Project Notes

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A small summary of the things I learnt during the project's duration updated chronologically. Should attach the papers I read in the bibliography, but that's for later

March 31, 2024

Contents

1. The CMB Power Spectrum

The CMB power spectrum is represented as the variation of power (in μK^2) with respect to the multipole.

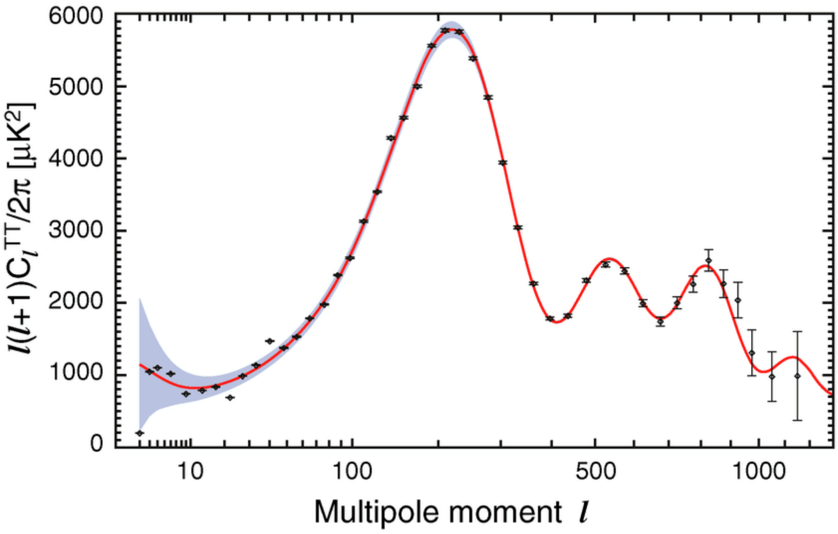

Figure 1: The CMB Power Spectrum

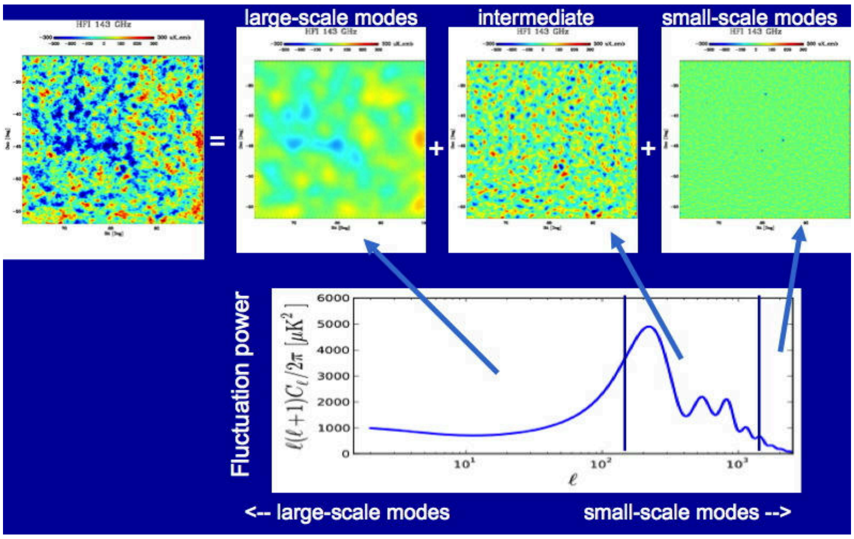

Figure 2: Different scales corresponding to the different l

The power spectrum can be classified into three different regions, corresponding to the multipole moments (i.e. based on what angular scales are we observing the CMB Maps, smaller l means larger angular scales and vice-versa, described in Figure 2). The three regions would be:

1. Small l (Large angle correlations): These correlations are majorly sourced by fluctuations that did not evolve before photon decoupling and are therefore a direct probe of the initial conditions.

2. Intermediate l (Intermediate angle correlations): These correlations are caused by the perturbations in the tightly coupled photon-baryon fluid that propagate as sound waves supported by the large photon pressure. The different modes are captured at different moments in their evolution.

