Lab 2: Basic Probability

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In this lab we will work through two basic probability problems, and in the process practice more with RMarkdown.

Packages needed: knitr, xtable, pander

R Markdown: Automated intro text when creating a new .Rmd file in RStudio This is an R Markdown document. Markdown is a simple formatting syntax for authoring HTML, PDF, and MS Word documents. For more details on using R Markdown see http://rmarkdown.rstudio.com.

When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

Task 1: Coin flipping

This task illustrates the interpretation of a probability as the long run relative frequency of an event after a large number of trials.

Dobrow presents R code on pages 29 for simulating coin tosses. We perform the experiment of observing the number of heads after tossing a fair coin 100 times (probability of a heads on any one toss is 50%). Just like rolling a die, we can use the R function `sample` to flip a coin. Though recognize there are only two outcomes: heads (1) and tails (0). We will report the number of heads after 50 tosses and intermediary output. We will also graphically display the cumulative proportion of heads again the coin toss (1 to 100). The `type="l"` parameter in the R function `plot` will draw a solid line. *Always label your axes! In RMarkdown, a graph title is useful too.*

Code set-up

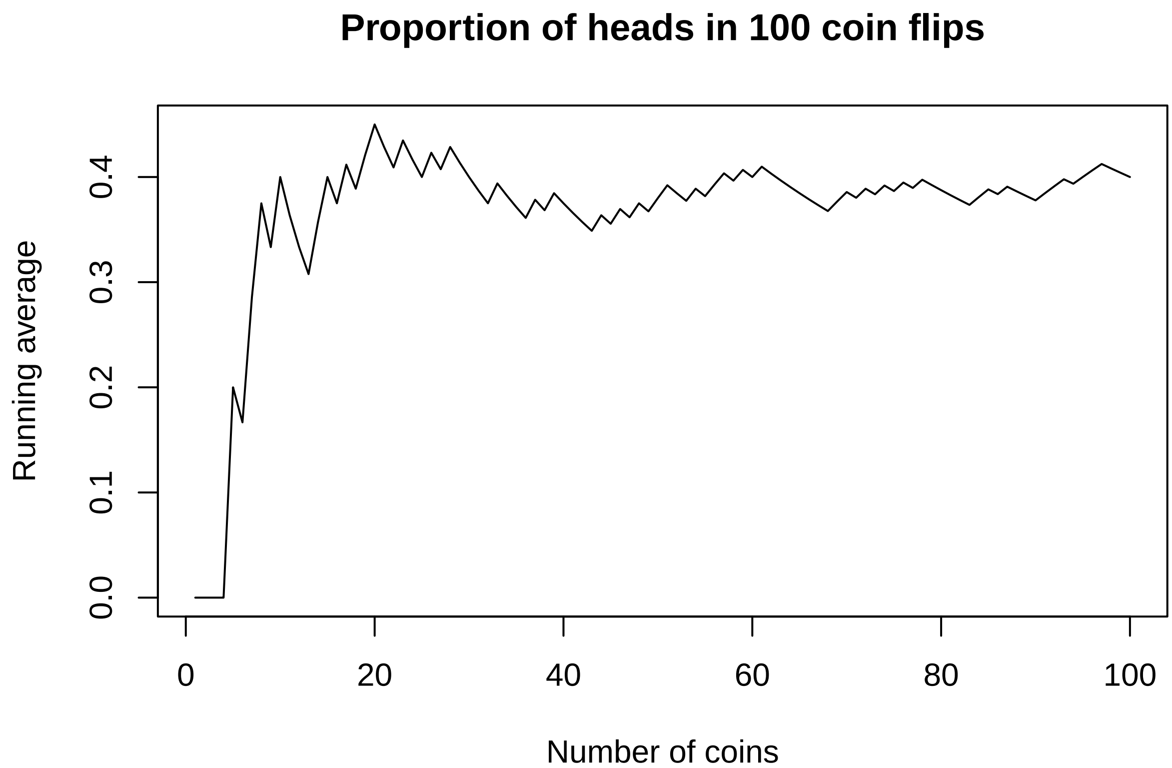
```
simnum = 100 # number of coin flips
coinflips = sample(0:1, simnum, replace = TRUE) # flip the coin: heads = 1, tails = 0
heads = cumsum(coinflips) # cumulative sum of number of heads after each coin toss
prop = heads/(1:simnum) # running proportion of heads after each coin toss
head(heads) # report cumulative number of heads after each of the first 6 flips
```

```
## [1] 0 0 0 0 1 1
```

```
heads[50]
```

```
## [1] 19
```

```
# running mean plot for proportion of heads.
plot(1:simnum, prop, type="l", xlab="Number of coins", ylab="Running average", main="Proportion of heads
abline(h=0.5) # add a line at 50%
```



