

# metaSEM: Meta-Analysis using Structural Equation Modeling

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May 22, 2011

## 1 Introduction

**metaSEM** is an R package that conducts univariate and multivariate meta-analysis using a structural equation modeling (SEM) approach (Cheung, 2008, 2011) via the **OpenMx** package (Boker et al., 2011). It also implements the two-stage structural equation modeling (TSSEM) approach (Cheung and Chan, 2005, 2009) to conducting a meta-analytic structural equation modeling (MASEM) on correlation/covariance matrices. Main functions in this package are:

- **tssem1()**: It conducts the first stage analysis of TSSEM by pooling correlation/covariance matrices with a fixed-effects model.
- **tssem2()**: It conducts the second stage analysis of TSSEM by calling **wls()**.
- **wls()**: It fits a correlation/covariance structure analysis with weighted least squares.
- **meta()** and **reml()**: **meta()** conducts univariate and multivariate meta-analysis with maximum likelihood estimation method while **reml()** estimates the variance components of the random-effects with restricted (residual) maximum likelihood estimation method. Mixed-effects meta-analysis can be conducted by including study characteristics as predictors. Equality constraints on intercepts, regression coefficients and variance components can be imposed.

Besides using Wald confidence intervals (CIs) based on z statistic, likelihood-based CIs on the parameter estimates may also be requested (Cheung, 2009a; Neale and Miller, 1997). Several generic functions such as **anova()**, **coef()**, **vcov()**, **print()**, **summary()** and **plot()**, have also been implemented. The current version is 0.6-0. Please send any bugs and comments to me at <mikewlcheung (at) nus.edu.sg>.

## 2 Installation

First of all, you need R to run it. Since **metaSEM** uses **OpenMx** as the workhorse, **OpenMx** has to be installed first. To install **OpenMx**, run the following command inside an R session:

```
source('http://openmx.psyc.virginia.edu/getOpenMx.R')
```

See <http://openmx.psyc.virginia.edu/installing-openmx> for the details on how to install **OpenMx**. Moreover, **metaSEM** also depends on the **ellipse** package that can be installed by the following command inside an R session:

```
install.packages('ellipse')
```

## 2.1 Windows platform

Download the [Windows binary](#) of metaSEM. If the file is saved at d:\. Run the following command inside an R session:

```
install.packages(pkgs="d:/metaSEM_0.6-0.zip", repos=NULL)
```

Please note that d:\ in Windows is represented by either d:/ or d:\\ in R.

## 2.2 Linux platform

Download the [source package](#) of metaSEM. Run the following command as Root inside a terminal:

```
R CMD INSTALL metaSEM_0.6-0.tar.gz
```

## 2.3 Mac OS X platform

The current version does not contain binaries for Mac OS X. Mac OS X users may need to build from the [source](#).

# 3 Examples

## 3.1 Two-stage SEM

An example from [Cheung \(2009b\)](#) was used to illustrate the two-stage structural equation modeling (TSSEM) procedure. `tssem1()` is used to pool the correlation matrices with a fixed-effects model in the first stage. `tssem2()` is then used to fit a structural model on the pooled correlation matrix with the inverse of its asymptotic covariance matrix as the weight matrix (see [Cheung and Chan \(2005, 2009\)](#) for the details).

```
R> ## Load the metaSEM library
```

```
R> library(metaSEM)
```

```
R> ## Sample correlation matrices with missing values
```

```
R> Cheung09$data
```

```
$`1`
```

	x1	x2	x3	x4	x5	x6	x7	x8
x1	0.77298	0.26975	0.24009	0.23778	0.20869	0.22377	0.18801	0.07055
x2	0.26975	0.91307	0.44374	0.26083	0.28387	0.20660	0.12764	0.22892
x3	0.24009	0.44374	1.11292	0.29440	0.26262	0.27320	0.18548	0.20417
x4	0.23778	0.26083	0.29440	0.80501	0.47489	0.45939	0.40998	0.09104
x5	0.20869	0.28387	0.26262	0.47489	0.89692	0.41972	0.31541	0.33907
x6	0.22377	0.20660	0.27320	0.45939	0.41972	1.36089	0.74274	0.18137
x7	0.18801	0.12764	0.18548	0.40998	0.31541	0.74274	1.01075	0.13724
x8	0.07055	0.22892	0.20417	0.09104	0.33907	0.18137	0.13724	1.81805
x9	0.10051	0.09590	0.18243	0.10142	0.06561	0.12973	0.12776	-0.01980

```
      x9
```

```
x1  0.10051
```

```
x2  0.09590
```

```
x3  0.18243
```

```
x4  0.10142
```

```
x5  0.06561
```

```
x6  0.12973
```

```
x7  0.12776
```

```
x8 -0.01980
```

