- Please complete this test by September 18. I will use the results to better calibrate the course to student background.
- Please don't spend more than 1 hour.
- Please don't consult outside resources. The goal is to see what you know, not what you can find answers to. (Looking up the definition of a term you've heard before is fine, but try to stick to the spirit of the test. Use your best judgment.)
- This is graded but on completion only (full points if submitted, none if not).

1. Please list the graduate level statistics/machine learning/data mining/computer science/econometrics courses you have taken. Include the names of the course with the course number if possible.

2. (Maximum likelihood) Let $X_1, \ldots, X_n$ be iid Gaussian random variables each with probability density function
$$f_X(x; s) = \frac{1}{\sqrt{2\pi s}} \exp\left\{-\frac{x^2}{2s}\right\}.$$
Find the maximum likelihood estimator for s.

3. (Estimation) Let $\widehat{\theta}$ be an estimator for θ. Write the formula for the bias of $\widehat{\theta}$. Is the MLE in the previous problem unbiased? Is it asymptotically unbiased?

4. (Risk analysis) Explain why we might prefer a biased estimator to an unbiased estimator.

5. (Convex optimization) Write down the Langrage Dual for the following optimization problem and explain the necessary conditions for the stationary point to be an optimum (you do not need to find the stationary point or perform any calculations):
$$\begin{aligned} &\min_b L(y, \mathbf{X}b) \\ \text{subject to} \quad &\|Ab\|^2 \leq C \end{aligned}$$
for some convex function $L : \mathbb{R}^n \to \mathbb{R}^+$, vectors $y \in \mathbb{R}^n$ and $b \in \mathbb{R}^p$, matrices $A \in \mathbb{R}^{m \times p}$ and $\mathbf{X} \in \mathbb{R}^{n \times p}$, and constant C. Here $\|\cdot\|^2$ is the squared ℓ_2-norm of a vector. That is $\|z\|^2 = z^\top z$.

6. (Coding) What does the following `R` code do?

```
x = 2*pi*runif(100)
y = sin(x) + rnorm(100)
B = splines::bs(x,20)
logSeq <- function(from=1e6, to=1, len=100){
  exp(seq(log(from),log(to),length.out = len))
}
lam = logSeq()
mod = MASS::lm.ridge(y~B, lambda=lam)
preds = cbind(1,B) %*% coef(mod)[which.min(mod$GCV),]
df = data.frame(y=y, x=x, truth=sin(x), preds=preds)
library(tidyverse)
df %>% gather(key="key",value="value",-x) %>%
  ggplot(aes(x=x,y=value,col=key)) + geom_line() + ylab('') +
  theme_minimal()
```