metaSEM: An R Package for Meta-Analysis Using Structural Equation Modeling

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Abstract

The **metaSEM** package provides functions to conducting univariate and multivariate meta-analyses using a structural equation modeling approach via the **OpenMx** package. It also implements the two-stage structural equation modeling (TSSEM) approach to conducting fixed- and random-effects meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. This paper outlines the basic theories and some applications of them. Examples are used to illustrate the procedures.

Keywords:~meta-analysis, structural equation modeling, meta-analytic structural equation modeling, **metaSEM**, R.

1. Introduction

Meta-analysis is a popular technique to synthesize research findings in the social, behavioral, educational and medical sciences (Borenstein, Hedges, Higgins, and Rothstein 2009; Hedges and Olkin 1985; Hunter and Schmidt 2004; Whitehead 2002). There are several standalone packages to conduct meta-analysis, e.g., Comprehensive Meta-Analysis and RevMan. Many standard statistical packages have macros, for instance, SPSS (Lipsey and Wilson 2000), SAS (Arthur, Bennett, and Huffcutt 2001) and STATA (Sterne 2009), have macros or packages to fit some meta-analytic models. Even in the R community, there are already several packages for meta-analysis, for instance, **meta** (Schwarzer 2010), **rmeta** (Lumley 2009), **mvmeta** (Gasparrini 2012), **metaLik** (Guolo and Varin 2011) and **metafor** (Viechtbauer 2010).

The **metaSEM** is yet another R package to conducting univariate and multivariate meta-analysis. It formulates meta-analytic models as structural equation models (Cheung 2008, 2011b) via the OpenMx package (Boker, Neale, Maes, Wilde, Spiegel, Brick, Spies, Estabrook, Kenny, Bates, Mehta, and Fox 2011). It also implements the two-stage structural equation modeling (TSSEM) approach (Cheung and Chan 2005b, 2009) to conducting fixed- and random-effects meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. The main functions in this package are:

- `meta()` and `reml()`: `meta()` conducts univariate and multivariate meta-analysis with maximum likelihood (ML) estimation method while `reml()` estimates the variance components of the random-effects with restricted (residual) maximum likelihood (REML) estimation method. Mixed-effects meta-analysis can be conducted by specifying study characteristics as predictors. Equality constraints on the intercepts, regression coefficients and variance components can be imposed.

- `tssem1()`: It conducts the first stage analysis of TSSEM by pooling correlation/covariance matrices with either a fixed- or random-effets model.
- `tssem2()`: It conducts the second stage analysis of TSSEM by fitting structural models on the pooled correlation/covariance matrix. It is a wrapper of `wls()`.
- `wls()`: It fits a correlation/covariance structure analysis with weighted least squares estimation method.

Besides reporting Wald confidence intervals (CIs) based on z statistic, likelihood-based CIs on the parameter estimates may also be requested (Cheung 2009a; Neale and Miller 1997). Several generic functions, such as `anova()`, `coef()`, `vcov()`, `print()`, `summary()` and `plot()`, have been implemented.

The current version of the **metaSEM** package is 0.7-1. This paper is organized as follows. The next section introduces general meta-analytic models. Basic theory of the TSSEM are then presented. Several examples are used to illustrate these procedures.

2. Structural Equation Modeling Based Meta-analysis

In this section, basic structural equation models are introduced. Univariate and multivariate meta-analysis are treated as special cases of SEM.

2.1. Structural equation model

Structural equation modeling is a multivariate technique to fitting and testing hypothesized models. Let $\mathbf{y}$ be a $p \times 1$ vector of the sample data where p is the number of variables. It is hypothesized that the model for the first and second moments are $\mu = \mu(\theta)$ and $\mathbf{\Sigma} = \mathbf{\Sigma}(\theta)$, respectively, where θ is a vector of parameters.

The -2*log-likelihood of the ith case is:

$$-2 * logL_i(\theta; \mathbf{y}_i) = p_i * log(2\pi) + log|\mathbf{\Sigma_i}(\theta)| + (\mathbf{y}_i - \mu_\mathbf{i}(\theta))' \mathbf{\Sigma_i}(\theta)^{-1} (\mathbf{y}_i - \mu_\mathbf{i}(\theta)), \quad (1)$$

where p_i is the number of variables in the ith case, $\mu_\mathbf{i}(\theta)$ and $\mathbf{\Sigma_i}(\theta)$ are the model implied mean vector and the model implied covariance matrix for the ith case, respectively. Since there is a subscript i in these quantities, the model implied mean vector and covariance matrix may vary across cases. In order words, this model handles incomplete data automatically by including only the complete variables in the log-likelihood function.

To obtain the parameter estimates, we may take the sum of the -2*log-likelihood over all cases and minimize it. This is known as the ML estimation method. After the optimization, the asymptotic covariance matrix (thus the standard errors) of the parameter estimates may be obtained from the inverse of the Hessian matrix. The parameter estimates divided by their standard errors follow a z distribution under the null hypothesis. Moreover, likelihood ratio statistic may be used to compare nested models.

2.2. Univariate fixed-effects model

When there is only one effect size, the univariate fixed-effects model for the ith study is:

$$y_i = \beta_{\text{fixed}} + e_i, \tag{2}$$

where β_{fixed} is the common effect under a fixed-effects model and $\text{var}(e_i) = v_i$ is the known sampling variance. To fit the unvariate fixed-effects meta-analysis with SEM, we may use the following model:

$$\mu_i(\theta) = \beta_{\text{fixed}} \tag{3}$$

and

$$\mathbf{\Sigma}_i(\theta) = v_i \tag{4}$$

Since v_i is known, the only parameter in the univariate fixed-effects model is β_{fixed} .

2.3. Univariate random-effects model

A random-effects model allows studies having their own study specific effect. The model for the ith study is:

$$y_i = \beta_{\text{random}} + u_i + e_i. \tag{5}$$

where β_{random} is the average effect under a random-effects model and $\text{var}(u_i) = \tau^2$ is the heterogeneity variance that has to be estimated. To fit the unvariate fixed-effects meta-analysis in SEM, we may use the following model:

$$\mu_i(\theta) = \beta_{\text{random}} \tag{6}$$

and

$$\mathbf{\Sigma}_i(\theta) = \tau^2 + v_i \tag{7}$$

In this model we have to estimate both β_{random} and τ^2.

2.4. Univariate mixed-effects model

The mixed-effects meta-analysis extends the random-effects meta-analysis by including predictors. Assuming that $\mathbf{x}$ is a $m \times 1$ vector of predictors where m is the number predictors, the model is:

$$y_i = \beta_0 + \beta'\mathbf{x} + u_i + e_i, \tag{8}$$

where β is a vector of regression coefficients.

To fit the univariate mixed-effects meta-analysis in SEM, we may use the following model:

$$\mu_i(\theta|\mathbf{x}) = \beta_0 + \beta'\mathbf{x} \tag{9}$$

and

$$\mathbf{\Sigma}_i(\theta|\mathbf{x}) = \tau^2 + v_i. \tag{10}$$

Since $\mathbf{x}$ is specified via the definition variables, the means and covariance matrix of $\mathbf{x}$ are not estimated.

2.5. Multivariate mixed-effects model

Let us assume that there are p effect sizes with m predictors in k studies. The model for the multivariate effect sizes in the ith study is:

$$\mathbf{y}_i = \mathbf{B}\mathbf{x}_i + \mathbf{u}_i + \mathbf{e}_i, \tag{11}$$

where $\mathbf{y}_i$ is a $p \times 1$ effect sizes, $\mathbf{B}$ is a $p \times (m+1)$ regression coefficients including the intercepts, $\mathbf{x}_i$ is a $(m+1) \times 1$ predictors including 1 as the first element, $\mathbf{u}_i$ is a $p \times 1$ study specific random effects, and $\mathbf{e}_i$ is a $p \times 1$ sampling error. We assume that $\text{var}(\mathbf{e}_i) = V_i$ is known and given in the ith study and $\text{var}(\mathbf{u}_i) = T^2$ is estimated from the data. T^2 is the variance component of the between-study heterogeneity.