x9 0.91252

\$`2`

	x1	x2	x3	x4	x5	x6	x7	x8	x9
x1	NA	NA	NA	NA	NA	NA	NA	NA	NA
x2	NA	NA	NA	NA	NA	NA	NA	NA	NA
x3	NA	NA	1.06293	0.27094	0.20331	0.16522	0.11922	0.25387	0.06877
x4	NA	NA	0.27094	0.73625	0.27053	0.33506	0.33495	0.16124	0.00912
x5	NA	NA	0.20331	0.27053	0.71718	0.19873	0.14582	0.21907	0.04089
x6	NA	NA	0.16522	0.33506	0.19873	0.92247	0.55128	0.17143	0.01380
x7	NA	NA	0.11922	0.33495	0.14582	0.55128	1.00462	0.16561	0.04322
x8	NA	NA	0.25387	0.16124	0.21907	0.17143	0.16561	1.49431	0.29094
x9	NA	NA	0.06877	0.00912	0.04089	0.01380	0.04322	0.29094	1.01960

\$`3`

	x1	x2	x3	x4	x5	x6	x7	x8	x9
x1	0.95825	0.32958	NA	NA	NA	0.13948	0.15463	0.15248	0.10405
x2	0.32958	1.02277	NA	NA	NA	0.07300	0.07002	0.17056	0.13502
x3	NA	NA	NA	NA	NA	NA	NA	NA	NA
x4	NA	NA	NA	NA	NA	NA	NA	NA	NA
x5	NA	NA	NA	NA	NA	NA	NA	NA	NA
x6	0.13948	0.07300	NA	NA	NA	0.82987	0.43769	0.23195	0.03856
x7	0.15463	0.07002	NA	NA	NA	0.43769	0.83476	0.19002	0.03986
x8	0.15248	0.17056	NA	NA	NA	0.23195	0.19002	1.42583	0.38343
x9	0.10405	0.13502	NA	NA	NA	0.03856	0.03986	0.38343	1.03062

\$`4`

	x1	x2	x3	x4	x5	x6	x7	x8	x9
x1	0.83995	0.21117	0.14249	0.13268	0.17861	0.22783	0.18991	NA	NA
x2	0.21117	0.93380	0.34383	0.19040	0.15068	0.12191	-0.04762	NA	NA
x3	0.14249	0.34383	1.33025	0.31041	0.10873	0.19756	0.12113	NA	NA
x4	0.13268	0.19040	0.31041	0.77512	0.36093	0.36519	0.22716	NA	NA
x5	0.17861	0.15068	0.10873	0.36093	0.91598	0.37035	0.19550	NA	NA
x6	0.22783	0.12191	0.19756	0.36519	0.37035	1.48445	0.62637	NA	NA
x7	0.18991	-0.04762	0.12113	0.22716	0.19550	0.62637	1.05049	NA	NA
x8	NA	NA	NA	NA	NA	NA	NA	NA	NA
x9	NA	NA	NA	NA	NA	NA	NA	NA	NA

R> ## Sample sizes

R> Cheung09\$n

[1] 591 656 832 823

R> ## Stage 1: Analysis of correlation matrices

R> ## A pooled correlation matrix will be estimated.

R> ## tssem1() is the function for stage 1 analysis.

R> cor1 <- tssem1(Cheung09\$data, Cheung09\$n)

Running TSSEM1 Analysis of Correlation Matrix

R> summary(cor1)

Call:

tssem1(my.df = Cheung09\$data, n = Cheung09\$n)

Coefficients:

	Estimate	Std.Error	z value	Pr(> z )
S1[1,2]	0.295204	0.019261	15.3267	< 2.2e-16 ***
S1[1,3]	0.195314	0.024753	7.8906	3.109e-15 ***
S1[1,4]	0.218765	0.024123	9.0687	< 2.2e-16 ***
S1[1,5]	0.219892	0.024410	9.0082	< 2.2e-16 ***
S1[1,6]	0.188085	0.020244	9.2910	< 2.2e-16 ***
S1[1,7]	0.192202	0.020246	9.4935	< 2.2e-16 ***
S1[1,8]	0.098346	0.025657	3.8331	0.0001265 ***
S1[1,9]	0.098793	0.025907	3.8134	0.0001371 ***
S1[2,3]	0.369539	0.022701	16.2786	< 2.2e-16 ***
S1[2,4]	0.254045	0.023529	10.7971	< 2.2e-16 ***
S1[2,5]	0.228516	0.024102	9.4813	< 2.2e-16 ***
S1[2,6]	0.114146	0.020524	5.5617	2.672e-08 ***
S1[2,7]	0.045226	0.020796	2.1747	0.0296520 *
S1[2,8]	0.148063	0.025249	5.8640	4.517e-09 ***
S1[2,9]	0.101851	0.025717	3.9605	7.479e-05 ***
S1[3,4]	0.307685	0.019829	15.5172	< 2.2e-16 ***
S1[3,5]	0.190536	0.021200	8.9877	< 2.2e-16 ***
S1[3,6]	0.168642	0.020895	8.0710	6.661e-16 ***
S1[3,7]	0.131197	0.021077	6.2246	4.827e-10 ***
S1[3,8]	0.165582	0.027014	6.1294	8.819e-10 ***
S1[3,9]	0.125626	0.027583	4.5545	5.251e-06 ***
S1[4,5]	0.451817	0.017509	25.8050	< 2.2e-16 ***
S1[4,6]	0.385261	0.018402	20.9362	< 2.2e-16 ***
S1[4,7]	0.356265	0.018830	18.9201	< 2.2e-16 ***
S1[4,8]	0.118677	0.026683	4.4477	8.681e-06 ***
S1[4,9]	0.056840	0.027195	2.0901	0.0366073 *
S1[5,6]	0.312551	0.019537	15.9978	< 2.2e-16 ***
S1[5,7]	0.231882	0.020370	11.3833	< 2.2e-16 ***
S1[5,8]	0.238514	0.026160	9.1175	< 2.2e-16 ***
S1[5,9]	0.064726	0.027417	2.3608	0.0182349 *
S1[6,7]	0.553723	0.012912	42.8834	< 2.2e-16 ***
S1[6,8]	0.162009	0.021035	7.7018	1.332e-14 ***
S1[6,9]	0.053050	0.021659	2.4493	0.0143139 *
S1[7,8]	0.135426	0.021212	6.3844	1.720e-10 ***
S1[7,9]	0.066476	0.021651	3.0704	0.0021377 **
S1[8,9]	0.198066	0.021254	9.3191	< 2.2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Goodness-of-fit indices:

	Value
Sample size	2902.0000
Chi-square of target model	172.7320
DF of target model	57.0000
p value of target model	0.0000
Chi-square of independent model	3246.6915
DF of independent model	93.0000
RMSEA	0.0529

SRMR	0.0549
TLI	0.9401
CFI	0.9633
AIC	58.7320
BIC	-281.7379

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:19 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)

See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

R> ## Stage 2: Fit a three-factor CFA model on the pooled correlation matrix

R>

R> ## Create a matrix for factor correlations

R> ## Values with "\*" are free parameters

```
R> P4 <- matrix(c(1,"0.2*p21*", "0.2*p31",
                 "0.2*p21", 1, "0.2*p32",
                 "0.2*p31", "0.2*p32", 1), ncol=3, nrow=3, byrow=TRUE)
```

R> ## Convert it into a mxMatrix class

```
R> P4 <- as.mxMatrix(P4)
```

R> ## Create a matrix for factor loadings

```
R> L4 <- matrix(c(rep(c("0.3*", 0, 0), 3),
                 rep(c(0, "0.3*", 0), 4),
                 rep(c(0, 0, "0.3*"), 2)), ncol=3, nrow=9, byrow=TRUE)
```

R> ## Convert it into a mxMatrix class

```
R> L4 <- as.mxMatrix(L4)
```

```
R> ## impliedR = L4 %*% P4 %*% t(L4)
```

```
R> impliedR4 <- mxAlgebra(L4 %*% P4, name="impliedR4")
```

R> ## tssem2() is the function for stage 2 analysis.

```
R> cor2 <- tssem2(cor1, impliedS=impliedR4, matrices=c(P4, L4))
```

Running TSSEM2 Analysis of Correlation Structure

```
R> summary(cor2)
```

Call:

```
wls(S = tssem1.obj$pooledS, acovS = tssem1.obj$acovS, n = tssem1.obj$total.n,
    impliedS = impliedS, matrices = matrices, cor.analysis = cor.analysis,
    intervals.type = intervals.type, model.name = model.name,
    suppressWarnings = suppressWarnings)
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
L4[1,1]	0.517061	0.023568	0.470869	0.563254	21.9389	< 2.2e-16 ***
L4[2,1]	0.575072	0.023120	0.529758	0.620387	24.8734	< 2.2e-16 ***
L4[3,1]	0.593858	0.025586	0.543711	0.644004	23.2106	< 2.2e-16 ***
L4[4,2]	0.705138	0.014698	0.676330	0.733946	47.9750	< 2.2e-16 ***
L4[5,2]	0.579019	0.016902	0.545892	0.612146	34.2580	< 2.2e-16 ***
L4[6,2]	0.746403	0.013060	0.720806	0.772000	57.1515	< 2.2e-16 ***
L4[7,2]	0.692214	0.013622	0.665515	0.718912	50.8157	< 2.2e-16 ***

