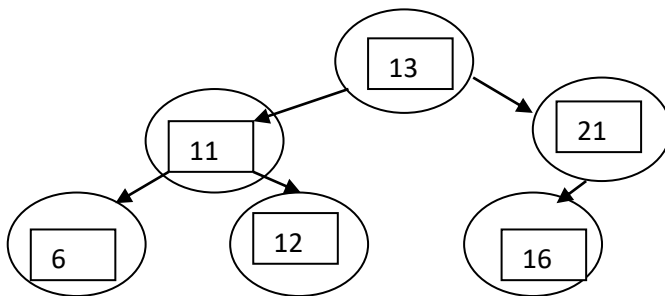
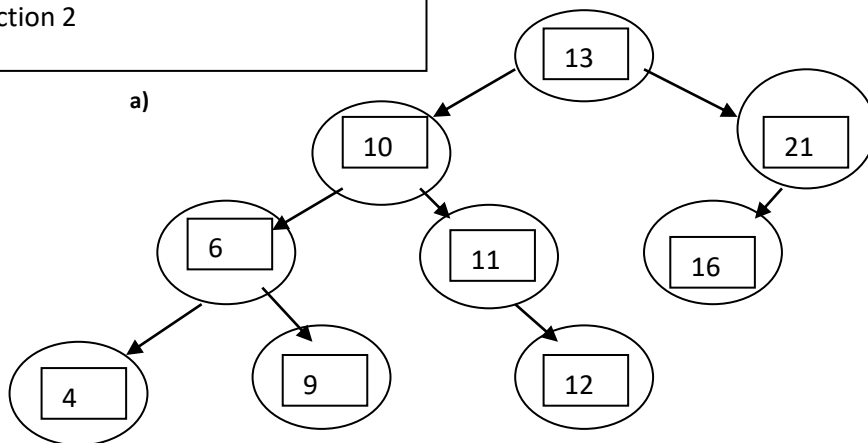
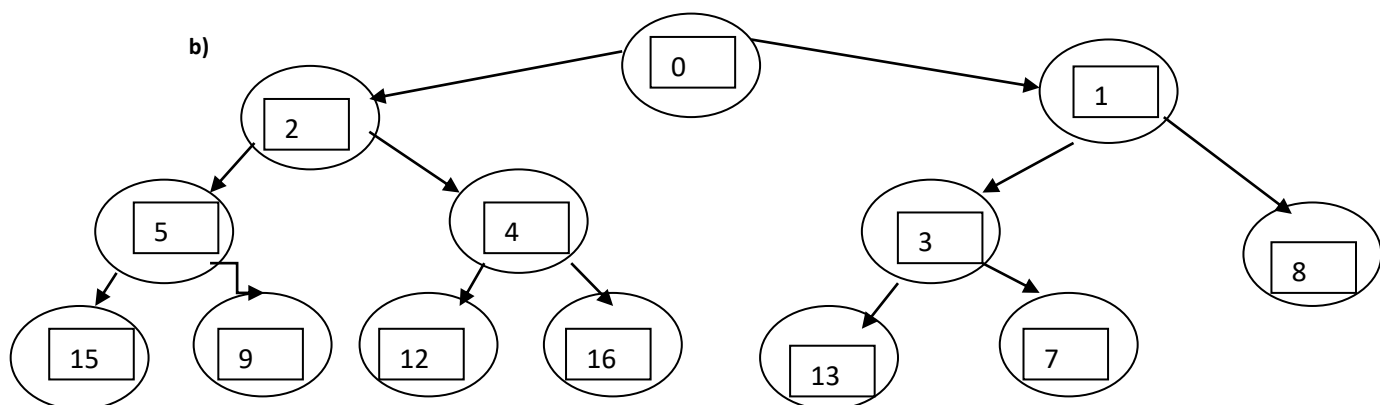


a)



b)



c) JOIN-AVL-TREES(T1, T2)

```

{
  int H1 = calcHeight(T1)
  int H2 = calcHeight(T2)
  if(H2 > H1)
  {
    While(calcHeight((T2->left) - H1 > 1)
    {
      T2Parent = T2
      T2 = T2->left
    }
    FindRigt(T1, RParent, TRight)
    TRight->right = T2
    T2Parent->left = TRight
    TRight->left = RParent
  }
}

```

```

Else
{
While(calcHeight((T1->right) - H2 > 1)
{
    T1Parent = T1
    T1 = T1->right
}
FindLeft(T2,LParent,TLeft)
T1Parent->right = T2
TLeft->left = T1
T1Parent->right = TLeft
TLeft->right = LParent

}
}

```

The idea is to put the tree with less height into the tree with greater height. Then in greater height tree, find the appropriate place depending on the height of the smaller tree, put the smaller tree there, then do the required rotation. Find the height of both of the trees with calcHeight, completes in $O(\log(T1.size + T2.size))$ time. If $T2$'s size is bigger, go to left, until a node's size of subtree is not greater than $T1$ at most (traversal to only one side in a balanced tree, $O(\log(T2.size))$). This is the proper place to insert, because inserting $T1$ here won't harm the balance here, because the other subtree is balanced with that height too. The important thing is to do the necessary rotations $O(1)$ here in order to keep the balance with the other subtree of the node. The algorithm is very similar when $T1$'s size is bigger, just use the opposite ways and rotations. Here, since $T1 + T2 = n$, since $T1$ and $T2$ are variants of n , the complexity is **not** $O(\log(T1))$ or any variant of that, it is $O(\log n)$, especially when you consider the other methods like calcHeight and traversal that I explained earlier.