metaSEM: An R Package for Meta-Analysis Using Structural Equation Modeling

Mike W.-L. Cheung
National University of Singapore

November 4, 2011

Abstract

The **metaSEM** package provides functions to conducting univariate and multivariate meta-analyses using a structural equation modeling approach via the **OpenMx** package. It also implements the two-stage structural equation modeling (TSSEM) approach to conducting meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. This paper outlines the basic theories and applications of these functions. Examples are used to illustrate the procedures.

Keywords: meta-analysis, structural equation modeling, meta-analytic structural equation modeling, **metaSEM**, R.

1. Introduction

The **metaSEM** is an R package to conducting univariate and multivariate meta-analysis using a structural equation modeling (SEM) approach (Cheung 2008, 2011) via the OpenMx package (Boker, Neale, Maes, Wilde, Spiegel, Brick, Spies, Estabrook, Kenny, Bates, Mehta, and Fox 2011). It also implements the two-stage structural equation modeling (TSSEM) approach (Cheung and Chan 2005b, 2009) to conducting meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. The main functions in this package are:

- `meta()` and `reml()`: `meta()` conducts univariate and multivariate meta-analysis with maximum likelihood (ML) estimation method while `reml()` estimates the variance components of the random-effects with restricted (residual) maximum likelihood estimation method (REML). Mixed-effects meta-analysis can be conducted by specifying study characteristics as predictors. Equality constraints on the intercepts, regression coefficients and variance components can be imposed.
- `tssem1()`: It conducts the stage one analysis of TSSEM by pooling correlation/covariance matrices with either a fixed- or random-effets model.
- `tssem2()`: It conducts the stage two analysis of TSSEM by fitting structural models on the pooled correlation/covariance matrix. It is a wrapper of `wls()`.
- `wls()`: It fits a correlation/covariance structure analysis with weighted least squares (WLS) estimation method.

Besides reporting Wald confidence intervals (CIs) based on z statistic, likelihood-based CIs on the parameter estimates may also be requested (Cheung 2009a; Neale and Miller 1997). Several generic functions, such as `anova()`, `coef()`, `vcov()`, `print()`, `summary()` and `plot()`, have been implemented. The current version of the **metaSEM** package is 0.7-0.

This paper is organized as follows. The next section introduces several meta-analytic models. Then basic theory of the TSSEM are presented. Several examples are used to illustrate the procedures.

2. Meta-analysis Models

In this section, the general multivariate mixed-effects meta-analysis with p effect sizes and m predictors are introduced. Univariate meta-analysis based on a mixed-, random- and fixed-effects are treated as special cases of the general multivariate model.

2.1. Mixed-effects model

Let us assume that there are p effect sizes with m predictors in k studies. The model for the multivariate effect sizes in the ith study is:

$$\mathbf{y}_i = \mathbf{B}\mathbf{x}_i + \mathbf{u}_i + \mathbf{e}_i, \tag{1}$$

where $\mathbf{y}_i$ is a $p \times 1$ effect sizes, $\mathbf{B}$ is a $p \times m$ regression coefficients, $\mathbf{x}_i$ is a $m \times 1$ predictors, $\mathbf{u}_i$ is a $p \times 1$ study specific random effects, and $\mathbf{e}_i$ is a $p \times 1$ sampling error. We assume that $\text{var}(\mathbf{e}_i) = V_i$ which is known and given in the studies and $\text{var}(\mathbf{u}_i) = T^2$ that is estimated from the data. T^2 is the variance component of the between-study heterogeneity.

The -2*log-likelihood of the above model is:

$$-2 * logL_i(\mathbf{B}, T^2; \mathbf{y}_i)_{\text{ML}} = p_i * log(2\pi) + log|T^2 + V_i| + (\mathbf{y}_i - \mathbf{B}\mathbf{x}_i)'(T^2 + V_i)^{-1}(\mathbf{y}_i - \mathbf{B}\mathbf{x}_i) \tag{2}$$

where p_i is the number of effect sizes in the ith study.

In applied research, different studies may report different effect sizes. The above -2*log-likelihood may handle missing effect sizes by using different dimenions of $\mathbf{y}_i$, T^2 and V_i across studies. It is expected that there is no missing data in $\mathbf{x}_i$. When there are missing data in $\mathbf{x}_i$, the whole study will be deleted before the analysis.

To obtain parameter estimates on $\mathbf{B}$ and T^2, we may take the sum of the -2*log-likelihood over all studies and minimize it. This is known as the ML estimation method. After the optimization, standard errors of the parameter estimates may also be obtained. The parameter estimate divided by the standard error follows a z distribution under the null hypothesis. Moreover, likelihood ratio statistic may also be used to test specific hypotheses. Since both the fixed- and random-effects are estimated simultaneously, it is well-known that $\hat{T}^2_{\text{ML}}$ is under-estimated because it does not take the uncertainty in estimating $\hat{\mathbf{B}}_{\text{ML}}$ into account.

2.2. Univariate meta-analysis

When there is only one effect size, i.e., $p = 1$, the general model becomes a univariate model. The model in the ith study is:

$$y_i = \beta\mathbf{x_i} + u_i + e_i. \tag{3}$$

where β is a $m \times 1$ regression coefficients including the intercept.

Now, $\text{var}(e_i) = v_i$ is the known sampling variance and $\text{var}(u_i) = \tau^2$ is the heterogeneity of the effect size.

2.3. Random-effects model

When there is no predictor, the mixed-effects model becomes a random-effects model. The model in the ith study is:

$$y_i = \beta_{\text{random}} + u_i + e_i. \tag{4}$$

where β_{random} is the average effect under a random-effects model.

2.4. Fixed-effects model

When there is no study specific random effect, the model becomes a fixed-effects model. The model in the ith study is:

$$y_i = \beta_{\text{fixed}} + e_i. \tag{5}$$

where β_{fixed} is the average effect under a fixed-effects model.

2.5. Examples

Two example data sets are used to demonstrate the procedures of conducting univariate and multivariate meta-analyses. Becker (1983) reported 10 studies on sex differences in conformity using the fictitious norm group paradigm. *di* and *vi* are the standardized mean difference and its sampling variance. Becker hypothesized that the logarithm of the number of items *items* may correlate with the effect size.

The second data set is from Berkey, Hoaglin, Antczak-Bouckoms, Mosteller, and Colditz (1998). They summarized five published trials comparing surgical and non-surgical treatments for medium-severity periodontal disease, one year after treatment. Publication year *pub_year* was hypothesized as a predictor.

Univariate random-effects model The function `meta()` is used to conduct the analysis. The arguments `y` and `v` are used to specify the effect sizes and its sampling variances (or covariances for multivariate meta-analysis), respectively. By default, a random-effects meta-analysis is performed. After running the analysis, `summary()` may be used to extract the results. The estimates of fixed- and random-effects are represented by **Intercept** and **Tau2**, respectively.

```
R> ## Load the library
R> library(metaSEM)
R> ## Show the first few studies of the data set
R> head(Becker83)

  study    di   vi percentage items
1     1 -0.33 0.03         25     2
2     2  0.07 0.03         25     2
3     3 -0.30 0.02         50     2
```

```
4     4  0.35 0.02        100    38
5     5  0.69 0.07        100    30
6     6  0.81 0.22        100    45

R> ## Random-effects meta-analysis with ML
R> summary( meta(y=di, v=vi, data=Becker83) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.174734  0.113378 -0.047482  0.396950  1.5412   0.1233
Tau2_1_1    0.077376  0.054108 -0.028674  0.183426  1.4300   0.1527

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies: 10
Number of observed statistics: 10
Number of parameter estimated: 2
Degrees of freedom: 8
-2 log likelihood: 7.928307

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:21 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Univariate mixed-effects model We may include a predictor to conduct a mixed-effects meta-analysis. The argument `x` may be used to specify the predictors. If there are more than one predictor, `cbind()` may be used to specify the predictors. The estimates are represented by **slope**.

