metaSEM: Meta-Analysis using Structural Equation Modeling

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1 Introduction

metaSEM is an R package that conducts univariate and multivariate meta-analysis using structural equation modeling (SEM) approach (Cheung, 2008) via the OpenMx package. It also implements the two-stage structural equation modeling (TSSEM) approach (Cheung and Chan, 2005, 2009) to conducting meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. The main functions in this package are:

- `tssem1()`: It conducts the first stage analysis of TSSEM by pooling correlation/covariance matrices with a fixed-effets model.
- `tssem2()`: It conducts the second stage analysis of TSSEM by calling `wls()`.
- `wls()`: It fits a correlation/covariance structure analysis with weighted least squares.
- `meta()` and `reml()`: `meta()` conducts univariate and multivariate meta-analysis with maximum likelihood estimation method while `reml()` estimates the variance components of the random-effects with restricted (residual) maximum likelihood estimation method. Mixed-effects meta-analysis can be conducted by including study characteristics as predictors. Equality constraints on intercepts, regression coefficients and variance components can be easily imposed.

Besides reporting approximate confidence intervals (CIs) based on z statistic, it is also possible to request likelihood-based CIs on the parameter estimates (Cheung, 2009a; Neale and Miller, 1997).

The current version is 0.5-2. Please send any bugs and comments to me at <mikewlcheung (at) nus.edu.sg>.

2 Installation

First of all, you need R to run it. Since metaSEM uses OpenMx as the workhorse, OpenMx has also to be installed. To install OpenMx, run the following command inside an R session:

```
source('http://openmx.psyc.virginia.edu/getOpenMx.R')
```

See http://openmx.psyc.virginia.edu/installing-openmx for the details on how to install OpenMx. If you are using Fedora and have problems in installing OpenMx, you may refer to the following post.

2.1 Windows platform

Download the Windows binary of metaSEM. If the file is saved at d:\. Run the following command inside an R session:

```
install.packages(pkgs="d:/metaSEM_0.5-2.zip", repos=NULL)
```

Please note that d:\ in Windows is represented by either d:/ or d:\\ in R.

2.2 Linux platform

Download the source package of `metaSEM`. Run the following command as Root in a terminal:

```
R CMD INSTALL metaSEM_0.5-2.tar.gz
```

