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## Electric vehicle routing problem

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### Abstract

This paper presents a general Electric Vehicle Routing Problem (EVRP) that finds the optimal routing strategy with minimal travel time cost and energy cost as well as number of EVs dispatched. This is the first EVRP model to consider the vehicle load effect on battery consumption. As demonstrated with a case study in Austin TX, the effect of vehicle load on routing strategy cannot be ignored. Compared to diesel truck VRP, EVRP has comparable travel time and distance but long en-route re-charging time, which translates into a considerable amount of additional labor cost. Lastly, the network topology greatly affects the routing strategies.

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**Keywords:** electric vehicle routing problem (EVRP); charging station location; battery capacity; charging technology; graph representation

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### 1. Introduction

As corporations are becoming more conscious about the environment and the associated externality costs, electric commercial vehicles are gaining tractions in firms that deliver products, goods, or service. For example, Staples Inc., PepsiCo, FedEx Corp, and AT&T Inc. are among those firms. The idea of being green has not only a long term effect on tackling climate change but also a short term business reward as fuel cost accounts for 39% to 60% of operating costs in the trucking sector (Sahin et al. 2009). EVs are much more energy efficient than the conventional petroleum-fuel powered and therefore are expected to bring considerable energy savings to urban freight. However, compared to the conventional petroleum-fuel powered vehicles, EVs have a shorter range - typically 40~100 miles -

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before being charged again. Due to the range limit, EVs may need to visit a charging station in between customer visits during its daily operation. Because of this unique feature, it makes the EVRP distinctively different from the traditional VRP in the literature as we will demonstrate throughout the paper.

In this paper, we investigate an Electric Vehicle Routing Problem (EVRP), in which electric commercial vehicles with a limited range may recharge at a charging station during their daily delivery (and pickup) operations, provided that the charging station facilities are already in place in the service area. The proposed EVRP formulation takes into account the costs associated with not only the travel time but also electricity consumption. The locations of EV charging stations are known and within the service area. Each charging station may be visited multiple times as needed by the same or different vehicles, or it may not be visited at all. There is a cost associated with electricity. At each customer location, there is a demand either to be delivered or picked up. Furthermore, there is a single depot at which all vehicle routes starts and ends. The proposed EVRP finds the optimal routing strategy in which the total cost (i.e., travel time cost + energy cost) is minimized such that: (1) each customer is visited exactly once by one vehicle on its route; (2) the total demand of the customers served on a route does not exceed the vehicle capacity; (3) each charging station may be visited more than once by the same and different vehicles, or not at all; and (4) minimum necessary number of EVs are needed to serve all the customers in the service area.

The major contributions of this research are as follows. First of all, it adds to the scientific literature of EVRP, which is in fact quite thin due to the fact that EVs as a delivery mode only started very recently, though there is much work on EV battery technology, batter swapping, and EV commuter routing, and charging location choice. Secondly, the proposed EVRP model considers the effect of vehicle load on energy (battery electricity) consumption. This is a unique feature that distinguishes this paper from the other EVRP papers. The significant effect of vehicle load on energy consumption has been documented in Suzuki (2011), Xiao et al. (2012), Chen and Lin (2014), and Zhou et al. (2015). Vehicle load is in turn affected by the customer demand (quantity and type – delivery or pickup) and the visiting order. As we will demonstrate through a case study later in the paper, the load effect cannot be ignored in EVRP routing strategies. Thirdly, the proposed EVRP formulation follows a simpler graph representation than other studies in the EVRP literature, and thus ours has an obvious computational advantage over the others. In addition, the proposed EVRP model considers both delivery and pick-up tasks, paired or unpaired, during routing, and makes no assumption about the vehicle size and battery capacity. In other words, the vehicle and the battery capacities can vary within the fleet mix.

The rest of the paper is organized as follows. First, a literature review of EVRP and related studies is given with the focus on model formulation. Then a general EVRP problem is defined and the model formulation is presented. This is followed by two small hypothetical numerical examples to demonstrate the impact of charging station topology on the routing strategy. Then a case study based on the real-world network setting in Austin, Texas is presented to compare EVRP with conventional VRP strategies. Lastly, study conclusions and future research directions are drawn in the conclusion section.

## 2. Literature review

The electric vehicle routing problem (EVRP) in the literature, albeit thin, can be viewed as a variant to the green vehicle routing problem (GVRP) proposed by Erdogan and Miller-Hooks (2012) in which they consider a vehicle routing problem involving alternative fuel vehicles (including EVs) that have limited travel range and must re-charge during routing. In the paper, GVRP minimizes the total distance traveled by AFVs. Each customer is visited exactly once as defined in the conventional VRP. On the other hand, the re-fueling stations may be visited by any vehicle in the fleet as many times as necessary (or none if not necessary). This represents a deviation from the conventional VRP. To accommodate potentially multiple visits to a charging station (including one at the depot) in the model formulation, the EVRP graph is augmented by creating a set of multiple copies of the vertices representing the charging stations (including one at the depot). The number of copies equals the number of potential visits to a charging station (and in the case of the depot the number equals the fleet size). By doing so, the conventional VRP formulation can be adopted for EVRP with minimal modification - the only change to the conventional VRP formulation is to add a constraint that each charging/re-fueling station is visited at most once. The paper does not consider the cost associated with energy consumption, nor does it take into account the vehicle capacity or time window constraints. In addition, the paper also assumes the refueling time is fixed.

More recently, Schneider et al. (2014) studied an EVRP with time windows and charging stations (E-VRPTW) that minimizes the total distance traveled by a homogenous EV fleet. The model considers the battery charging time depends on the remaining battery level upon arriving at the charging station. And the battery consumption is a function of travel distance (travel speed on arcs is assumed constant).

Following Erdogan and Miller-Hooks (2012) and Schneider et al. (2014), Felipe et al. (2014) presents an EVRP with multiple charging technologies and partial recharges. The model formulation follows closely that of Erdogan and Miller-Hooks (2012). Again, the energy consumption is simply a function of distance traveled. The authors find that the commercially available solvers (such as CPLEX) failed to provide solutions - the computational time was simply too long for even a small network of 10 customers - a finding shared by Schneider et al. (2014). Hence, the focus of the paper is on developing the heuristics and comparing their performance.