```
R> ## Mixed-effects meta-analysis with "log(items)" as the predictor
R> summary( meta(y=di, v=vi, x=log(items), data=Becker83) )

Running Meta analysis with ML

Call:
```

```
meta(y = di, v = vi, x = log(items), data = Becker83)

95% confidence intervals: z statistic approximation
Coefficients:
              Estimate   Std.Error      lbound      ubound z value
Slope1_1    2.1088e-01  4.5084e-02  1.2251e-01  2.9924e-01  4.6774
Intercept1 -3.2015e-01  1.0981e-01 -5.3539e-01 -1.0492e-01 -2.9154
Tau2_1_1    1.0000e-10  2.0095e-02 -3.9386e-02  3.9386e-02  0.0000
            Pr(>|z|)
Slope1_1   2.905e-06 ***
Intercept1  0.003552 **
Tau2_1_1    1.000000
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies: 10
Number of observed statistics: 20
Number of parameter estimated: 5
Degrees of freedom: 15
-2 log likelihood: 30.29783

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:21 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Univariate fixed-effects model Mathematically, fixed-effects meta-analysis is a special case of the random-effects meta-analysis by fixing the variance of the random-effects at 0. The argument `RE.constraints`, which expects a matrix as input, may be used to constrain the variance component of the random effects.

```
R> ## Fixed-effects meta-analysis
R> summary( meta(y=di, v=vi, data=Becker83, RE.constraints=matrix(0, ncol=1, nrow=1)) )

Running Meta analysis with ML

Call:
meta(y = di, v = vi, data = Becker83, RE.constraints = matrix(0,
    ncol = 1, nrow = 1))
```

```
95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Intercept1  0.100640  0.060510 -0.017957  0.219237  1.6632  0.09627 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 30.64949
Degrees of freedom of the Q statistic: 9
P value of the Q statistic: 0.0003399239

Number of studies: 10
Number of observed statistics: 10
Number of parameter estimated: 1
Degrees of freedom: 9
-2 log likelihood: 17.86043

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:21 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Multivariate random-effects model Multivariate meta-analysis can be performed by specifying the multivariate effect sizes and its sampling covariance matrix in arguments `y` and `v` with `cbind()`, respectively. Only the lower triangle of the sampling covariance matrix arranged by the column major is used in `v`.

```
R> ## Show the data set
R> Berkey98

  trial pub_year no_of_patients   PD    AL var_PD cov_PD_AL var_AL
1     1     1983             14 0.47 -0.32 0.0075    0.0030 0.0077
2     2     1982             15 0.20 -0.60 0.0057    0.0009 0.0008
3     3     1979             78 0.40 -0.12 0.0021    0.0007 0.0014
4     4     1987             89 0.26 -0.31 0.0029    0.0009 0.0015
5     5     1988             16 0.56 -0.39 0.0148    0.0072 0.0304

R> ## Multivariate meta-analysis with a random-effects model
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98) )

Running Meta analysis with ML

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
```

```
    data = Berkey98)

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value
Intercept1  0.3448390  0.0536312  0.2397238  0.4499542  6.4298
Intercept2 -0.3379383  0.0812479 -0.4971813 -0.1786952 -4.1593
Tau2_1_1    0.0070020  0.0090497 -0.0107351  0.0247391  0.7737
Tau2_2_1    0.0094607  0.0099698 -0.0100797  0.0290010  0.9489
Tau2_2_2    0.0261445  0.0177409 -0.0086270  0.0609161  1.4737
            Pr(>|z|)
Intercept1 1.278e-10 ***
Intercept2 3.192e-05 ***
Tau2_1_1      0.4391
Tau2_2_1      0.3427
Tau2_2_2      0.1406
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 10
Number of parameter estimated: 5
Degrees of freedom: 5
-2 log likelihood: -11.68131

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:21 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Multivariate mixed-effects model We then include *pub_year* as a predictor. To make the intercept more interpretable, we may center the publication year at 1979, the publication year of the first record in the data set. Moreover, we may test whether the regression coefficients are the same by constraining the regression coefficients from *pub_year* to PD and AL equally with the argument `coeff.constraints`. Since these two models are nested, we may compare them by a likelihood ratio statistic by calling the `anova()` function.

```
R> ## Multivariate meta-analysis with "publication year-1979" as a predictor
R> mul1 <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
               x=scale(pub_year, center=1979), model.name="No equality constraint")
```

```
Running No equality constraint

R> summary(mul1)

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    x = scale(pub_year, center = 1979), data = Berkey98, model.name = "No equality constra

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value
Slope1_1    0.0063540  0.1078235 -0.2049762  0.2176842  0.0589
Slope2_1   -0.0705888  0.1620965 -0.3882922  0.2471146 -0.4355
Intercept1  0.3440001  0.0857659  0.1759020  0.5120982  4.0109
Intercept2 -0.2918174  0.1312796 -0.5491208 -0.0345141 -2.2229
Tau2_1_1    0.0080405  0.0101206 -0.0117955  0.0278766  0.7945
Tau2_2_1    0.0093413  0.0105515 -0.0113392  0.0300218  0.8853
Tau2_2_2    0.0250135  0.0170788 -0.0084603  0.0584873  1.4646
            Pr(>|z|)
Slope1_1     0.95301
Slope2_1     0.66322
Intercept1 6.048e-05 ***
Intercept2   0.02622 *
Tau2_1_1     0.42692
Tau2_2_1     0.37599
Tau2_2_2     0.14303
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 15
Number of parameter estimated: 9
Degrees of freedom: 6
-2 log likelihood: -4.595466

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:21 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Multivariate meta-analysis with an equality constraint on the slopes
R> mul2 <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
```

```
                x=scale(pub_year, center=1979), model.name="With equality constraint",
                coeff.constraints=matrix(c("0.3*Equal_slope", "0.3*Equal_slope"), nrow=2))

Running With equality constraint

R> summary(mul2)

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    x = scale(pub_year, center = 1979), data = Berkey98, coeff.constraints = matrix(c("0.3
        "0.3*Equal_slope"), nrow = 2), model.name = "With equality constraint")

95% confidence intervals: z statistic approximation
Coefficients:
              Estimate  Std.Error     lbound     ubound z value
Equal_slope  0.0016748  0.1024443 -0.1991123  0.2024619  0.0163
Intercept1   0.3437612  0.0849828  0.1771979  0.5103245  4.0451
Intercept2  -0.3390010  0.1041005 -0.5430344 -0.1349677 -3.2565
Tau2_1_1     0.0070474  0.0094638 -0.0115013  0.0255962  0.7447
Tau2_2_1     0.0095165  0.0105668 -0.0111940  0.0302269  0.9006
Tau2_2_2     0.0261979  0.0180773 -0.0092330  0.0616288  1.4492
             Pr(>|z|)
Equal_slope  0.986956
Intercept1  5.231e-05 ***
Intercept2   0.001128 **
Tau2_1_1     0.456471
Tau2_2_1     0.367800
Tau2_2_2     0.147278
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 15
Number of parameter estimated: 8
Degrees of freedom: 7
-2 log likelihood: -4.268456

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:21 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
R> ## Likelihood ratio test on comparing these two models
R> anova(mul1, mul2)

                   base               comparison ep  minus2LL df
1 No equality constraint                    <NA>  9 -4.595466  6
2 No equality constraint With equality constraint  8 -4.268456  7
        AIC    diffLL diffdf         p
1 -16.59547        NA     NA        NA
2 -18.26846 0.3270107      1 0.5674246
```