3 Examples

3.1 Two-stage SEM

An example on two-stage structural equation modeling (TSSEM) from Cheung (2009b):

```
> library(metaSEM)
> ## Sample correlation matrices with missing values
> Cheung09$data

$`1`
        x1      x2      x3      x4      x5      x6      x7       x8       x9
x1 0.77298 0.26975 0.24009 0.23778 0.20869 0.22377 0.18801  0.07055  0.10051
x2 0.26975 0.91307 0.44374 0.26083 0.28387 0.20660 0.12764  0.22892  0.09590
x3 0.24009 0.44374 1.11292 0.29440 0.26262 0.27320 0.18548  0.20417  0.18243
x4 0.23778 0.26083 0.29440 0.80501 0.47489 0.45939 0.40998  0.09104  0.10142
x5 0.20869 0.28387 0.26262 0.47489 0.89692 0.41972 0.31541  0.33907  0.06561
x6 0.22377 0.20660 0.27320 0.45939 0.41972 1.36089 0.74274  0.18137  0.12973
x7 0.18801 0.12764 0.18548 0.40998 0.31541 0.74274 1.01075  0.13724  0.12776
x8 0.07055 0.22892 0.20417 0.09104 0.33907 0.18137 0.13724  1.81805 -0.01980
x9 0.10051 0.09590 0.18243 0.10142 0.06561 0.12973 0.12776 -0.01980  0.91252

$`2`
   x1 x2      x3      x4      x5      x6      x7      x8      x9
x1 NA NA      NA      NA      NA      NA      NA      NA      NA
x2 NA NA      NA      NA      NA      NA      NA      NA      NA
x3 NA NA 1.06293 0.27094 0.20331 0.16522 0.11922 0.25387 0.06877
x4 NA NA 0.27094 0.73625 0.27053 0.33506 0.33495 0.16124 0.00912
x5 NA NA 0.20331 0.27053 0.71718 0.19873 0.14582 0.21907 0.04089
x6 NA NA 0.16522 0.33506 0.19873 0.92247 0.55128 0.17143 0.01380
x7 NA NA 0.11922 0.33495 0.14582 0.55128 1.00462 0.16561 0.04322
x8 NA NA 0.25387 0.16124 0.21907 0.17143 0.16561 1.49431 0.29094
x9 NA NA 0.06877 0.00912 0.04089 0.01380 0.04322 0.29094 1.01960

$`3`
        x1      x2 x3 x4 x5      x6      x7      x8      x9
x1 0.95825 0.32958 NA NA NA 0.13948 0.15463 0.15248 0.10405
x2 0.32958 1.02277 NA NA NA 0.07300 0.07002 0.17056 0.13502
x3      NA      NA NA NA NA      NA      NA      NA      NA
x4      NA      NA NA NA NA      NA      NA      NA      NA
x5      NA      NA NA NA NA      NA      NA      NA      NA
x6 0.13948 0.07300 NA NA NA 0.82987 0.43769 0.23195 0.03856
x7 0.15463 0.07002 NA NA NA 0.43769 0.83476 0.19002 0.03986
x8 0.15248 0.17056 NA NA NA 0.23195 0.19002 1.42583 0.38343
x9 0.10405 0.13502 NA NA NA 0.03856 0.03986 0.38343 1.03062

$`4`
```

```
        x1       x2      x3      x4      x5      x6       x7 x8 x9
x1 0.83995  0.21117 0.14249 0.13268 0.17861 0.22783  0.18991 NA NA
x2 0.21117  0.93380 0.34383 0.19040 0.15068 0.12191 -0.04762 NA NA
x3 0.14249  0.34383 1.33025 0.31041 0.10873 0.19756  0.12113 NA NA
x4 0.13268  0.19040 0.31041 0.77512 0.36093 0.36519  0.22716 NA NA
x5 0.17861  0.15068 0.10873 0.36093 0.91598 0.37035  0.19550 NA NA
x6 0.22783  0.12191 0.19756 0.36519 0.37035 1.48445  0.62637 NA NA
x7 0.18991 -0.04762 0.12113 0.22716 0.19550 0.62637  1.05049 NA NA
x8      NA       NA      NA      NA      NA      NA       NA NA NA
x9      NA       NA      NA      NA      NA      NA       NA NA NA
```

```
> ## Sample sizes
> Cheung09$n

[1] 591 656 832 823

> ## Stage 1: Analysis of correlation matrices
> ## A pooled correlation matrix will be estimated.
> ## tssem1() is the function for stage 1 analysis.
> cor1 <- tssem1(Cheung09$data, Cheung09$n)

Running TSSEM1 Analysis of Correlation Matrix

> summary(cor1)

Call:
tssem1(my.df = Cheung09$data, n = Cheung09$n)

Coefficients:
        Estimate Std.Error z value  Pr(>|z|)
S1[1,2] 0.295204  0.019261 15.3267 < 2.2e-16 ***
S1[1,3] 0.195314  0.024753  7.8906 3.109e-15 ***
S1[1,4] 0.218765  0.024123  9.0687 < 2.2e-16 ***
S1[1,5] 0.219892  0.024410  9.0082 < 2.2e-16 ***
S1[1,6] 0.188085  0.020244  9.2910 < 2.2e-16 ***
S1[1,7] 0.192202  0.020246  9.4935 < 2.2e-16 ***
S1[1,8] 0.098346  0.025657  3.8331 0.0001265 ***
S1[1,9] 0.098793  0.025907  3.8134 0.0001371 ***
S1[2,3] 0.369539  0.022701 16.2786 < 2.2e-16 ***
S1[2,4] 0.254045  0.023529 10.7971 < 2.2e-16 ***
S1[2,5] 0.228516  0.024102  9.4813 < 2.2e-16 ***
S1[2,6] 0.114146  0.020524  5.5617 2.672e-08 ***
S1[2,7] 0.045226  0.020796  2.1747 0.0296520 *
S1[2,8] 0.148063  0.025249  5.8640 4.517e-09 ***
S1[2,9] 0.101851  0.025717  3.9605 7.479e-05 ***
S1[3,4] 0.307685  0.019829 15.5172 < 2.2e-16 ***
S1[3,5] 0.190536  0.021200  8.9877 < 2.2e-16 ***
S1[3,6] 0.168642  0.020895  8.0710 6.661e-16 ***
