metaSEM: Meta-Analysis using Structural Equation Modeling

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1 Introduction

metaSEM is an R package that conducts univariate and multivariate meta-analysis using structural equation modeling (SEM) approach (Cheung, 2008) via the OpenMx package (Boker et al., 2011). It also implements the two-stage structural equation modeling (TSSEM) approach (Cheung and Chan, 2005, 2009) to conducting a meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. The main functions in this package are:

- `tssem1()`: It conducts the first stage analysis of TSSEM by pooling correlation/covariance matrices with a fixed-effets model.
- `tssem2()`: It conducts the second stage analysis of TSSEM by calling `wls()`.
- `wls()`: It fits a correlation/covariance structure analysis with weighted least squares.
- `meta()` and `reml()`: `meta()` conducts univariate and multivariate meta-analysis with maximum likelihood estimation method while `reml()` estimates the variance components of the random-effects with restricted (residual) maximum likelihood estimation method. Mixed-effects meta-analysis can be conducted by including study characteristics as predictors. Equality constraints on intercepts, regression coefficients and variance components can be imposed and tested.

Besides reporting approximate confidence intervals (CIs) based on z statistic, likelihood-based CIs on the parameter estimates may also be requested (Cheung, 2009a; Neale and Miller, 1997). Several generic functions such as, `anova`, `coef`, `summary` and `vcov`, have also been implemented. The current version is 0.6-0. Please send any bugs and comments to me at <mikewlcheung (at) nus.edu.sg>.

2 Installation

First of all, you need R to run it. Since metaSEM uses OpenMx as the workhorse, OpenMx has also to be installed. To install OpenMx, run the following command inside an R session:

```
source('http://openmx.psyc.virginia.edu/getOpenMx.R')
```

See http://openmx.psyc.virginia.edu/installing-openmx for the details on how to install OpenMx.

2.1 Windows platform

Download the Windows binary of metaSEM. If the file is saved at d:\. Run the following command inside an R session:

```
install.packages(pkgs="d:/metaSEM_0.6-0.zip", repos=NULL)
```

Please note that d:\ in Windows is represented by either d:/ or d:\\ in R.

2.2 Linux platform

Download the source package of `metaSEM`. Run the following command as Root inside a terminal:

```
R CMD INSTALL metaSEM_0.6-0.tar.gz
```