The -2*log-likelihood of the above model is:

$$-2 * logL_i(\mathbf{B}, T^2; \mathbf{y}_i)_{\text{ML}} = p_i * log(2\pi) + log|T^2 + V_i| + (\mathbf{y}_i - \mathbf{B}\mathbf{x}_i)'(T^2 + V_i)^{-1}(\mathbf{y}_i - \mathbf{B}\mathbf{x}_i), \tag{12}$$

where p_i is the number of effect sizes in the ith study.

In applied research, different studies may report different effect sizes, that is, p_i may vary across studies. The above -2*log-likelihood may handle missing effect sizes by using different dimenions of the elements in the above equation. It is expected that there is no missing data in $\mathbf{x}_i$. When there are missing data in $\mathbf{x}_i$, the whole study will be deleted before the analysis.

2.6. Restricted Maximum Likelihood (REML) Estimation Method

Since both the fixed- and random-effects are estimated simultaneously, it is well-known that $\hat{T}^2_{\text{ML}}$ based on the ML estimation is under-estimated. It is because it does not take the uncertainty in estimating $\hat{\mathbf{B}}_{\text{ML}}$ into account. If the unbiasness of the variance component is crucial in the research questions, it is possible to obtain the variance component $\hat{T}^2_{\text{REML}}$ based on the REML estimation method (Cheung 2011a; Harville 1977; Patterson and Thompson 1971).

The -2log-likelihood of the model with a constant term is:

$$-2logL_i(T^2; \mathbf{y}_i)_{\text{REML}} = p_i * log(2\pi) + log|T^2 + V_i| + (\mathbf{y}_i - \alpha\mathbf{X}_i)'(T^2 + V_i)^{-1}(\mathbf{y}_i - \alpha\mathbf{X}_i) + |X_i' V_i^{-1} X_i|, \tag{13}$$

where $\alpha = (X'V^{-1}X)^{-1}X'V^{-1}\mathbf{y}$.

2.7. Examples

Two example data sets are used to demonstrate the procedures of univariate and multivariate meta-analyses. The first data set was taken from Becker (1983) who reported 10 studies on sex differences in conformity using the fictitious norm group paradigm. *di* and *vi* are the standardized mean difference and its sampling variance, respectively. Becker hypothesized that the logarithm of the number of items (*items*) predicted the effect size.

The second data set is adapted from Berkey, Hoaglin, Antczak-Bouckoms, Mosteller, and Colditz (1998). They summarized five published trials comparing surgical and non-surgical treatments for medium-severity periodontal disease, one year after treatment. Publication year *pub_year* was hypothesized as a predictor.

Univariate random-effects model The function `meta()` is used to conduct the analyses. The arguments `y` and `v` are used to specify the effect sizes and its sampling variances (and covariances for multivariate meta-analysis), respectively. By default, a random-effects meta-analysis is fitted. After running the analysis, `summary()` may be used to extract the results. The estimated fixed- and random-effects are represented by the **Intercept** and **Tau2** parameters.

From the following analyses, the Q statistic ($df = 9$) is 30.6495, $p < .001$. The pooled effect size with its 95% Wald confidence interval (CI) based on the random-effects model is 0.1747 (-0.0475, 0.3970). The estimated heterogeneity variance is 0.0774.

```
R> ## Load the library
R> library(metaSEM)
R> ## Show the first few studies of the data set
R> head(Becker83)

  study    di   vi percentage items
1     1 -0.33 0.03         25     2
2     2  0.07 0.03         25     2
3     3 -0.30 0.02         50     2
4     4  0.35 0.02        100    38
5     5  0.69 0.07        100    30
6     6  0.81 0.22        100    45

R> ## Random-effects meta-analysis with ML
R> summary( meta(y=di, v=vi, data=Becker83) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.174734  0.113378 -0.047482  0.396950  1.5412   0.1233
Tau2_1_1    0.077376  0.054108 -0.028674  0.183426  1.4300   0.1527

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 2
Degrees of freedom: 8
-2 log likelihood: 7.928307
```

```
R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:58 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Univariate mixed-effects model We may include a predictor to conduct a mixed-effects meta-analysis. The argument `x` is used to specify the predictors. If there are more than one predictor, `cbind()` may be used to specify them. The estimated regression coefficients are represented by the **slope** parameter. The result suggests that $log(items)$ is a significant predictor with the estimated regression coefficient and its 95% CI of 0.2109 (0.1225, 2.9924).

```
R> ## Mixed-effects meta-analysis with "log(items)" as the predictor
R> summary( meta(y=di, v=vi, x=log(items), data=Becker83) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, x = log(items), data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
              Estimate   Std.Error      lbound      ubound z value
Intercept1 -3.2015e-01  1.0981e-01 -5.3539e-01 -1.0492e-01 -2.9154
Slope1_1    2.1088e-01  4.5084e-02  1.2251e-01  2.9924e-01  4.6774
Tau2_1_1    1.0000e-10  2.0095e-02 -3.9386e-02  3.9386e-02  0.0000
            Pr(>|z|)
Intercept1  0.003552 **
Slope1_1   2.905e-06 ***
Tau2_1_1    1.000000
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 3
Degrees of freedom: 7
-2 log likelihood: -4.208024

R version: 2.14.1
```

```
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:58 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Univariate fixed-effects model Mathematically, fixed-effects meta-analysis is a special case of the random-effects meta-analysis by fixing the variance of the random-effects at 0. The argument `RE.constraints`, which expects a matrix as input, is used to constrain the variance component of the random effects.

```
R> ## Fixed-effects meta-analysis
R> summary( meta(y=di, v=vi, data=Becker83, RE.constraints=matrix(0, ncol=1, nrow=1)) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83, RE.constraints = matrix(0,
    ncol = 1, nrow = 1))

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.100640  0.060510 -0.017957  0.219237  1.6632  0.09627 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 1
Degrees of freedom: 9
-2 log likelihood: 17.86043

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:58 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Multivariate random-effects model Multivariate meta-analysis can be performed by specifying the multivariate effect sizes and its sampling covariance matrix in the arguments `y` and v with `cbind()`, respectively. Only the lower triangle of the sampling covariance matrix arranged by the column major is used in v.

The Q statistic ($df = 8$) is 128.2267, $p < .001$. The pooled effect sizes with their 95% Wald CIs based on the random-effects model for PD and AL are 0.3448 (0.2397, 0.4500) and -0.3379 (-0.4972, -0.1787), respectively. The estimated variance component is $\begin{bmatrix} 0.0070 & \\ 0.0095 & 0.02614 \end{bmatrix}$.

```
R> ## Show the data set
R> Berkey98

  trial pub_year no_of_patients   PD    AL var_PD cov_PD_AL var_AL
1     1     1983             14 0.47 -0.32 0.0075    0.0030 0.0077
2     2     1982             15 0.20 -0.60 0.0057    0.0009 0.0008
3     3     1979             78 0.40 -0.12 0.0021    0.0007 0.0014
4     4     1987             89 0.26 -0.31 0.0029    0.0009 0.0015
5     5     1988             16 0.56 -0.39 0.0148    0.0072 0.0304

R> ## Multivariate meta-analysis with a random-effects model
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98) )

Running Meta analysis with ML

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    data = Berkey98)

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value
Intercept1  0.3448392  0.0536312  0.2397239  0.4499544  6.4298
Intercept2 -0.3379381  0.0812480 -0.4971812 -0.1786951 -4.1593
Tau2_1_1    0.0070020  0.0090497 -0.0107351  0.0247391  0.7737
Tau2_2_1    0.0094607  0.0099698 -0.0100797  0.0290010  0.9489
Tau2_2_2    0.0261445  0.0177409 -0.0086270  0.0609161  1.4737
            Pr(>|z|)
Intercept1 1.278e-10 ***
Intercept2 3.192e-05 ***
Tau2_1_1      0.4391
Tau2_2_1      0.3427
Tau2_2_2      0.1406
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0
```

```
Number of studies (or clusters): 5
Number of observed statistics: 10
Number of estimated parameters: 5
Degrees of freedom: 5
-2 log likelihood: -11.68131

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:58 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Multivariate mixed-effects model As an illustration, we use *pub_year* as a predictor. To make the intercept more interpretable, we may center the publication year at 1979, the first record of publication year in the data set. The estimated regression coefficients and their 95% CIs on PD and AL are 0.0064 (-0.2048, 0.2177) and -0.0706 (-0.3883, 0.2471), respectively. The likelihood ratio statistic on testing both regression coefficient is $\chi^2(df = 2) = 0.3273, p = .8490$. Thus, both regression coefficients are non-significant.