3. Large l (Small angle correlations): Effects like damping can be understood by studying the CMB at these scales.

1.1. CMB Anisotropies

1.2. Project Details

Where to find the cache files and clear them when using camb

```
C:\Users\boori\AppData\Local\Programs\Python\Python313\Lib\site-
packages\camb\__pycache__
```

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c

Details about the CMB Maps and the other stuff:

- $10^{\circ} \times 10^{\circ}$ patches of the simulated CMB sky used with 0.5 arcmin resolution
- 148 GHz maps
- N_side = 8192

1.2.0.1. Maps with not all foregrounds

- **Constant Radii Bubbles:**
  - Radius: $40'$
  - Amplitude: $-6\mu K$
  - Unlensed:
    - All bubbles detected
    - Slight error in the amplitude
  - Lensed:
    - 9/10 bubbles detected
  - **No false positives in both cases**
- **Varrying Radii Bubbles:**
  - Bubbles can vary randomly from 38 to 42 arcmin
  - Apply matched filters with radius θ_c as $\{38', 39', 40', 41', 42'\}$
  - Amplitude was still $-6\mu K$
  - Unlensed:
    - 6/8 bubbles detected
    - No spurious detections
  - Lensed:
    - 5/8 bubbles detected
    - 6 spurious detections

$Radius_{(\theta_x^\circ,\theta_y^\circ)}$	MF Radius ($'$)	Recovered Amplitude
$38_{(5.217,4.858)}$	38	-6.403549
$38_{(4.442,7.158)}$	38	-5.914817
$39_{(3.275,3.742)}$	39	-7.799565
$40_{(5.500,3.225)}$	40	-8.303618
$40_{(2.192,2.875)}$	40	-7.272582
$42_{(6.900,5.817)}$	42	-6.607460

Table A1. Source Detection with the Matched Filter applied on Unlensed CMB + simulated reionization kSZ + HII Ionization bubbles + instrumental noise. $(\theta_x^\circ, \theta_y^\circ)$ are the positions of the bubble sources. Actual amplitude for all sources is $-6\mu K$.

$Radius_{(\theta_x^\circ,\theta_y^\circ)}$	MF Radius ($'$)	Recovered Amplitude
$38_{(5.217,4.858)}$	38	-6.834603
$39_{(3.275,3.742)}$	39	-7.636465
$40_{(5.500,3.225)}$	40	-8.511117
$40_{(2.192,2.875)}$	40	-7.348668
$42_{(6.900,5.817)}$	42	-6.313925

Table A2. Same as above but with lensed CMB.

1.2.0.2. Maps with all foregrounds

- Total number of sources: 10
- Random radii between 36 and 44 arcmin
- Unlensed:
  - 8/10 bubbles recovered
  - 2 spurious detections
- Lensed:
  - 7/10 bubbles recovered
  - 22 spurious detections

$Radius_{(RA^\circ,Dec^\circ)}$	MF Radius ($'$)	Recovered Amplitude
$36_{(8.1754.800)}$	36	-6.838021
$38_{(5.6835.883)}$	38	-7.619793
$39_{(7.0335.708)}$	38	-7.268827
$39_{(7.1586.500)}$	39	-6.789971
$40_{(7.1255.100)}$	39	-6.936532
$42_{(2.9002.542)}$	40	-7.030653
$43_{(7.8502.450)}$	42	-5.856849
$44_{(1.7082.717)}$	43	-7.693487

Table A3. Source Detection with the Matched Filter applied on Unlensed CMB + all foregrounds including masked kSZ of S10 maps+ HII Ionization bubbles + white noise. Actual amplitude for all sources is $-6\mu K$.

$Radius_{(RA^\circ,Dec^\circ)}$	MF Radius ($'$)	Recovered Amplitude
$36_{(8.175,4.800)}$	36	-6.975150
$38_{(5.683,5.883)}$	38	-7.779363
$39_{(7.033,5.708)}$	39	-6.347103
$39_{(7.158,6.500)}$	39	-6.756921
$40_{(7.125,5.100)}$	40	-6.955670
$42_{(2.900,2.542)}$	42	-6.722138
$44_{(1.708,2.717)}$	44	-7.381340

Table A4. Same as above but with lensed CMB.

1.2.0.3. Everything + Gaussian Noise

- Add some non uniformity by introducing some gaussian noise into the bubbles
- Total number of bubbles: 10
- Gaussian noise percentages: 1%, 10%, 30%
- True amplitude: $-6\mu K$

Noise level in the bubbles	No. of correctly recovered sources	Median deviation from true amplitude (%)	No. of spurious detections
No noise	8	16.39 ± 3.99	2
1 %	9	16.07 ± 4.79	2
10 %	9	18.3 ± 4.8	2
30 %	8	17.24 ± 4.65	43

Table 1. Effect of non-uniformity on detected amplitude for the bubble. We use the foregrounds described in Section 6 for the the unlensed CMB case.