3 Examples

3.1 Two-stage SEM

An example from Cheung (2009b) was used to illustrate the two-stage structural equation modeling (TSSEM) procedure. `tssem1()` is used to pool the correlation matrices in the first stage. `tssem2()` is then used to fit a structural model on the pooled correlation matrix with its asymptotic covariance matrix.

```
R> ## Load the metaSEM library
R> library(metaSEM)
R> ## Sample correlation matrices with missing values
R> Cheung09$data
```

```
$`1`
        x1      x2      x3      x4      x5      x6      x7       x8
x1 0.77298 0.26975 0.24009 0.23778 0.20869 0.22377 0.18801  0.07055
x2 0.26975 0.91307 0.44374 0.26083 0.28387 0.20660 0.12764  0.22892
x3 0.24009 0.44374 1.11292 0.29440 0.26262 0.27320 0.18548  0.20417
x4 0.23778 0.26083 0.29440 0.80501 0.47489 0.45939 0.40998  0.09104
x5 0.20869 0.28387 0.26262 0.47489 0.89692 0.41972 0.31541  0.33907
x6 0.22377 0.20660 0.27320 0.45939 0.41972 1.36089 0.74274  0.18137
x7 0.18801 0.12764 0.18548 0.40998 0.31541 0.74274 1.01075  0.13724
x8 0.07055 0.22892 0.20417 0.09104 0.33907 0.18137 0.13724  1.81805
x9 0.10051 0.09590 0.18243 0.10142 0.06561 0.12973 0.12776 -0.01980
         x9
x1  0.10051
x2  0.09590
x3  0.18243
x4  0.10142
x5  0.06561
x6  0.12973
x7  0.12776
x8 -0.01980
x9  0.91252

$`2`
   x1 x2      x3      x4      x5      x6      x7      x8      x9
x1 NA NA      NA      NA      NA      NA      NA      NA      NA
x2 NA NA      NA      NA      NA      NA      NA      NA      NA
x3 NA NA 1.06293 0.27094 0.20331 0.16522 0.11922 0.25387 0.06877
x4 NA NA 0.27094 0.73625 0.27053 0.33506 0.33495 0.16124 0.00912
x5 NA NA 0.20331 0.27053 0.71718 0.19873 0.14582 0.21907 0.04089
x6 NA NA 0.16522 0.33506 0.19873 0.92247 0.55128 0.17143 0.01380
x7 NA NA 0.11922 0.33495 0.14582 0.55128 1.00462 0.16561 0.04322
x8 NA NA 0.25387 0.16124 0.21907 0.17143 0.16561 1.49431 0.29094
x9 NA NA 0.06877 0.00912 0.04089 0.01380 0.04322 0.29094 1.01960
```

```
$`3`
        x1      x2 x3 x4 x5      x6      x7      x8      x9
x1 0.95825 0.32958 NA NA NA 0.13948 0.15463 0.15248 0.10405
x2 0.32958 1.02277 NA NA NA 0.07300 0.07002 0.17056 0.13502
x3      NA      NA NA NA NA      NA      NA      NA      NA
x4      NA      NA NA NA NA      NA      NA      NA      NA
x5      NA      NA NA NA NA      NA      NA      NA      NA
x6 0.13948 0.07300 NA NA NA 0.82987 0.43769 0.23195 0.03856
x7 0.15463 0.07002 NA NA NA 0.43769 0.83476 0.19002 0.03986
x8 0.15248 0.17056 NA NA NA 0.23195 0.19002 1.42583 0.38343
x9 0.10405 0.13502 NA NA NA 0.03856 0.03986 0.38343 1.03062

$`4`
        x1       x2      x3      x4      x5      x6       x7 x8 x9
x1 0.83995  0.21117 0.14249 0.13268 0.17861 0.22783  0.18991 NA NA
x2 0.21117  0.93380 0.34383 0.19040 0.15068 0.12191 -0.04762 NA NA
x3 0.14249  0.34383 1.33025 0.31041 0.10873 0.19756  0.12113 NA NA
x4 0.13268  0.19040 0.31041 0.77512 0.36093 0.36519  0.22716 NA NA
x5 0.17861  0.15068 0.10873 0.36093 0.91598 0.37035  0.19550 NA NA
x6 0.22783  0.12191 0.19756 0.36519 0.37035 1.48445  0.62637 NA NA
x7 0.18991 -0.04762 0.12113 0.22716 0.19550 0.62637  1.05049 NA NA
x8      NA       NA      NA      NA      NA      NA       NA NA NA
x9      NA       NA      NA      NA      NA      NA       NA NA NA

R> ## Sample sizes
R> Cheung09$n

[1] 591 656 832 823

R> ## Stage 1: Analysis of correlation matrices
R> ## A pooled correlation matrix will be estimated.
R> ## tssem1() is the function for stage 1 analysis.
R> cor1 <- tssem1(Cheung09$data, Cheung09$n)

Running TSSEM1 Analysis of Correlation Matrix

R> summary(cor1)

Call:
tssem1(my.df = Cheung09$data, n = Cheung09$n)

Coefficients:
        Estimate Std.Error z value  Pr(>|z|)
S1[1,2] 0.295204  0.019261 15.3267 < 2.2e-16 ***
S1[1,3] 0.195314  0.024753  7.8906 3.109e-15 ***
S1[1,4] 0.218765  0.024123  9.0687 < 2.2e-16 ***
S1[1,5] 0.219892  0.024410  9.0082 < 2.2e-16 ***
S1[1,6] 0.188085  0.020244  9.2910 < 2.2e-16 ***
S1[1,7] 0.192202  0.020246  9.4935 < 2.2e-16 ***
S1[1,8] 0.098346  0.025657  3.8331 0.0001265 ***
S1[1,9] 0.098793  0.025907  3.8134 0.0001371 ***
S1[2,3] 0.369539  0.022701 16.2786 < 2.2e-16 ***
S1[2,4] 0.254045  0.023529 10.7971 < 2.2e-16 ***
```

```
S1[2,5] 0.228516  0.024102  9.4813 < 2.2e-16 ***
S1[2,6] 0.114146  0.020524  5.5617 2.672e-08 ***
S1[2,7] 0.045226  0.020796  2.1747 0.0296520 *
S1[2,8] 0.148063  0.025249  5.8640 4.517e-09 ***
S1[2,9] 0.101851  0.025717  3.9605 7.479e-05 ***
S1[3,4] 0.307685  0.019829 15.5172 < 2.2e-16 ***