Sometimes, we may want to test the equality of the regression coefficients and see if they are differnt. We may impose the equality constraint on the regression coefficients with the argument `coef.constraints`. The average regression coefficient is 0.0017 (-0.1991, 0.2025). The likelihood ratio statistic on testing the equality of the regression coefficients is $\chi^2(df = 1) = 0.3270, p = .5674$. There is no evidence that one regression coefficient is stronger from the other.

```
R> ## Multivariate meta-analysis with "publication year-1979" as a predictor
R> mul1 <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
               x=scale(pub_year, center=1979), model.name="No constraint")

Running No constraint

R> summary(mul1)

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    x = scale(pub_year, center = 1979), data = Berkey98, model.name = "No constraint")

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value
Intercept1  0.3440001  0.0857659  0.1759020  0.5120982  4.0109
Slope1_1    0.0063540  0.1078235 -0.2049762  0.2176842  0.0589
Intercept2 -0.2918175  0.1312797 -0.5491209 -0.0345140 -2.2229
Slope2_1   -0.0705888  0.1620966 -0.3882922  0.2471147 -0.4355
```

```
Tau2_1_1    0.0080405  0.0101206 -0.0117955  0.0278766  0.7945
Tau2_2_1    0.0093413  0.0105515 -0.0113392  0.0300218  0.8853
Tau2_2_2    0.0250135  0.0170788 -0.0084603  0.0584873  1.4646
            Pr(>|z|)
Intercept1 6.048e-05 ***
Slope1_1     0.95301
Intercept2   0.02622 *
Slope2_1     0.66322
Tau2_1_1     0.42692
Tau2_2_1     0.37599
Tau2_2_2     0.14303
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies (or clusters): 5
Number of observed statistics: 10
Number of estimated parameters: 7
Degrees of freedom: 3
-2 log likelihood: -12.00859

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:59 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Multivariate meta-analysis with the constraint that both regression coefficients are
R> mul0 <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
               x=scale(pub_year, center=1979), model.name="Both reg coef are 0",
               coef.constraints=matrix(c("0", "0"), nrow=2))

Running Both reg coef are 0

R> summary(mul0)

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    x = scale(pub_year, center = 1979), data = Berkey98, coef.constraints = matrix(c("0",
        "0"), nrow = 2), model.name = "Both reg coef are 0")

95% confidence intervals: z statistic approximation
Coefficients:
```

```
              Estimate  Std.Error     lbound     ubound z value
Intercept1  0.3448392  0.0536312  0.2397239  0.4499544  6.4298
Intercept2 -0.3379381  0.0812480 -0.4971812 -0.1786951 -4.1593
Tau2_1_1    0.0070020  0.0090497 -0.0107351  0.0247391  0.7737
Tau2_2_1    0.0094607  0.0099698 -0.0100797  0.0290010  0.9489
Tau2_2_2    0.0261445  0.0177409 -0.0086270  0.0609161  1.4737
            Pr(>|z|)
Intercept1 1.278e-10 ***
Intercept2 3.192e-05 ***
Tau2_1_1      0.4391
Tau2_2_1      0.3427
Tau2_2_2      0.1406
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies (or clusters): 5
Number of observed statistics: 10
Number of estimated parameters: 5
Degrees of freedom: 5
-2 log likelihood: -11.68131

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:59 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Likelihood ratio test on testing both regression coefficients are 0
R> anova(mul1, mul0)

           base          comparison ep  minus2LL df       AIC
1 No constraint                <NA>  7 -12.00859  3 -18.00859
2 No constraint Both reg coef are 0  5 -11.68131  5 -21.68131
     diffLL diffdf         p
1        NA     NA        NA
2 0.3272789      2 0.8490481

R> ## Multivariate meta-analysis with an equality constraint on the slopes
R> mul2 <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
               x=scale(pub_year, center=1979), model.name="With equality constraint",
               coef.constraints=matrix(c("0.3*Equal_slope", "0.3*Equal_slope"), nrow=2))
```

```
Running With equality constraint

R> summary(mul2)

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    x = scale(pub_year, center = 1979), data = Berkey98, coef.constraints = matrix(c("0.3*
        "0.3*Equal_slope"), nrow = 2), model.name = "With equality constraint")

95% confidence intervals: z statistic approximation
Coefficients:
              Estimate  Std.Error     lbound     ubound z value
Intercept1   0.3437612  0.0849828  0.1771979  0.5103245  4.0451
Equal_slope  0.0016748  0.1024443 -0.1991122  0.2024619  0.0163
Intercept2  -0.3390010  0.1041005 -0.5430343 -0.1349677 -3.2565
Tau2_1_1     0.0070474  0.0094638 -0.0115013  0.0255962  0.7447
Tau2_2_1     0.0095165  0.0105668 -0.0111940  0.0302269  0.9006
Tau2_2_2     0.0261979  0.0180773 -0.0092330  0.0616288  1.4492
             Pr(>|z|)
Intercept1  5.231e-05 ***
Equal_slope  0.986956
Intercept2   0.001128 **
Tau2_1_1     0.456471
Tau2_2_1     0.367800
Tau2_2_2     0.147278
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies (or clusters): 5
Number of observed statistics: 10
Number of estimated parameters: 6
Degrees of freedom: 4
-2 log likelihood: -11.68158

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:59 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Likelihood ratio test on the equality of regression coefficients
R> anova(mul1, mul2)
```

```
           base              comparison ep  minus2LL df       AIC
1 No constraint                    <NA>  7 -12.00859  3 -18.00859
2 No constraint With equality constraint  6 -11.68158  4 -19.68158
     diffLL diffdf         p
1        NA     NA        NA
2 0.3270107      1 0.5674246
```

Multivariate fixed-effects model A multivariate fixed-effects meta-analysis can be conducted by fixing the variance component at a zero matrix. The following code illustrates the syntax.

```
R> ## Multivariate meta-analysis with a fixed-effects model
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
           RE.constraints=matrix(0, nrow=2, ncol=2)) )

Running Meta analysis with ML

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    data = Berkey98, RE.constraints = matrix(0, nrow = 2, ncol = 2))

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value  Pr(>|z|)
Intercept1  0.307219  0.028575  0.251212  0.363225  10.751 < 2.2e-16
Intercept2 -0.394377  0.018649 -0.430929 -0.357825 -21.147 < 2.2e-16

Intercept1 ***
Intercept2 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies (or clusters): 5
Number of observed statistics: 10
Number of estimated parameters: 2
Degrees of freedom: 8
-2 log likelihood: 90.88326

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:59 2012
```

```
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

REML The `reml()` function may be used to estimate the variance component with the REML estimation method. It should be noted that it does not estimate the fixed-effects. The fixed-effects estimates can be obtained via the `meta()` function by specifying the variance component in the `RE.constraints` argument. This approach is consistent with the idea of REML that removes the fixed-effects parameter when estimating the variance component.