Multivariate fixed-effects model We may conduct a multivariate fixed-effects meta-analysis by fixing the variance component at a zero matrix.

```
R> ## Multivariate meta-analysis with a fixed-effects model
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98,
           RE.constraints=matrix(0, nrow=2, ncol=2)) )

Running Meta analysis with ML

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    data = Berkey98, RE.constraints = matrix(0, nrow = 2, ncol = 2))

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value  Pr(>|z|)
Intercept1  0.307219  0.028575  0.251212  0.363225  10.751 < 2.2e-16
Intercept2 -0.394377  0.018649 -0.430929 -0.357825 -21.147 < 2.2e-16

Intercept1 ***
Intercept2 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 10
Number of parameter estimated: 2
Degrees of freedom: 8
-2 log likelihood: 90.88326

R version: 2.14.0
OpenMx version: 1.1.2-1818
```

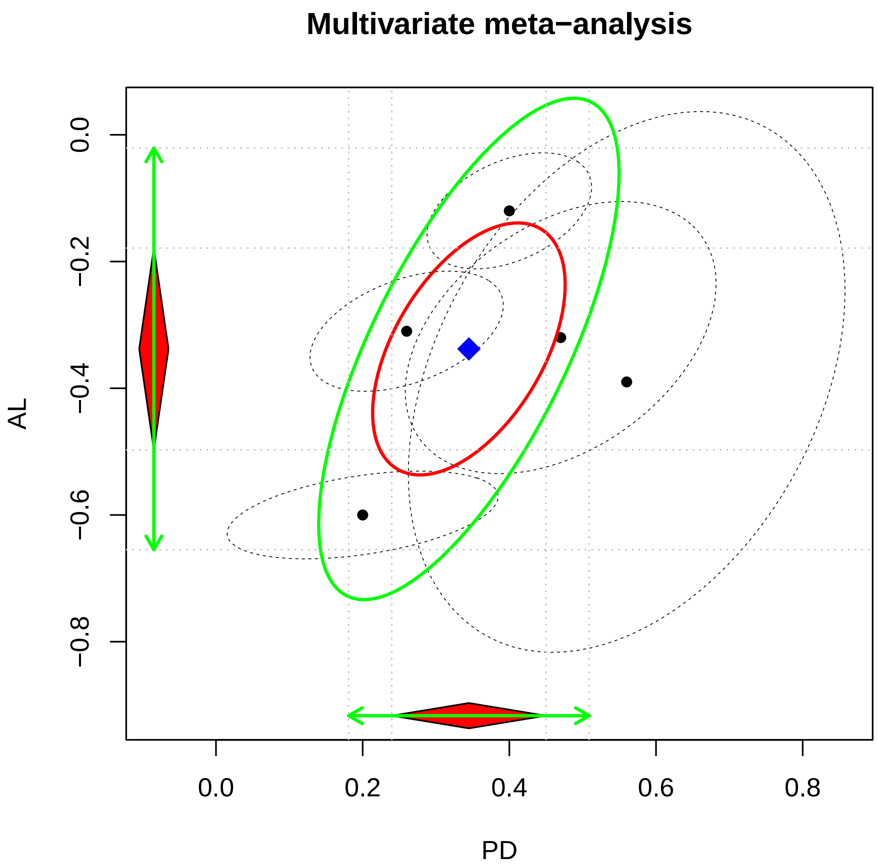
```
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:22 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Plots of multivariate effect sizes If multivariate meta-analysis is conducted, pairwise plots on the pooled effect sizes and their confidence ellipses may be requested via the `plot()` function. By default, 95% confidence intervals and confidence ellipses on the average effect sizes and the random effects are plotted. For example,

```
R> Berkey98.ma <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98)
```

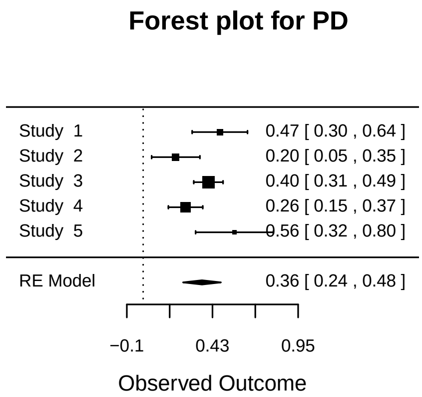
```
Running Meta analysis with ML
```

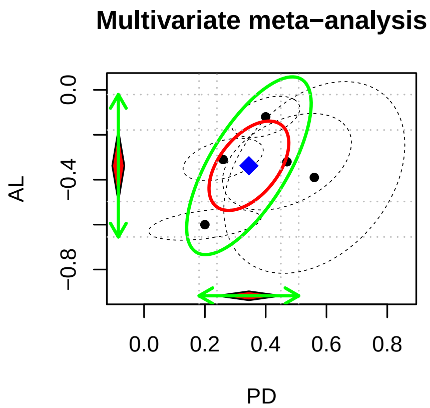
```
R> plot(Berkey98.ma, main="Multivariate meta-analysis", axis.label=c("PD", "AL"))
```

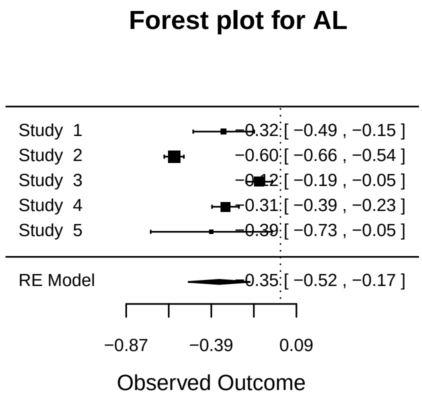


By combining with the forest plots based on the **metafor** package, more information can be displayed.

```
R> ## Load the metafor package to display forest plots
R> library(metafor)
R> plot(Berkey98.ma, diag.panel=TRUE, main="Multivariate meta-analysis",
       axis.label=c("PD", "AL"))
R> forest( rma(yi=PD, vi=var_PD, data=Berkey98) )
R> title("Forest plot for PD")
R> forest( rma(yi=AL, vi=var_AL, data=Berkey98) )
R> title("Forest plot for AL")
```







3. Meta-analytic structural equation modeling

MASEM combines the idea of SEM and meta-analysis by pooling studies to test structural equation models. Generally speaking, there are two stages in conducting a MASEM. In the first stage the correlation/covariance matrices are pool together. In the second stage, the pooled correlation/covariance matrix can be used to fit structural equation models.

Cheung and Chan (2005b, 2009) proposed a two-stage structural equation modeling (TSSEM) based on a fixed-effects model. The `metaSEM` package has implemented the TSSEM approach. Moreover, the TSSEM approach has been extended to the random-effects model by using a multivariate meta-analysis (Cheung 2011). Regardless of whether a fixed- or random-effects model is used, the `tssem2()` function will handle this automatically. In other words, parameter estimates, standard errors and goodness-of-fit indices in the stage 2 analysis has already taken whether the stage 1 model is fixed or random into account.

An example from Cheung (2009b) is used to illustrate the two-stage structural equation modeling (TSSEM) procedure. In this example, Digman (1997) reported a second-order factor analysis on a five-factor model with 14 studies. He proposed that there were two second-order factors for the five-factor model: an alpha factor for agreeableness, conscientiousness, and emotional stability, and a beta factor for extroversion and intellect.

3.1. Fixed-effects model

`tssem1()` is used to pool the correlation matrices with a fixed-effects model in the first stage. `tssem2()` is then used to fit a factor analytic model on the pooled correlation matrix with the inverse of its asymptotic covariance matrix as the weight matrix (see Cheung and Chan (2005b, 2009) for the details).