S1[3,5] 0.190536  0.021200  8.9877 < 2.2e-16 ***
S1[3,6] 0.168642  0.020895  8.0710 6.661e-16 ***
S1[3,7] 0.131197  0.021077  6.2246 4.827e-10 ***
S1[3,8] 0.165582  0.027014  6.1294 8.819e-10 ***
S1[3,9] 0.125626  0.027583  4.5545 5.251e-06 ***
S1[4,5] 0.451817  0.017509 25.8050 < 2.2e-16 ***
S1[4,6] 0.385261  0.018402 20.9362 < 2.2e-16 ***
S1[4,7] 0.356265  0.018830 18.9201 < 2.2e-16 ***
S1[4,8] 0.118677  0.026683  4.4477 8.681e-06 ***
S1[4,9] 0.056840  0.027195  2.0901 0.0366073 *
S1[5,6] 0.312551  0.019537 15.9978 < 2.2e-16 ***
S1[5,7] 0.231882  0.020370 11.3833 < 2.2e-16 ***
S1[5,8] 0.238514  0.026160  9.1175 < 2.2e-16 ***
S1[5,9] 0.064726  0.027417  2.3608 0.0182349 *
S1[6,7] 0.553723  0.012912 42.8834 < 2.2e-16 ***
S1[6,8] 0.162009  0.021035  7.7018 1.332e-14 ***
S1[6,9] 0.053050  0.021659  2.4493 0.0143139 *
S1[7,8] 0.135426  0.021212  6.3844 1.720e-10 ***
S1[7,9] 0.066476  0.021651  3.0704 0.0021377 **
S1[8,9] 0.198066  0.021254  9.3191 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      2902.0000
Chi-square of target model        172.7320
DF of target model                 57.0000
p value of target model             0.0000
Chi-square of independent model 3246.6915
DF of independent model           93.0000
RMSEA                              0.0529
SRMR                               0.0549
TLI                                0.9401
CFI                                0.9633
AIC                               58.7320
BIC                             -281.7379

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:30 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Stage 2: Fit a three-factor CFA model on the pooled correlation matrix
R>
R> ## Create a matrix for factor correlations
R> ## Values with "*" are free parameters
R> P4 <-  matrix(c(1,"0.2*p21*","0.2*p31",
                  "0.2*p21",1,"0.2*p32",
                  "0.2*p31","0.2*p32",1), ncol=3, nrow=3, byrow=TRUE)
R> ## Convert it into a mxMatrix class
R> P4 <- as.mxMatrix(P4)
R> ## Create a matrix for factor loadings
R> L4 <- matrix(c(rep(c("0.3*",0,0),3),
                  rep(c(0, "0.3*",0),4), rep(c(0,0,"0.3*"),2)), ncol=3, nrow=9, byrow=TRUE)
R> ## Convert it into a mxMatrix class
R> L4 <- as.mxMatrix(L4)
R> ## impliedR = L4 %*% P4 %*% t(L4)
R> impliedR4 <- mxAlgebra(L4 %&% P4, name="impliedR4")
R> ## tssem2() is the function for stage 2 analysis.
R> cor2 <- tssem2(cor1, impliedS=impliedR4, matrices=c(P4, L4))

Running TSSEM2 Analysis of Correlation Structure

R> summary(cor2)

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,1] 0.517061  0.023568 0.470869 0.563254 21.9389 < 2.2e-16 ***
L4[2,1] 0.575072  0.023120 0.529758 0.620387 24.8734 < 2.2e-16 ***
L4[3,1] 0.593858  0.025586 0.543711 0.644004 23.2106 < 2.2e-16 ***
L4[4,2] 0.705138  0.014698 0.676330 0.733946 47.9750 < 2.2e-16 ***
L4[5,2] 0.579019  0.016902 0.545892 0.612146 34.2580 < 2.2e-16 ***
L4[6,2] 0.746403  0.013060 0.720806 0.772000 57.1515 < 2.2e-16 ***
L4[7,2] 0.692214  0.013622 0.665515 0.718912 50.8157 < 2.2e-16 ***
L4[8,3] 0.621998  0.052462 0.519175 0.724822 11.8561 < 2.2e-16 ***
L4[9,3] 0.332553  0.032670 0.268521 0.396584 10.1792 < 2.2e-16 ***
P4[2,1] 0.544769  0.025617 0.494560 0.594977 21.2659 < 2.2e-16 ***
P4[3,1] 0.488471  0.055862 0.378984 0.597958  8.7443 < 2.2e-16 ***
P4[3,2] 0.392611  0.040345 0.313536 0.471685  9.7314 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                   Value
Sample size                     2902.0000
Chi-square of target model       384.9041
DF of target model                24.0000
```

```
p value of target model            0.0000
Chi-square of independent model 4636.5843
DF of independent model           36.0000
RMSEA                              0.0720
SRMR                               0.2953
TLI                                0.8823
CFI                                0.9216
AIC                              336.9041
BIC                              193.5484

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:31 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.2 Reading External Data Files

Data sets are most likely stored externally. `metaSEM` reads three types of data formats. The first type is full correlation/covariance matrices, for example, fullmat.dat is the same as the built-in data set `Cheung09`. Missing values are represented by `NA` (the default option). Suppose you have saved it at d:\fullmat.dat, you may read it by using the following command in `R`:

```
my.df <- readFullMat(file="d:/fullmat.dat")
```

The second type is lower triangle correlation/covariance matrices, for example, lowertriangle.dat. Missing values are represented by the strings 1.00000 and 0.00000. Suppose you have saved it at d:\lowertriangle.dat, you may read it by using the following command in `R`:

```
my.df <- readLowTriMat(file = "d:/lowertriangle.dat", no.var = 9,
                       na.strings=c("1.00000", "0.00000"))
```

The third type is vectors of correlation/covariance elements based on column vectorization. One row represents one study, for example, stackvec.dat. Suppose you have saved it at d:\stackvec.dat, you may read it by using the following `R` command:

```
my.df <- readStackVec(file="d:/stackvec.dat")
```

3.3 Analysis of Correlation/Covariance Structure with Weighted Least Squares

Besides fitting a TSSEM, `wls()` may be used to fit a correlation/covariance structure with weighted least squares (WLS) as the estimation method. Likelihood-based CIs may also be calculated. The following example fits a one-factor CFA model on the correlation matrix with WLS estimation method.

```
R> ## Sample correlation matrix
R> R1 <- matrix(c(1.00, 0.22, 0.24, 0.18,
                  0.22, 1.00, 0.30, 0.22,
                  0.24, 0.30, 1.00, 0.24,
                  0.18, 0.22, 0.24, 1.00), ncol=4, nrow=4)
R> ## Sample size
R> n <- 1000
R> ## Estimate the asymptotic covariance matrix of the sample correlation matrix
```