```
R> ## Random-effects meta-analysis with ML
R> summary( meta(y=di, v=vi, data=Becker83) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.174734  0.113378 -0.047482  0.396950  1.5412   0.1233
Tau2_1_1    0.077376  0.054108 -0.028674  0.183426  1.4300   0.1527

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 2
Degrees of freedom: 8
-2 log likelihood: 7.928307

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:20:59 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Random-effects meta-analysis with REML
R> summary( VarComp <- reml(y=di, v=vi, data=Becker83) )

Running Variance component with REML

Call:
```

```
reml(y = di, v = vi, data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.091445  0.064228 -0.034439  0.217329  1.4238   0.1545

Number of studies (or clusters): 10
Number of observed statistics: 9
Number of estimated parameters: 1
Degrees of freedom: 8
-2 log likelihood: -6.110579

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:00 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Extract the variance component
R> VarComp_REML <- matrix( coef(VarComp), ncol=1, nrow=1 )
R> ## Meta-analysis by treating the variance component as fixed
R> summary( meta(y=di, v=vi, data=Becker83, RE.constraints=VarComp_REML) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83, RE.constraints = VarComp_REML)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.180189  0.117535 -0.050176  0.410555  1.5331   0.1253

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 1
Degrees of freedom: 9
-2 log likelihood: 7.986161

R version: 2.14.1
OpenMx version: 1.2.0-1926
```

```
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:00 2012
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( reml(y=di, v=vi, x=log(items), data=Becker83) )

Running Variance component with REML

Call:
reml(y = di, v = vi, x = log(items), data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
           Estimate  Std.Error     lbound     ubound z value Pr(>|z|)
Tau2_1_1  0.0052656  0.0212014 -0.0362884  0.0468196  0.2484   0.8039

Number of studies (or clusters): 10
Number of observed statistics: 8
Number of estimated parameters: 1
Degrees of freedom: 7
-2 log likelihood: -10.84567

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:00 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98) )

Running Variance component with REML

Call:
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    data = Berkey98)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.011733  0.013645 -0.015011  0.038477  0.8599   0.3899
Tau2_2_1  0.011916  0.014416 -0.016340  0.040172  0.8266   0.4085
Tau2_2_2  0.032651  0.024402 -0.015176  0.080479  1.3380   0.1809
```

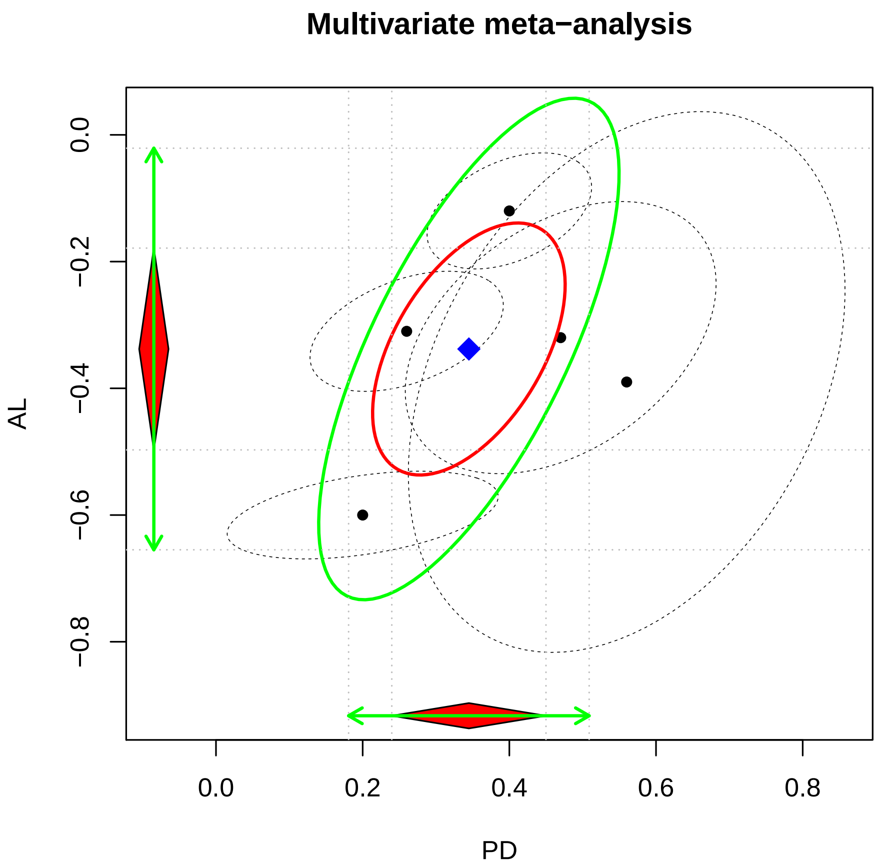
```
Number of studies (or clusters): 5
Number of observed statistics: 8
Number of estimated parameters: 3
Degrees of freedom: 5
-2 log likelihood: -18.86768

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:01 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Plots of multivariate effect sizes If multivariate meta-analysis is conducted, pairwise plots on the pooled effect sizes and their confidence ellipses can be obtained via the `plot()` function. By default, 95% confidence intervals on the average effect sizes and confidence ellipses on the random effects are plotted. For example,

```
R> Berkey98.ma <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98)
```

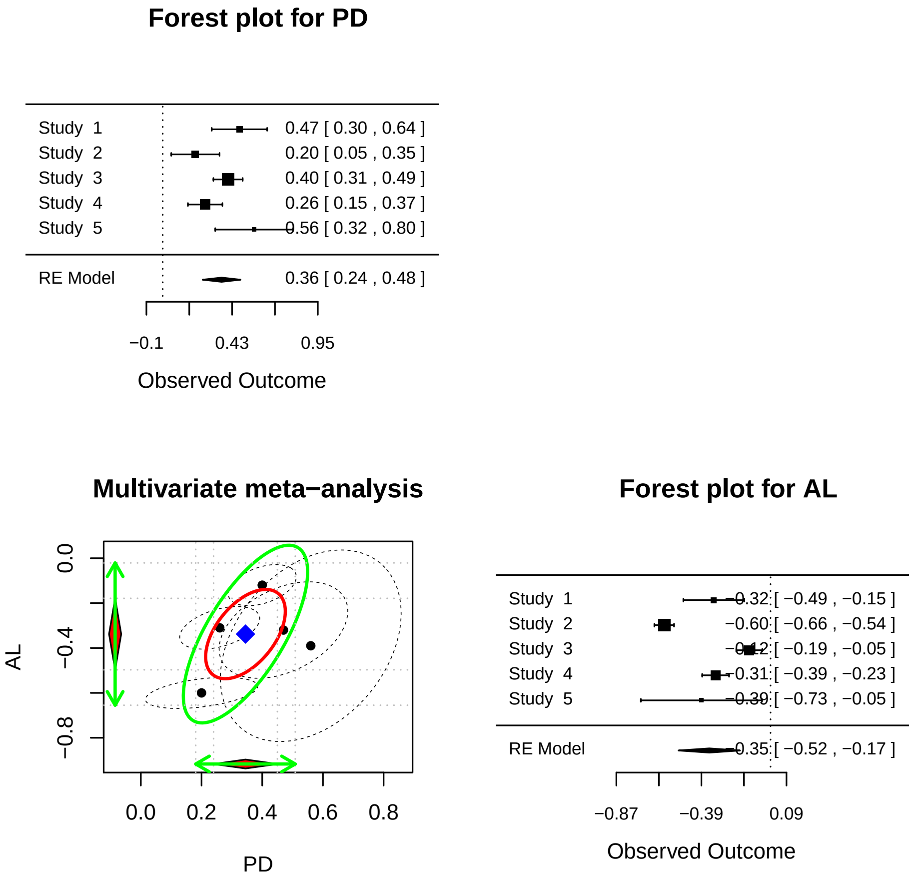
```
Running Meta analysis with ML
```

```
R> plot(Berkey98.ma, main="Multivariate meta-analysis", axis.label=c("PD", "AL"))
```



By combining with the forest plots from the **metafor** package, we may combine the univariate and multivariate natures of the effect sizes in a single figure. This will be very useful for multivariate meta-analysis.

```
R> ## Load the metafor package to display forest plots
R> library(metafor)
R> plot(Berkey98.ma, diag.panel=TRUE, main="Multivariate meta-analysis",
        axis.label=c("PD", "AL"))
R> forest( rma(yi=PD, vi=var_PD, data=Berkey98) )
R> title("Forest plot for PD")
R> forest( rma(yi=AL, vi=var_AL, data=Berkey98) )
R> title("Forest plot for AL")
```



3. Meta-analytic structural equation modeling

MASEM combines the idea of meta-analysis and SEM by pooling correlation/covariance matrices and testing structural equation models on the pooled matrix. There are two stages in conducting a MASEM. In the first stage the correlation/covariance matrices are pooled together. In the second stage, the pooled correlation/covariance matrix is used to fit structural equation models.