In summary, the current EVRP literature is limited to a homogenous fleet vehicle routing problem, in which the re-charging time is either assumed constant or as a function of travel distance only. In our study to be presented below, we propose a general EVRP formulation that considers (1) a heterogeneous EV fleet, (2) any topology of the charging stations, (3) any number of visits to a charging station by a single vehicle or multiple vehicles, (4) the joint effect of vehicle load and speed on battery consumption, and (5) the total cost of travel time and energy consumption. On the other hand, this study does not consider time-window constraints. That can be readily extended from our base model.

### 3. Problem definition and formulation

#### 3.1. Problem Definition

The proposed EVRP model formulation follows a classic VRP formulation: let  $\mathcal{G} = (\mathbf{N} \cup \mathbf{F}, \mathbf{A})$  be a graph where vertex set  $\mathbf{N}$  is a combination of the customer set  $\mathbf{N}_0 = \{1, 2, \dots, i, \dots, j, \dots, n\}$  and the depot  $\{O\}$ ;  $\mathbf{F} = \{n+1, n+2, \dots, n+s, O_s \in O\}$  is a set of charging stations. The set of charging stations  $\mathbf{F}$  includes a charging station  $O_1$  located at the depot. The set  $\mathbf{A} = \{(i, j), \forall i, j \in \mathbf{N}, i \neq j\}$  corresponds to all the possible arcs connecting vertices of  $\mathbf{N}$ . Note the differences in graph representation between a traditional VRP and the EVRP. In the traditional VRP, the vertices are all serviced customer points plus the depot and each pair of vertices is connected exactly once (one and only one arc), i.e. a complete graph. In the EVRP, the vertices also include all the charging stations in the study area, some of which may not be visited at all and the others may be visited multiple times in a given strategy. Each arc  $(i, j)$  is associated with a non-negative travel time  $t_{ij}$  and distance  $d_{ij}$ . Travel speeds  $v_{ij}$  are assumed to be constant over an arc. There are at most  $\mathbf{M}$  number of EVs that can be dispatched to perform the delivery/pickup tasks. Fig. 1 graphically describes the EVRP.

The battery re-charging rule is defined as follows: when an EV starts the route (daily operation) at the depot ( $O$ ), its battery is fully charged at  $O_1$ ; the EV can be re-charged once or more at any of the charging stations in  $\mathbf{F}$  during routing; and when it returns to the depot after accomplishing all the tasks, it is recharged to the full battery capacity at  $O_1$  at the end of the daily operation.

The proposed EVRP must satisfy the following additional conditions or assumptions.

1. There is one single depot at which all vehicle routes begin and end.
2. Travel speed on each arc is constant and may vary across arcs;
3. Battery re-charging rate (in joules/hr) is constant;
4. The battery is re-charged to full each time after visiting a charging station;
5. The total work hour limit of an EV is 8 hours;
6. There is no idling time on an arc or at stops (either customer or charging station);
7. No time window constraint is considered for the customers.
8. There is a mix of delivery and pickup service on a vehicle route.

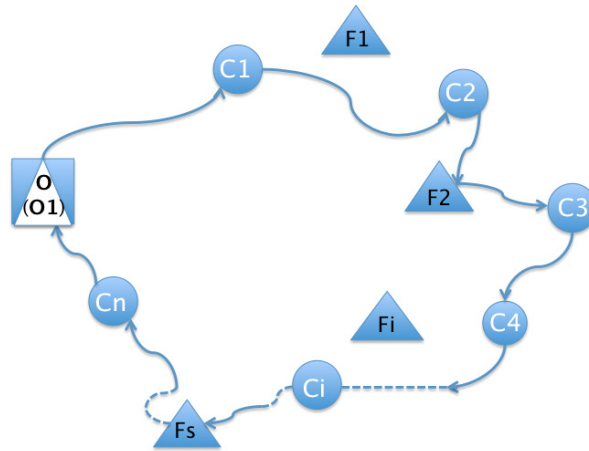


Fig. 1. Graph definition of electric vehicle routing problem

It is worth noting that the graph representation of the proposed model is consistent with its true topology without the augmentation used in the other studies noted in the literature review above, and is simpler than the others. First, our graph representation clearly has a smaller network size in terms of number of vertices and arcs than the other studies in the literature. Second, in our graph the total number of vertices, visited or not, is known but the number of used arcs is not, whereas in the other studies neither the total number of vertices nor the total number of arcs is known because the augmented graph depends on how many visits are paid to which charging stations. That number of visits is a decision variable. Thus, theoretically speaking our model formulation has a considerable advantage over the others in terms of computation time. In fact, we are able to use the existing solver to find the exact solution for a network of thirteen customers in our case study to be described later in the paper, while Schneider et al. (2014) and Felipe et al. (2014) have noted that existing solvers failed to deliver solutions even for a network of ten customers.

### 3.2. Model Formulation (P0)

The proposed EVRP seeks to find a minimum set of EVs that visit each customer once and only once such that the total cost (travel time + energy) is minimized. In doing so, EVs are allowed to be re-charged once or more at any charging stations in the study area. The following letter notations are used in the model:

#### (I) Graph notation:

- $Z_{tt}$ : travel time cost as a function of travel time.
- $Z_e$ : battery charging cost as a function of energy consumption.
- $Z_r$ : battery charging waiting time cost as a function of re-charging time.
- $N_0$ : set of customer vertices to be visited.
- $O$ : Depot.
- $O_1$ : charging station at the depot.
- $N = N_0 \cup \{O\}$ .
- $F$ : set of available charging stations, including  $O_1$ .
- $G = N \cup F$
- $A = \{(i,j)\}, \forall i,j \in G$ .
- $M = \{1,2,...,M\}$ : vehicle fleet with a size of  $M$ .

#### (II) Model input parameters:

- $d_{ij}$ : travel distance (miles) on arc  $(i,j)$ ,  $\forall i,j \in G$ .
- $t_{ij}$ : travel time (hours) on arc  $(i,j)$ ,  $\forall i,j \in G$ .

$v_{ij}$ : travel speed (mph) on arc  $(i,j)$ ,  $\forall i,j \in \mathbf{G}$ .

$D_i$ : demand (tons) of customer  $i$ ,  $\forall i \in \mathbf{G}$ . In particular,  $D_i = 0$  when  $i \in \mathbf{F}$ .