```
R> acovR <- asyCov(R1, n)
R> ## P1: Factor variance is fixed at 1.0
R> P1 <- as.mxMatrix( matrix(1), name="P1")
R> ## L1: Factor loadings
R> L1 <- as.mxMatrix( matrix( rep("0.3*", 4), nrow=4, ncol=1), name="L1" )
R> ## Model implied correlation matrix
R> ## Please note that error variances are not involved in correlation structure analysis
R> impliedR1 <- mxAlgebra(L1 %&% P1, name="impliedR1")
R> ## wls() is the function to fitting correlation/covariance structure with WLS
R> wls.fit1 <- wls(S=R1, acovS=acovR, n=n, impliedS=impliedR1,
                  matrices=c(P1, L1), cor.analysis=TRUE, intervals.type="LB")

Running WLS Analysis of Correlation Structure

R> summary(wls.fit1)

Call:
wls(S = R1, acovS = acovR, n = n, impliedS = impliedR1, matrices = c(P1,
    L1), cor.analysis = TRUE, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L1[1,1] 0.421592  0.038727 0.346320 0.498692  10.886 < 2.2e-16 ***
L1[2,1] 0.523764  0.039257 0.448295 0.603091  13.342 < 2.2e-16 ***
L1[3,1] 0.570921  0.040144 0.494310 0.652919  14.222 < 2.2e-16 ***
L1[4,1] 0.421592  0.038727 0.346320 0.498692  10.886 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      1000.0000
Chi-square of target model          0.0134
DF of target model                  2.0000
p value of target model             0.9933
Chi-square of independent model   243.9802
DF of independent model             6.0000
RMSEA                               0.0000
SRMR                                0.4830
TLI                                 1.0250
CFI                                 1.0000
AIC                                -3.9866
BIC                               -13.8021

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:32 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.4 Univariate and Multivariate Meta-Analysis