```

L4[8,3] 0.621998 0.052462 0.519175 0.724822 11.8561 < 2.2e-16 ***
L4[9,3] 0.332553 0.032670 0.268521 0.396584 10.1792 < 2.2e-16 ***
P4[2,1] 0.544769 0.025617 0.494560 0.594977 21.2659 < 2.2e-16 ***
P4[3,1] 0.488471 0.055862 0.378984 0.597958 8.7443 < 2.2e-16 ***
P4[3,2] 0.392611 0.040345 0.313536 0.471685 9.7314 < 2.2e-16 ***

```

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Goodness-of-fit indices:

	Value
Sample size	2902.0000
Chi-square of target model	384.9041
DF of target model	24.0000
p value of target model	0.0000
Chi-square of independent model	4636.5843
DF of independent model	36.0000
RMSEA	0.0720
SRMR	0.2953
TLI	0.8823
CFI	0.9216
AIC	336.9041
BIC	193.5484

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:20 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)

See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

## 3.2 Reading External Data Files

Data sets are most likely stored externally. `metaSEM` reads three types of data formats. The first type is full correlation/covariance matrices, for example, [fullmat.dat](#) is the same as the built-in data set `Cheung09`. Missing values are represented by `NA` (the default option). Suppose you have saved it at `d:\fullmat.dat`, you may read it by using the following command in R:

```
my.df <- readFullMat(file="d:/fullmat.dat")
```

The second type is lower triangle correlation/covariance matrices, for example, [lowertriangle.dat](#). Missing values are represented by the strings `NA`. Suppose you have saved it at `d:\lowertriangle.dat`, you may read it by using the following command in R:

```
my.df <- readLowTriMat(file = "d:/lowertriangle.dat", no.var = 9, na.strings="NA")
```

The third type is vectors of correlation/covariance elements based on column vectorization. One row represents one study, for example, [stackvec.dat](#). Suppose you have saved it at `d:\stackvec.dat`, you may read it by using the following R command:

```
my.df <- readStackVec(file="d:/stackvec.dat")
```

### 3.3 Analysis of Correlation/Covariance Structure with Weighted Least Squares

Besides fitting a TSSEM, `wls()` may be used to fit a correlation/covariance structure with weighted least squares (WLS) as the estimation method. Likelihood-based CIs may also be calculated. The following example fits a one-factor CFA model on the correlation matrix with WLS estimation method.

```
R> ## Sample correlation matrix
R> R1 <- matrix(c(1.00, 0.22, 0.24, 0.18,
                  0.22, 1.00, 0.30, 0.22,
                  0.24, 0.30, 1.00, 0.24,
                  0.18, 0.22, 0.24, 1.00), ncol=4, nrow=4)
R> ## Sample size
R> n <- 1000
R> ## Estimate the asymptotic covariance matrix of the sample correlation matrix
R> acovR <- asyCov(R1, n)
R> ## P1: Factor variance is fixed at 1.0
R> P1 <- as.mxMatrix( matrix(1), name="P1")
R> ## L1: Factor loadings
R> L1 <- as.mxMatrix( matrix( rep("0.3*", 4), nrow=4, ncol=1), name="L1" )
R> ## Model implied correlation matrix
R> ## Please note that error variances are not involved in correlation structure analysis
R> impliedR1 <- mxAlgebra(L1 %&% P1, name="impliedR1")
R> ## wls() is the function to fitting correlation/covariance structure with WLS
R> wls.fit1 <- wls(S=R1, acovS=acovR, n=n, impliedS=impliedR1,
                  matrices=c(P1, L1), cor.analysis=TRUE, intervals.type="LB")
```

Running WLS Analysis of Correlation Structure

```
R> summary(wls.fit1)
```

Call:

```
wls(S = R1, acovS = acovR, n = n, impliedS = impliedR1, matrices = c(P1,
    L1), cor.analysis = TRUE, intervals.type = "LB")
```

95% confidence intervals: Likelihood-based statistic

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
L1[1,1]	0.421592	0.038727	0.346320	0.498692	10.886	< 2.2e-16 ***
L1[2,1]	0.523764	0.039257	0.448295	0.603091	13.342	< 2.2e-16 ***
L1[3,1]	0.570921	0.040144	0.494310	0.652919	14.222	< 2.2e-16 ***
L1[4,1]	0.421592	0.038727	0.346320	0.498692	10.886	< 2.2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Goodness-of-fit indices:

	Value
Sample size	1000.0000
Chi-square of target model	0.0134
DF of target model	2.0000
p value of target model	0.9933
Chi-square of independent model	243.9789
DF of independent model	6.0000
RMSEA	0.0000

```
SRMR                0.4830
TLI                 1.0250
CFI                 1.0000
AIC                 -3.9866
BIC                 -13.8021
```

```
R version: 2.13.0
OpenMx version: 999.0.0-1661
metaSEM version: 0.6-0
Date of analysis: Sun May 22 16:19:20 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.
```

### 3.4 Univariate and Multivariate Meta-Analysis

Other useful functions are `meta()` and `reml()`. They conduct fixed-, random-, and mixed-effects meta-analysis. Please note that there is no estimate on the fixed-effects when `reml()` is used. The followings are some examples.