$u_i$ : service type at vertex  $i$ ,  $\begin{cases} u_i = 1 & \text{for pickup, } \forall i \in \mathbf{N}, \\ u_i = -1 & \text{for delivery, } \forall i \in \mathbf{N} \\ u_i = 0, & \forall i \in \mathbf{F} \end{cases}$

$h_i$ : handling time (hours) at vertex  $i$ ,  $\forall i \in \mathbf{G}$ . In particular,  $h_i = 0$  when  $i \in \mathbf{F}$ .

$C_u$ : driver's hourly wage (\$/hr)

$C_e$ : battery monetary charging rate as a function of energy consumption (dollars/Joules).

$b_{ij}$ : energy consumption on arc  $(i,j)$  (Joules).

$C_m$ : battery capacity (Joules) of vehicle  $m$ .

$Q_m$ : vehicle capacity (tons) of vehicle  $m$ .

$r$ : re-charging rate (Joules/hour), a constant.

$H$ : Daily work hour limit, a constant.

### (III) Decision variables (output):

$x_{ijm}$ : binary variable representing vehicle flow on arc  $(i,j)$ ,  $\forall i,j \in \mathbf{G}, \forall m \in \mathbf{M}$ . If vehicle  $m$  leaves vertex  $i$  for  $j$ ,  $x_{ijm} = 1$ ; otherwise,  $x_{ijm} = 0$ .

$y_{ijm}$ : remaining battery capacity of vehicle  $m$  on arrival at vertex  $j$  from vertex  $i$ ,  $\forall i,j \in \mathbf{G}, \forall m \in \mathbf{M}$ .

$l_{ij}$ : vehicle load (tons) on arc  $(i,j)$ ,  $\forall i,j \in \mathbf{G}$ .

$\tau_{ij}$ : vehicle departure time at vertex  $j$  directly connected from vertex  $i$ ,  $\forall i,j \in \mathbf{G}$ . Departure time at depot  $\tau_0 = 0$ .

Because the model considers the effect of load on energy consumption, and arc loads depend on the visiting order, the model must track the visiting order of each individual vehicle. Then a third subscript of vehicle ID is introduced. So the EVRP model takes the following form:

$$\text{Min} Z = \sum_{i \in \mathbf{F}, j \in \mathbf{G}} \sum_{m \in \mathbf{M}} Z_e x_{imj} + \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} Z_u x_{ijm} + \sum_{i \in \mathbf{G}, j \in \mathbf{F}} \sum_{m \in \mathbf{M}} Z_r x_{ijm} \quad (1)$$

Subject to:

$$\sum_{j \in \mathbf{N}} x_{0jm} = 1, \forall m \in \mathbf{M} \quad (2)$$

$$\sum_{j \in \mathbf{G}} \sum_{m \in \mathbf{M}} x_{ijm} = 1, \forall i \in \mathbf{N}_0 \quad (3)$$

$$\sum_{i \in \mathbf{G}} x_{ijm} = \sum_{j \in \mathbf{G}} x_{jim}, \forall j \in \mathbf{G}, \forall m \in \mathbf{M} \quad (4)$$

$$\sum_{j \in \mathbf{G}} l_{ji} - \sum_{j \in \mathbf{G}} l_{ij} = D_i u_i, \forall i \in \mathbf{G} \quad (5)$$

$$\frac{1}{2}(u_j - 1)D_j u_j x_{ijm} \leq l_{ij} \leq Q_m x_{ij} - \frac{1}{2}(u_j + 1)D_j u_j x_{ijm}, \forall i \in \mathbf{G}, \forall j \in \mathbf{G}, \forall m \in \mathbf{M} \quad (6)$$

$$0 \leq l_{ij} \leq Q_m - \frac{1}{2}(u_i - 1)D_i u_i, \forall i, j \in \mathbf{G}, \forall m \in \mathbf{M} \quad (7)$$

$$H(x_{ijm} - 1) \leq \tau_{ki} + t_{ij} + h_j - \tau_{ij} \leq H(1 - x_{ijm}), \forall k, i \in \mathbf{G}, \forall j \in \mathbf{N}, \forall m \in \mathbf{M} \quad (8)$$

$$H(x_{ijm} - 1) \leq \tau_{ki} + t_{ij} + \frac{(C_m - y_{ijm})}{r} - \tau_{ij} \leq H(1 - x_{ijm}), \forall k, i \in \mathbf{G}, \forall j \in \mathbf{F}, \forall m \in \mathbf{M} \quad (9)$$

$$\sum_{(i,j) \in \mathbf{A}} t_{ij} x_{ijm} + \sum_{i \in \mathbf{G}, j \in \mathbf{N}} h_j x_{ijm} + \sum_{i \in \mathbf{G}, j \in \mathbf{F}} \frac{C_m - y_{ijm}}{r} x_{ijm} \leq H, \forall m \in \mathbf{M} \quad (10)$$

$$C_m(x_{ijm} - 1) \leq y_{ijm} - (C_m - b_{ij}) \leq C_m(1 - x_{ijm}), \quad \forall i \in \mathbf{F} \cup \{\mathbf{O}\}, \forall j \in \mathbf{N}, \forall m \in \mathbf{M} \quad (11)$$

$$C_m(x_{ijm} - 1) \leq y_{ijm} - (y_{kim} - b_{ij}) \leq C_m(1 - x_{ijm}), \quad \forall i \in \mathbf{N}_0, \forall k, j \in \mathbf{G}, \forall m \in \mathbf{M} \quad (12)$$

$$0 \leq y_{ijm} \leq C_m, \quad \forall i, j \in \mathbf{G}, \forall m \in \mathbf{M} \quad (13)$$

$$_0 = 0 \quad (14)$$

$$d_{ij} = v_{ij}t_{ij}, \quad \forall i, j \in \mathbf{G} \quad (15)$$

$$x_{ijm} \in \{0,1\}, \quad \forall i, j \in \mathbf{G}, \forall m \in \mathbf{M} \quad (16)$$

