高等数学（下）

南京信息工程大学 数学与统计学院

大学数学部 高等数学教学团队

第五节 多元复合函数的求导法则

多元复合函数的求导法则

教学目标

- 多元复合函数求导的链式法则
- 全微分的形式不变性

重难点

重点： 多元复合函数求导法则

难点： 多元复合函数求导法则

一、多元复合函数的求导法则

1、一元函数与多元函数的复合（中间变量为一元函数）

定理1： 若函数 $u=\phi(t)$ 及 $v=\psi(t)$ 都在点 t 处可导，函数 $z=f(u,v)$ 在对应点 (u,v) 具有连续偏导数，则复合函数 $z=f[\phi(t),\psi(t)]$ 在对应点 t 可导，且有

$$\boxed{\frac{\mathrm{d}z}{\mathrm{d}t}=\frac{\partial z}{\partial u}\cdot\frac{\mathrm{d}u}{\mathrm{d}t}+\frac{\partial z}{\partial v}\cdot\frac{\mathrm{d}v}{\mathrm{d}t}}$$

证明 设t获得增量 Δt，则

$$\Delta u=\phi(t+\Delta t)-\phi(t),\quad \Delta v=\psi(t+\Delta t)-\psi(t).$$

由于函数 $z=f(u,v)$ 在点 (u,v) 有连续偏导数，

则函数在该点可微分，从而

$$\Delta z=\frac{\partial z}{\partial u}\Delta u+\frac{\partial z}{\partial v}\Delta v+o(\rho),\ 其中\ \rho=\sqrt{(\Delta u)^2+(\Delta v)^2}.$$

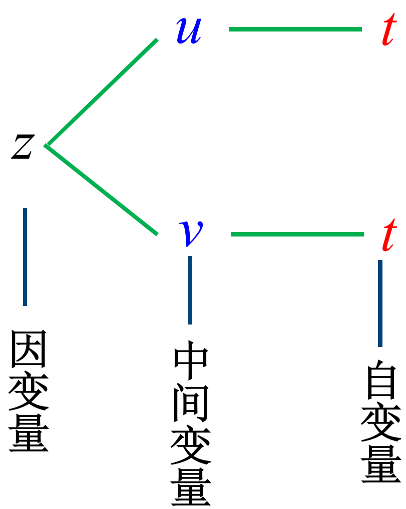
当 $\Delta u\to 0,\Delta v\to 0$ 时，$\rho\to 0$，$\dfrac{\Delta z}{\Delta t}=\dfrac{\partial z}{\partial u}\cdot\dfrac{\Delta u}{\Delta t}+\dfrac{\partial z}{\partial v}\cdot\dfrac{\Delta v}{\Delta t}+\dfrac{o(\rho)}{\Delta t}$,

故

$$\boxed{\frac{dz}{dt}=\lim_{\Delta t\to 0}\frac{\Delta z}{\Delta t}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}.}$$

多元复合函数的求导法则

- 公式可表示为



$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

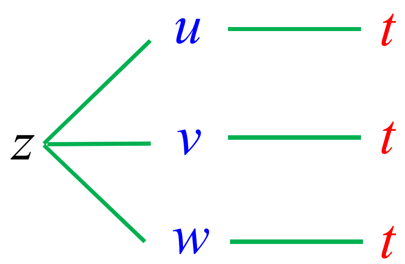
按线相乘,分线相加

复合函数的求导法则，亦称为**链式法则**.

多元复合函数的求导法则

- 公式可推广到中间变量为三个或更多的情况：

推广 设 $z=f(u,v,w)$ 是可微的，$u=u(t),v=v(t),w=w(t)$ 是可导的，则



$$\boxed{\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}+\frac{\partial z}{\partial w}\cdot\frac{dw}{dt}}$$

以上公式中的导数 $\dfrac{dz}{dt}$ 称为**全导数**.

例1 设 $z=\dfrac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求 $\dfrac{dz}{dt}$，$\dfrac{d^2z}{dt^2}$.

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot\left(-2e^{2t}\right)$$

$$=-e^{-t}-e^t,$$

$$\frac{d^2z}{dt^2}=e^{-t}-e^t.$$

多元复合函数的求导法则

例2 设 $z=uv+w$，而 $u=e^t, v=\cos t, v=\cos t$, 求 $\frac{dz}{dt}$.

