

MATH 225
Linear Algebra and Differential Equations
2019-2020 Spring
Project 2

DUE DATE: April 21, Tuesday before 23:55 p.m.

QUESTIONS:

1)(40 pts.)Let

$$A = \begin{pmatrix} 1 & -24 & 8 \\ 0 & -11 & 4 \\ 0 & -24 & 9 \end{pmatrix}$$

be given.

(a)(5 pts.)Find all eigenvalues of A .

(b)(10 pts.)Find a basis for each eigenspace and determine the associated eigenvectors.

(c)(5 pts.)Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

(d)(10 pts.)Find the inverse of the matrix P in part (c) by using **Gaussian elimination** method. Verify your answer i.e. show that $PP^{-1} = I = P^{-1}P$.

(e)(5 pts.)Calculate A^{2020} .

(f)(5 pts.)Find A^{-1} by using **Cayley-Hamilton** Theorem.

2) (a)(10 pts.) The characteristic polynomial of a certain 3×3 matrix A is $p(x) = x^3 - 7x^2 + 5x - 9$. Use this fact to express $\text{adj}(A)$ as a linear combination of A^2 , A and I .

(b)(10 pts.) If A is an $n \times n$ non-singular matrix, show that $\text{adj}(A)$ can be expressed as a linear combination of $A^{n-1}, A^{n-2}, \dots, A, I$.

3) (10 pts.)The $n \times n$ matrix A is said to be *idempotent* if $A^2 = A$. If λ is an eigenvalue of such a matrix, show that λ is either 0 or 1. What can be said about a non-singular idempotent matrix?

IMPORTANT:

1.Don't forget to write your Name, Lastname, Department, Section and Student ID on the 1st page of your project.

2.You must show all your work in well-organized English or mathematical sentences, and explain your reasoning carefully.

3.Your project must be hand written. The projects written by latex or word etc. will not be accepted.

4.You must submit your project as a 1 pdf file. Before submission check the file you convert to pdf. If you cannot read it, try to do the best. The name of the pdf file must contain your name. e.g. **yosum.project2.pdf** would be the name of my project.

5.You must **SUBMIT** your project. The **DRAFTS** will not be accepted because **DRAFT** means **INCOMPLETE**. You are **responsible** to complete the submission process. It is not our job.

6. Late submissions will not be accepted