```
R> #### Show the first 2 studies in Digman97
R> head(Digman97$data, n=2)

$`Digman 1 (1994)`
       E     A     C   ES    I
E   1.00 -0.48 -0.10 0.27 0.37
A  -0.48  1.00  0.62 0.41 0.00
C  -0.10  0.62  1.00 0.59 0.35
ES  0.27  0.41  0.59 1.00 0.41
I   0.37  0.00  0.35 0.41 1.00

$`Digman 2 (1994)`
       E     A    C   ES     I
E   1.00 -0.30 0.07 0.09  0.45
A  -0.30  1.00 0.39 0.53 -0.05
C   0.07  0.39 1.00 0.59  0.44
ES  0.09  0.53 0.59 1.00  0.22
I   0.45 -0.05 0.44 0.22  1.00

R> #### Show the first 2 sample sizes in Digman97
R> head(Digman97$n, n=2)

[1] 102 149

R> #### Example of Fixed-effects TSSEM
R> fixed1 <- tssem1(Digman97$data, Digman97$n)

Running TSSEM1 Analysis of Correlation Matrix

R> summary(fixed1)
```

```
Call:
tssem1FE(my.df = my.df, n = n, cor.analysis = cor.analysis, model.name = model.name,
    cluster = cluster, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.103751  0.015070  6.8848 5.787e-12 ***
S[1,3] 0.135208  0.014799  9.1365 < 2.2e-16 ***
S[1,4] 0.244505  0.014175 17.2493 < 2.2e-16 ***
S[1,5] 0.424514  0.012395 34.2475 < 2.2e-16 ***
S[2,3] 0.363116  0.013390 27.1178 < 2.2e-16 ***
S[2,4] 0.390176  0.012903 30.2400 < 2.2e-16 ***
S[2,5] 0.092246  0.015071  6.1208 9.310e-10 ***
S[3,4] 0.415999  0.012539 33.1751 < 2.2e-16 ***
S[3,5] 0.141213  0.014890  9.4835 < 2.2e-16 ***
S[4,5] 0.138167  0.014858  9.2994 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     4496.0000
Chi-square of target model      1499.7340
DF of target model               130.0000
p value of target model            0.0000
Chi-square of independent model 4454.5995
DF of independent model          140.0000
RMSEA                              0.1812
SRMR                               0.1750
TLI                                0.6581
CFI                                0.6825
AIC                             1239.7340
BIC                              406.3114

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:29 2011
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> #### Extract the pooled correlation matrix
R> coef(fixed1)

          x1         x2        x3        x4         x5
x1 1.0000000 0.10375112 0.1352076 0.2445051 0.42451421
x2 0.1037511 1.00000000 0.3631157 0.3901765 0.09224586
```

```
x3 0.1352076 0.36311572 1.0000000 0.4159987 0.14121296
x4 0.2445051 0.39017648 0.4159987 1.0000000 0.13816668
x5 0.4245142 0.09224586 0.1412130 0.1381667 1.00000000

R> #### Prepare a CFA model to be fitted
R> P4 <- mxMatrix("Stand", ncol=2, nrow=2, value=.2, free=TRUE, name="P4")
R> L4 <- mxMatrix("Full", ncol=2, nrow=5, value=c(0,.3,.3,.3,0,.3,0,0,0,.3),
  free=c(FALSE,TRUE,TRUE,TRUE,FALSE,TRUE,FALSE,FALSE,FALSE,TRUE),
  name="L4")
R> ## Model to be fitted
R> impliedR4 <- mxAlgebra(L4 %&% P4, name="impliedR4")
R> fixed2 <- tssem2(fixed1, impliedS=impliedR4, matrices=c(P4, L4),
                    model.name="TSSEM2 Digman97")

Running TSSEM2 Digman97

R> summary(fixed2)

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,2] 0.779567  0.033174 0.714548 0.844586  23.500 < 2.2e-16 ***
L4[2,1] 0.564486  0.014282 0.536494 0.592477  39.525 < 2.2e-16 ***
L4[3,1] 0.607144  0.014297 0.579121 0.635166  42.465 < 2.2e-16 ***
L4[4,1] 0.717134  0.014772 0.688182 0.746086  48.548 < 2.2e-16 ***
L4[5,2] 0.554205  0.025113 0.504984 0.603426  22.068 < 2.2e-16 ***
P4[1,2] 0.362686  0.021954 0.319656 0.405716  16.520 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Goodness-of-fit indices:
                                    Value
Sample size                     4496.0000
Chi-square of target model        67.8895
DF of target model                 4.0000
p value of target model            0.0000
Chi-square of independent model 4132.8948
DF of independent model           10.0000
RMSEA                              0.0596
SRMR                               0.0285
TLI                                0.9613
```

```
CFI                                          0.9845
AIC                                         59.8895
BIC                                         34.2457

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:29 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

Example: Fixed-effects model with cluster Studies may not share the same population correlation matrix when conduting a MASEM on a pool of correlation matrices. If the studies can be grouped into various subgroups, it is possible to pool the correlation matrices by the subgroups (Cheung and Chan 2005a). This is similar to the subgroup analysis in conventional meta-analysis. For example, Digman (1997) groups the 14 studies into several groups with different samples. These include children, adolescents, young adults, and mature adults. This variable is stored in the variable `Digman97$cluster`.

