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Q1)

$u_1=(1,2,3)$ $u_2=(0,1,1)$ $u_3=(1,3,4)$

$v = c_1u_1 + c_2u_2 + c_3u_3$

$(k^2,-3k,-2) = (c_1+c_3,\ 2c_1+c_2+3c_3,\ 3c_1+c_2+4c_3)$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc:c} 1 & 0 & 1 & k^2 \\ 2 & 1 & 3 & -3k \\ 3 & 1 & 4 & -2 \end{array}\right] \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc:c} 1 & 0 & 1 & k^2 \\ 0 & 1 & 1 & -2k^2-3k \\ 0 & 1 & 1 & -3k^2-2 \end{array}\right]$$

$$\xrightarrow{-R_2+R_3} \left[\begin{array}{ccc:c} 1 & 0 & 1 & k^2 \\ 0 & 1 & 1 & -2k^2-3k \\ 0 & 0 & 0 & -k^2+3k-2 \end{array}\right]$$

$k^2-3k+2=0$

$(k-2)(k-1)=0$

$k=2$ or $k=1$

Q2) a) $A = \begin{bmatrix} 1 & 3 & 15 & 7 & -2 & 0 \\ 2 & 4 & 22 & 8 & 3 & 1 \\ 2 & 7 & 34 & 17 & -1 & 3 \end{bmatrix} \xrightarrow[-2R_1+R_3]{-2R_1+R_2}$

$$\begin{bmatrix} 1 & 3 & 15 & 7 & -2 & 0 \\ 0 & -2 & -8 & -6 & 7 & 1 \\ 0 & 1 & 4 & 3 & 3 & 3 \end{bmatrix} \xrightarrow{SWAP(R_2,R_3)} \begin{bmatrix} 1 & 3 & 15 & 7 & -2 & 0 \\ 0 & 1 & 4 & 3 & 3 & 3 \\ 0 & -2 & -8 & -6 & 7 & 1 \end{bmatrix}$$

$$\xrightarrow[-3R_2+R_1]{2R_2+R_3} \begin{bmatrix} 1 & 0 & 3 & -2 & -11 & -9 \\ 0 & 1 & 4 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 13 & 7 \end{bmatrix} \xrightarrow{\frac{R_3}{13}} \begin{bmatrix} 1 & 0 & 3 & -2 & -11 & -9 \\ 0 & 1 & 4 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 & \frac{7}{13} \end{bmatrix}$$

$$\xrightarrow[11R_3+R_1]{-3R_3+R_2} \begin{bmatrix} 1 & 0 & 3 & -2 & 0 & -40/13 \\ 0 & 1 & 4 & 3 & 0 & 18/13 \\ 0 & 0 & 0 & 0 & 1 & 7/13 \end{bmatrix}$$

Subject :

b) $B_R = \{(1, 0, 3, -2, 0, -40/13),$
$(0, 1, 4, 3, 0, 18/13),$
$(0, 0, 0, 0, 1, 7/13)\}$

c) Leading 1's are first, second, 5th columns.
come back to A and choose the corresponding columns.

$B_C = \{(1, 2, 2), (3, 4, 7), (-2, 3, -1)\}$

d) The row equivalent matrices have the same solution set.
Solve for $AX = 0$. x_3, x_4, x_6 are free variables.
$x_3 = r, x_4 = t, x_6 = s.$
$x_5 = -7/13s \quad x_2 + 4r + 3t + 18/13s = 0$
$x_2 = -4r - 3t - 18/13s$
$x_1 + 3r - 2t - \frac{40}{13}s = 0 \quad x_1 = \frac{40}{13}s + 2t - 3r$
$Null(A) = \{r(-3, -4, 1, 0, 0, 0), t(2, -3, 0, 1, 0, 0),$
$s(40/13, -18/13, 0, 0, -7/13, 1)\} \; s, t \in R\}$

e) $rank(A) = 3, null(A) = 3$

Q3) Write as row vectors and find a basis for the Row(A).

a)

$$A=\begin{bmatrix}1&3&-1&2&3\\2&7&-2&5&2\\1&4&-1&3&-1\end{bmatrix} \xrightarrow[-R_1+R_3]{-2R_1+R_2} \begin{bmatrix}1&3&-1&2&3\\0&1&0&1&-4\\0&1&0&1&-4\end{bmatrix}$$

$$\xrightarrow{-R_2+R_3} \begin{bmatrix}1&3&-1&2&3\\0&1&0&1&-4\\0&0&0&0&0\end{bmatrix} \xrightarrow{-3R_2+R_1} \begin{bmatrix}1&0&-1&-1&15\\0&1&0&1&-4\\0&0&0&0&0\end{bmatrix}$$

$B=\{(1,0,-1,-1,15),(0,1,0,1,-4)\}$ is a basis for W.

$\dim(W)=2$.

b) Let $V=(a,b,c,d,e)$

$$A'=\begin{bmatrix}-V_1-\\-V_2-\end{bmatrix}=\begin{array}{c}\begin{array}{ccccc}x_1&x_2&a&b&c\end{array}\\\begin{bmatrix}1&0&-1&-1&15\\0&1&0&1&-4\end{bmatrix}\end{array}\approx$$

$x_1=a+b-15c \quad x_2=4c-b$

$$W'=\{a\underbrace{(1,0,1,0,0)}_{u_1},\ b\underbrace{(1,-1,0,1,0)}_{u_2},\ c\underbrace{(-15,4,0,0,1)}_{u_3}\}$$

$=\text{span}\{u_1,u_2,u_3\}$ $\quad$ u_1 & u_2 & u_3 independent

$\dim(W')=3$