Other useful functions are `meta()` and `reml()`. They conduct fixed-, random-, and mixed-effects univariate and multivariate meta-analysis. Please note that there is no estimate on the fixed-effects when `reml()` is used. The followings are some examples.

```
R> ## Random-effects meta-analysis
R> summary( with(Hox02, meta(y=yi, v=vi)) )

Running Meta analysis with ML
Call:
meta(y = yi, v = vi)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value  Pr(>|z|)
Intercept1  0.579035  0.105100  0.373042  0.785028  5.5093 3.602e-08
Tau2_1_1    0.131520  0.073536 -0.012608  0.275648  1.7885   0.07369

Intercept1 ***
Tau2_1_1   .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 20
Number of parameter estimated: 2
Degrees of freedom: 18
-2 log likelihood: 27.79916

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:32 2011
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( with(Hox02, reml(y=yi, v=vi)) )

Running Variance component with REML
Call:
reml(y = yi, v = vi)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.144609  0.079766 -0.011729  0.300947  1.8129  0.06984 .
---
```

```
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Number of studies: 20
Number of observed statistics: 19
Number of parameter estimated: 1
Degrees of freedom: 18
-2 log likelihood: -4.477744

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:33 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Fixed-effects meta-analysis
R> summary( with(Hox02, meta(y=yi, v=vi, RE.constraints=matrix(0, ncol=1, nrow=1))) )
```

```
Running Meta analysis with ML
Call:
meta(y = yi, v = vi, RE.constraints = matrix(0, ncol = 1, nrow = 1))

95% confidence intervals: z statistic approximation
Coefficients:
           Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
Intercept1 0.550206  0.064998 0.422813 0.677599   8.465 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 20
Number of parameter estimated: 1
Degrees of freedom: 19
-2 log likelihood: 37.70073

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:33 2011
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Mixed-effects meta-analysis with "weeks" as a predictor
R> ## Request likelihood-based CI
R> summary( with(Hox02, meta(y=yi, v=vi, x=weeks, intervals.type="LB")) )
```

```
Running Meta analysis with ML
Call:
meta(y = yi, v = vi, x = weeks, intervals.type = "LB")
```

```
95% confidence intervals: Likelihood-based statistic
Coefficients:
              Estimate   Std.Error      lbound      ubound z value
Slope1_1    1.3866e-01  3.2089e-02  7.4635e-02  2.0695e-01  4.3210
Intercept1 -2.1356e-01  1.9284e-01 -6.1977e-01  1.8104e-01 -1.1075
Tau2_1_1    2.3252e-02  3.5481e-02  1.0000e-10  1.3790e-01  0.6553
            Pr(>|z|)
Slope1_1   1.553e-05 ***
Intercept1    0.2681
Tau2_1_1      0.5123
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 40
Number of parameter estimated: 5
Degrees of freedom: 35
-2 log likelihood: 104.9018

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:34 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( with(Hox02, reml(y=yi, v=vi, x=weeks, intervals.type="LB")) )

Running Variance component with REML
Call:
reml(y = yi, v = vi, x = weeks, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
           Estimate  Std.Error     lbound     ubound z value Pr(>|z|)
Tau2_1_1 3.6582e-02 4.2208e-02 4.0001e-06 1.7650e-01  0.8667   0.3861

Number of studies: 20
Number of observed statistics: 18
Number of parameter estimated: 1
Degrees of freedom: 17
-2 log likelihood: -10.86705

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
```

```
Date of analysis: Sat Apr 30 22:51:36 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Multivariate meta-analysis
R> summary( with(Berkey98, meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL))) )
```

```
Running Meta analysis with ML
Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL))

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value
Intercept1  0.3448392  0.0536312  0.2397239  0.4499544  6.4298