Cheung and Chan (2005b, 2009) proposed a two-stage structural equation modeling (TSSEM) based on a fixed-effects model. The `metaSEM` package has implemented the TSSEM approach. Moreover, the TSSEM approach has been extended to the random-effects model by using a multivariate meta-analysis (Cheung 2011b) in the first stage analysis. Regardless of whether a fixed- or random-effects model is used, the `tssem2()` function will handle this automatically. In other words, parameter estimates, standard errors and goodness-of-fit indices in the stage 2 analysis has already taken the stage 1 model into account.

An example from Cheung (2009b) is used to illustrate the procedures. In this example, Digman (1997) reported a second-order factor analysis on a five-factor model with 14 studies. He proposed that there were two second-order factors for the five-factor model: an alpha factor for agreeableness, conscientiousness, and emotional stability, and a beta factor for extroversion and intellect.

3.1. Fixed-effects model

The `tssem1()` function is used to pool the correlation matrices with a fixed-effects model in the first stage by specifying `method='FEM'` in the argument. `tssem2()` is then used to fit a factor analytic model on the pooled correlation matrix with the inverse of its asymptotic covariance matrix as the weight matrix (Cheung and Chan 2005b, 2009).

The fit indices for testing the homogeneity of the correlation matrices in Stage 1 analysis are $\chi^2(130, N = 4496) = 1499.73, p < .001$, CFI=0.6825, TLI=0.6581, SRMR=0.1750 and RMSEA=0.1812. This indicates that it is not reasonable to assume that the correlation matrices are homogenous. Sub-group analysis or random-effects model that will be illustrated later are more appropriate. As an exercise, we continute to fit the stage 2 model. The fit indices for fitting the structural model in Stage 2 are $\chi^2(4, N = 4496) = 67.89, p < .001$, CFI=0.9845, TLI=0.9613, SRMR=0.0285 and RMSEA=0.0596.

```
R> #### Show the first 2 studies in Digman97
R> head(Digman97$data, n=2)

$`Digman 1 (1994)`
       E     A     C   ES    I
E   1.00 -0.48 -0.10 0.27 0.37
A  -0.48  1.00  0.62 0.41 0.00
C  -0.10  0.62  1.00 0.59 0.35
ES  0.27  0.41  0.59 1.00 0.41
I   0.37  0.00  0.35 0.41 1.00

$`Digman 2 (1994)`
       E     A    C   ES     I
E   1.00 -0.30 0.07 0.09  0.45
A  -0.30  1.00 0.39 0.53 -0.05
C   0.07  0.39 1.00 0.59  0.44
ES  0.09  0.53 0.59 1.00  0.22
I   0.45 -0.05 0.44 0.22  1.00

R> #### Show the first 2 sample sizes in Digman97
R> head(Digman97$n, n=2)

[1] 102 149

R> #### Example of Fixed-effects TSSEM
R> fixed1 <- tssem1(Digman97$data, Digman97$n, method="FEM")

Running TSSEM1 Analysis of Correlation Matrix

R> summary(fixed1)
```

```
Call:
tssem1FEM(my.df = my.df, n = n, cor.analysis = cor.analysis,
    model.name = model.name, cluster = cluster, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.103751  0.015070  6.8846 5.796e-12 ***
S[1,3] 0.135208  0.014799  9.1363 < 2.2e-16 ***
S[1,4] 0.244505  0.014175 17.2487 < 2.2e-16 ***
S[1,5] 0.424514  0.012396 34.2463 < 2.2e-16 ***
S[2,3] 0.363116  0.013391 27.1169 < 2.2e-16 ***
S[2,4] 0.390176  0.012903 30.2387 < 2.2e-16 ***
S[2,5] 0.092246  0.015071  6.1207 9.319e-10 ***
S[3,4] 0.415999  0.012540 33.1736 < 2.2e-16 ***
S[3,5] 0.141213  0.014891  9.4834 < 2.2e-16 ***
S[4,5] 0.138167  0.014858  9.2991 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     4496.0000
Chi-square of target model      1499.7340
DF of target model               130.0000
p value of target model            0.0000
Chi-square of independent model 4454.5995
DF of independent model          140.0000
RMSEA                              0.1812
SRMR                               0.1750
TLI                                0.6581
CFI                                0.6825
AIC                             1239.7340
BIC                              406.3114

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:08 2012
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> #### Extract the pooled correlation matrix
R> coef(fixed1)

          x1         x2        x3        x4         x5
x1 1.0000000 0.10375113 0.1352076 0.2445051 0.42451422
x2 0.1037511 1.00000000 0.3631157 0.3901765 0.09224586
```

```
x3 0.1352076 0.36311571 1.0000000 0.4159987 0.14121294
x4 0.2445051 0.39017646 0.4159987 1.0000000 0.13816667
x5 0.4245142 0.09224586 0.1412129 0.1381667 1.00000000

R> #### Prepare a CFA model to be fitted
R> #### The syntax is based on OpenMx
R> P <- mxMatrix("Stand", ncol=2, nrow=2, value=.2, free=TRUE, name="P")
R> L <- mxMatrix("Full", ncol=2, nrow=5, value=c(0,.3,.3,.3,0,.3,0,0,0,.3),
                free=c(FALSE,TRUE,TRUE,TRUE,FALSE,TRUE,FALSE,FALSE,FALSE,TRUE),
                name="L")
R> ## Model to be fitted
R> impliedR <- mxAlgebra(L %&% P, name="impliedR")
R> fixed2 <- tssem2(fixed1, impliedS=impliedR, matrices=c(P, L),
                    model.name="TSSEM2 Digman97")

Running TSSEM2 Digman97

R> summary(fixed2)

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
       Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L[1,2] 0.779567  0.033173 0.714548 0.844586  23.500 < 2.2e-16 ***
L[2,1] 0.564486  0.014282 0.536494 0.592477  39.525 < 2.2e-16 ***
L[3,1] 0.607144  0.014297 0.579121 0.635166  42.465 < 2.2e-16 ***
L[4,1] 0.717134  0.014772 0.688182 0.746086  48.548 < 2.2e-16 ***
L[5,2] 0.554206  0.025113 0.504985 0.603427  22.068 < 2.2e-16 ***
P[1,2] 0.362687  0.021955 0.319657 0.405717  16.520 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     4496.0000
Chi-square of target model        67.8897
DF of target model                 4.0000
p value of target model            0.0000
Chi-square of independent model 4132.8505
DF of independent model           10.0000
RMSEA                              0.0596
SRMR                               0.0285
```

```
TLI                                      0.9613
CFI                                      0.9845
AIC                                     59.8897
BIC                                     34.2459

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:08 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Example: Fixed-effects model with sub-group analysis Studies may not share the same population correlation matrix. If the studies can be grouped into various subgroups, we may pool the correlation matrices separately by the subgroups (Cheung and Chan 2005a). This is similar to the subgroup analysis in conventional meta-analysis (Hedges and Olkin 1985). For example, Digman (1997) groups the 14 studies into several groups according to their sample characteristics. These include children, adolescents, young adults, and mature adults. This variable is stored in the variable `Digman97$cluster`.

The following R code may be used to conduct the analysis. Users have to supply the cluster (a vector of labels) to the `cluster` argument in `tssem1()`. The correlation/covariance matrices will be pooled separately for each cluster. The structural models will also be fitted separately for each cluster.