The total cost  $Z$  in the objective function (1) has three terms: battery charging cost  $Z_e$ , travel time cost  $Z_t$ , over all arcs, and battery charging waiting time cost  $Z_r$ . The battery charging cost is equal to the cost of energy consumed on the arc, denoted as  $b_{ij}$ ,  $\forall i, j \in \mathbf{G}$ . Then the battery charging cost can be expressed as  $\sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_e b_{ij} x_{ijm}$ . The travel

time cost  $Z_t$  is a function of travel time  $t_{ij}$ . The battery charging waiting time cost is a function of en-route re-charging time – final re-charging to full at the depot at the end of the daily operation does not incur waiting time cost. Thus, the objective function is re-written as:

$$\min Z = \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_e b_{ij} x_{ijm} + \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_t t_{ij} x_{ijm} + \sum_{i \in \mathbf{G}, j \in \mathbf{F}} \sum_{m \in \mathbf{M}} C_r \frac{C_m - y_{ijm}}{r} x_{ijm} \quad (1a)$$

Adopted from Barth et al. (2005, 2009) and Bektas and Laporte (2011), a simplified form of energy cost function is shown in equation (17), which is a linear function of vehicle weight and a quadratic form of vehicle speed.

$$b_{ij} = (v_{ij}(w + l_{ij})d_{ij} + (v_{ij})^2 d_{ij}) / e_f \quad (17)$$

where  $\alpha_{ij} = a + g \sin \theta_{ij} + g C_r \cos \theta_{ij}$  is an arc specific constant, and  $\beta = 0.5 C_d A \rho$  is a vehicle specific constant. The rest of the notations are:

- $e_f$ : engine efficiency
- $w$ : vehicle curb weight (tons),
- $a$ : acceleration ( $\text{m/s}^2$ ),
- $g$ : gravitational constant ( $\text{m/s}^2$ ),
- $\theta$ : road angle,
- $A$ : frontal surface area of a vehicle ( $\text{m}^2$ ),
- $\rho$ : air density ( $\text{kg/m}^3$ ),
- $C_r$ : coefficient of rolling resistance
- $C_d$ : coefficient of rolling drag.

The energy cost on arc  $(i,j)$  is then calculated as:

$$C_e b_{ij} = C_e \alpha_{ij} (w + l_{ij}) d_{ij} / e_f + C_e \beta (v_{ij})^2 d_{ij} / e_f \quad (18)$$

Hence, the total cost  $Z$  can be further expressed as:

$$\begin{aligned} \min Z = & \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_e \alpha_{ij} w d_{ij} x_{ijm} / e_f + \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_e \alpha_{ij} l_{ij} d_{ij} x_{ijm} / e_f + \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_e \beta (v_{ij})^2 d_{ij} x_{ijm} / e_f \\ & + \sum_{(i,j) \in \mathbf{A}} \sum_{m \in \mathbf{M}} C_t t_{ij} x_{ijm} + \sum_{i \in \mathbf{G}, j \in \mathbf{F}} \sum_{m \in \mathbf{M}} C_r \frac{C_m - y_{ijm}}{r} x_{ijm} \end{aligned} \quad (1b)$$

Equation (2) restricts at most  $M$  number of vehicles depart from the depot. Equation (3) guarantees that each customer is visited exactly once. Equation (4) is a flow conservation constraint at each vertex, be it a customer or a charging station. Equation (5) preserves the demand conservation, i.e., the difference between the outbound load and the inbound load at vertex  $i$  equals the demand at  $i$ . Equations (6) and (7) are vehicle load constraints.

Specifically, equation (6) ensures that (a) if the demand at vertex  $j$  is positive (i.e., a pickup order) then the vehicle load on arc  $(i,j)$  is bounded by zero and the vehicle capacity less the demand at  $j$  so that the vehicle can accommodate the demand at  $j$ ; and (b) if the demand at vertex  $j$  is negative (i.e., a delivery order) then the vehicle load on arc  $(i,j)$  is bounded by the demand at  $j$  and the vehicle capacity  $Q_m$ . Equation (7) guarantees that the vehicle load on each arc must be non negative and no greater than the vehicle capacity. And in the case that the demand at vertex  $i$  is negative (i.e., a delivery order), the remaining vehicle load on arc  $(i,j)$  should not exceed the vehicle capacity less the amount unloaded at  $i$ . Equation (8) ensures that the departure time at customer  $j$  equals the arrival time to  $j$  plus the handling time at  $j$ . And equation (9) indicates the departure time at charging station  $j$  equals the arrival time to  $j$  plus the charging time at  $j$ . Equation (10) enforces the total work hour (travel time + handling time + re-charging time) limit of 8 hours assumed in this study. Equations (11) and (12) are related to the battery consumption that says the remaining battery upon reaching vertex  $j$  after visiting vertex  $i$  should be reduced by  $b_{ij}$ . In equation (11), if vertex  $i$  is a charging station, the battery level always goes back to full capacity when leaving  $i$ . Equation (13) enforces the remaining battery  $y_{ijm}$  should be non-negative and less than the battery capacity  $C_m$ . Equation (14) specifies the start time from depot is 0. Equation (15) simply expresses the relationship among travel distance and travel time, speed. Lastly, equation (16) defines the binary decision variable  $x_{ijm}$ .

### 3.3. Linearized Model Formulation (P1)

The above EVRP model formulation P0 is non-linear because in the objective function (1b), the energy cost contains the product of two decision variables  $l_{ij}$  and  $x_{ijm}$ . Similarly in the battery charging waiting time cost term and equation (10), there exist a product of two decision variable  $y_{ijm}$  and  $x_{ijm}$ . To reduce the computational complexity and improve the computation time, we linearize the model by transforming those two equations (1b) and (10) as follows.

Note that the load effect term on energy cost (  $\sum_{(i,j) \in A} \sum_{m \in M} C_e \alpha_{ij} l_{ij} d_{ij} x_{ijm} / e_f$  ) can be simplified as  $\sum_{(i,j) \in A} C_e \alpha_{ij} l_{ij} d_{ij} / e_f$ . That is because  $x_{ijm}$  is a binary variable (1 or 0), so the product  $l_{ij} x_{ijm}$  takes the value of  $l_{ij}$  in either case, i.e.,

$$l_{ij} x_{ijm} = \begin{cases} l_{ij} > 0, & \text{when } x_{ijm} = 1 \\ l_{ij} = 0, & \text{when } x_{ijm} = 0 \end{cases} \quad (19)$$

Then the objective function (1b) can be rewritten as:

$$\begin{aligned} \min Z = & \sum_{(i,j) \in A} \sum_{m \in M} C_e \alpha_{ij} w d_{ij} x_{ijm} / e_f + \sum_{(i,j) \in A} \sum_{m \in M} C_e \alpha_{ij} l_{ij} d_{ij} / e_f + \sum_{(i,j) \in A} \sum_{m \in M} C_e \beta (v_{ij})^2 d_{ij} x_{ijm} / e_f \\ & + \sum_{(i,j) \in A} \sum_{m \in M} C_u t_{ij} x_{ijm} + \sum_{i \in G, j \in F} \sum_{m \in M} C_u \frac{C_m - y_{ijm}}{r} x_{ijm} \end{aligned} \quad (1c)$$

Similarly, equation (10) can be re-written into equations (20) and (21) as shown below.