The following R code may be used to conduct the analysis. Users have to supply the cluster (a vector of labels) to the `cluster` argument in `tssem1`. The correlation matrices will be pooled within each cluster. And the structural models will also be fitted within each cluster.

```
R> #### Show the clusters in Digman97
R> Digman97$cluster

 [1] "Children"      "Children"      "Children"      "Children"
 [5] "Adolescents"   "Young adults"  "Young adults"  "Young adults"
 [9] "Mature adults" "Mature adults" "Mature adults" "Mature adults"
[13] "Mature adults" "Mature adults"

R> ##### Example of Fixed-effects TSSEM with several clusters
R> fixed1.cluster <- tssem1(Digman97$data, Digman97$n, cluster=Digman97$cluster)

Running TSSEM1 Analysis of Correlation Matrix
Running TSSEM1 Analysis of Correlation Matrix
Running TSSEM1 Analysis of Correlation Matrix
Running TSSEM1 Analysis of Correlation Matrix

R> summary(fixed1.cluster)

$Adolescents

Call:
tssem1FE(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)
```

```
Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.290000  0.096544  3.0038 0.0026662 **
S[1,3] 0.160000  0.102710  1.5578 0.1192854
S[1,4] 0.320000  0.094615  3.3821 0.0007193 ***
S[1,5] 0.530000  0.075800  6.9921 2.708e-12 ***
S[2,3] 0.640000  0.062233 10.2839 < 2.2e-16 ***
S[2,4] 0.350000  0.092496  3.7839 0.0001544 ***
S[2,5] 0.220000  0.100307  2.1933 0.0282880 *
S[3,4] 0.270000  0.097724  2.7629 0.0057296 **
S[3,5] 0.220000  0.100307  2.1933 0.0282882 *
S[4,5] 0.360000  0.091748  3.9238 8.717e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                 Value
Sample size                      91.00
Chi-square of target model        0.00
DF of target model                0.00
p value of target model           0.00
Chi-square of independent model 109.63
DF of independent model          10.00
RMSEA                              Inf
SRMR                              0.00
TLI                               -Inf
CFI                               1.00
AIC                               0.00
BIC                               0.00

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:32 2011
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$Children

Call:
tssem1FE(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)

Coefficients:
        Estimate Std.Error z value  Pr(>|z|)
S[1,2] -0.071259  0.037983 -1.8761   0.06065 .
```

```
S[1,3] -0.084678  0.036506 -2.3196   0.02036 *
S[1,4]  0.158313  0.035949  4.4038 1.064e-05 ***
S[1,5]  0.473158  0.028765 16.4489 < 2.2e-16 ***
S[2,3]  0.600192  0.023695 25.3302 < 2.2e-16 ***
S[2,4]  0.479811  0.028723 16.7048 < 2.2e-16 ***
S[2,5]  0.043055  0.036728  1.1723   0.24110
S[3,4]  0.526708  0.026960 19.5365 < 2.2e-16 ***
S[3,5]  0.331623  0.032727 10.1332 < 2.2e-16 ***
S[4,5]  0.298135  0.033719  8.8419 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      747.0000
Chi-square of target model       311.3516
DF of target model                30.0000
p value of target model            0.0000
Chi-square of independent model 1352.0398
DF of independent model           40.0000
RMSEA                              0.2242
SRMR                               0.1401
TLI                                0.7141
CFI                                0.7856
AIC                              251.3516
BIC                              112.8696

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:32 2011
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Mature adults`

Call:
tssem1FE(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.076227  0.018725  4.0708 4.685e-05 ***
S[1,3] 0.170404  0.018269  9.3275 < 2.2e-16 ***
S[1,4] 0.191577  0.018047 10.6157 < 2.2e-16 ***
S[1,5] 0.366062  0.016326 22.4217 < 2.2e-16 ***
```

```
S[2,3] 0.196629  0.018031 10.9051 < 2.2e-16 ***
S[2,4] 0.305076  0.017177 17.7605 < 2.2e-16 ***
S[2,5] 0.013859  0.018955  0.7312    0.4647
S[3,4] 0.385234  0.016189 23.7956 < 2.2e-16 ***
S[3,5] 0.030844  0.018907  1.6314    0.1028
S[4,5] 0.037283  0.018830  1.9800    0.0477 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                      2862.0000
Chi-square of target model        420.5247
DF of target model                 50.0000
p value of target model             0.0000
Chi-square of independent model 1707.9108
DF of independent model            60.0000
RMSEA                               0.1247
SRMR                                0.1522
TLI                                 0.7302
CFI                                 0.7752
AIC                               320.5247
BIC                                22.5609

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:33 2011
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Young adults`

Call:
tssem1FE(my.df = data.cluster[[i]], n = n.cluster[[i]], cor.analysis = cor.analysis,
    model.name = model.name, suppressWarnings = suppressWarnings)

Coefficients:
       Estimate Std.Error z value  Pr(>|z|)
S[1,2] 0.322154  0.031894 10.1008 < 2.2e-16 ***
S[1,3] 0.219371  0.033847  6.4813 9.092e-11 ***
S[1,4] 0.471710  0.027645 17.0629 < 2.2e-16 ***
S[1,5] 0.554691  0.024744 22.4175 < 2.2e-16 ***
S[2,3] 0.613646  0.022452 27.3310 < 2.2e-16 ***
S[2,4] 0.560195  0.024404 22.9555 < 2.2e-16 ***
S[2,5] 0.351222  0.031207 11.2545 < 2.2e-16 ***
```

```
S[3,4] 0.424434  0.029180 14.5454 < 2.2e-16 ***
S[3,5] 0.286843  0.032639  8.7884 < 2.2e-16 ***
S[4,5] 0.276926  0.032878  8.4227 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      796.0000
Chi-square of target model        66.8333
DF of target model                20.0000
p value of target model            0.0000
Chi-square of independent model 1285.0187
DF of independent model           30.0000
RMSEA                              0.0940
SRMR                               0.1511
TLI                                0.9440
CFI                                0.9627
AIC                               26.8333
BIC                              -66.7587

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:33 2011
OpenMx status: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ##### Extract the pooled correlation matrices
R> coef(fixed1.cluster)

$Adolescents
     x1   x2   x3   x4   x5
x1 1.00 0.29 0.16 0.32 0.53
x2 0.29 1.00 0.64 0.35 0.22
x3 0.16 0.64 1.00 0.27 0.22
x4 0.32 0.35 0.27 1.00 0.36
x5 0.53 0.22 0.22 0.36 1.00

$Children
            x1          x2          x3        x4         x5
x1  1.00000000 -0.07125881 -0.08467822 0.1583127 0.47315830
x2 -0.07125881  1.00000000  0.60019239 0.4798110 0.04305501
x3 -0.08467822  0.60019239  1.00000000 0.5267076 0.33162339
x4  0.15831268  0.47981099  0.52670763 1.0000000 0.29813501
x5  0.47315830  0.04305501  0.33162339 0.2981350 1.00000000
```

```
$`Mature adults`
           x1         x2         x3         x4         x5
x1 1.00000000 0.07622689 0.17040393 0.19157745 0.36606178
x2 0.07622689 1.00000000 0.19662879 0.30507564 0.01385939
x3 0.17040393 0.19662879 1.00000000 0.38523397 0.03084428
x4 0.19157745 0.30507564 0.38523397 1.00000000 0.03728294
x5 0.36606178 0.01385939 0.03084428 0.03728294 1.00000000

$`Young adults`
          x1        x2        x3        x4        x5
x1 1.0000000 0.3221536 0.2193714 0.4717095 0.5546912
x2 0.3221536 1.0000000 0.6136457 0.5601947 0.3512219
x3 0.2193714 0.6136457 1.0000000 0.4244337 0.2868427
x4 0.4717095 0.5601947 0.4244337 1.0000000 0.2769260
x5 0.5546912 0.3512219 0.2868427 0.2769260 1.0000000

R> fixed2.cluster <- tssem2(fixed1.cluster, impliedS=impliedR4, matrices=c(P4, L4))

Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure
Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure
Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure
Running TSSEM2 (Fixed Effects Model) Analysis of Correlation Structure

R> summary(fixed2.cluster)

$Adolescents

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,2] 0.738279  0.104533 0.533399 0.943159  7.0627 1.633e-12 ***
L4[2,1] 0.867342  0.075056 0.720234 1.014450 11.5559 < 2.2e-16 ***
L4[3,1] 0.742502  0.075055 0.595397 0.889606  9.8928 < 2.2e-16 ***
L4[4,1] 0.526038  0.090630 0.348407 0.703670  5.8042 6.466e-09 ***
L4[5,2] 0.734002  0.106559 0.525149 0.942855  6.8882 5.651e-12 ***
P4[1,2] 0.548082  0.114039 0.324569 0.771595  4.8061 1.539e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
```

```
Sample size                          91.0000
Chi-square of target model           10.7341
DF of target model                    4.0000
p value of target model               0.0297
Chi-square of independent model     270.6745
DF of independent model              10.0000
RMSEA                                 0.1368
SRMR                                  0.1028
TLI                                   0.9354
CFI                                   0.9742
AIC                                   2.7341
BIC                                  -7.3093

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:33 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$Children

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate  Std.Error     lbound     ubound z value  Pr(>|z|)
L4[1,2] 4.2447e-03         NA         NA         NA      NA        NA
L4[2,1] 7.3718e-01 2.2482e-02 6.9311e-01 7.8124e-01  32.790 < 2.2e-16
L4[3,1] 9.1394e-01 1.8543e-02 8.7759e-01 9.5028e-01  49.287 < 2.2e-16
L4[4,1] 6.8942e-01 2.2164e-02 6.4598e-01 7.3286e-01  31.105 < 2.2e-16
L4[5,2] 1.2109e+02         NA         NA         NA      NA        NA
P4[1,2] 3.1813e-03         NA         NA         NA      NA        NA

L4[1,2]
L4[2,1] ***
L4[3,1] ***
L4[4,1] ***
L4[5,2]
P4[1,2]
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Goodness-of-fit indices:
                                    Value
Sample size                      747.0000
Chi-square of target model       150.9107
DF of target model                 4.0000
p value of target model            0.0000
Chi-square of independent model 3583.7651
DF of independent model           10.0000
RMSEA                              0.2219
SRMR                               0.1074
TLI                                0.8972
CFI                                0.9589
AIC                              142.9107
BIC                              124.4464

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:33 2011
OpenMx status1: 6 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Mature adults`

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,2] 1.386476  0.294764 0.808748 1.964203  4.7037 2.555e-06 ***
L4[2,1] 0.397877  0.021232 0.356263 0.439491 18.7393 < 2.2e-16 ***
L4[3,1] 0.523006  0.022691 0.478533 0.567480 23.0490 < 2.2e-16 ***
L4[4,1] 0.746117  0.027054 0.693093 0.799141 27.5791 < 2.2e-16 ***
L4[5,2] 0.264596  0.058484 0.149970 0.379222  4.5243 6.060e-06 ***
P4[1,2] 0.192413  0.046615 0.101048 0.283777  4.1277 3.665e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     2862.0000
```

```
Chi-square of target model          8.9335
DF of target model                  4.0000
p value of target model             0.0628
Chi-square of independent model 1704.2696
DF of independent model            10.0000
RMSEA                               0.0208
SRMR                                0.0148
TLI                                 0.9927
CFI                                 0.9971
AIC                                 0.9335
BIC                               -22.9036

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:33 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.


$`Young adults`

Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,2] 0.865183  0.031067 0.804294 0.926072  27.849 < 2.2e-16 ***
L4[2,1] 0.844737  0.017826 0.809799 0.879674  47.389 < 2.2e-16 ***
L4[3,1] 0.699981  0.021093 0.658640 0.741322  33.186 < 2.2e-16 ***
L4[4,1] 0.765366  0.020161 0.725851 0.804882  37.962 < 2.2e-16 ***
L4[5,2] 0.708990  0.028447 0.653235 0.764745  24.923 < 2.2e-16 ***
P4[1,2] 0.595879  0.031500 0.534140 0.657617  18.917 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                     Value
Sample size                       796.0000
Chi-square of target model         85.9696
DF of target model                  4.0000
p value of target model             0.0000
Chi-square of independent model 3125.1750
```

```
DF of independent model          10.0000
RMSEA                             0.1606
SRMR                              0.0805
TLI                               0.9342
CFI                               0.9737
AIC                              77.9696
BIC                              59.2512

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:33 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.2. Random-effects model

TSSEM using a random-effects model may be conducted by using `method='RE'` argument in `tssem1`. It is assumed that a positive definite covariance matrix among the random-effects is used by default (`RE_diag=FALSE`). For some reasons, e.g., there are not enough studies, a diagonal matrix on the random-effects may also be used by specifying `RE_diag=TRUE`.