Intercept2 -0.3379381  0.0812479 -0.4971812 -0.1786951 -4.1593
Tau2_1_1    0.0070020  0.0090497 -0.0107351  0.0247391  0.7737
Tau2_2_1    0.0094607  0.0099698 -0.0100797  0.0290010  0.9489
Tau2_2_2    0.0261445  0.0177409 -0.0086270  0.0609161  1.4737
            Pr(>|z|)
Intercept1 1.278e-10 ***
Intercept2 3.192e-05 ***
Tau2_1_1      0.4391
Tau2_2_1      0.3427
Tau2_2_2      0.1406
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 10
Number of parameter estimated: 5
Degrees of freedom: 5
-2 log likelihood: -11.68131

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:36 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Estimate variance components with REML
R> summary( with(Berkey98, reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL))) )
```

```
Running Variance component with REML
Call:
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL))

95% confidence intervals: z statistic approximation
```

```
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.011733  0.013645 -0.015011  0.038477  0.8599   0.3899
Tau2_2_1  0.011916  0.014416 -0.016340  0.040172  0.8266   0.4085
Tau2_2_2  0.032651  0.024402 -0.015176  0.080479  1.3380   0.1809

Number of studies: 5
Number of observed statistics: 8
Number of parameter estimated: 3
Degrees of freedom: 5
-2 log likelihood: -18.86768

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:37 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Multivariate meta-analysis with "publication year-1979" as a predictor
R> summary( with(Berkey98, meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
                                x=scale(pub_year, center=1979))) )

Running Meta analysis with ML
Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),      x = scale(pub_year, center = 1979))

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value
Slope1_1    0.0063540  0.1078235 -0.2049761  0.2176842  0.0589
Slope2_1   -0.0705888  0.1620965 -0.3882921  0.2471146 -0.4355
Intercept1  0.3440001  0.0857659  0.1759021  0.5120982  4.0109
Intercept2 -0.2918175  0.1312796 -0.5491208 -0.0345141 -2.2229
Tau2_1_1    0.0080405  0.0101206 -0.0117955  0.0278766  0.7945
Tau2_2_1    0.0093413  0.0105515 -0.0113392  0.0300218  0.8853
Tau2_2_2    0.0250135  0.0170788 -0.0084603  0.0584873  1.4646
            Pr(>|z|)
Slope1_1     0.95301
Slope2_1     0.66322
Intercept1 6.048e-05 ***
Intercept2   0.02622 *
Tau2_1_1     0.42692
Tau2_2_1     0.37599
Tau2_2_2     0.14303
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0
```

```
Number of studies: 5
Number of observed statistics: 15
Number of parameter estimated: 9
Degrees of freedom: 6
-2 log likelihood: -4.595466

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:37 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Estimate variance components with REML
R> summary( with(Berkey98, reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
                                x=scale(pub_year, center=1979))) )
```

```
Running Variance component with REML
Call:
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),      x = scale(pub_year, center = 1979))

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.020447  0.022523 -0.023697  0.064590  0.9078   0.3640
Tau2_2_1  0.016226  0.022696 -0.028258  0.060710  0.7149   0.4747
Tau2_2_2  0.040857  0.034707 -0.027168  0.108882  1.1772   0.2391

Number of studies: 5
Number of observed statistics: 6
Number of parameter estimated: 3
Degrees of freedom: 3
-2 log likelihood: -11.44636

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:38 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

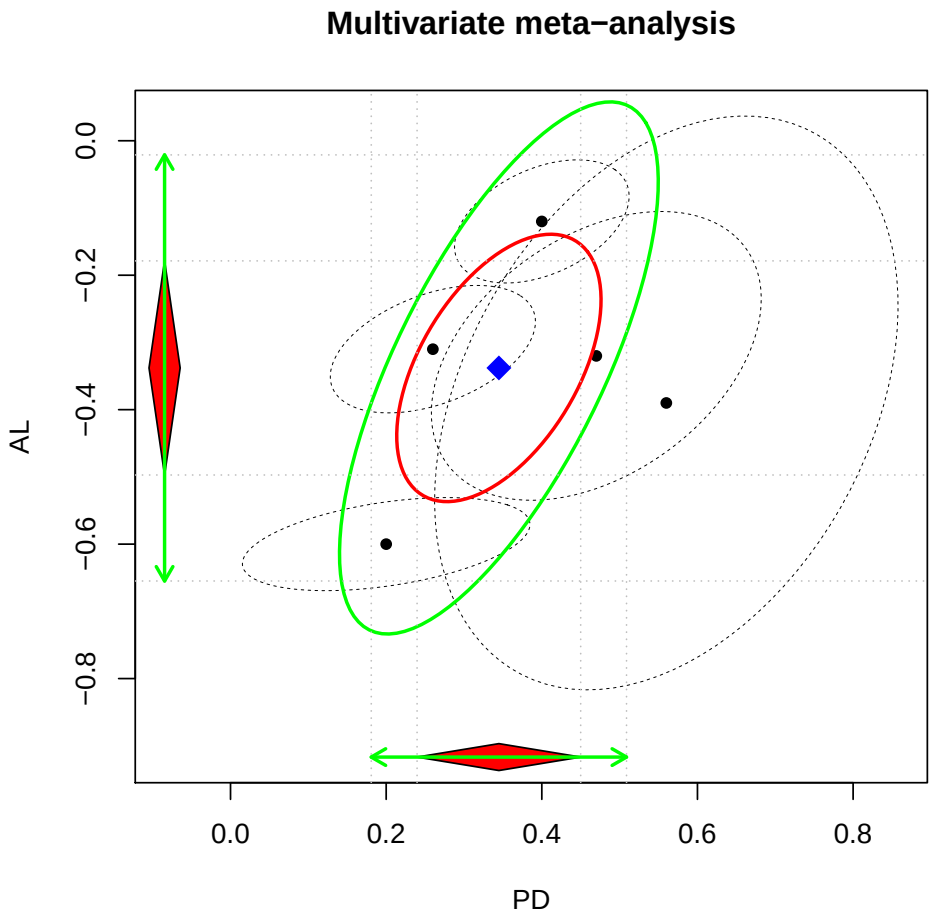
```
R> ## Multivariate meta-analysis with an equality constraint on the slopes
R> summary( with(Berkey98, meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
                                x=scale(pub_year, center=1979),
                                coeff.constraints=
                                matrix(c("0.3*Eq_slope", "0.3*Eq_slope"), nrow=2))) )
```