S1[3,7] 0.131197  0.021077  6.2246 4.827e-10 ***
S1[3,8] 0.165582  0.027014  6.1294 8.819e-10 ***
S1[3,9] 0.125626  0.027583  4.5545 5.251e-06 ***
S1[4,5] 0.451817  0.017509 25.8050 < 2.2e-16 ***
S1[4,6] 0.385261  0.018402 20.9362 < 2.2e-16 ***
```

```
S1[4,7] 0.356265  0.018830 18.9201 < 2.2e-16 ***
S1[4,8] 0.118677  0.026683  4.4477 8.681e-06 ***
S1[4,9] 0.056840  0.027195  2.0901 0.0366073 *
S1[5,6] 0.312551  0.019537 15.9978 < 2.2e-16 ***
S1[5,7] 0.231882  0.020370 11.3833 < 2.2e-16 ***
S1[5,8] 0.238514  0.026160  9.1175 < 2.2e-16 ***
S1[5,9] 0.064726  0.027417  2.3608 0.0182349 *
S1[6,7] 0.553723  0.012912 42.8834 < 2.2e-16 ***
S1[6,8] 0.162009  0.021035  7.7018 1.332e-14 ***
S1[6,9] 0.053050  0.021659  2.4493 0.0143139 *
S1[7,8] 0.135426  0.021212  6.3844 1.720e-10 ***
S1[7,9] 0.066476  0.021651  3.0704 0.0021377 **
S1[8,9] 0.198066  0.021254  9.3191 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      2902.0000
Chi-square of target model        172.7320
DF of target model                 57.0000
p value of target model             0.0000
Chi-square of independent model 3246.6915
DF of independent model           93.0000
RMSEA                              0.0529
SRMR                               0.0549
TLI                                0.9401
CFI                                0.9633
AIC                               58.7320
BIC                             -281.7379

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:20 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Stage 2: Fit a three-factor CFA model on the pooled correlation matrix
> ## See http://openmx.psyc.virginia.edu/documentation on the OpenMx syntax
> P4 <- mxMatrix("Stand", ncol=3, nrow=3, value=.2, free=TRUE, name="P4")
> L4 <- mxMatrix("Full", ncol=3, nrow=9, value=c( rep(c(0.3,0,0),3),
+                 rep(c(0, 0.3,0),4), rep(c(0,0,0.3),2)),
+                 free=c( rep(c(T,F,F),3), rep(c(F,T,F),4),
+                 rep(c(F,F,T),2)), byrow=TRUE, name="L4")
> ## impliedR = L4 %*% P4 %*% t(L4)
> impliedR4 <- mxAlgebra(L4 %&% P4, name="impliedR4")
> ## tssem2() is the function for stage 2 analysis.
> cor2 <- tssem2(cor1, impliedS=impliedR4, matrices=c(P4, L4))

Running Correlation structure

> summary(cor2)
```

```
Call:
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, suppressWarnings = suppressWarnings)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L4[1,1] 0.517061  0.023568 0.470869 0.563254 21.9389 < 2.2e-16 ***
L4[2,1] 0.575072  0.023120 0.529758 0.620387 24.8734 < 2.2e-16 ***
L4[3,1] 0.593858  0.025586 0.543711 0.644004 23.2106 < 2.2e-16 ***
L4[4,2] 0.705138  0.014698 0.676330 0.733946 47.9750 < 2.2e-16 ***
L4[5,2] 0.579019  0.016902 0.545892 0.612146 34.2580 < 2.2e-16 ***
L4[6,2] 0.746403  0.013060 0.720806 0.772000 57.1515 < 2.2e-16 ***
L4[7,2] 0.692214  0.013622 0.665515 0.718912 50.8157 < 2.2e-16 ***
L4[8,3] 0.621998  0.052462 0.519175 0.724822 11.8561 < 2.2e-16 ***
L4[9,3] 0.332553  0.032670 0.268521 0.396584 10.1792 < 2.2e-16 ***
P4[1,2] 0.544769  0.025617 0.494560 0.594977 21.2659 < 2.2e-16 ***
P4[1,3] 0.488471  0.055862 0.378984 0.597958  8.7443 < 2.2e-16 ***
P4[2,3] 0.392611  0.040345 0.313536 0.471685  9.7314 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      2902.0000
Chi-square of target model        384.9041
DF of target model                 24.0000
p value of target model             0.0000
Chi-square of independent model 4636.5843
DF of independent model           36.0000
RMSEA                              0.0720
SRMR                               0.0729
TLI                                0.8823
CFI                                0.9216
AIC                              336.9041
BIC                              193.5484

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:21 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.2 Reading External Data Files

Data sets are most likely stored externally. `metaSEM` reads three types of formats. The first type is full correlation/covariance matrices, for example, fullmat.dat is the same as the built-in data set `Cheung09`. Missing values are represented by `NA` (the default option). Suppose you save it at d:\fullmat.dat, you may read it by using the following command in `R`:

```
my.df <- readFullMat(file="d:/fullmat.dat")
```

The second type is lower triangle correlation/covariance matrices, for example, lowertriangle.dat. Missing values are represented by the strings 1.00000 and 0.00000. Suppose you save it at d:\lowertriangle.dat, you may read it by using the following command in R:

```
my.df <- readLowTriMat(file = "d:/lowertriangle.dat", no.var = 9,
                       na.strings=c("1.00000", "0.00000"))
```

The third type is vectors of correlation/covariance elements based on column vectorization. One row is for one study, for example, stackvec.dat. Suppose you save it at d:\stackvec.dat, you may read it by using the following R command:

```
my2 <- readStackVec(file="d:/stackvec.dat")
```

3.3 Analysis of Correlation/Covariance Structure with Weighted Least Squares

Besides fitting a TSSEM, `wls()` may be used to fit a correlation/covariance structure with weighted least squares as the estimation method. Likelihood-based CIs may also be calculated. The following is an example.

```
> R1 <- matrix(c(1.00, 0.22, 0.24, 0.18,
+                0.22, 1.00, 0.30, 0.22,
+                0.24, 0.30, 1.00, 0.24,
+                0.18, 0.22, 0.24, 1.00), ncol=4, nrow=4)
> ## Sample size
> n <- 1000
> ## Calculate the asymptotic covariance matrix of the sample correlation matrix
> acovR <- asyCov(R1, n)
> ## P1: Factor variance
> P1 <- mxMatrix("Full", ncol=1, nrow=1, value=1, free=FALSE, name="P1")
> ## L1: Factor loadings
> L1 <- mxMatrix("Full", ncol=1, nrow=4, value=c(0.3, 0.4, 0.5, 0.4),
+                free=TRUE, name="L1")
> ## Model implied correlation matrix
> ## Please note that error variances are not involved in correlation structure analysis
> impliedR1 <- mxAlgebra(L1 %&% P1, name="impliedR1")
> ## wls() is the function to fitting correlation/covariance structure with WLS
> wls.fit1 <- wls(S=R1, acovS=acovR, n=n, impliedS=impliedR1,
+                 matrices=c(P1, L1), cor.analysis=TRUE, intervals.type="LB")

Running Correlation structure

> summary(wls.fit1)

Call:
wls(S = R1, acovS = acovR, n = n, impliedS = impliedR1, matrices = c(P1,
    L1), cor.analysis = TRUE, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
        Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
L1[1,1] 0.421592  0.038727 0.346320 0.498692  10.886 < 2.2e-16 ***
L1[2,1] 0.523764  0.039257 0.448295 0.603091  13.342 < 2.2e-16 ***
L1[3,1] 0.570921  0.040144 0.494310 0.652919  14.222 < 2.2e-16 ***
```

```
L1[4,1] 0.421592  0.038727 0.346326 0.498692  10.886 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Goodness-of-fit indices:
                                    Value
Sample size                      1000.0000
Chi-square of target model          0.0134
DF of target model                  2.0000
p value of target model             0.9933
Chi-square of independent model   243.9826
DF of independent model             6.0000
RMSEA                               0.0000
SRMR                                0.0012
TLI                                 1.0250
CFI                                 1.0000
AIC                                -3.9866
BIC                               -13.8021

