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Rutherford Scattering

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1 Abstract

In this lab, we verify the Rutherford formula for nuclear scattering in 1909. Their experiment led to the confirmation of the existence of an atomic nucleus, thus destroying the plum pudding model developed by J. J. Thomson. Rutherford's team successfully disproved Thomson's model by observing a significant amount of charged alpha particles scattering back from a thin sheet of gold foil rather than passing through it with only a few deflections. Therefore, they were able to confirm that an atom consists of a dense nucleus surrounded by a less dense cloud of electrons surrounding the heavy nucleus, instead of an even distribution of mass throughout the atom. The distribution of alpha particles scattered by gold foil can be observed as a function of the scattering angle, which relies on the distance between the foil and the gold source.

2 Theory

Geiger and Marsden formed the first preliminary theory for the Rutherford model of the atom. They were able to gather that most α-particles, He nuclei, passed right through a thin foil without being deflected. However, there were many particles that experienced large angle scattering. The differential scattering cross section will be determined in this lab:

$$\frac{d\sigma}{d\Omega}(\theta)\Delta\Omega = \frac{number\ of\ particles\ per\ unit\ time\ into\ a\ solid\ angle\ \Delta\Omega(\theta)}{(number\ of\ scatterers) \times (incident\ flux)} \tag{1}$$

Based on the previous definition, Rutherford was able to show that a positively charged nucleus has a differential scattering cross-section for charged particles that is given by:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{Z^2z^2e^4}{16E^2}\frac{1}{sin^4(\theta/2)} \tag{2}$$

where Ze is nuclear charge, ze is charge of the scattered particle and E is the energy of the scattered particle. θ is the scattering angle. However, we can see that this equation does not give appropriate units for a number of particles per area, instead we have $\frac{C^4}{J^2}$. In order to correct this dimensional analysis disagreement, we look at the kinetic energy of the alpha particle, E, which is:

$$E = \frac{1}{2}mv^2. \tag{3}$$

However, when colliding alpha particles head on with the nucleus, all of the kinetic energy is turned into potential energy and the particle is at rest, so E is also:

$$E = \frac{q_1q_2}{4\pi\epsilon_0 b}. \tag{4}$$

In the previous equation, $q_1 = Ze$ and $q_2 = ze$. We can solve for b, the distance between the center of the alpha particle and the center of the nucleus, where

$$b = \frac{Zze^2}{2\pi\epsilon_0 mv^2}. \tag{5}$$

Therefore, Equation 2 can be rewritten as

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{1}{4b}\right)^2 \frac{1}{sin^4(\theta/2)}. \tag{6}$$

The scattering cross section can then be used to find the rate of detected particles, N_2, which is expressed in terms of N_0, the total number of particles emitted by the source per unit solid angle. Using the geometry of the set up and the Rutherford formula, the rate of detected particles is determined by the following formula

$$N_2 = n\frac{d\sigma}{d\Omega}(\theta)\Delta\Omega_2 \times (incident\ flux) \tag{7}$$

where n is the number of scattering centers. The incident flux is given by $\frac{N_0}{R_1^2}$, which correspond to the figures in the next section. Then, N_2 is:

$$N_2 = \frac{nN_0}{R_1^2}\frac{d\sigma}{d\Omega}\Delta\Omega_2. \tag{8}$$

The scattering angle, θ, varies with the changing distance Y in the apparatus. Using the geometry of the setup, this formula can be given as

$$N_2 = N_0 \times \left(\frac{nA_2Z^2z^2e^4}{R_1^2r_1^2 16E^2}\right) \times \left(\frac{cos\theta_2 sin\theta_2^2}{sin(\theta/2)^4}\right) \tag{9}$$

or more simply as

$$N_2 = N_0 \times G \times f(Y). \tag{10}$$

Here G is the atomic factor that accounts for the force between the gold atoms and the scattered alpha particles as they pass through the gold foil. Here, we take for granted that the solid angles we mention are averages and that the scattering center is an average scattering center since we do not know where on the surface area of the gold foil each, or most, of the alpha particles will pass through.