The problem

Let us now flip a "biased" coin. Perform the experiment of observing the number of heads after tossing a coin 1000 times, with the probability of getting a heads on any one toss being 40%. To change the probability in the R function `sample` use the parameter `prob=c(0.6,0.4)`; note that we need to specify the probability of a tails (0) and a heads (1) in this parameter.

Report the following:

- Proportion of heads after 10, 50, 100, 200, and 500 tosses (see table code chunk below under "RMarkdown presenting output"!)
- Plot of the cumulative proportion of heads vs. coin toss number (1 to 1000); label the axes and title the graphic appropriately!
- On the plot, draw a horizontal line at $y=0.40$, the probability of tossing a head for this coin

```
n=1000
coinflips=sample(0:1, n, prob=c(0.6,0.4), replace=T)
heads=cumsum(coinflips)
prop=heads/(1:n)
head(heads)
```

```
## [1] 1 1 2 3 3 4
```

```
heads[10]
```

```
## [1] 6
```

```r
heads[50]
```

```
## [1] 24
```

```r
heads[100]
```

```
## [1] 41
```

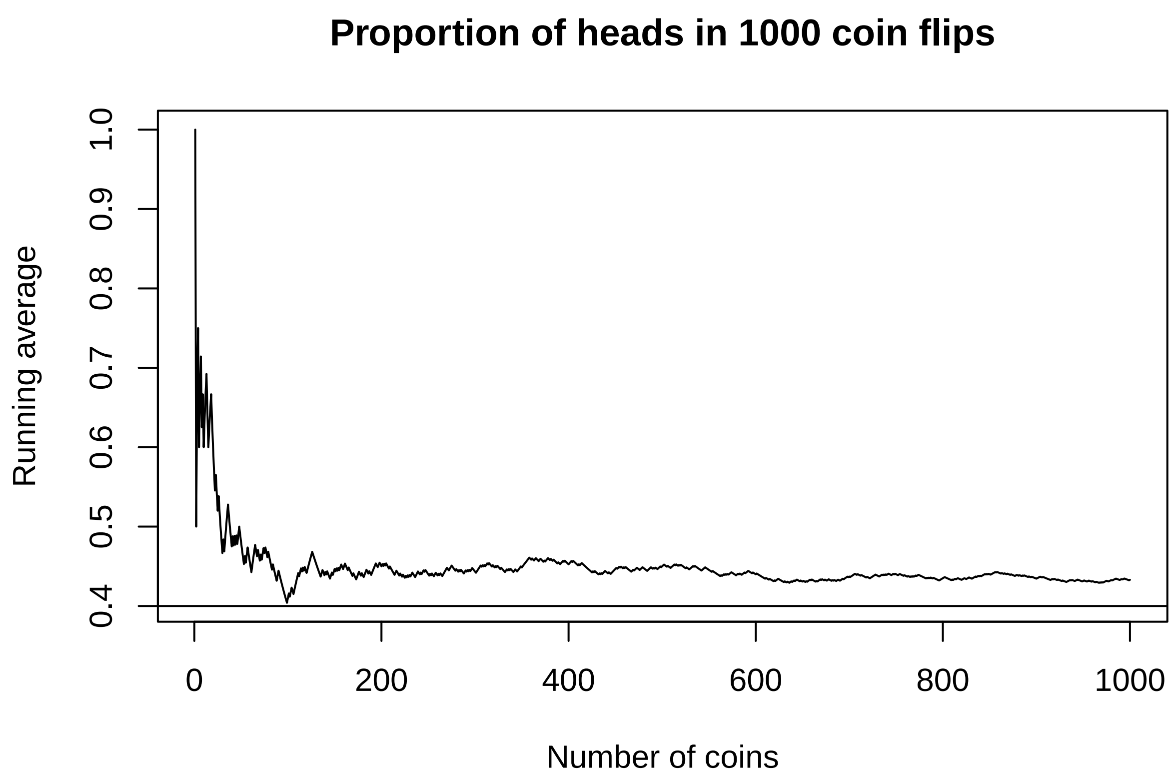
```r
heads[200]
```

```
## [1] 90
```

```r
heads[500]
```

```
## [1] 225
```

```r
plot(1:n, prop, type="l", xlab="Number of coins", ylab="Running average", main="Proportion of heads in 1
abline(h=0.4)
```