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:22 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

3.4 Univariate and Multivariate Meta-Analysis

Another useful functions are `meta()` and `reml()`. They conduct fixed-, random-, and mixed-effects univariate and multivariate meta-analysis. The followings are some examples.

```
> attach(Hox02)
> summary( meta(y=yi, v=vi) )

Running Meta analysis
Call:
meta(y = yi, v = vi)

95% confidence intervals: z statistic approximation
Coefficients:
            Estimate Std.Error    lbound    ubound z value  Pr(>|z|)
Intercept1  0.579035  0.105100  0.373042  0.785028  5.5093 3.602e-08 ***
Tau1_1      0.131520  0.073536 -0.012608  0.275648  1.7885    0.0737 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.0001508010

Number of studies: 20
Number of observed statistics: 20
```

```
Number of parameter estimated: 2
Degrees of freedom: 18
-2 log likelihood: 27.79916

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:22 2010
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Estimate variance components with REML
> summary( reml(y=yi, v=vi) )

Running REML
Call:
reml(y = yi, v = vi)

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau1_1  0.144609  0.079766 -0.011729  0.300947  1.8129  0.06984 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Number of studies: 0
Number of observed statistics: 0
Number of parameter estimated: 1
Degrees of freedom: -1
-2 log likelihood: -4.477744

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:23 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Fixed-effects meta-analysis
> summary( meta(y=yi, v=vi, RE.constraints=matrix(0, ncol=1, nrow=1)) )

Running Meta analysis
Call:
meta(y = yi, v = vi, RE.constraints = matrix(0, ncol = 1, nrow = 1))

95% confidence intervals: z statistic approximation
Coefficients:
           Estimate Std.Error   lbound   ubound z value  Pr(>|z|)
Intercept1 0.550206  0.064998 0.422813 0.677599   8.465 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
```

```
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.0001508010

Number of studies: 20
Number of observed statistics: 20
Number of parameter estimated: 1
Degrees of freedom: 19
-2 log likelihood: 37.70073

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:23 2010
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
> ## Mixed-effects meta-analysis with "weeks" as the predictor
> ## Use likelihood-based CI
> summary( meta(y=yi, v=vi, x=weeks, intervals.type="LB") )
```

```
Running Meta analysis
Call:
meta(y = yi, v = vi, x = weeks, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
              Estimate   Std.Error      lbound      ubound z value  Pr(>|z|)
Slope1_1    1.3866e-01  3.2089e-02  7.4635e-02  2.0695e-01  4.3210 1.553e-05
Intercept1 -2.1356e-01  1.9284e-01 -6.1977e-01  1.8104e-01 -1.1075    0.2681
Tau1_1      2.3252e-02  3.5481e-02  1.0000e-10  1.3790e-01  0.6553    0.5123

Slope1_1   ***
Intercept1
Tau1_1
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.0001508010

Number of studies: 20
Number of observed statistics: 40
Number of parameter estimated: 5
Degrees of freedom: 35
-2 log likelihood: 104.9018

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:24 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

```
> ## Estimate variance components with REML
> summary( reml(y=yi, v=vi, x=weeks, intervals.type="LB") )

Running REML
Call:
reml(y = yi, v = vi, x = weeks, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
         Estimate  Std.Error     lbound     ubound z value Pr(>|z|)
Tau1_1 3.6582e-02 4.2208e-02 4.0001e-06 1.7650e-01  0.8667   0.3861

Number of studies: 0
Number of observed statistics: 0
Number of parameter estimated: 1
Degrees of freedom: -1
-2 log likelihood: -10.86705

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:26 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> detach(Hox02)
> ## Multivariate meta-analysis
> attach(Berkey98)
> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL)) )

Running Meta analysis
Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL))

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value  Pr(>|z|)
Intercept1  0.3448392  0.0536312  0.2397239  0.4499544  6.4298 1.278e-10 ***
Intercept2 -0.3379381  0.0812479 -0.4971812 -0.1786951 -4.1593 3.192e-05 ***
Tau1_1      0.0070020  0.0090497 -0.0107351  0.0247391  0.7737    0.4391
Tau2_1      0.0094607  0.0099698 -0.0100797  0.0290010  0.9489    0.3427
Tau2_2      0.0261445  0.0177409 -0.0086270  0.0609161  1.4737    0.1406
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 10
Number of parameter estimated: 5
```