3 Experimental Methods

3.1 Apparatus

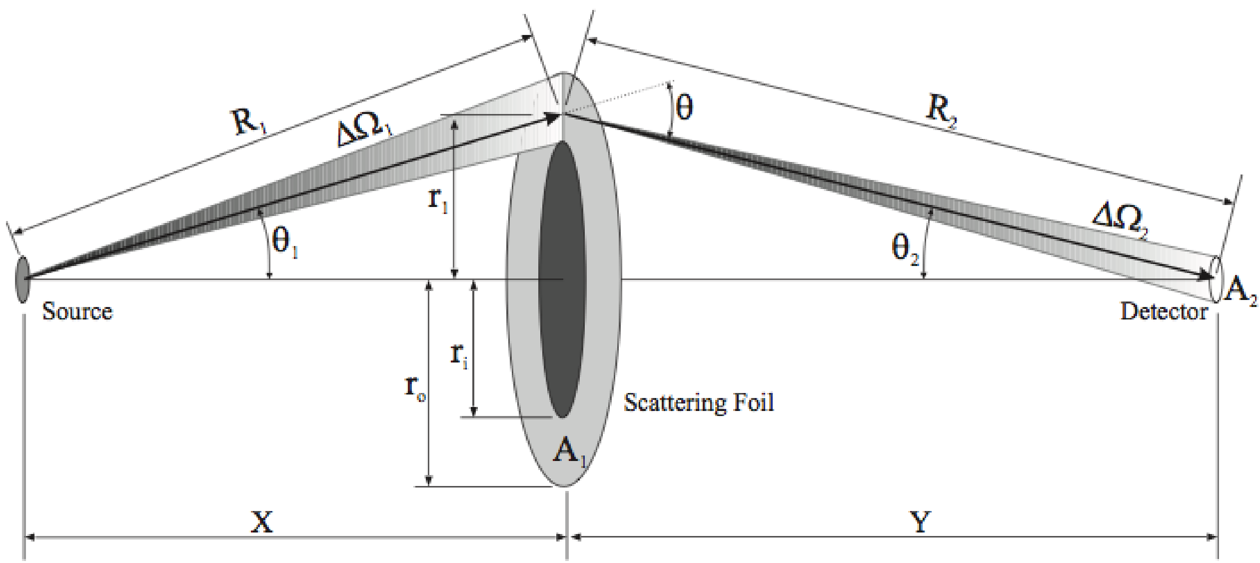

Figure 1: Experimental geometry.

3.2 Procedures

3.3 Calibration

3.4 Recording Data

4 Results and Discussion

4.1 The Scattering Angle Distribution

Using Equations 9 and 10, solving for $f(Y)$ gives the angular distribution as a function of Y. We can plot Y vs. $f(Y)$ for a range of 0 cm to 20 cm to get a curve that we can use to fit our experimental results. We solve for f(Y) in the following steps:

$$f(Y) = \frac{cos\theta_2 sin\theta_2^2}{sin(\theta/2)^4} \tag{11}$$

where

$$\theta_1 = tan^{-1}\left(\frac{r_1}{X}\right) \tag{12}$$

and

$$\theta_2 = tan^{-1}\left(\frac{r_1}{Y}\right) \tag{13}$$

and

$$cos\theta_2 = \frac{Y}{\sqrt{Y^2 + r_1^2}} \tag{14}$$

and

$$sin\theta_2^2 = \frac{r_1^2}{Y^2 + r_1^2}, \tag{15}$$

therefore:

$$f(Y) = \frac{\frac{Y}{\sqrt{Y^2+r_1^2}} \frac{r_1^2}{Y^2+r_1^2}}{sin(0.5(tan^{-1}\left(\frac{r_1}{X}\right) + tan^{-1}\left(\frac{r_1}{Y}\right)))^4}. \tag{16}$$

We know the following quantities as they are given in the lab manual because we cannot open the apparatus and check them for ourselves:

Quantity	Description
$r_s = 0.32 \pm 0.01cm$	radius of source
$X = 7.22 \pm 0.01cm$	detector to plane of scattering foil
$r_i = 2.30cm$	radius of the inside of the scattering foil
$r_o = 2.70cm$	radius of the outside of the scattering foil
$r_d = 0.48cm$	radius of the detector

Table 1: Several components of the apparatus that have been measured for us so that we do not have to disassemble the radioactive source.