When  $x_{ijm} = 1$ ,

$$\sum_{(i,j) \in A} t_{ij} x_{ijm} + \sum_{i \in G, j \in N} h_j x_{ijm} + \sum_{i \in G, j \in F} \frac{C_m - y_{ijm}}{r} \leq H, \quad \forall m \in M \quad (20)$$

When  $x_{ijm} = 0$ ,

$$0 \leq \frac{C_m - y_{ijm}}{r} \leq H \sum_{i \in G} x_{ijm}, \quad \forall j \in F, \forall m \in M \quad (21)$$

There are four groups of decision variables in the model:  $x_{ijm}$ ,  $l_{ij}$ ,  $\tau_{ij}$  and  $y_{ijm}$ . Among them,  $x_{ijm}$  is a binary integer variable and the rest are continuous variables. Therefore P1 is a mixed integer linear problem, which could be solved with existing solvers such as CPLEX 12.1 and Matlab. In this study, we use Matlab to solve for the numerical examples in the following sections.

#### 4. Demonstration of effect of network topology through numerical examples

As seen in the model formulation section, our model formulation (P0 and P1) applies to any network topology in terms of the location of customers and charging stations and any EV battery capacity. Clearly, the relative location between customers and charging stations as well as the battery capacity will greatly impact the EV routing strategies. We use two simple numerical examples to demonstrate the effects.

Fig. 1 shows the network representation of numerical examples 1 and 2. Both have the similar problem setting: there are two customers (C1 and C2) and three charging stations (O1, F1 and F2). O1 is at the depot O. The only difference is the location of charging stations (F1 and F2). In example 2, the charging stations have farther distances from the customers than in example 1. Suppose the demand profile is as seen in Table 1 and the rest of the model parameter values are shown in Table 2 for both numerical examples.

Table 1. Customer demand profile

Node	Demand (1000lbs)	Service Type <sup>1</sup> ( $u$ )	Dwell time h (mins)
Customer1(C1)	1	-1	5
Customer2(C2)	3	-1	15

<sup>1</sup>Service type  $u = -1$  is a delivery service

Table 2. Model parameter values

Parameter	Description	Values	Source
$C_H$	Hourly driver wage (\$)	16.43	Payscale (2009)
$C_e$	Battery charging cost(\$/kwh)	0.12	Bureau of Labor Statistics (2015)
$C_f$	Diesel fuel price (\$/gallon)	4	Market price
$r$	Charging rate (kW)	15 <sup>2</sup>	US DOE (2015)
$C_m$	Battery capacity(kwh)	25	Wikipedia (2015)
$C_d$	Unitless coefficient of rolling drag	0.7	Akçelik et al. (2003)
$e_f$	Engine efficiency (input energy/output energy)	48% (diesel truck)	Giannelli and Nam (2004), Tesla (2015)
		70% (electric truck)	
$A$	Frontal surface area of a vehicle (m <sup>2</sup> )	5	Akçelik et al. (2003)
$a$	Acceleration (m/s <sup>2</sup> )	0	Genta (1997)
$\theta_{ij}$	Road angle (degree)	0°	Genta (1997)
$\rho$	Air density (kg/m <sup>3</sup> )	1.2041	Genta (1997)
$C_r$	Unitless rolling resistance	0.01	Genta (1997)
$g$	Gravitational constant (m/s <sup>2</sup> )	9.81	
$w$	Vehicle curb weight (tons)	3.629 (or 8,000 lbs)	Smith's Electric Vehicles (2015)
$Q_m$	Vehicle capacity (tons)	9.073 (or 20,000 lbs)	Smith's Electric vehicles (2015)

<sup>2</sup> This is equivalent to a level II EV charging technology currently available in the market. Level I is primarily used for home EV charging with a charging rate up to 3 kW. Levels II and III are for commercial charging stations. Level II charging rate may be as high as 20 kW. Level III is a fast charging technology up to 10 times faster than level II. However its market share is very limited.



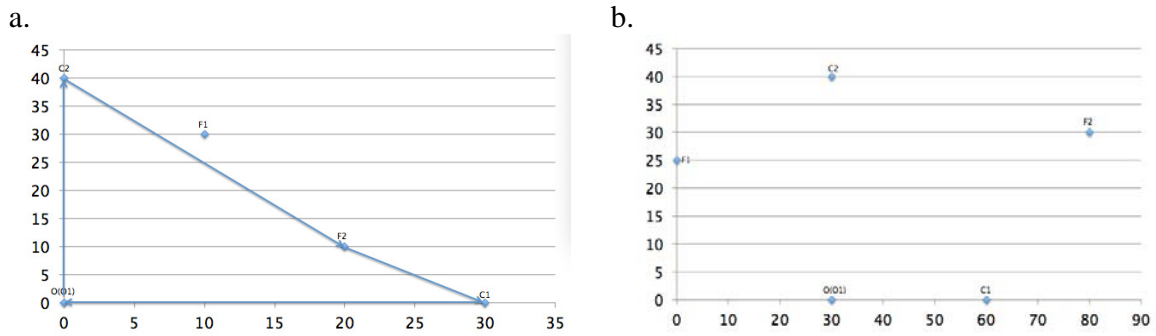


Fig. 1. Network representations of (a) numerical example 1 and (b) numerical example 2  
(Customer points: C1 and C2; charging stations: O, F1, and F2)

By solving model P1 for both examples, we find that example 1 requires only one EV to complete all the tasks: O-C2-F2-C1-O, while in example 2 there is no feasible solution because the charging stations/depot and customers are too far apart and the given battery capacity does not have sufficient power to move the vehicle to a charging station to re-charge before it can carry on. Therefore, the two simple numerical examples reveal the very different EV routing strategies with the only difference in relative location between customers and charging stations. Currently we are conducting more detailed, in-depth analyses of the network topology effects.