Noise level in the bubbles	No. of correctly recovered sources	Median deviation from true amplitude (%)	No. of spurious detections
No noise	7	15.93 ± 3.89	22
1 %	7	16.24 ± 3.91	22
10 %	8	15.54 ± 5.03	22
30 %	7	19.32 ± 3.41	44

Table 2. Same as above but for lensed case.

1.3. Files Nomenclature

Note that the files in the folders "papres" and "papres_main" are literally the same

Example: cmbksz40af_9filters_30.0

- Single filter: Matched filtering done with only one radius
- 9 filters: Matched filtering done with 9 radii: {36, 37, 38, 39, 40, 41, 42, 43, 44}
- the 30.0 here represents the gaussian noise percentage used in the bubbles

another example: cmbksz40af_kink_multi_filters

- The kink here stands for the more accurate theoretical profile used for generating the models, the bubbles are more rough in this case
- No kink uses a more basic profile, the bubbles are smooth

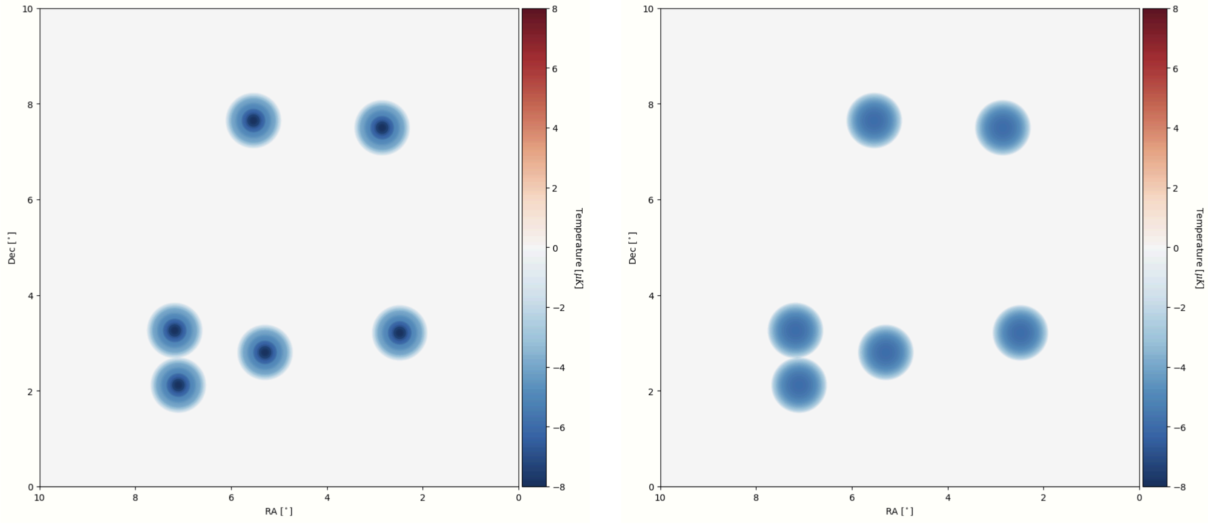

Figure 5: Kinks (left) vs No Kink (right)

2. A brief description about my work

I essentially have the raddi and amplitude (which is in μK so essentially the temperature on the map) of the detected quasar bubbles. Now I need to figure out the rest of the properties of the quasar using these two quantities.

For this I am going to perform MCMC sampling and get the best estimate for each of the parameters involved in the model. Which model you ask? Well that's the part I need to figure out next. I need to go through the papers and isolate the model that is being used to generate these bubbles and backtrack them essentially till I get the theoretical model. Then I can proceed forwards with doing the MCMC sampling.

More precisely I need to obtain the parameters of the ionization bubble surrounding the quasar and the model that governs these parameters.

2.1. Figuring out the model

Now the time evolution of the size of the ionized bubble is given by:

$$R(t) = R_S\left[1 - \exp\left(-\frac{t}{\mathrm{f_H} t_{\mathrm{rec}}}\right)\right]^{\frac{1}{3}} \tag{1}$$

where:

- R_S is the Stromgren radius given by:

$$R_S = \left(\frac{3\dot{N}}{4\pi\alpha_{\mathrm{HII}} n_H^2}\right)^{\frac{1}{3}} \tag{2}$$

  where
  - $\dot{N}$ is the ionizing photon emission rate related to the luminosity (L_ν) by:

$$\dot{N} = \int_{13.6}^{\infty} \frac{L_\nu}{h\nu}\,\mathrm{d}\nu \tag{3}$$

  - n_{H} is the mean cosmological density of neutral hydrogen at a given redshift
  - α_{HII} is the recombination coefficient (rate of recombination of $H^+ + e^- \to H + \gamma$ per unit volume) which is a function of temperature. ($\alpha_{\mathrm{HII}} \sim 10^{-13}\mathrm{cm}^3\mathrm{s}^{-1}$)

- $\mathrm{f_H}$ is the fraction of neutral hydrogen having number density n_H

- t_{rec} is defined as:

$$t_{\mathrm{rec}} = \frac{1}{\alpha_{\mathrm{HII}} n_H} \tag{4}$$

In most cases, the ionization bubble around the quasar stops growing after a certain time t_Q which is its "lifetime". Now assuming $t = t_Q \ll t_{\mathrm{rec}}$, we get a simplified version of Equation 1:

$$R(t_Q) \approx \left(\frac{3\dot{N}t_Q}{4\pi n_{H(z=0)}}\right)^{\frac{1}{3}} \cdot (1+z)^{-1} \tag{5}$$

Now, for our case, we use the following to generate them bubbles:

- $z \sim 8$
- $\Delta\theta \approx 40'$
- $\frac{\Delta T}{T} \sim 2 - 3 \times 10^{-6}\ K$

and that's why in all the bubbles we have used the amplitude $-6\mu K$ and the radius of the bubble to be around 40 arcminutes.

Now, the question arises, are we not calculating the actual radius of the bubble in Equation 5, how do we translate this to angular size?

This is where all the cosmology theory comes in. For this, I referred to "Distance measures in cosmology" by David Hogg. From there we have the following relations:

(Before that, few basic constants:

- The Hubble's constant H_0 is usually written as: $H_0 = 100h$ km s^{-1} Mpc^{-1} where h is a dimensionless parameter.
- The hubble distance D_H is defined as:

$$D_H = \frac{c}{H_0} = 3000h^{-1}\ \ \text{Mpc} \tag{6}$$

- The mass density ρ of the universe and the value of the cosmological constant Λ are usually characterized by the dimensional parameters:

$$\Omega_M = \frac{8\pi G\rho_0}{3H_0^2} \quad \text{and} \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2} \tag{7}$$

- Another density parameter, Ω_k, which measures the curvature of space is defined using the relation:

$$\Omega_M + \Omega_\Lambda + \Omega_k = 1 \tag{8}$$

Defined by this relation, there are three major world models:

name	Ω_M	Ω_Λ
Einstein–de-Sitter	1	0
low density	0.05	0
high lambda	0.2	0.8

We define something called the co-moving line of sight distance to be :

$$D_C = D_H \int_0^z \frac{\mathrm{d}z'}{E(z')} \tag{9}$$

where

$$E(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda} \tag{10}$$

Similarly, the co-moving transverse distance is given by:

$$D_M = \begin{cases} D_H \frac{1}{\sqrt{\Omega_k}} \sinh\left[\sqrt{\Omega_k}\, D_C/D_H\right] & \text{for } \Omega_k > 0 \\ D_C & \text{for } \Omega_k = 0 \\ D_H \frac{1}{\sqrt{|\Omega_k|}} \sin\left[\sqrt{|\Omega_k|}\, D_C/D_H\right] & \text{for } \Omega_k < 0 \end{cases}$$

And finally, we can calculate our angular diameter distance (D_A) which is nothing but

$$D_A = \frac{\text{Physical Diameter}}{\text{Angular Diameter}} = \frac{D}{\Delta\theta} = \frac{D_M}{1+z} \tag{11}$$

2.2. Questions

1. What are the formulas that I need to use and what's the plan?

I think this is the overall plan to follow:

1. Use formulas Equation 9 through Equation 11 to convert the obtained Angular size to the actual physical diameter, $R(t)$.
2. Use Equation 5 to estimate the parameters.

2. What are the parameters that I would be doing the MCMC sampling for?

My guess for the parameters are:

$$n_{H(z=0)}, \quad \dot{N}, \quad t_Q, \quad z \tag{12}$$

3. What are the values for the 'constants' that I would be using

From what I've read, the right model would be that of setting $\Omega_k = 0$, but I don't have much idea regarding the rest of the parameters values. What values were used for the following parameters:

$$h \ \text{ or } \ H_0, \quad \Omega_M, \quad \Omega_\Lambda, \quad z \tag{13}$$

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