Questions:

- Describe the behavior of the graphic (cumulative proportion of heads) during the first 150 tosses (1-150), next 150 tosses (151-300), and then later tosses. *Because the proportion is cumulative, it has massive spikes in the beginning, but in the later tosses, it slowly begins flattening.*
- What do you notice about the limiting value of the curve in your plot? *The limiting value is where the line flattens down to.*
- Why would you expect the behavior you discuss in the previous two bullets? *Because we changed the probability from 50/50 to 40/60, we should expect heads to only occur 40% of the time, given a*

sufficiently large enough amount of trials

RMarkdown presenting output: Below is code to present a table for the proportion of heads after 10, 50, and 100 tosses.

Reminders:

The `echo=FALSE` parameter prevents printing of code from a code chunk. The `include=FALSE` parameter prevents printing of output from a code chunk. The `results=asis` allows the LaTeX code produced by xtable to be compiled and output.

```
# we will create a table using xtable and pander
library(knitr)
library(xtable)
library(pander)
# output desired summary statistics
# formatC used so integer coin tosses do not have a decimal place in the figure!
numtoss = formatC(c(10, 50, 100), digits=0, format="d", flag="#")
num.heads = c(heads[10], heads[50], heads[100])
num.heads = formatC(num.heads, digits=0, format="d", flag="#")
# formatC used here so proportions have exactly two decimal places (including zeros at the end)!
prop.heads = c(prop[10], prop[50], prop[100])
prop.heads = formatC(signif(prop.heads,digits=6), digits=2, format="f", flag="#")
table.elts = rbind(numtoss, num.heads, prop.heads)
row.names(table.elts) = cbind("# coin tosses", "Number of heads", "Proportion of heads")
lab1.table = xtable(table.elts, caption = "Proportion of heads for a given number of tosses of a fair c
pander(lab1.table)
```

Table 1: Proportion of heads for a given number of tosses of a fair coin.

	1	2	3
# coin tosses	10	50	100
Number of heads	6	24	41
Proportion of heads	0.60	0.48	0.41

The table shows the proportion of heads after a certain number of tosses in a single simulation experiment. These proportions provide empirical estimates of the probability of a head for specified simulation sample sizes.

The problem

Directly in the code chunk above, add columns for 200 tosses and 500 tosses. Hint: you will need to append these two elements to `numtoss`, `num.heads`, and `prop.heads`. Also, note that in the `xtable` function, the align is only for 3 right-justified columns; need to augment that to 5 right-justified columns.

Task 2: Divisibility probability

This task provides a probability problem to explore if-then statements and functions in R. Consider an interger drawn uniformly at random from the numbers $\{1, 2, \ldots, 1000\}$ such that each number is equally likely. We wish to simulate the probability that the number drawn is divisible by 3, 5, or 6.