## 5. Comparison Of alternative routing strategies using case study in Austin Texas

### 5.1. Case Study Setting

Model P1 is applied to a case study based on the real-world network setting in Austin, Texas. Fig. 2 shows the case study network, in which there are 13 customers labeled as C1, C2,...,C13, two charging stations F1 and F2, and a single depot O. The thirteen customers covers as far north as Serenada, as far south as Shady Hollow, as far east as Manor, and as far west as Cedar Park. The distance from south to north is 46.8 miles and 34.9 miles from east to west.

The customer and cargo information is extracted from the 2005-2006 Texas Commercial Vehicle Surveys administered by the Texas Department of Transportation. It includes, customer location, cargo type, industry type, loading/unloading weight (demand), and handling time at customer. Please refer to Ruan et al. (2012) for detailed description of the data. Table 3 summarizes the demand, service type as well as handling time at each customer. In addition, the survey data contains information about the depot location and arc distances. The charging station location is obtained from the Department of Energy website (USDOE, 2015). In this case study, the travel speed on each arc is assumed to be a constant value of 25 mph. The other model parameters have the same values as in Table 2.

Table 3. Summary of Customer Information

Vertex	Demand (lbs)	Service Type <sup>1</sup> ( $u$ )	Handling time (mins)
Customer1(C1)	1000	+1	10
Customer2(C2)	500	-1	5
Customer3(C3)	1000	-1	10
Customer4(C4)	500	+1	5
Customer5(C5)	1500	-1	15
Customer6(C6)	1000	-1	10
Customer7(C7)	1000	-1	10
Customer8(C8)	1500	-1	15
Customer9(C9)	2500	-1	25
Customer10(C10)	1500	+1	15
Customer11(C11)	1500	+1	15
Customer12(C12)	1000	-1	10
Customer13(C13)	3000	-1	30

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<sup>1</sup> Service type  $u=+1$  is a pickup,  $u=-1$  is a delivery

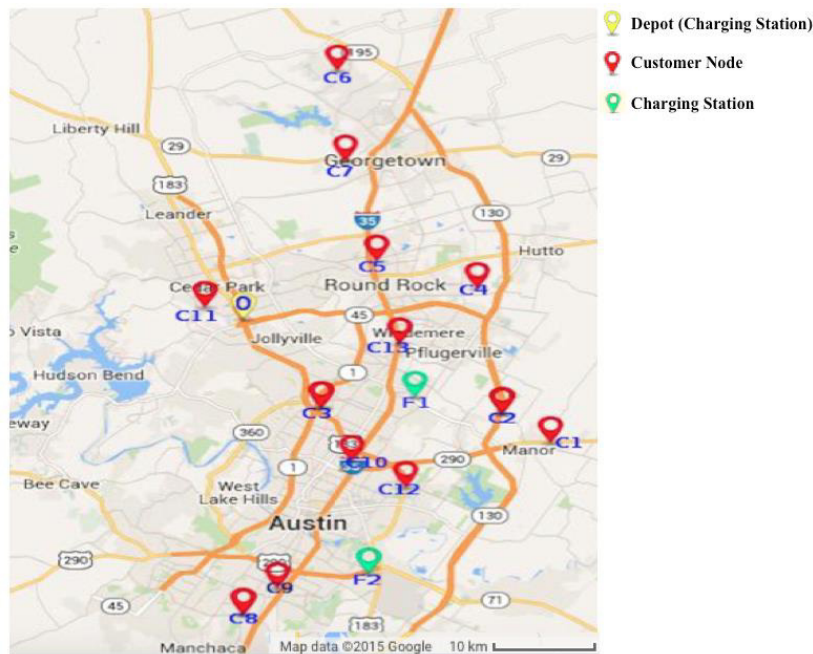


Fig. 2. The Austin TX case study network

### 5.2. Comparison of Alternative Routing Strategies

In the Texas Commercial Vehicle Surveys, four diesel trucks were observed to perform the pick-up/delivery operations of manufactured equipment at the thirteen customers shown in Fig. 2. The observed routing strategy ( $Z_a$ ) is: (1) O-C1-C2-O, (2) O-C3-C4-C5-C6-O, (3) O-C7-C8-C9-C10-C11-O, and (4) O-C12-C13-O.

If the above four-truck operation could be consolidated and minimized in terms of total cost (travel time cost + energy cost), keeping everything else the same, the optimal consolidated routing strategy ( $Z_c$ ) would require only

one diesel truck to complete all the tasks within the constraints and the visiting order would become: O-C11-C3-C10-C8-C9-C12-C1-C2-C13-C4-C5-C7-C6-O.

Alternatively, if the four trucks were to switch to EVs (with the vehicle capacity and size kept constant) to perform the tasks, the EVRP strategy (Z) obtained from Model P1 would require only one EV to complete all the tasks: O-C11-C3-C10-C9-C8-F2-C12-C1-C2-C13-C4-C5-C7-C6-O (see Fig. 3). During the route, the EV would need to visit one charging stations (F2) in order to carry on the rest of the tasks.

Observe that if we remove the visit to charging station F2 from the EVRP strategy (Z), that represents a feasible solution to the consolidated routing strategy using a diesel truck. We denote that as  $Z_c'$ . The visiting order is O-C11-C3-C10-C9-C8-C12-C1-C2-C13-C4-C5-C7-C6-O. It is clear that the travel distance and travel time of  $Z_c'$  would be reduced from Z.

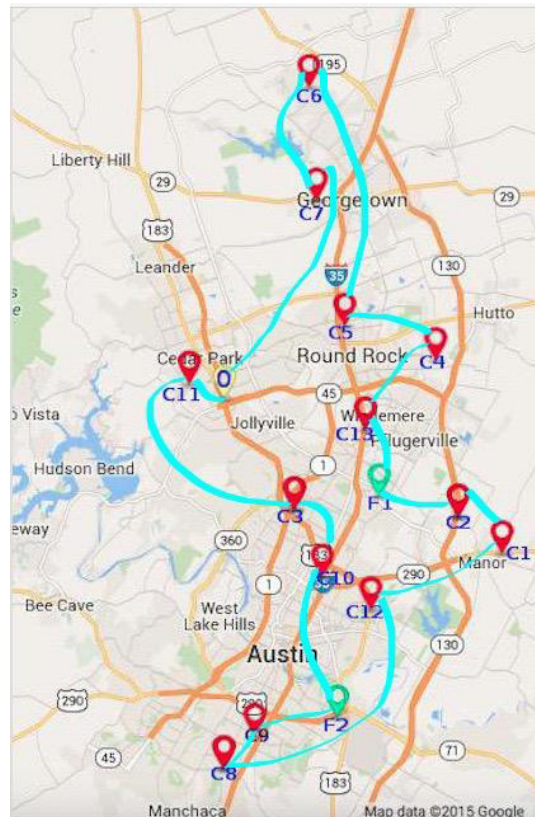


Fig. 3. EVRP strategy (Z)

Table 4 summarizes the performance measures of the above four alternative strategies, namely, the observed ( $Z_a$ ), the optimal consolidated VRP ( $Z_c$ ), the EVRP (Z), and the feasible consolidated ( $Z_c'$ ). A few observations are made.