```
R> #### Show the clusters in Digman97
R> Digman97$cluster

 [1] "Children"      "Children"      "Children"      "Children"
 [5] "Adolescents"   "Young adults"  "Young adults"  "Young adults"
 [9] "Mature adults" "Mature adults" "Mature adults" "Mature adults"
[13] "Mature adults" "Mature adults"

R> ##### Example of Fixed-effects TSSEM with several clusters
R> fixed1.cluster <- tssem1(Digman97$data, Digman97$n, method="FEM",
                            cluster=Digman97$cluster)

Running TSSEM1 Analysis of Correlation Matrix
Running TSSEM1 Analysis of Correlation Matrix
Running TSSEM1 Analysis of Correlation Matrix
Running TSSEM1 Analysis of Correlation Matrix

R> summary(fixed1.cluster)

$Adolescents

Call:
```

```
tssem1FEM(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.290000  0.096544  3.0038 0.0026661 **
S[1,3] 0.160000  0.102710  1.5578 0.1192859
S[1,4] 0.320000  0.094615  3.3821 0.0007193 ***
S[1,5] 0.530000  0.075800  6.9921 2.708e-12 ***
S[2,3] 0.640000  0.062234 10.2838 < 2.2e-16 ***
S[2,4] 0.350000  0.092496  3.7839 0.0001544 ***
S[2,5] 0.220000  0.100307  2.1933 0.0282882 *
S[3,4] 0.270000  0.097725  2.7629 0.0057296 **
S[3,5] 0.220000  0.100307  2.1933 0.0282887 *
S[4,5] 0.360000  0.091748  3.9238 8.717e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                 Value
Sample size                      91.00
Chi-square of target model        0.00
DF of target model                0.00
p value of target model           0.00
Chi-square of independent model 109.63
DF of independent model          10.00
RMSEA                              Inf
SRMR                              0.00
TLI                               -Inf
CFI                               1.00
AIC                               0.00
BIC                               0.00

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:13 2012
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$Children

Call:
tssem1FEM(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)
```

```
Coefficients:
        Estimate Std.Error z value  Pr(>|z|)
S[1,2] -0.071259  0.037983 -1.8761   0.06065 .
S[1,3] -0.084678  0.036505 -2.3196   0.02036 *
S[1,4]  0.158313  0.035949  4.4038 1.064e-05 ***
S[1,5]  0.473158  0.028765 16.4489 < 2.2e-16 ***
S[2,3]  0.600192  0.023695 25.3302 < 2.2e-16 ***
S[2,4]  0.479811  0.028723 16.7049 < 2.2e-16 ***
S[2,5]  0.043055  0.036728  1.1723   0.24109
S[3,4]  0.526708  0.026960 19.5366 < 2.2e-16 ***
S[3,5]  0.331623  0.032726 10.1333 < 2.2e-16 ***
S[4,5]  0.298135  0.033718  8.8419 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                       747.0000
Chi-square of target model        311.3516
DF of target model                 30.0000
p value of target model             0.0000
Chi-square of independent model 1352.0398
DF of independent model            40.0000
RMSEA                               0.2242
SRMR                                0.1401
TLI                                 0.7141
CFI                                 0.7856
AIC                               251.3516
BIC                               112.8696

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:13 2012
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Mature adults`

Call:
tssem1FEM(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.076227  0.018725  4.0708 4.686e-05 ***
```

```
S[1,3] 0.170404  0.018269  9.3274 < 2.2e-16 ***
S[1,4] 0.191577  0.018047 10.6156 < 2.2e-16 ***
S[1,5] 0.366062  0.016326 22.4217 < 2.2e-16 ***
S[2,3] 0.196629  0.018031 10.9051 < 2.2e-16 ***
S[2,4] 0.305076  0.017177 17.7604 < 2.2e-16 ***
S[2,5] 0.013859  0.018955  0.7312    0.4647
S[3,4] 0.385234  0.016189 23.7955 < 2.2e-16 ***
S[3,5] 0.030844  0.018907  1.6314    0.1028
S[4,5] 0.037283  0.018830  1.9800    0.0477 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     2862.0000
Chi-square of target model       420.5247
DF of target model                50.0000
p value of target model            0.0000
Chi-square of independent model 1707.9108
DF of independent model           60.0000
RMSEA                              0.1247
SRMR                               0.1522
TLI                                0.7302
CFI                                0.7752
AIC                              320.5247
BIC                               22.5609

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:13 2012
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Young adults`

Call:
tssem1FEM(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.322154  0.031894 10.1009 < 2.2e-16 ***
S[1,3] 0.219371  0.033847  6.4813 9.093e-11 ***
S[1,4] 0.471710  0.027645 17.0630 < 2.2e-16 ***
S[1,5] 0.554691  0.024744 22.4175 < 2.2e-16 ***
```

```
S[2,3] 0.613646  0.022452 27.3309 < 2.2e-16 ***
S[2,4] 0.560195  0.024403 22.9555 < 2.2e-16 ***
S[2,5] 0.351222  0.031207 11.2545 < 2.2e-16 ***
S[3,4] 0.424434  0.029180 14.5454 < 2.2e-16 ***
S[3,5] 0.286843  0.032639  8.7883 < 2.2e-16 ***
S[4,5] 0.276926  0.032878  8.4227 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                       796.0000
Chi-square of target model         66.8333
DF of target model                 20.0000
p value of target model             0.0000
Chi-square of independent model 1285.0187
DF of independent model            30.0000
RMSEA                               0.0940
SRMR                                0.1511
TLI                                 0.9440
CFI                                 0.9627
AIC                                26.8333
BIC                               -66.7587

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:13 2012
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ##### Extract the pooled correlation matrices
R> coef(fixed1.cluster)

$Adolescents
     x1   x2   x3   x4   x5
x1 1.00 0.29 0.16 0.32 0.53
x2 0.29 1.00 0.64 0.35 0.22
x3 0.16 0.64 1.00 0.27 0.22
x4 0.32 0.35 0.27 1.00 0.36
x5 0.53 0.22 0.22 0.36 1.00

$Children
            x1          x2          x3        x4         x5
x1  1.00000000 -0.07125880 -0.08467822 0.1583127 0.47315830
x2 -0.07125880  1.00000000  0.60019239 0.4798110 0.04305501
x3 -0.08467822  0.60019239  1.00000000 0.5267076 0.33162339
```

```
x4  0.15831269  0.47981099  0.52670763 1.0000000 0.29813501
x5  0.47315830  0.04305501  0.33162339 0.2981350 1.00000000

$`Mature adults`
          x1        x2         x3         x4         x5
x1 1.0000000 0.0762269 0.17040394 0.19157746 0.36606178
x2 0.0762269 1.0000000 0.19662879 0.30507564 0.01385940
x3 0.1704039 0.1966288 1.00000000 0.38523398 0.03084429
x4 0.1915775 0.3050756 0.38523398 1.00000000 0.03728294
x5 0.3660618 0.0138594 0.03084429 0.03728294 1.00000000

$`Young adults`
          x1        x2        x3        x4        x5
x1 1.0000000 0.3221536 0.2193714 0.4717095 0.5546912
x2 0.3221536 1.0000000 0.6136457 0.5601947 0.3512219
x3 0.2193714 0.6136457 1.0000000 0.4244336 0.2868427
x4 0.4717095 0.5601947 0.4244336 1.0000000 0.2769260
x5 0.5546912 0.3512219 0.2868427 0.2769260 1.0000000

R> fixed2.cluster <- tssem2(fixed1.cluster, impliedS=impliedR, matrices=c(P, L))

Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure
Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure
Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure
Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure

R> summary(fixed2.cluster)

$Adolescents

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
       Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L[1,2] 0.738279  0.104533 0.533399 0.943159  7.0627 1.633e-12 ***
L[2,1] 0.867342  0.075056 0.720234 1.014450 11.5559 < 2.2e-16 ***
L[3,1] 0.742502  0.075055 0.595397 0.889606  9.8928 < 2.2e-16 ***
L[4,1] 0.526038  0.090630 0.348407 0.703668  5.8043 6.465e-09 ***
L[5,2] 0.734002  0.106560 0.525149 0.942855  6.8882 5.651e-12 ***
P[1,2] 0.548082  0.114039 0.324570 0.771595  4.8061 1.539e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Goodness-of-fit indices:
                                        Value
Sample size                           91.0000
Chi-square of target model            10.7341
DF of target model                     4.0000
p value of target model                0.0297
Chi-square of independent model     270.6747
DF of independent model               10.0000
RMSEA                                  0.1368
SRMR                                   0.1028
TLI                                    0.9354
CFI                                    0.9742
AIC                                    2.7341
BIC                                   -7.3094

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:14 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$Children