```
Degrees of freedom: 5
-2 log likelihood: -11.68131

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:26 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Estimate variance components with REML
> summary( reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL)) )

Running REML
Call:
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL))

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau1_1  0.011733  0.013645 -0.015011  0.038477  0.8599   0.3899
Tau2_1  0.011916  0.014416 -0.016340  0.040172  0.8266   0.4085
Tau2_2  0.032651  0.024402 -0.015176  0.080479  1.3380   0.1809

Number of studies: 0
Number of observed statistics: 0
Number of parameter estimated: 3
Degrees of freedom: -3
-2 log likelihood: -18.86768

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:27 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Multivariate meta-analysis with "publication year-1979" as the predictor
> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
+               x=scale(pub_year, center=1979)) )

Running Meta analysis
Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),      x = scale(pub_year, center = 1979))

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value  Pr(>|z|)
Slope1_1    0.0063540  0.1078235 -0.2049761  0.2176842  0.0589   0.95301
Slope2_1   -0.0705888  0.1620965 -0.3882921  0.2471146 -0.4355   0.66322
Intercept1  0.3440001  0.0857659  0.1759021  0.5120982  4.0109 6.048e-05 ***
Intercept2 -0.2918175  0.1312796 -0.5491208 -0.0345141 -2.2229   0.02622 *
Tau1_1      0.0080405  0.0101206 -0.0117955  0.0278766  0.7945   0.42692
```

```
Tau2_1      0.0093413  0.0105515 -0.0113392  0.0300218  0.8853   0.37599
Tau2_2      0.0250135  0.0170788 -0.0084603  0.0584873  1.4646   0.14303
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 15
Number of parameter estimated: 9
Degrees of freedom: 6
-2 log likelihood: -4.595466

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:27 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Estimate variance components with REML
> summary( reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
+               x=scale(pub_year, center=1979)) )

Running REML
Call:
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),      x = scale(pub_year, center = 1979))

95% confidence intervals: z statistic approximation
Coefficients:
        Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau1_1  0.020447  0.022523 -0.023697  0.064590  0.9078   0.3640
Tau2_1  0.016226  0.022696 -0.028258  0.060710  0.7149   0.4747
Tau2_2  0.040857  0.034707 -0.027168  0.108882  1.1772   0.2391

Number of studies: 0
Number of observed statistics: 0
Number of parameter estimated: 3
Degrees of freedom: -3
-2 log likelihood: -11.44636

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:28 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> ## Multivariate meta-analysis with an equality constraint on the slopes
> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
```

```
+               x=scale(pub_year, center=1979),
+               coeff.constraints=
+               matrix(c("0.3*Eq_slope", "0.3*Eq_slope"), nrow=2)) )

Running Meta analysis
Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),      x = scale(pub_year, center = 1979), c

95% confidence intervals: z statistic approximation
Coefficients:
             Estimate  Std.Error     lbound     ubound z value  Pr(>|z|)
Eq_slope    0.0016748  0.1024443 -0.1991123  0.2024619  0.0163  0.986956
Intercept1  0.3437612  0.0849828  0.1771979  0.5103245  4.0451 5.231e-05 ***
Intercept2 -0.3390010  0.1041005 -0.5430344 -0.1349677 -3.2565  0.001128 **
Tau1_1      0.0070474  0.0094638 -0.0115013  0.0255962  0.7447  0.456471
Tau2_1      0.0095165  0.0105668 -0.0111940  0.0302269  0.9006  0.367800
Tau2_2      0.0261979  0.0180773 -0.0092330  0.0616288  1.4492  0.147278
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 15
Number of parameter estimated: 8
Degrees of freedom: 7
-2 log likelihood: -4.268456

R version: 2.12.0
OpenMx version: 1.0.3-1505
metaSEM version: 0.5-2
Date of analysis: Sat Dec 11 22:34:28 2010
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

> detach(Berkey98)
```

4 Acknowledgements

This package cannot be written without `R` and `OpenMx`. Contributions by the R Development Core Team and the OpenMx Core Development Team are highly appreciated.

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