First, the optimal consolidated strategy ( $Z_c$ ) could bring considerable improvements to the real-world practice ( $Z_a$ ), i.e., 31.9% less distance traveled, 31.9% shorter travel time spent, 30.2% less energy used, and overall 31.7% lower total cost incurred. In addition, it would require only one truck to perform the job rather than four. There would be a large reduction in capital and operational costs associated with the fleet.

Second, between the feasible and optimal consolidated strategies, namely  $Z_c'$  and  $Z_c$ ,  $Z_c'$  represents 0.1% increase in travel distance and travel time from  $Z_c$ , but 0.7% savings in energy consumption; overall  $Z_c'$  yields a 0.2% increase in total cost from  $Z_c$ . Clearly there exists a trade-off between travel time cost and energy cost in the optimal strategy where labor cost and energy cost are jointly considered. That is, less travel time does not necessarily yield

less energy consumption, because energy consumption is also a function of the vehicle load, which depends on the visiting order. In other words, the effect of vehicle load on the optimal strategy cannot be ignored.

Third, if the couriers chose to switch to EVs, there would also be significant improvements in all of the performance measures but the battery charging time. Specifically, one would see over 31.0% savings in total travel distance and total travel time, 51.6% in energy, and 16.3% in total cost. And only one EV would be needed. The disadvantage of using EV is of course associated with the long battery charging time. In this example, with a relatively small battery capacity of 25 kWh and a level II charging rate of 15 kW, it would require a total of 2.57 hours of re-charging time including 1.52 hours taking place at the depot after the daily operation. Nonetheless, 1.05 hours of en-route charging time is still a large amount of time spent for charging.

Lastly, between the two alternatives,  $Z_c$  and  $Z$ , the optimal consolidated strategy  $Z_c$  outperforms the EV alternative ( $Z$ ) very slightly in terms of distance and travel time (by 1.3%). On the other hand,  $Z$  would save 30.7% energy from  $Z_c$ . This is mainly due to the engine efficiency performance between diesel and electric trucks - according to the US EPA (2004), diesel trucks have on average 48% engine efficiency, and 70% for the EV based on the market statistics of Tesla and Nissan Leaf. However,  $Z$  would incur 1.05 hours of en-route battery re-charging time. Even though there is a large saving in energy consumption (and cost due to cheaper electricity than diesel) with EV, the additional labor cost due to the amount of time spent waiting for the battery to be re-charged is so much higher that the overall cost is 19.8% higher than  $Z_c$ . Viewed from a different perspective, the energy cost accounts for about 11% of the total cost in  $Z_c$  and only about 7.2% in  $Z$ . If we could reduce the labor cost, the EV alternative would yield greater reduction in total cost. Furthermore, EV would be less susceptible to energy price hike. In addition, if we bring in externalities such as vehicle emissions, EVs are zero-emission vehicles and diesel trucks are major contributors to greenhouse gas emissions (GHGs), particulate matter (PM) and nitrogen oxides (NOx) - for example, PM can be as much as 40 times higher than passenger vehicles in terms of grams per mile (U.S. EPA, 2009). Therefore, from the green vehicle routing perspective, EVs certainly represent an appealing alternative in the vehicle operating stage<sup>3</sup>.

Table 4. Comparison of alternative routing strategies

Strategy	Optimal Route	# vehicles dispatched	Total distance (miles)	Total travel time (hrs)	Battery re-charging time (hrs)	Energy used ( $\times 10^8$ joules)	Total cost (\$)
Observed ( $Z_a$ )	(1) O-C1-C2-O	4	147.54	5.90	n/a	4.11	108.57
	(2) O-C3-C4-C5-C6-O						
	(3) O-C7-C8-C9-C10-C11-O						
	(4) O-C12-C13-O						
Optimal Consolidated ( $Z_c$ )	O-C11-C3-C10-C8-C9-C12-C1-C2-C13-C4-C5-C7-C6-O	1	100.50	4.02	n/a	2.87	74.17
EVRP ( $Z$ )	O-C11-C3-C10-C9-C8-F2-C12-C1-C2-C13-C4-C5-C7-C6-O	1	101.82	4.07	2.57 <sup>a</sup>	1.99	90.85
Feasible Consolidated <sup>b</sup> ( $Z_c'$ )	O-C11-C3-C10-C9-C8-C12-C1-C2-C13-C4-C5-C7-C6-O	1	100.63	4.03	n/a	2.85	74.29

<sup>a</sup>This includes a total of 1.05 hrs en-route re-charging time and 1.52 hrs at the depot at the end of the daily operation.

<sup>b</sup>This feasible consolidated route ( $Z_c'$ ) has the same visiting order to customers as  $Z$ , after removing the visit to charging station F2.

<sup>3</sup> From the life-cycle perspective of energy from production to consumption, the green prospect of EV over diesel trucks becomes much murkier.

The above case study is noticeably a small one. Currently we are working on a much larger network of Austin, TX over a thousand customers, 80 EV charging stations, and 138 EVs. We have noticed that the existing solvers fail to provide solution for an EVRP of that network size. We are currently working on heuristics to expedite the search for solution.

## 6. Conclusion

This paper has presented a general formulation of EVRP for determining the minimal total cost routing strategies using EVs. Total cost includes both travel time cost and energy cost. The proposed EVRP considers a limited battery capacity and unrestricted re-charging activities at charging stations. A unique feature of our EVRP is that the battery consumption is affected by not only travel speed but also vehicle load that is in turn affected by the customer demand and visiting order. As demonstrated in the Austin TX case study, the effect of vehicle load on routing strategy cannot be ignored. It is also found that compared to diesel truck VRP, EVRP has comparable travel time and distance, but long en-route re-charging time of an EV translates into a considerable amount of additional labor cost. Though EV has its appeal to the idea of green routing for being a zero-emission vehicle. Lastly, the relative distribution of charging stations to customer points greatly affects the routing strategies.