Plotting this function for f(Y) over an interval of 0 to 20cm, we have:

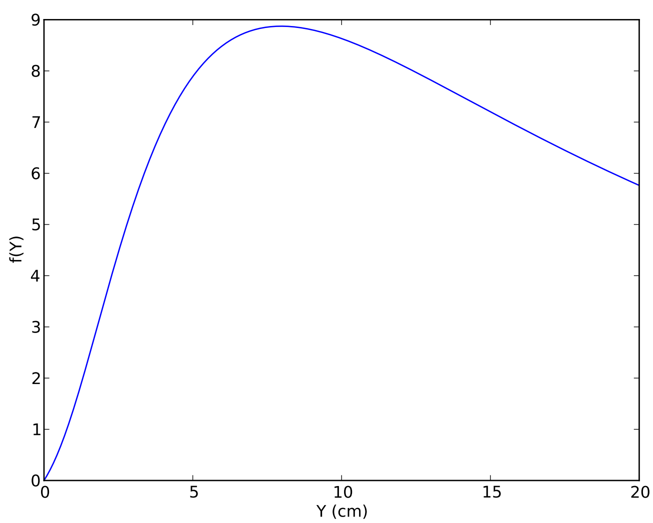

Figure 2: The angular distribution for the third term in the rate of detected particles(N_2) that varies with the scattering angle as a function of length. The peak occurs in the vicinity of 8 cm.

4.2 Determining G & N_0

Figure 3 below shows our preliminary data set taken over a range from 3cm to 18cm in 1cm intervals. At each value for Y, the distance from the gold foil to the source, we counted for 20 minutes. Our data shows a peak around 10cm as opposed to the 8cm prediction given in the angular distribution plot above. It can be seen however, that the peak distance is relative to where on the apparatus one begins to record data. The axis on which we can move the source in an out of the apparatus chamber was measured to be 19.7cm$\pm$.01cm long. The length along this axis at which we choose to begin measurements however, is dependent on our gains for the linear amplifier in the electronic schematic of the apparatus, and therefore arbitrary to the length at which we make our zero reference point.

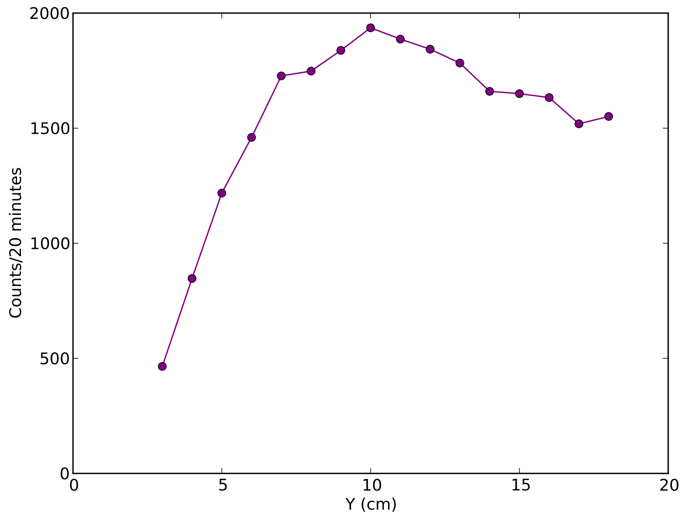

Figure 3: The first run of data taken from 3cm to 18cm in intervals of 1cm.

After seeing a curve that represented the same shape as that of $f(Y)$, we went back and collected data in between the intervals we previously recorded so that we could have a full set of data from 3cm to 18cm in 0.5cm steps.

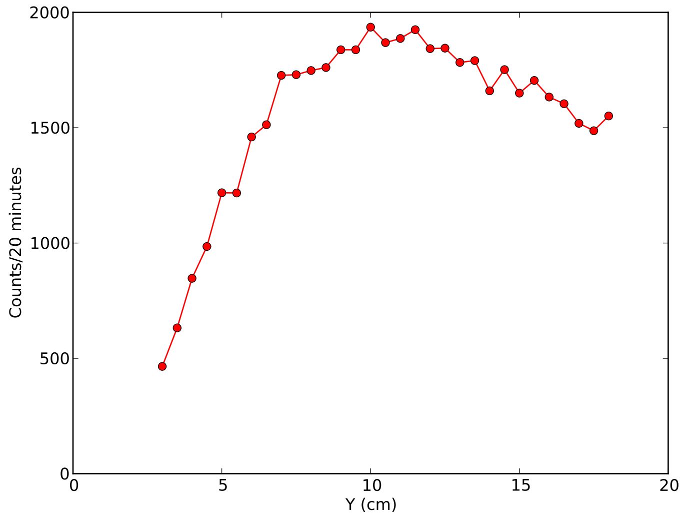

Figure 4: The first run of data taken from 3cm to 18cm in intervals of 1cm combined with an additional run with points taken in intervals of 0.5cm to obtain more statistics.