```
Running Meta analysis with ML
Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),      x = scale(pub_year, center = 1979), c

95% confidence intervals: z statistic approximation
Coefficients:
```

```
             Estimate  Std.Error     lbound     ubound z value
Eq_slope    0.0016748  0.1024443 -0.1991123  0.2024619  0.0163
Intercept1  0.3437612  0.0849828  0.1771979  0.5103245  4.0451
Intercept2 -0.3390010  0.1041005 -0.5430344 -0.1349677 -3.2565
Tau2_1_1    0.0070474  0.0094638 -0.0115013  0.0255962  0.7447
Tau2_2_1    0.0095165  0.0105668 -0.0111940  0.0302269  0.9006
Tau2_2_2    0.0261979  0.0180773 -0.0092330  0.0616288  1.4492
            Pr(>|z|)
Eq_slope    0.986956
Intercept1 5.231e-05 ***
Intercept2  0.001128 **
Tau2_1_1    0.456471
Tau2_2_1    0.367800
Tau2_2_2    0.147278
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 15
Number of parameter estimated: 8
Degrees of freedom: 7
-2 log likelihood: -4.268456

R version: 2.13.0
OpenMx version: 999.0.0-1609
metaSEM version: 0.6-0
Date of analysis: Sat Apr 30 22:51:38 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

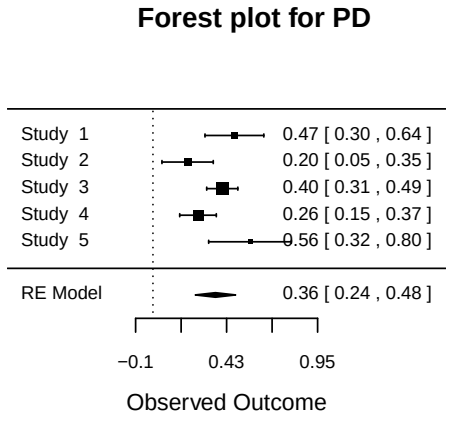
If multivariate meta-analysis is conducted, pairwise plots on the pooled effect sizes and their confidence ellipses may be requested. For example,

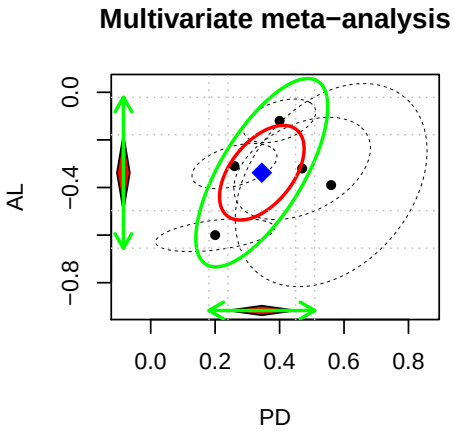
```
R> Berkey98.ma <- with(Berkey98, meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL)))

Running Meta analysis with ML

R> plot(Berkey98.ma, main="Multivariate meta-analysis", axis.label=c("PD", "AL"))
```

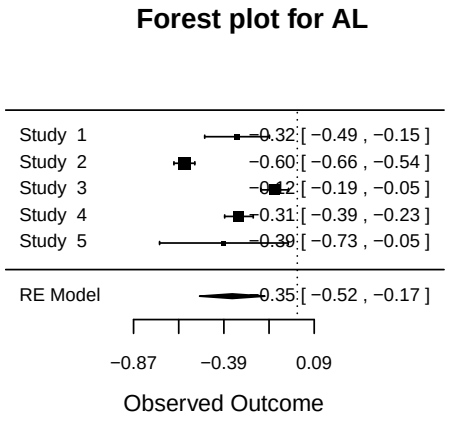


By combining with the forest plots based on the `metafor` package, more information can be displayed.

```
R> library(metafor)
R> plot(Berkey98.ma, diag.panel=TRUE, main="Multivariate meta-analysis", axis.label=c("PD", "AL"))
R> forest( rma(yi=PD, vi=var_PD, data=Berkey98) )
R> title("Forest plot for PD")
R> forest( rma(yi=AL, vi=var_AL, data=Berkey98) )
R> title("Forest plot for AL")
```







4 Acknowledgements

This package cannot be written without `R` and `OpenMx`. Contributions by the R Development Core Team and the OpenMx Core Development Team are highly appreciated.

References

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