We also noticed that computational time increased exponentially when the network size increases. Therefore, we are currently constructing heuristics to expedite the search to be tested with a large network.

There are obviously many extensions of the proposed EVRP. For example, it would be interesting to see how EVs perform under the time window constraint compared to conventional trucks. Based on the findings of this study, the answer seems to be a discouraging one; however more rigorous scientific investigation is needed. Also based on the findings of this study, it seems that a mixed fleet of conventional trucks and EVs may provide a good balance between operating cost and externalities. The question is "is it?" and "if so, what is the right mix?" Lastly, in a highly dynamic traffic environment EV routing is more susceptible to real-time unexpected disruptions such as heavy traffic congestion, road closure, and other special events. A robust EV routing strategy will be more the important for EVs to become more widely adopted for commercial use.

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## References

- Akçelik, R., Besley, M., 2003. Operating cost, fuel consumption, and emission models in aaSIDRA and aaMOTION, the 25th Conference of Australian Institutes of Transport Research (CAITR 2003). University of South Australia, Adelaide, Australia.
- Barth, M., Boriboonsomsin, K., 2009. Energy and emissions impacts of a freeway-based dynamic eco-driving system. *Transportation Research Part D: Transportation and Environment* 14 (6), 400–410.
- Barth, M., Younglove, T., Scora, G., 2005. Development of a heavy-duty diesel modal emissions and fuel consumption model. Tech. rep. UCB-ITS-PRR-2005-1, California PATH Program, Institute of Transportation Studies, University of California at Berkeley.
- Bektaş, T., Laporte, G., 2011. The pollution-routing problem. *Transportation Research Part B: Methodological* 45(8), 1232–1250.
- Bureau of Labor Statistics. [http://www.bls.gov/regions/midwest/data/averageenergyprices\\_selectedareas\\_table.htm](http://www.bls.gov/regions/midwest/data/averageenergyprices_selectedareas_table.htm), last accessed April 9, 2015.
- Chen, Q., Lin, J., 2014. A Preliminary Investigation of Sustainable Urban Truck Routing Strategies Considering Cargo Weight and Vehicle Speed, the 2014 Transportation Research Board Annual Meeting, Washington D.C., paper no. 14-3300.
- Electric Truck Wiki. [http://en.wikipedia.org/wiki/Electric\\_truck#Pickup\\_trucks](http://en.wikipedia.org/wiki/Electric_truck#Pickup_trucks), last accessed on April 9, 2015
- Erdogan, S., Miller-Hooks, E., 2012. A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review* 48(1), 100–114.
- Felipe, A., Ortuno, M.T., Righini, G., Tirado, G., 2014. A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. *Transportation Research Part E: Logistics and Transportation Review* 71(1), 111–128.
- Genta, G., 1997. *Motor Vehicle Dynamics: Modelling and Simulation*. World Scientific Publishing, Singapore.
- Giannelli, R.A., Nam, E., 2004. Medium and Heavy Duty Diesel Vehicle Modeling Using A Fuel Consumption Methodology. National Vehicle Fuel and Emission Laboratory, US Environmental Protection Agency, Ann Arbor, MI. Available at [www.epa.gov/oms/models/ngm/may04/crc0304c.pdf](http://www.epa.gov/oms/models/ngm/may04/crc0304c.pdf), last accessed April 3rd, 2015.
- Payscale, 2009. [http://www.payscale.com/research/US/Job=Truck\\_Driver,\\_Heavy\\_%2F\\_Tractor%-Trailer/Hourly\\_Rate](http://www.payscale.com/research/US/Job=Truck_Driver,_Heavy_%2F_Tractor%-Trailer/Hourly_Rate), last accessed April 9,

2015.

- Ruan, M., Lin, J., Kawamura, K., 2012. Modeling Commercial Vehicle Daily Tour Chaining. *Transportation Research Part E: Transportation Review and Logistics*, Vol 48 (6), 1169-1184
- Sahin, B., Yilmaz, H., Ust, Y., Guneri, A. F., Gulsun, B., 2009. An approach for analysing transportation costs and a case study. *European Journal of Operational Research* 193(1), 1-11.
- Schneider, M., Stenger, A., and Goeke, D., 2014. The Electric Vehicle-Routing Problem with Time Windows and Recharging Stations. *Transportation Science* 48 (4), 500-520.
- Smith Electric Vehicles website. <http://en.wikipedia.org/wiki/Smith%2520Electric%2520Vehicles?oldid=643003539>, last accessed April 9, 2015.
- Suzuki, Y., 2011. A new truck-routing approach for reducing fuel consumption and pollutants emission. *Transportation Research Part D: Transport and Environment* 16(1), 73-77.
- Tesla website. [http://my.teslamotors.com/it\\_IT/forum/forums/battery-charging-and-discharging-losses](http://my.teslamotors.com/it_IT/forum/forums/battery-charging-and-discharging-losses), last accessed April 9, 2015.
- U.S. Department of Energy. Energy Efficiency and Renewable Energy: EV charging station locations, available at [http://www.afdc.energy.gov/fuels/electricity\\_locations.html](http://www.afdc.energy.gov/fuels/electricity_locations.html), last accessed January 5, 2015.
- U.S. Department of Energy. [http://www.afdc.energy.gov/fuels/electricity\\_infrastructure.html](http://www.afdc.energy.gov/fuels/electricity_infrastructure.html), last accessed on April 9, 2015
- US Environmental Protection Agency, 2009. Motor Vehicle Emission Simulator. MOVES 2010 User Guide. EPA report EPA-420-B-09-041, Office of Transportation and Air Quality, December 2009
- Xiao, Y., Zhao, Q., Kaku, I., Xu, Y., 2012. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & Operations Research* 39(7), 1419-1431.
- Zhou, W., Chen, Q., Lin, J., 2015. Green Vehicle Routing Problem Considering Joint Effect of Vehicle Load and Speed, the 2015 Transportation Research Board Annual Meeting, Washington D.C., paper no. 15-1112.