Since we cannot find a value for N_0 directly because it would require dissembling the apparatus chamber and placing the detector directly next to the source, we can indirectly calculated it using the three values that we know or have measured, G, N_2, and $f(Y)$. First, G is simple a factor that is a result of combination of known values in this apparatus.

$$G = \frac{nA_2Z^2z^2e^4k^2}{R_1^2r_1^2E^2} \tag{17}$$

That factors that are included in G are listed below:

Quantity	Description	Value
$n = \left(\frac{\rho N_A}{A_g}\right) A_1 t$	Total number of scattering centers	1.491×10^{19}
ρ	Density of Gold	$1.93 \times 10^4 kg/m^3$
N_A	Avogadro's Number	6.022×10^{23}
A_g	Atomic weight of Gold	$196.97u = 0.197kg/mol$
$A_1 = \pi r_o^2 - \pi r_i^2$	Area of scatting foil	$1.257 \times 10^{-4} m^2$
t	Thickness of scattering foil	$2 \times 10^{-6} m$
$A_2 = \pi r_d^2$	Area of the detector	$1.508 \times 10^{-4} m^2$
Ze	Nuclear charge	$79 \times (1.6 \times^1 0^{-19} C)$
ze	Alpha particle charge	$2 \times (1.6 \times^1 0^{-19} C)$
E	Energy of scattered particle	$4.4MeV = 7.0496 \times 10^{-13} J$
R_1	Hypotenuse from source to scattering foil	$0.03118m$
r_1	Average scattering center	$0.025m$
k	Coulomb's constant	$9 \times 10^9 N \cdot m/C^2$

Table 2: All values needed to calculate the factor G as given by the lab manual and the period table.

Therefore, $G = 8.932 \times 10^{-6}$. We cannot propagate error on the value for G because the values that we are using are taken for granted since we cannot make our measurements of the apparatus that are inside the chamber. We can now use the calculated value for G, the angular distribution $f(Y)$ and the experimentally collected count rates, N_2 to determine N_0.

$$N_0 = \frac{N_2}{f(Y)G} = 2.2509 \times 10^7 \pm 4.8028 \times 10^6 \; particles/20 \; minutes \tag{18}$$

Here we calculate the standard deviation by the following formula:

$$\sigma_{N_0} = \sqrt{\frac{1}{N}\Sigma(N_{average} - N_i)} = 4.8028 \times 10^6 \; particles/20 \; minutes \tag{19}$$

The value for N_0 should hypothetically be much higher than the values for N_2 that we are experimentally measuring because N_0 is the number of particles that the source is emitting directly, which is much more than what we measure for the counts as a function of distance from the source. Since the zero reference point of the length is arbitrary, we shift our length values by two centimeters to the left to arrive at results that correspond to our calculated curve. The most important comparison is how optimally our data follows the calculated curve, not how in or out of phase the curves are themselves.