```
R> ## Random-effects meta-analysis with ML
R> summary( meta(y=yi, v=vi, data=Hox02) )
```

Running Meta analysis with ML

```
Call:
meta(y = yi, v = vi, data = Hox02)
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Intercept1	0.579035	0.105100	0.373042	0.785028	5.5093	3.602e-08
Tau2_1_1	0.131520	0.073536	-0.012608	0.275648	1.7885	0.07369

```
Intercept1 ***
```

```
Tau2_1_1 .
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Q statistic on homogeneity of effect sizes: 49.5852

Degrees of freedom of the Q statistic: 19

P value of the Q statistic: 0.000150801

Number of studies: 20

Number of observed statistics: 20

Number of parameter estimated: 2

Degrees of freedom: 18

-2 log likelihood: 27.79916

```
R version: 2.13.0
OpenMx version: 999.0.0-1661
metaSEM version: 0.6-0
Date of analysis: Sun May 22 16:19:20 2011
```



OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)  
See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

```
R> ## Random-effects meta-analysis with REML
R> summary( VarComp <- reml(y=yi, v=vi, data=Hox02) )
```

Running Variance component with REML

Call:  
reml(y = yi, v = vi, data = Hox02)

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Tau2_1_1	0.144609	0.079766	-0.011729	0.300947	1.8129	0.06984 .

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Number of studies: 20  
Number of observed statistics: 19  
Number of parameter estimated: 1  
Degrees of freedom: 18  
-2 log likelihood: -4.477744

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:21 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)  
See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

```
R> ## Extract the variance component
R> VarComp_REML <- matrix( coef(VarComp), ncol=1, nrow=1 )
R> ## Meta-analysis by treating the variance component as fixed
R> summary( meta(y=yi, v=vi, data=Hox02, RE.constraints=VarComp_REML) )
```

Running Meta analysis with ML

Call:  
meta(y = yi, v = vi, data = Hox02, RE.constraints = VarComp\_REML)

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Intercept1	0.58015	0.10800	0.36847	0.79182	5.3716	7.802e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 49.5852  
Degrees of freedom of the Q statistic: 19  
P value of the Q statistic: 0.000150801

Number of studies: 20

Number of observed statistics: 20  
Number of parameter estimated: 1  
Degrees of freedom: 19  
-2 log likelihood: 27.82858

R version: 2.13.0  
OpenMx version: 999.0.0-1661  
metaSEM version: 0.6-0  
Date of analysis: Sun May 22 16:19:22 2011  
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)  
See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

```
R> ## Fixed-effects meta-analysis
R> summary( meta(y=yi, v=vi, data=Hox02, RE.constraints=matrix(0, ncol=1, nrow=1),
              model.name="Fixed effects model") )
```

Running Fixed effects model

Call:

```
meta(y = yi, v = vi, data = Hox02, RE.constraints = matrix(0,
  ncol = 1, nrow = 1), model.name = "Fixed effects model")
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Intercept1	0.550206	0.064998	0.422813	0.677599	8.465	< 2.2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 49.5852

Degrees of freedom of the Q statistic: 19

P value of the Q statistic: 0.000150801

Number of studies: 20  
Number of observed statistics: 20  
Number of parameter estimated: 1  
Degrees of freedom: 19  
-2 log likelihood: 37.70073

R version: 2.13.0  
OpenMx version: 999.0.0-1661  
metaSEM version: 0.6-0  
Date of analysis: Sun May 22 16:19:22 2011  
OpenMx status1: 1 ("0" and "1": considered fine; other values indicate problems)  
See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

```
R> ## Mixed-effects meta-analysis with "weeks" as a predictor
R> ## Request likelihood-based CI
R> summary( meta(y=yi, v=vi, x=weeks, data=Hox02, intervals.type="LB") )
```