```
R> random1 <- tssem1(Digman97$data, Digman97$n, method="RE", RE_diag=TRUE)

Running TSSEM1 (Random Effects Model) Analysis of Correlation Matrix

R> summary(random1)

Call:
meta(y = ES, v = acovR, RE.constraints = diag(x = paste(RE.startvalues,
    "*Tau2_", 1:no.es, "_", 1:no.es, sep = ""), nrow = no.es,
    ncol = no.es), model.name = model.name)

95% confidence intervals: z statistic approximation
Coefficients:
               Estimate   Std.Error      lbound      ubound z value
Intercept1   0.05444808  0.06316915 -0.06936117  0.17825733  0.8619
Intercept10  0.19296050  0.04340498  0.10788831  0.27803269  4.4456
Intercept2   0.12867833  0.04174108  0.04686732  0.21048933  3.0828
Intercept3   0.24064412  0.03220613  0.17752127  0.30376698  7.4720
Intercept4   0.44713455  0.03211665  0.38418707  0.51008203 13.9222
Intercept5   0.39981597  0.05455528  0.29288959  0.50674235  7.3286
Intercept6   0.44433484  0.04168030  0.36264295  0.52602673 10.6605
Intercept7   0.10138323  0.04681347  0.00963051  0.19313595  2.1657
Intercept8   0.43415271  0.04000890  0.35573671  0.51256871 10.8514
Intercept9   0.20732484  0.04973239  0.10985114  0.30479853  4.1688
```

```
Tau2_1_1     0.05115982  0.02059752  0.01078943  0.09153022  2.4838
Tau2_2_2     0.01977639  0.00914600  0.00185056  0.03770222  2.1623
Tau2_3_3     0.01030043  0.00505946  0.00038407  0.02021680  2.0359
Tau2_4_4     0.01122093  0.00494563  0.00152767  0.02091420  2.2689
Tau2_5_5     0.03815850  0.01523930  0.00829002  0.06802697  2.5040
Tau2_6_6     0.02132566  0.00868726  0.00429894  0.03835238  2.4548
Tau2_7_7     0.02571725  0.01094040  0.00427446  0.04716005  2.3507
Tau2_8_8     0.01901269  0.00820037  0.00294027  0.03508511  2.3185
Tau2_9_9     0.02995569  0.01234178  0.00576625  0.05414514  2.4272
Tau2_10_10   0.02172539  0.00934584  0.00340788  0.04004289  2.3246
             Pr(>|z|)
Intercept1   0.388720
Intercept10 8.765e-06 ***
Intercept2   0.002051 **
Intercept3  7.905e-14 ***
Intercept4  < 2.2e-16 ***
Intercept5  2.325e-13 ***
Intercept6  < 2.2e-16 ***
Intercept7   0.030335 *
Intercept8  < 2.2e-16 ***
Intercept9  3.062e-05 ***
Tau2_1_1     0.012999 *
Tau2_2_2     0.030595 *
Tau2_3_3     0.041763 *
Tau2_4_4     0.023277 *
Tau2_5_5     0.012281 *
Tau2_6_6     0.014096 *
Tau2_7_7     0.018740 *
Tau2_8_8     0.020421 *
Tau2_9_9     0.015217 *
Tau2_10_10   0.020093 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 1968.963
Degrees of freedom of the Q statistic: 130
P value of the Q statistic: 0

Number of studies: 14
Number of observed statistics: 140
Number of parameter estimated: 20
Degrees of freedom: 120
-2 log likelihood: -109.6845

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
```

```
Date of analysis: Fri Nov  4 13:47:36 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ##### Extract the fixed-effects (pooled correlation matrix)
R> coef(random1, select="fixed")

 Intercept1  Intercept2  Intercept3  Intercept4  Intercept5
 0.05444808  0.12867833  0.24064412  0.44713455  0.39981597
 Intercept6  Intercept7  Intercept8  Intercept9 Intercept10
 0.44433484  0.10138323  0.43415271  0.20732484  0.19296050

R> ##### Extract the random-effects (variance component)
R> coef(random1, select="random")

  Tau2_1_1   Tau2_2_2   Tau2_3_3   Tau2_4_4   Tau2_5_5   Tau2_6_6
0.05115982 0.01977639 0.01030043 0.01122093 0.03815850 0.02132566
  Tau2_7_7   Tau2_8_8   Tau2_9_9 Tau2_10_10
0.02571725 0.01901269 0.02995569 0.02172539

R> random2 <- tssem2(random1, impliedS=impliedR4, matrices=c(P4, L4))