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate   Std.Error      lbound      ubound z value
L[1,2]  4.4830e-03  6.0321e-01 -1.1778e+00  1.1867e+00  0.0074
L[2,1]  7.3717e-01  2.2482e-02  6.9311e-01  7.8124e-01 32.7896
L[3,1]  9.1394e-01  1.8545e-02  8.7759e-01  9.5029e-01 49.2821
L[4,1]  6.8942e-01  2.2180e-02  6.4595e-01  7.3289e-01 31.0831
L[5,2]  1.1466e+02  1.5428e+04 -3.0124e+04  3.0353e+04  0.0074
P[1,2]  3.3598e-03  4.5209e-01 -8.8271e-01  8.8943e-01  0.0074
       Pr(>|z|)
L[1,2]   0.9941
L[2,1]   <2e-16 ***
L[3,1]   <2e-16 ***
L[4,1]   <2e-16 ***
L[5,2]   0.9941
```

```
P[1,2]   0.9941
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      747.0000
Chi-square of target model       150.9105
DF of target model                 4.0000
p value of target model            0.0000
Chi-square of independent model 3583.7700
DF of independent model           10.0000
RMSEA                              0.2219
SRMR                               0.1074
TLI                                0.8972
CFI                                0.9589
AIC                              142.9105
BIC                              124.4462

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:14 2012
OpenMx status1: 6 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Mature adults`

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
       Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L[1,2] 1.386477  0.294766 0.808746 1.964207  4.7037 2.555e-06 ***
L[2,1] 0.397877  0.021232 0.356262 0.439491 18.7393 < 2.2e-16 ***
L[3,1] 0.523007  0.022691 0.478533 0.567480 23.0489 < 2.2e-16 ***
L[4,1] 0.746117  0.027054 0.693093 0.799141 27.5792 < 2.2e-16 ***
L[5,2] 0.264596  0.058484 0.149970 0.379222  4.5243 6.061e-06 ***
P[1,2] 0.192413  0.046616 0.101047 0.283778  4.1276 3.665e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Goodness-of-fit indices:
                                     Value
Sample size                      2862.0000
Chi-square of target model          8.9336
DF of target model                  4.0000
p value of target model             0.0628
Chi-square of independent model 1704.2578
DF of independent model            10.0000
RMSEA                               0.0208
SRMR                                0.0148
TLI                                 0.9927
CFI                                 0.9971
AIC                                 0.9336
BIC                               -22.9036

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:14 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Young adults`

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
       Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L[1,2] 0.865183  0.031067 0.804293 0.926072  27.849 < 2.2e-16 ***
L[2,1] 0.844737  0.017826 0.809799 0.879675  47.389 < 2.2e-16 ***
L[3,1] 0.699981  0.021093 0.658640 0.741322  33.186 < 2.2e-16 ***
L[4,1] 0.765366  0.020161 0.725851 0.804881  37.962 < 2.2e-16 ***
L[5,2] 0.708991  0.028447 0.653236 0.764746  24.923 < 2.2e-16 ***
P[1,2] 0.595879  0.031500 0.534140 0.657617  18.917 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                       796.0000
Chi-square of target model         85.9696
```

```
DF of target model                   4.0000
p value of target model              0.0000
Chi-square of independent model 3125.1714
DF of independent model             10.0000
RMSEA                                0.1606
SRMR                                 0.0805
TLI                                  0.9342
CFI                                  0.9737
AIC                                 77.9696
BIC                                 59.2512

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:14 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.2. Random-effects model

TSSEM using a random-effects model may be requested by specifying the `method='REM'` argument in `tssem1()`. By default (`RE.diag.only=FALSE`), a positive definite covariance matrix among the random-effects is used. For some practical reasons, e.g., there are not enough studies, a diagonal matrix on the random-effects may also be used by specifying `RE.diag.only=TRUE`.

The fit indices for fitting the structural model in Stage 2 are $\chi^2(4, N = 4496) = 8.28, p < .001$, CFI=0.9920, TLI=0.9801, SRMR=0.0154 and RMSEA=0.0154. This indicates that the model fits the data quite well.

```
R> random1 <- tssem1(Digman97$data, Digman97$n, method="REM", RE.diag.only=TRUE)

Running TSSEM1 (Random Effects Model) Analysis of Correlation Matrix

R> summary(random1)

Call:
meta(y = ES, v = acovR, RE.constraints = diag(x = paste(RE.startvalues,
    "*Tau2_", 1:no.es, "_", 1:no.es, sep = ""), nrow = no.es,
    ncol = no.es), model.name = model.name)

95% confidence intervals: z statistic approximation
Coefficients:
               Estimate   Std.Error      lbound      ubound z value
Intercept1   0.05444839  0.06316913 -0.06936083  0.17825760  0.8619
Intercept2   0.12867852  0.04174104  0.04686758  0.21048945  3.0828
Intercept3   0.24064427  0.03220612  0.17752145  0.30376710  7.4720
```

```
Intercept4   0.44713471  0.03211667  0.38418719  0.51008223 13.9222
Intercept5   0.39981625  0.05455527  0.29288989  0.50674260  7.3286
Intercept6   0.44433517  0.04168028  0.36264332  0.52602703 10.6606
Intercept7   0.10138347  0.04681342  0.00963085  0.19313609  2.1657
Intercept8   0.43415303  0.04000888  0.35573706  0.51256900 10.8514
Intercept9   0.20732505  0.04973241  0.10985132  0.30479878  4.1688
Intercept10  0.19296081  0.04340500  0.10788858  0.27803305  4.4456
Tau2_1_1     0.05115980  0.02059752  0.01078940  0.09153021  2.4838
Tau2_2_2     0.01977634  0.00914599  0.00185053  0.03770216  2.1623
Tau2_3_3     0.01030042  0.00505946  0.00038405  0.02021679  2.0359
Tau2_4_4     0.01122095  0.00494564  0.00152766  0.02091423  2.2689
Tau2_5_5     0.03815848  0.01523930  0.00829001  0.06802695  2.5040
Tau2_6_6     0.02132565  0.00868726  0.00429893  0.03835236  2.4548
Tau2_7_7     0.02571719  0.01094038  0.00427444  0.04715994  2.3507
Tau2_8_8     0.01901268  0.00820036  0.00294026  0.03508509  2.3185
Tau2_9_9     0.02995573  0.01234181  0.00576623  0.05414522  2.4272
Tau2_10_10   0.02172542  0.00934586  0.00340787  0.04004297  2.3246
             Pr(>|z|)
Intercept1   0.388717
Intercept2   0.002051 **
Intercept3  7.905e-14 ***
Intercept4  < 2.2e-16 ***
Intercept5  2.325e-13 ***
Intercept6  < 2.2e-16 ***
Intercept7   0.030335 *
Intercept8  < 2.2e-16 ***
Intercept9  3.062e-05 ***
Intercept10 8.765e-06 ***
Tau2_1_1     0.012999 *
Tau2_2_2     0.030595 *
Tau2_3_3     0.041763 *
Tau2_4_4     0.023277 *
Tau2_5_5     0.012281 *
Tau2_6_6     0.014096 *
Tau2_7_7     0.018740 *
Tau2_8_8     0.020421 *
Tau2_9_9     0.015217 *
Tau2_10_10   0.020093 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 1968.958
Degrees of freedom of the Q statistic: 130
P value of the Q statistic: 0

Number of studies (or clusters): 14
Number of observed statistics: 140
```

```
Number of estimated parameters: 20
Degrees of freedom: 120
-2 log likelihood: -109.6847

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:17 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ##### Extract the fixed-effects (pooled correlation matrix)
R> coef(random1, select="fixed")

 Intercept1  Intercept2  Intercept3  Intercept4  Intercept5
 0.05444839  0.12867852  0.24064427  0.44713471  0.39981625
 Intercept6  Intercept7  Intercept8  Intercept9 Intercept10
 0.44433517  0.10138347  0.43415303  0.20732505  0.19296081

R> ##### Extract the random-effects (variance component)
R> coef(random1, select="random")

  Tau2_1_1   Tau2_2_2   Tau2_3_3   Tau2_4_4   Tau2_5_5   Tau2_6_6
0.05115980 0.01977634 0.01030042 0.01122095 0.03815848 0.02132565
  Tau2_7_7   Tau2_8_8   Tau2_9_9 Tau2_10_10
0.02571719 0.01901268 0.02995573 0.02172542

R> random2 <- tssem2(random1, impliedS=impliedR, matrices=c(P, L))

Running TSSEM2 (Random Effects Model) Analysis of Correlation Structure

R> summary(random2)

Call:
wls(S = pooledS, acovS = acovS, n = tssem1.obj$total.n, impliedS = impliedS,
    matrices = matrices, cor.analysis = cor.analysis, intervals.type = intervals.type,
    model.name = model.name, suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
       Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L[1,2] 0.690304  0.074084 0.545103 0.835505  9.3179 < 2.2e-16 ***
L[2,1] 0.577074  0.051758 0.475629 0.678518 11.1494 < 2.2e-16 ***
L[3,1] 0.594979  0.051924 0.493209 0.696749 11.4586 < 2.2e-16 ***
L[4,1] 0.770870  0.061201 0.650919 0.890821 12.5958 < 2.2e-16 ***
L[5,2] 0.647773  0.069433 0.511687 0.783860  9.3294 < 2.2e-16 ***
```

```
P[1,2] 0.394763  0.047190 0.302271 0.487254  8.3653 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                      4496.0000
Chi-square of target model          8.2809
DF of target model                  4.0000
p value of target model             0.0818
Chi-square of independent model   546.8073
DF of independent model            10.0000
RMSEA                               0.0154
SRMR                                0.0465
TLI                                 0.9801
CFI                                 0.9920
AIC                                 0.2809
BIC                               -25.3629

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:18 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

4. Other Useful Functions

4.1. Analysis of Correlation/Covariance Structure with Weighted Least Squares

The `wls()` function may be used to fit a correlation/covariance structure with weighted least squares (WLS) estimation method. The following example fits a one-factor CFA model on the correlation matrix with WLS estimation method. It should be noted that the only off-diagonal elements are used when a correlation structure is fitted.