Running Meta analysis with ML

Call:

```

meta(y = yi, v = vi, x = weeks, data = Hox02, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
      Estimate Std.Error    lbound    ubound z value
Slope1_1 1.3866e-01 3.2089e-02 7.4635e-02 2.0695e-01 4.3210
Tau2_1_1 2.3252e-02 3.5481e-02 1.0000e-10 1.3790e-01 0.6553
      Pr(>|z|)
Slope1_1 1.553e-05 ***
Tau2_1_1 0.5123
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 49.5852
Degrees of freedom of the Q statistic: 19
P value of the Q statistic: 0.000150801

Number of studies: 20
Number of observed statistics: 40
Number of parameter estimated: 5
Degrees of freedom: 35
-2 log likelihood: 104.9018

R version: 2.13.0
OpenMx version: 999.0.0-1661
metaSEM version: 0.6-0
Date of analysis: Sun May 22 16:19:22 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( reml(y=yi, v=vi, x=weeks, data=Hox02, intervals.type="LB") )

Running Variance component with REML

Call:
reml(y = yi, v = vi, x = weeks, data = Hox02, intervals.type = "LB")

95% confidence intervals: Likelihood-based statistic
Coefficients:
      Estimate Std.Error    lbound    ubound z value Pr(>|z|)
Tau2_1_1 3.6582e-02 4.2208e-02 4.0001e-06 1.7650e-01 0.8667 0.3861

Number of studies: 20
Number of observed statistics: 18
Number of parameter estimated: 1
Degrees of freedom: 17
-2 log likelihood: -10.86705

R version: 2.13.0
OpenMx version: 999.0.0-1661
metaSEM version: 0.6-0

```

```

Date of analysis: Sun May 22 16:19:24 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Multivariate meta-analysis
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98) )

Running Meta analysis with ML

Call:
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
      data = Berkey98)

95% confidence intervals: z statistic approximation
Coefficients:
      Estimate Std.Error    lbound    ubound z value
Intercept1  0.3448392  0.0536312  0.2397239  0.4499544  6.4298
Intercept2 -0.3379381  0.0812479 -0.4971812 -0.1786951 -4.1593
Tau2_1_1    0.0070020  0.0090497 -0.0107351  0.0247391  0.7737
Tau2_2_1    0.0094607  0.0099698 -0.0100797  0.0290010  0.9489
Tau2_2_2    0.0261445  0.0177409 -0.0086270  0.0609161  1.4737
Pr(>|z|)
Intercept1 1.278e-10 ***
Intercept2 3.192e-05 ***
Tau2_1_1    0.4391
Tau2_2_1    0.3427
Tau2_2_2    0.1406
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 128.2267
Degrees of freedom of the Q statistic: 8
P value of the Q statistic: 0

Number of studies: 5
Number of observed statistics: 10
Number of parameter estimated: 5
Degrees of freedom: 5
-2 log likelihood: -11.68131

R version: 2.13.0
OpenMx version: 999.0.0-1661
metaSEM version: 0.6-0
Date of analysis: Sun May 22 16:19:25 2011
OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)
See http://openmx.psyc.virginia.edu/wiki/errors for the details.

R> ## Estimate variance components with REML
R> summary( reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98) )

Running Variance component with REML

Call:

```

```
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
     data = Berkey98)
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Tau2_1_1	0.011733	0.013645	-0.015011	0.038477	0.8599	0.3899
Tau2_2_1	0.011916	0.014416	-0.016340	0.040172	0.8266	0.4085
Tau2_2_2	0.032651	0.024402	-0.015176	0.080479	1.3380	0.1809

Number of studies: 5

Number of observed statistics: 8

Number of parameter estimated: 3

Degrees of freedom: 5

-2 log likelihood: -18.86768

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:25 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)

See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

*R> ## Multivariate meta-analysis with "publication year-1979" as a predictor*

```
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),
              x=scale(pub_year, center=1979), data=Berkey98) )
```

Running Meta analysis with ML

Call:

```
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),
     x = scale(pub_year, center = 1979), data = Berkey98)
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Slope1_1	0.0063540	0.1078235	-0.2049761	0.2176842	0.0589	
Slope2_1	-0.0705888	0.1620965	-0.3882921	0.2471146	-0.4355	
Intercept1	0.3440001	0.0857659	0.1759021	0.5120982	4.0109	
Intercept2	-0.2918175	0.1312796	-0.5491208	-0.0345141	-2.2229	
Tau2_1_1	0.0080405	0.0101206	-0.0117955	0.0278766	0.7945	
Tau2_2_1	0.0093413	0.0105515	-0.0113392	0.0300218	0.8853	
Tau2_2_2	0.0250135	0.0170788	-0.0084603	0.0584873	1.4646	
						Pr(> z )
Slope1_1					0.95301	
Slope2_1					0.66322	
Intercept1	6.048e-05	***				
Intercept2	0.02622	*				
Tau2_1_1	0.42692					
Tau2_2_1	0.37599					
Tau2_2_2	0.14303					
---						