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}+\frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$$

$$=v\cdot e^t-u\cdot\sin t+\cos t$$

$$=e^t\left(\cos t-\sin t\right)+\cos t.$$

2、多元函数与多元函数的复合（中间变量为多元函数）

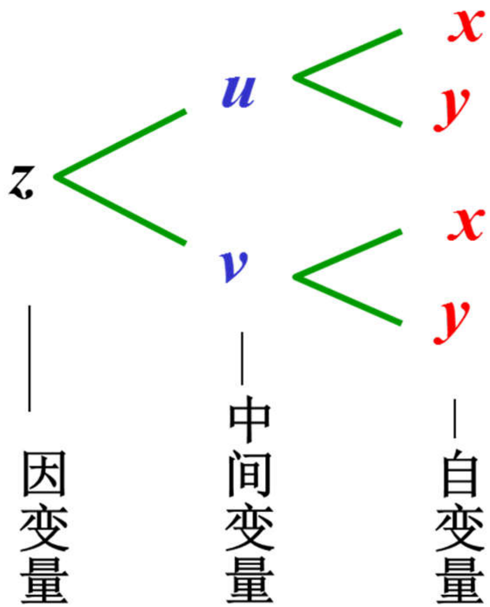
定理2：若函数 $u=\varphi(x,y)$ 及 $v=\psi(x,y)$ 都在点 (x,y) 有对 x 和 y 的偏导数，且函数 $z=f(u,v)$ 在对应点 (u,v) 具有连续偏导数，则复合函数 $z=f(\varphi(x,y),\psi(x,y))$ 在点 (x,y) 的两个偏导数存在，且有

$$\boxed{\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}}\qquad\boxed{\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}}$$

注：1) 公式可表示为:

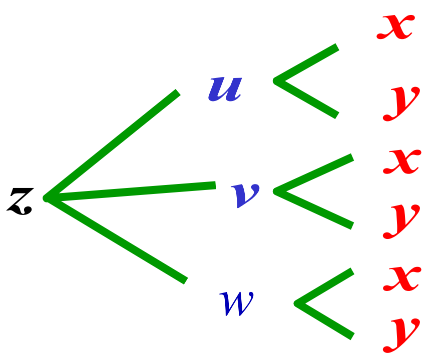
$$\boxed{\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}}$$

$$\boxed{\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}}$$



2) 公式可推广到中间变量为三个或更多的情况：

推广：设 $u=\varphi(x,y)$、$v=\psi(x,y)$、$w=w(x,y)$都在点 (x,y) 有对 x 和 y 的偏导数，复合函数 $z=f[\varphi(x,y),\psi(x,y),w(x,y)]$在点 (x,y) 的两个偏导数存在，且



$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}+\frac{\partial z}{\partial w}\frac{\partial w}{\partial x}$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}+\frac{\partial z}{\partial w}\frac{\partial w}{\partial y}.$$

例3 设$z=e^u\sin v$，而$u=xy, v=x+y$，求$\dfrac{\partial z}{\partial x}$和$\dfrac{\partial z}{\partial y}$.

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}=e^u\sin v\cdot y+e^u\cos v\cdot 1$$

$$=e^u(y\sin v+\cos v)=e^{xy}[y\sin(x+y)+\cos(x+y)]$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}=e^u\sin v\cdot x+e^u\cos v\cdot 1$$

$$=e^u(x\sin v+\cos v)=e^{xy}[x\sin(x+y)+\cos(x+y)]$$

例4 设 $z = ue^{\frac{v}{u}}$，而 $u = x^2 + y^2, v = xy$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$.

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= \left(e^{\frac{v}{u}} - \frac{v}{u} e^{\frac{v}{u}} \right) \cdot 2x + e^{\frac{v}{u}} \cdot y = \left(2x + y - \frac{2x^2 y}{x^2 + y^2} \right) e^{\frac{xy}{x^2 + y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$= \left(e^{\frac{v}{u}} - \frac{v}{u} e^{\frac{v}{u}} \right) \cdot 2y + e^{\frac{v}{u}} \cdot x = \left(2y + x - \frac{2xy^2}{x^2 + y^2} \right) e^{\frac{xy}{x^2 + y^2}}$$

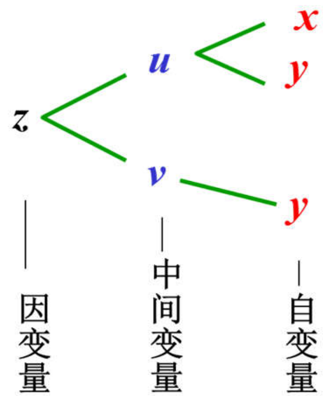
3、复合函数的中间变量既有一元函数,又有多元函数情形

定理3： 如果 $u=\phi(x,y)$ 在 (x,y) 具有对 x 和 y 的偏导数, 函数 $v=\psi(y)$ 在点 y 可导, 函数 $z=f(u,v)$ 在对应点 (u,v) 有连续偏导数, 则复合函数 $z=f[\phi(x,y),\psi(y)]$ 在对应点 (x,y) 的两个偏导存在，且有

$$\boxed{\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x},\quad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{dv}{dy}.}$$

注： 1) 公式可记为：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{dv}{dy}.$$



2) 该情形为情形2的一种特例：

在情形2中,若变量v与x无关,从而$\frac{\partial v}{\partial x}=0$； 在$v$对$y$求导时，

由于v是y的一元函数，故$\frac{\partial v}{\partial y}$换成了$\frac{dv}{dy}$

如函数$z=f(u,x,y)$可微，其中$u=\varphi(x,y)$可偏导，则$\dfrac{\partial z}{\partial x},\dfrac{\partial z}{\partial y}$如何？