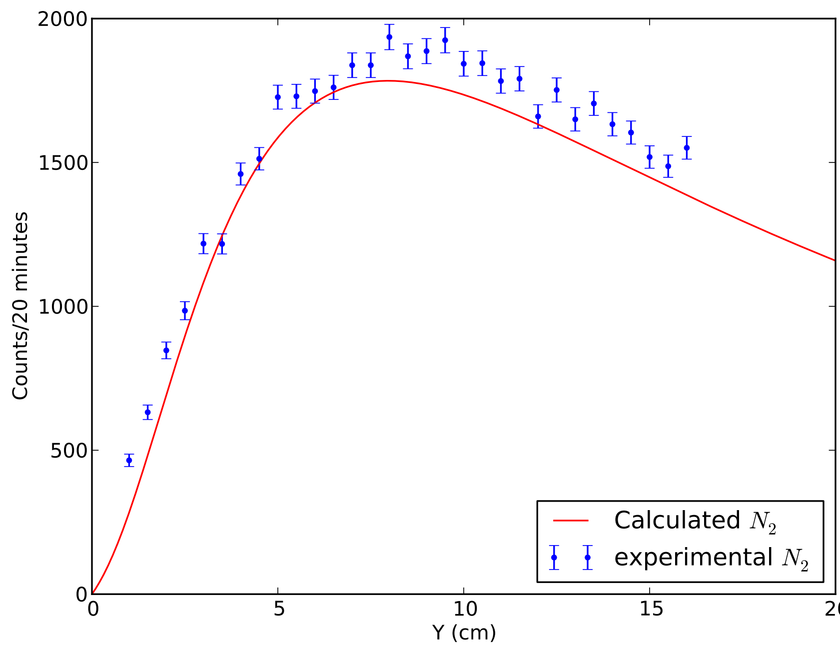

Figure 5: The calculated value for N_2 is shown in red, while our experimental value for N_2 is given in the blue data points with error bars corresponding to $\sqrt{\# \; of \; counts}$ per data point.

Our experimental values tend to vary from the calculated rate of scattered particles mostly after the peak of the curves, which are about the same. The reason for this discrepancy at higher values for Y is due to the fact that there is a higher chance of particles scattering off of the walls of the chamber and still making it though to the detector. Therefore, there is some over-counting that we see at values of Y greater than about 8cm.

4.3 Finding the Atomic Number (Z)

The atomic number of gold, given by Z, cannot be found using our data because we would need to disassemble the apparatus and place the source right next to the detector in order to determine the proper value for N_0. Z is a factor in G, so when we rearrange Equation 10 appropriately, we have:

$$G(Z) = \frac{N_2}{N_0 \times f(Y)}. \tag{20}$$

In the above equation, we have values for N_2 and a distribution to represent $f(Y)$, but if we cannot experimentally determine N_0, we do not have enough known variables to solve for Z. Therefore, we accept that the atomic number of Gold is 79 as given by the periodic table.

4.4 Calculating the Nuclear Radius Upper Limit

The upper limit of the nucleus radius can be determined from the impact parameter, b, which we derived earlier in section 2. It is the distance from the center of the alpha particle to the center of the nucleus. The alpha particles do not have enough force to penetrate the nucleus a significant amount compared to the radius of the nucleus, so even though b is the sum of the alpha particle radius and the nucleus radius, we can assume that it is a good approximation for the upper limit of the nuclear radius. Therefore, we know that b is:

$$b = \frac{Zze^2}{2\pi\epsilon_0 mv^2}. \tag{21}$$

Here, we are also making the assumption that the alpha particles hits the nucleus head on and deflects back along a straight line 180° from its initial impact. The quantities in Equation 21 are:

- $Ze = 79 \times (1.6 \times^1 0^{-19} C)$
- $ze = 2 \times (1.6 \times^1 0^{-19} C)$
- $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 N \cdot m/C^2$
- $m = 6.7 \times 10^{-27} kg$
- $v = 2 \times 10^7 m/s$

Therefore,

$$b = \frac{(79 \times (1.6 \times^1 0^{-19} C))(2 \times (1.6 \times^1 0^{-19} C))(9 \times 10^9 N \cdot m/C^2)}{(6.7 \times 10^{-27} kg)(2 \times 10^7 m/s)^2} = 2.7 \times 10^{-14} m. \tag{22}$$

The upper limit on the nuclear radius of Gold is 2.7×10^{-14}m as calculated from the parameters given the setup that we have for this experiment.

5 Conclusion

References

[1] Sleator, Tycho, and Windt, David, *Rutherford Scattering.* Experimental Physics. V85.0112. Spring, 2011.

6 Appendix

Y (cm)	Counts/ 20 min
3.0	465
4.0	847
5.0	1218
6.0	1460
7.0	1727
8.0	1748
9.0	1838
10.0	1936
11.0	1867
12.0	1843
13.0	1783
14.0	1660
15.0	1650
16.0	1633
17.0	1519
18.0	1551

Table 3: First run of data taken from 3cm to 18cm in intervals of 1cm.

Y (cm)	Counts/ 20 min
3.5	632
4.5	985
5.5	1217
6.5	1513
7.5	1730
8.5	1741
9.5	1838
10.5	1869
11.5	1925
12.5	1845
13.5	1791
14.5	1752
15.5	1705
16.5	1604
17.5	1487

Table 4: Second run of data taken from 3.5 cm to 17.5cm in intervals of 1cm.