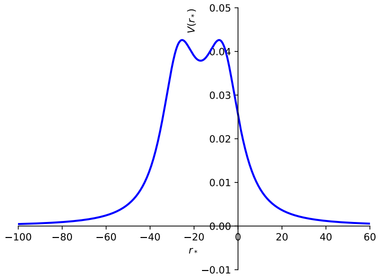

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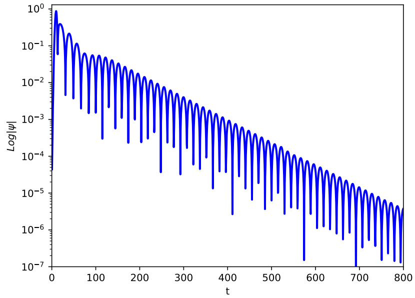

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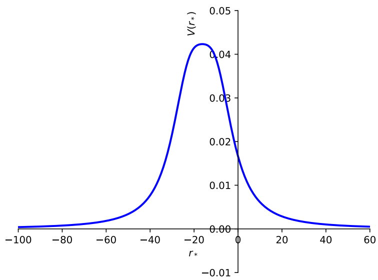

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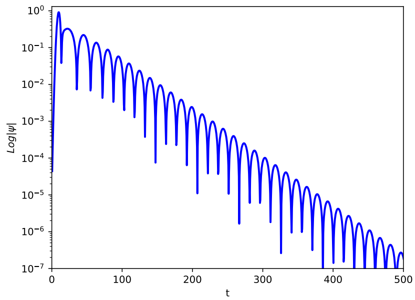

(f)

Fig. 4

5 The QNM frequencies of the regularized dilatonic black bounces spacetimes

6 Conclusions

In summary, we study the regularized dilatonic black bounces spacetimes and observe clear echoes signal. By considering the perturbation of electromagnetic field and scalar field, we derived the motion equation and the effective potential.

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