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 128.2267

Degrees of freedom of the Q statistic: 8

P value of the Q statistic: 0

Number of studies: 5

Number of observed statistics: 15

Number of parameter estimated: 9

Degrees of freedom: 6

-2 log likelihood: -4.595466

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:25 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)

See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

*R> ## Estimate variance components with REML*

```
R> summary( reml(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),  
              x=scale(pub_year, center=1979), data=Berkey98) )
```

Running Variance component with REML

Call:

```
reml(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),  
      x = scale(pub_year, center = 1979), data = Berkey98)
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value	Pr(> z )
Tau2_1_1	0.020447	0.022523	-0.023697	0.064590	0.9078	0.3640
Tau2_2_1	0.016226	0.022696	-0.028258	0.060710	0.7149	0.4747
Tau2_2_2	0.040857	0.034707	-0.027168	0.108882	1.1772	0.2391

Number of studies: 5

Number of observed statistics: 6

Number of parameter estimated: 3

Degrees of freedom: 3

-2 log likelihood: -11.44636

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:26 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)

See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

*R> ## Multivariate meta-analysis with an equality constraint on the slopes*

```
R> summary( meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL),  
               x=scale(pub_year, center=1979), data=Berkey98,  
               coeff.constraints=matrix(c("0.3*Eq_slope", "0.3*Eq_slope"), nrow=2)) )
```

Running Meta analysis with ML

Call:

```
meta(y = cbind(PD, AL), v = cbind(var_PD, cov_PD_AL, var_AL),  
      x = scale(pub_year, center = 1979), data = Berkey98, coeff.constraints = matrix(c("0.3*Eq_slope",  
                                              "0.3*Eq_slope"), nrow = 2))
```

95% confidence intervals: z statistic approximation

Coefficients:

	Estimate	Std.Error	lbound	ubound	z value
Eq_slope	0.0016748	0.1024443	-0.1991123	0.2024619	0.0163
Intercept1	0.3437612	0.0849828	0.1771979	0.5103245	4.0451
Intercept2	-0.3390010	0.1041005	-0.5430344	-0.1349677	-3.2565
Tau2_1_1	0.0070474	0.0094638	-0.0115013	0.0255962	0.7447
Tau2_2_1	0.0095165	0.0105668	-0.0111940	0.0302269	0.9006
Tau2_2_2	0.0261979	0.0180773	-0.0092330	0.0616288	1.4492
Pr(> z )					
Eq_slope	0.986956				
Intercept1	5.231e-05	***			
Intercept2	0.001128	**			
Tau2_1_1	0.456471				
Tau2_2_1	0.367800				
Tau2_2_2	0.147278				

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Q statistic on homogeneity of effect sizes: 128.2267

Degrees of freedom of the Q statistic: 8

P value of the Q statistic: 0

Number of studies: 5

Number of observed statistics: 15

Number of parameter estimated: 8

Degrees of freedom: 7

-2 log likelihood: -4.268456

R version: 2.13.0

OpenMx version: 999.0.0-1661

metaSEM version: 0.6-0

Date of analysis: Sun May 22 16:19:26 2011

OpenMx status1: 0 ("0" and "1": considered fine; other values indicate problems)

See <http://openmx.psyc.virginia.edu/wiki/errors> for the details.

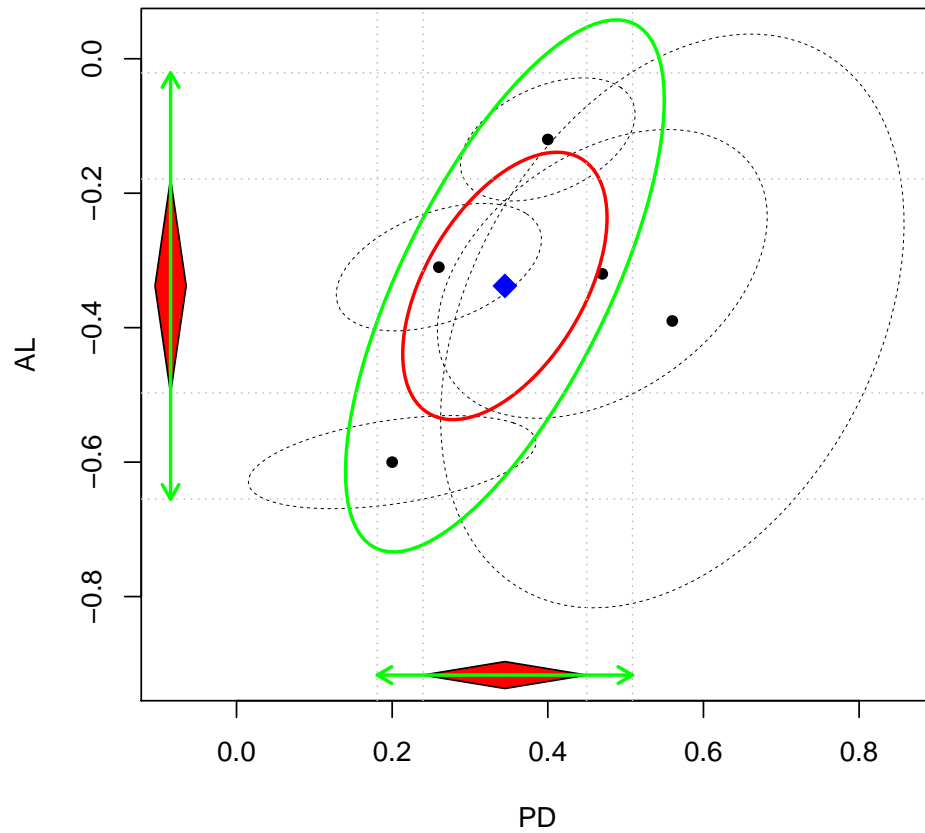
If multivariate meta-analysis is conducted, pairwise plots on the pooled effect sizes and their confidence ellipses may be requested. For example,