```
R> ## Sample correlation matrix
R> R1 <- matrix(c(1.00, 0.22, 0.24, 0.18,
                 0.22, 1.00, 0.30, 0.22,
                 0.24, 0.30, 1.00, 0.24,
                 0.18, 0.22, 0.24, 1.00), ncol=4, nrow=4)
R> ## Sample size
R> n <- 1000
R> ## Estimate the asymptotic covariance matrix of the sample correlation matrix
R> acovR <- asyCov(R1, n)
```

```
R> ## P1: Factor variance is fixed at 1.0
R> P1 <- as.mxMatrix( matrix(1), name="P1")
R> ## L1: Factor loadings
R> L1 <- as.mxMatrix( matrix( rep("0.3*", 4), nrow=4, ncol=1), name="L1" )
R> ## Model implied correlation matrix
R> ## Please note that error variances are not involved in correlation structure analysis
R> impliedR1 <- mxAlgebra(L1 %&% P1, name="impliedR1")
R> ## wls() is the function to fitting correlation/covariance structure with WLS
R> wls.fit1 <- wls(S=R1, acovS=acovR, n=n, impliedS=impliedR1,
                  matrices=c(P1, L1), cor.analysis=TRUE)

Running WLS Analysis of Correlation Structure

R> summary(wls.fit1)

Call:
wls(S = R1, acovS = acovR, n = n, impliedS = impliedR1, matrices = c(P1,
    L1), cor.analysis = TRUE)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L1[1,1] 0.421592  0.038727 0.345688 0.497496  10.886 < 2.2e-16 ***
L1[2,1] 0.523764  0.039256 0.446823 0.600706  13.342 < 2.2e-16 ***
L1[3,1] 0.570921  0.040144 0.492241 0.649601  14.222 < 2.2e-16 ***
L1[4,1] 0.421592  0.038727 0.345688 0.497496  10.886 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                      1000.0000
Chi-square of target model          0.0134
DF of target model                  2.0000
p value of target model             0.9933
Chi-square of independent model   243.9789
DF of independent model             6.0000
RMSEA                               0.0000
SRMR                                0.0012
TLI                                 1.0250
CFI                                 1.0000
AIC                                -3.9866
BIC                               -13.8021

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
```

```
Date of analysis: Tue Feb 14 14:21:18 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

4.2. Likelihood-based Confidence Intervals

Most CIs are based on the estimated standard errors. These are known as Wald CIs. Wald CIs are symmetric around the estimates. The Wald CIs might be outside of the meaningful boundaries, for example, a negative lower limit for the variance or larger than 1 for a correlation coefficient. A preferable approach is to construct the CIs based on the likelihood. This is known as the likelihood based CI (Cheung 2009a; Neale and Miller 1997). Likelihood based CIs on the parameter estimates can be required by specifying `intervals.type='LB'` argument.

```
R> ## Random-effects meta-analysis with ML
R> summary( meta(y=di, v=vi, data=Becker83, intervals.type="LB") )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.174734  0.113378 -0.052165  0.437627  1.5412   0.1233
Tau2_1_1    0.077376  0.054108  0.015124  0.302999  1.4300   0.1527

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 2
Degrees of freedom: 8
-2 log likelihood: 7.928307

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:18 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Mixed-effects meta-analysis with "log(items)" as a predictor
R> summary( meta(y=di, v=vi, x=log(items), data=Becker83, intervals.type="LB") )
```

```
Running Meta analysis with ML

Call:
meta(y = di, v = vi, x = log(items), data = Becker83, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
              Estimate   Std.Error      lbound      ubound z value
Intercept1 -3.2015e-01  1.0981e-01 -5.4408e-01 -7.7598e-02 -2.9154
Slope1_1    2.1088e-01  4.5084e-02  1.1838e-01  3.0789e-01  4.6774
Tau2_1_1    1.0000e-10  2.0095e-02  1.0000e-10  5.7947e-02  0.0000
            Pr(>|z|)
Intercept1  0.003552 **
Slope1_1   2.905e-06 ***
Tau2_1_1    1.000000
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies (or clusters): 10
Number of observed statistics: 10
Number of estimated parameters: 3
Degrees of freedom: 7
-2 log likelihood: -4.208024

R version: 2.14.1
OpenMx version: 1.2.0-1926
metaSEM version: 0.7-1
Date of analysis: Tue Feb 14 14:21:19 2012
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

4.3. Reading External Data Files

Data sets are most likely stored externally. `metaSEM` reads three types of data formats. The first type is full correlation/covariance matrices, for example, fullmat.dat is the same as the built-in data set `Cheung09`. Missing values are represented by `NA` (the default option). Suppose you have saved it at d:\fullmat.dat, you may read it by using the following command in `R`:

```
my.df <- readFullMat(file="d:/fullmat.dat")
```

The second type is lower triangle correlation/covariance matrices, for example, lowertriangle.dat. Missing values are represented by the strings NA. Suppose you have saved it at d:\lowertriangle.dat, you may read it by using the following command in `R`:

```
my.df <- readLowTriMat(file = "d:/lowertriangle.dat", no.var = 9, na.strings="NA")
```

The third type is vectors of correlation/covariance elements based on column vectorization. One row represents one study, for example, stackvec.dat. Suppose you have saved it at d:\stackvec.dat, you may read it by using the following R command:

```
my.df <- readStackVec(file="d:/stackvec.dat")
```

5. Installation

First of all, you need R to run it. Since metaSEM uses OpenMx as the workhorse, OpenMx has to be installed first. To install OpenMx, run the following command inside an R session:

```
install.packages('OpenMx', repos='http://openmx.psyc.virginia.edu/packages/')
```

See http://openmx.psyc.virginia.edu/installing-openmx for the details on how to install OpenMx. Moreover, metaSEM also depends on the ellipse package that can be installed by the following command inside an R session:

```
install.packages('ellipse')
```

5.1. Windows platform

Download the Windows binary of metaSEM. If the file is saved at d:\. Run the following command inside an R session:

```
install.packages(pkgs="d:/metaSEM_0.7-1.zip", repos=NULL)
```

Please note that d:\ in Windows is represented by either d:/ or d:\\ in R.

5.2. Linux platform

Download the source package of metaSEM. Run the following command as Root inside a terminal:

```
R CMD INSTALL metaSEM_0.7-1.tar.gz
```

5.3. Mac OS X platform

The current version does not contain binaries for Mac OS X. Mac OS X users may need to build from the source.

6. Acknowledgements

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