Code set-up

Dobrow presents R code on page 31 for simulating this experiment, and provides the exact probability calculation in Example 1.33. The code presents a slick application of the `replicate` R function, one we used in the R Introduction lab. In particular, a function is written which draws the number at random from the integers 1 to 1000 and then checks if it is divisible by 3, 5, or 6. It uses modular arithmetic, `x%%n` being x *mod* n in R. For example, if the remainder of the number divided by 3 (modulus) is 0, then the number is divisible by 3! We will then repeat the function (experiment) 1000 times to get the empirical probability.

```
# simdivis() simulates one trial
simdivis = function(){
  num = sample(1:1000, 1) # draw a number at random from the integers 1 to 1000
  # determine if the number is divisible by 3, 5 or 6 by checking if the remainder is 0
  if (num%%3==0 || num%%5==0 | num%%6==0) 1 else 0
}
simlist = replicate(1000, simdivis()) # replicate the experiment 1000 times
mean(simlist)
```

The true probability that a randomly drawn integer between 1 and 1000 is divisible by 3, 5, or 6 is 0.467.

```
## [1] 0.499
```

The problem

Simulate the probablity that a random integer between 1 and 5000 is divisible by 4, 7, or 10.

```
simdivis = function() {
  num = sample(1:5000, 1)
  if (num%%4==0 || num%%7==0 || num%%10==0) 1 else 0
}
n=100000
simlist = replicate(n, simdivis())
intdivis = cumsum(simlist)
prop = intdivis/(1:n)

numtoss = formatC(c(100, 1000, 10000, 100000), digits=0, format="d", flag="#")
num.ints = c(intdivis[100], intdivis[1000], intdivis[10000], intdivis[100000])
num.ints = formatC(num.ints, digits=0, format="d", flag="#")
prop.ints = c(prop[100], prop[1000], prop[10000], prop[100000])
prop.ints = formatC(signif(prop.ints,digits=6), digits=2, format="f", flag="#")
table.elts = rbind(numtoss, num.ints, prop.ints)
row.names(table.elts) = cbind("Number of integers", "Number of ints divisible", "Proportion of ints divi
lab1.table = xtable(table.elts, caption = "Proportion of integers from 1 to 5000 being divisible by 4, 
pander(lab1.table)
```

The true probability that a randomly drawn integer between 1 and 5000 is divisible by 4, 7, or 10 is 0.40.

Table 2: Proportion of integers from 1 to 5000 being divisible by 4, 7, or 10

	1	2	3	4
Number of integers	100	1000	10000	100000
Number of ints divisible	44	418	4030	40185
Proportion of ints divisible	0.44	0.42	0.40	0.40

Questions:

- Present the empirical probability based on repeating the experiment 100, 1000, 10000, and 100000 times. Consider using the `xtable` code chunk from the first task to build a table for these values.

The probability of an integer from 1 to 5000 being divisible by 4, 7, or 10 seems to be 0.40

- How do these values compare to the truth?

After 10000 iterations, the sample proportion was true to true proportion

- Extra credit: show that the true probability that a random integer between 1 and 5000 is divisible by 4, 7, or 10 is 40%?

$$\begin{aligned}
A &= \text{set of ints divisible by 4}, \quad A \cap (B, C) = \text{set of ints divisible by } \operatorname{lcm}(A, B) \text{ and } \operatorname{lcm}(A, C) \\
B &= \text{set of ints divisible by 7}, \quad B \cap C = \text{set of ints divisible by } \operatorname{lcm}(B, C) \\
C &= \text{set of ints divisible by 10}, \quad A \cap B \cap C = \text{set of ints divisible by } \operatorname{lcm}(A, B, C)
\end{aligned}$$

$$\begin{aligned}
P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \\
P(A) &= \frac{\frac{5000}{4}}{5000} = 0.25, \quad P(B) = \frac{\frac{5000}{7}}{5000} \approx 0.143, \quad P(C) = \frac{\frac{5000}{10}}{5000} = 0.10 \\
P(A \cap B) &= \frac{\frac{5000}{28}}{5000} \approx 0.036, \quad P(A \cap C) = \frac{\frac{5000}{20}}{5000} = 0.05, \\
P(B \cap C) &= \frac{\frac{5000}{70}}{5000} \approx 0.014, \quad P(A \cap B \cap C) = \frac{\frac{5000}{140}}{5000} \approx 0.007 \\
P(A \cup B \cup C) &= 0.25 + 0.143 + 0.10 - 0.036 - 0.05 - 0.014 + 0.007 \\
&= 0.399 \approx 0.4 \text{ due to rounding}
\end{aligned}$$