```
R> Berkey98.ma <- meta(y=cbind(PD, AL), v=cbind(var_PD, cov_PD_AL, var_AL), data=Berkey98)
```

Running Meta analysis with ML

```
R> plot(Berkey98.ma, main="Multivariate meta-analysis", axis.label=c("PD", "AL"))
```

## Multivariate meta-analysis

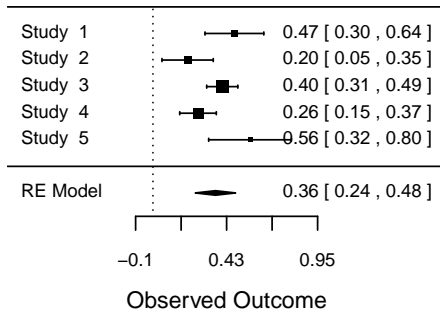


By combining with the forest plots based on the `metafor` package, more information can be displayed.

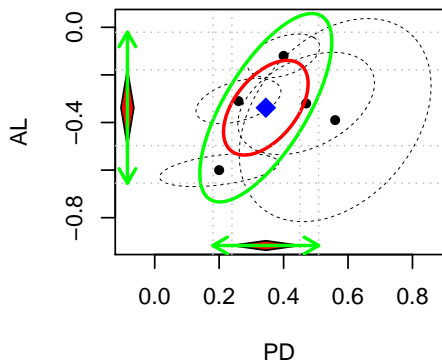
```
R> library(metafor)
R> plot(Berkey98.ma, diag.panel=TRUE, main="Multivariate meta-analysis", axis.label=c("PD", "AL"))
R> forest( rma(yi=PD, vi=var_PD, data=Berkey98) )
R> title("Forest plot for PD")
R> forest( rma(yi=AL, vi=var_AL, data=Berkey98) )
R> title("Forest plot for AL")
```



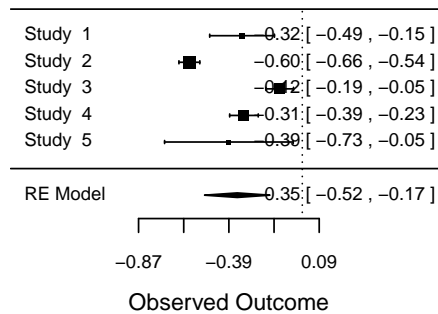
### Forest plot for PD



### Multivariate meta-analysis



### Forest plot for AL



## 4 Acknowledgements

This package cannot be written without R and OpenMx. Contributions by the R Development Core Team and the OpenMx Core Development Team are highly appreciated.

## References

- Steven Boker, Michael Neale, Hermine Maes, Michael Wilde, Michael Spiegel, Timothy Brick, Jeffrey Spies, Ryne Estabrook, Sarah Kenny, Timothy Bates, Paras Mehta, and John Fox. Openmx: An open source extended structural equation modeling framework. *Psychometrika*, pages 1–12, 2011. ISSN 0033-3123. URL <http://dx.doi.org/10.1007/s11336-010-9200-6>. 10.1007/s11336-010-9200-6.
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- M. W. L. Cheung and W. Chan. A two-stage approach to synthesizing covariance matrices in meta-analytic structural equation modeling. *Structural Equation Modeling: A Multidisciplinary Journal*, 16(1):28–53, 2009.
- M. C. Neale and M. B. Miller. The use of likelihood-based confidence intervals in genetic models. *Behavior Genetics*, 27(2):113–120, 1997.