Running TSSEM2 (Random Effects Model) Analysis of Correlation Structure

R> summary(random2)

Call:
wls(S = pooledS, acovS = acovS, n = tssem1.obj$total.n, impliedS = impliedS,
    matrices = matrices, cor.analysis = cor.analysis, intervals.type = intervals.type,
    model.name = model.name, suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,2] 0.690304  0.074084 0.545102 0.835505  9.3179 < 2.2e-16 ***
L4[2,1] 0.577073  0.051758 0.475629 0.678518 11.1494 < 2.2e-16 ***
L4[3,1] 0.594979  0.051924 0.493209 0.696748 11.4586 < 2.2e-16 ***
L4[4,1] 0.770869  0.061201 0.650918 0.890821 12.5958 < 2.2e-16 ***
L4[5,2] 0.647773  0.069433 0.511686 0.783860  9.3294 < 2.2e-16 ***
P4[1,2] 0.394762  0.047191 0.302271 0.487254  8.3653 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     4496.0000
```

```
Chi-square of target model        8.2809
DF of target model                4.0000
p value of target model           0.0818
Chi-square of independent model  546.8067
DF of independent model          10.0000
RMSEA                             0.0154
SRMR                              0.0465
TLI                               0.9801
CFI                               0.9920
AIC                               0.2809
BIC                             -25.3629

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:36 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.3. Analysis of Correlation/Covariance Structure with Weighted Least Squares

Besides fitting a TSSEM, `wls()` may be used to fit a correlation/covariance structure with weighted least squares (WLS) as the estimation method. Likelihood-based CIs may also be calculated. The following example fits a one-factor CFA model on the correlation matrix with WLS estimation method.

```
R> ## Sample correlation matrix
R> R1 <- matrix(c(1.00, 0.22, 0.24, 0.18,
                 0.22, 1.00, 0.30, 0.22,
                 0.24, 0.30, 1.00, 0.24,
                 0.18, 0.22, 0.24, 1.00), ncol=4, nrow=4)
R> ## Sample size
R> n <- 1000
R> ## Estimate the asymptotic covariance matrix of the sample correlation matrix
R> acovR <- asyCov(R1, n)
R> ## P1: Factor variance is fixed at 1.0
R> P1 <- as.mxMatrix( matrix(1), name="P1")
R> ## L1: Factor loadings
R> L1 <- as.mxMatrix( matrix( rep("0.3*", 4), nrow=4, ncol=1), name="L1" )
R> ## Model implied correlation matrix
R> ## Please note that error variances are not involved in correlation structure analysis
R> impliedR1 <- mxAlgebra(L1 %&% P1, name="impliedR1")
R> ## wls() is the function to fitting correlation/covariance structure with WLS
R> wls.fit1 <- wls(S=R1, acovS=acovR, n=n, impliedS=impliedR1,
                  matrices=c(P1, L1), cor.analysis=TRUE, intervals.type="LB")
```

```
Running WLS Analysis of Correlation Structure

R> summary(wls.fit1)

Call:
wls(S = R1, acovS = acovR, n = n, impliedS = impliedR1, matrices = c(P1,
    L1), cor.analysis = TRUE, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L1[1,1] 0.421592  0.038727 0.346320 0.498692  10.886 < 2.2e-16 ***
L1[2,1] 0.523764  0.039256 0.448295 0.603091  13.342 < 2.2e-16 ***
L1[3,1] 0.570921  0.040144 0.494311 0.652919  14.222 < 2.2e-16 ***
L1[4,1] 0.421592  0.038727 0.346320 0.498692  10.886 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                     1000.0000
Chi-square of target model         0.0134
DF of target model                 2.0000
p value of target model            0.9933
Chi-square of independent model  243.9763
DF of independent model            6.0000
RMSEA                              0.0000
SRMR                               0.0012
TLI                                1.0250
CFI                                1.0000
AIC                               -3.9866
BIC                              -13.8021

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:36 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

4. Other Useful Functions

4.1. Likelihood-based Confidence Intervals

Most confidence intervals (CIs) are based on the estimated standard errors. These are known as Wald CIs. Wald CIs are symmetric around the estimates. The Wald CIs might be outside of the meaningful boundaries, for example, a negative lower limit for the variance or larger than 1 for a correlation coefficient. A preferable approach is to construct the CIs based on the likelihood. This is known as the likelihood based CI (Cheung 2009a; Neale and Miller 1997). Likelihood based CIs on the parameter estimates can be required by using `intervals.type='LB'` argument.

```
R> ## Random-effects meta-analysis with ML
R> summary( meta(y=yi, v=vi, data=Hox02, intervals.type="LB") )

Running Meta analysis with ML

Call:
meta(y = yi, v = vi, data = Hox02, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
           Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
Intercept1 0.579035  0.105100 0.364949 0.800770  5.5093 3.602e-08 ***
Tau2_1_1   0.131520  0.073536 0.032359 0.365417  1.7885   0.07369 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 20
Number of parameter estimated: 2
Degrees of freedom: 18
-2 log likelihood: 27.79916

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:37 2011
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Mixed-effects meta-analysis with "weeks" as a predictor
R> summary( meta(y=yi, v=vi, x=weeks, data=Hox02, intervals.type="LB") )

Running Meta analysis with ML

Call:
meta(y = yi, v = vi, x = weeks, data = Hox02, intervals.type = "LB")
```

```
95% confidence intervals: Likelihood-based statistic
Coefficients:
              Estimate   Std.Error      lbound      ubound z value
Slope1_1    1.3866e-01  3.2089e-02  7.4635e-02  2.0695e-01  4.3211
Intercept1 -2.1356e-01  1.9284e-01 -6.1977e-01  1.8104e-01 -1.1075
Tau2_1_1    2.3252e-02  3.5481e-02  9.8467e-11  1.3790e-01  0.6553
            Pr(>|z|)
Slope1_1   1.553e-05 ***
Intercept1    0.2681
Tau2_1_1      0.5123
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 40
Number of parameter estimated: 5
Degrees of freedom: 35
-2 log likelihood: 104.9018

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:37 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

4.2. Restricted Maximum Likelihood (REML) Estimation Method

If the unbiasness of the variance component plays a crutial role in addressing the research questions, it is possible to obtain the parameter estimates based on the REML (Harville 1977; Patterson and Thompson 1971).

It should be noted that the `reml()` function does not estimate the fixed-effects. The fixed-effects estimates can be obtained via the `meta()` function by specifying the variance component in the `RE.constraints` argument. This approach is consistent with the idea of REML that removes the fixed-effects parameter when estimating the variance component.

```
R> ## Random-effects meta-analysis with ML
R> summary( meta(y=yi, v=vi, data=Hox02) )

Running Meta analysis with ML

Call:
```

```
meta(y = yi, v = vi, data = Hox02)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value  Pr(>|z|)
Intercept1  0.579035  0.105100  0.373042  0.785028  5.5093 3.602e-08
Tau2_1_1    0.131520  0.073536 -0.012608  0.275648  1.7885   0.07369

Intercept1 ***
Tau2_1_1   .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 20
Number of parameter estimated: 2
Degrees of freedom: 18
-2 log likelihood: 27.79916

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:37 2011
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Random-effects meta-analysis with REML
R> summary( VarComp <- reml(y=yi, v=vi, data=Hox02) )

Running Variance component with REML

Call:
reml(y = yi, v = vi, data = Hox02)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.144609  0.079766 -0.011729  0.300947  1.8129  0.06984 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Number of studies: 20
Number of observed statistics: 19
```

```
Number of parameter estimated: 1
Degrees of freedom: 18
-2 log likelihood: -4.477744

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:38 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Extract the variance component
R> VarComp_REML <- matrix( coef(VarComp), ncol=1, nrow=1 )
R> ## Meta-analysis by treating the variance component as fixed
R> summary( meta(y=yi, v=vi, data=Hox02, RE.constraints=VarComp_REML) )

Running Meta analysis with ML

Call:
meta(y = yi, v = vi, data = Hox02, RE.constraints = VarComp_REML)

95% confidence intervals: z statistic approximation
Coefficients:
           Estimate Std.Error  lbound  ubound z value  Pr(>|z|)
Intercept1  0.58015   0.10800 0.36847 0.79182  5.3716 7.802e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 20
Number of parameter estimated: 1
Degrees of freedom: 19
-2 log likelihood: 27.82858

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:38 2011
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( reml(y=yi, v=vi, x=weeks, data=Hox02) )
```

```
Running Variance component with REML

Call:
reml(y = yi, v = vi, x = weeks, data = Hox02)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.036582  0.042208 -0.046143  0.119308  0.8667   0.3861

Number of studies: 20
Number of observed statistics: 18
Number of parameter estimated: 1
Degrees of freedom: 17
-2 log likelihood: -10.86705

R version: 2.14.0
OpenMx version: 1.1.2-1818
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:38 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98) )

Running Variance component with REML

Call:
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
    data = Berkey98)

95% confidence intervals: z statistic approximation
Coefficients:
          Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1  0.011733  0.013645 -0.015011  0.038477  0.8599   0.3899
Tau2_2_1  0.011916  0.014416 -0.016340  0.040172  0.8266   0.4085
Tau2_2_2  0.032651  0.024402 -0.015176  0.080479  1.3380   0.1809

Number of studies: 5
Number of observed statistics: 8
Number of parameter estimated: 3
Degrees of freedom: 5
-2 log likelihood: -18.86768

R version: 2.14.0
OpenMx version: 1.1.2-1818
```

```
metaSEM version: 0.7-0
Date of analysis: Fri Nov  4 13:47:38 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

4.3. Reading External Data Files

Data sets are most likely stored externally. metaSEM reads three types of data formats. The first type is full correlation/covariance matrices, for example, fullmat.dat is the same as the built-in data set Cheung09. Missing values are represented by NA (the default option). Suppose you have saved it at d:\fullmat.dat, you may read it by using the following command in R:

```
my.df <- readFullMat(file="d:/fullmat.dat")
```

The second type is lower triangle correlation/covariance matrices, for example, lowertriangle.dat. Missing values are represented by the strings NA. Suppose you have saved it at d:\lowertriangle.dat, you may read it by using the following command in R:

```
my.df <- readLowTriMat(file = "d:/lowertriangle.dat", no.var = 9, na.strings="NA")
```

The third type is vectors of correlation/covariance elements based on column vectorization. One row represents one study, for example, stackvec.dat. Suppose you have saved it at d:\stackvec.dat, you may read it by using the following R command:

```
my.df <- readStackVec(file="d:/stackvec.dat")
```

5. Installation

First of all, you need R to run it. Since metaSEM uses OpenMx as the workhorse, OpenMx has to be installed first. To install OpenMx, run the following command inside an R session:

```
source('http://openmx.psyc.virginia.edu/getOpenMx.R')
```

See http://openmx.psyc.virginia.edu/installing-openmx for the details on how to install OpenMx. Moreover, metaSEM also depends on the ellipse package that can be installed by the following command inside an R session:

```
install.packages('ellipse')
```

5.1. Windows platform

Download the Windows binary of metaSEM. If the file is saved at d:\. Run the following command inside an R session:

```
install.packages(pkgs="d:/metaSEM_0.7-0.zip", repos=NULL)
```

Please note that d:\ in Windows is represented by either d:/ or d:\\ in R.

5.2. Linux platform

Download the source package of `metaSEM`. Run the following command as Root inside a terminal:

```
R CMD INSTALL metaSEM_0.7-0.tar.gz
```

5.3. Mac OS X platform

The current version does not contain binaries for Mac OS X. Mac OS X users may need to build from the source.

6. Acknowledgements

This package cannot be written without `R` and `OpenMx`. Contributions by the R Development Core Team and the OpenMx Core Development Team are highly appreciated.

References

Becker BJ (1983). "Influence again: A comparison of methods for meta-analysis." In *Paper presented at the annual meeting of the American Educational Research Association*. Montreal.

Berkey CS, Hoaglin DC, Antczak-Bouckoms A, Mosteller F, Colditz GA (1998). "Meta-analysis of multiple outcomes by regression with random effects." *Statistics in Medicine*, **17**(22), 2537–2550. `doi:10.1002/(SICI)1097-0258(19981130)17:22<2537::AID-SIM953>3.0.CO;2-C`.

Boker S, Neale M, Maes H, Wilde M, Spiegel M, Brick T, Spies J, Estabrook R, Kenny S, Bates T, Mehta P, Fox J (2011). "OpenMx: An Open Source Extended Structural Equation Modeling Framework." *Psychometrika*, pp. 1–12. URL `http://dx.doi.org/10.1007/s11336-010-9200-6`.

Cheung MWL (2008). "A model for integrating fixed-, random-, and mixed-effects meta-analyses into structural equation modeling." *Psychological Methods*, **13**(3), 182–202. `doi:10.1037/a0013163`.

Cheung MWL (2009a). "Constructing approximate confidence intervals for parameters with structural equation models." *Structural Equation Modeling: A Multidisciplinary Journal*, **16**(2), 267–294. `doi:10.1080/10705510902751291`.

Cheung MWL (2009b). "TSSEM: A LISREL syntax generator for two-stage structural equation modeling (Version 1.11)." URL `http://courses.nus.edu.sg/course/psycwlm/internet/tssem.zip`.

Cheung MWL (2011). "Multivariate meta-analysis as structural equation models." Manuscript submitted for publication.

Cheung MWL, Chan W (2005a). "Classifying correlation matrices into relatively homogeneous subgroups: a cluster analytic approach." *Educational and Psychological Measurement*, **65**(6), 954–979. doi:10.1177/0013164404273946.

Cheung MWL, Chan W (2005b). "Meta-analytic structural equation modeling: a two-stage approach." *Psychological Methods*, **10**(1), 40–64. doi:10.1037/1082-989X.10.1.40.

Cheung MWL, Chan W (2009). "A two-stage approach to synthesizing covariance matrices in meta-analytic structural equation modeling." *Structural Equation Modeling: A Multidisciplinary Journal*, **16**(1), 28–53. doi:10.1080/10705510802561295.

Digman JM (1997). "Higher-order factors of the Big Five." *Journal of Personality and Social Psychology*, **73**(6), 1246–1256. doi:10.1037/0022-3514.73.6.1246.

Harville DA (1977). "Maximum Likelihood Approaches to Variance Component Estimation and to Related Problems." *Journal of the American Statistical Association*, **72**(358), 320–338. doi:10.2307/2286796.

Neale MC, Miller MB (1997). "The use of likelihood-based confidence intervals in genetic models." *Behavior Genetics*, **27**(2), 113–120. doi:10.1023/A:1025681223921.

Patterson HD, Thompson R (1971). "Recovery of inter-block information when block sizes are unequal." *Biometrika*, **58**(3), 545 –554. doi:10.1093/biomet/58.3.545.

Affiliation:

Mike W.-L. Cheung
Department of Psychology
Faculty of Arts and Social Sciences
National University of Singapore
Singapore 117570
E-mail: mikewlcheung@nus.edu.sg
URL: http://courses.nus.edu.sg/course/psycwlm/internet/metaSEM/