利用复合函数求导公式，令$v=x,\ w=y$，则

$$\frac{\partial z}{\partial x}=1,\quad \frac{\partial w}{\partial x}=0,\quad \frac{\partial v}{\partial y}=0,\quad \frac{\partial w}{\partial y}=1,$$

得复合函数$z=f[\varphi(x,y),x,y]$的偏导数：

$$\boxed{\frac{\partial z}{\partial x}}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\boxed{\frac{\partial z}{\partial x}}\cdot 1+\frac{\partial z}{\partial y}\cdot 0.$$

对自变量x的偏导数

对中间变量x的偏导数

相同，但所表示的意思不同！

为了避免混淆，一般地，

将中间变量的偏导数记为 $\dfrac{\partial f}{\partial x},\dfrac{\partial f}{\partial y},\dfrac{\partial f}{\partial u},\dfrac{\partial f}{\partial v}$

将函数的偏导数记为 $\dfrac{\partial z}{\partial x},\dfrac{\partial z}{\partial y},\dfrac{\partial z}{\partial u},\dfrac{\partial z}{\partial v}$

因此，上例中的复合函数的偏导数表示为：

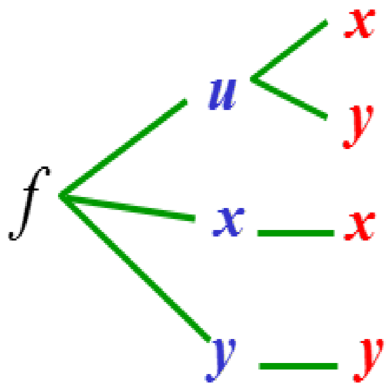
$$\boxed{\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial x}} \qquad \boxed{\frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f}{\partial y}}$$

3)中间变量本身又是复合函数的自变量的情况：

设函数$z=f(u,x,y)$可微,函数 $u=\varphi(x,y)$可偏导,则复合函数

$z=f[\varphi(x,y),x,y]$的偏导数为：

$$\begin{cases} \boxed{\dfrac{\partial z}{\partial x}=\dfrac{\partial f}{\partial u}\cdot\dfrac{\partial u}{\partial x}+\dfrac{\partial f}{\partial x},} \\ \boxed{\dfrac{\partial z}{\partial y}=\dfrac{\partial f}{\partial u}\cdot\dfrac{\partial u}{\partial y}+\dfrac{\partial f}{\partial y}.} \end{cases}$$



说明： 在上式中,符号:

(1) $\frac{\partial z}{\partial x}$是把复合函数$z=f[\varphi(x,y),x,y]$中的变量$y$看作不变量，而对自变量$x$的偏导数；

$\frac{\partial f}{\partial x}$是把$f(u,x,y)$中的$u$与$y$均看成是不变量，而对中的自变量$x$的偏导数；

(2) 符号$\frac{\partial z}{\partial y},\frac{\partial f}{\partial y}$也有类似的含义.

为了表达简便起见，引入以下记号：

$$f_1' = \frac{\partial f(u,v)}{\partial u}, f_2' = \frac{\partial f(u,v)}{\partial v}$$

这里,下标1和2分别表示对第一个中间变量 u和第二个中间变量v求偏导数，同理,可得如下记号：

$$f_{11}'',\quad f_{21}'',\quad f_{22}''.$$

使用上面的简单符号,上例中的结果可表示为:

$$\boxed{\frac{\partial z}{\partial x} = f_1' \cdot \frac{\partial u}{\partial x} + f_2'} \qquad \boxed{\frac{\partial z}{\partial y} = f_1' \cdot \frac{\partial u}{\partial y} + f_3'}$$

例5 设$u = f(x+y, xy)$, 求 $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial^2 u}{\partial x \partial y}$($f$ 具有二阶连续偏导数).

解 $\dfrac{\partial u}{\partial x} = f_1' \cdot 1 + f_2' \cdot y = f_1' + yf_2'$ $\dfrac{\partial u}{\partial y} = f_1' \cdot 1 + f_2' \cdot x = f_1' + xf_2'$

$$\frac{\partial^2 u}{\partial x \partial y} = (f_{11}'' \cdot 1 + f_{12}'' \cdot x) + f_2' + y(f_{21}'' \cdot 1 + f_{22}'' \cdot x)$$

$$= f_{11}'' + xf_{12}'' + f_2' + yf_{21}'' + xyf_{22}''$$

例6 设 $w=f(x+y+z,xyz)$, f 具有二阶连续偏导数，求 $\dfrac{\partial w}{\partial x}$ 和 $\dfrac{\partial^2 w}{\partial x\partial z}$.

解 令 $u=x+y+z,\ v=xyz$,

记 $f_1'=\dfrac{\partial f(u,v)}{\partial u}$, $f_{12}''=\dfrac{\partial^2 f(u,v)}{\partial u\partial v}$, 同理有 f_2', f_{11}'', f_{22}''.

$$\frac{\partial w}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x}=f_1'+yzf_2';$$

$$\frac{\partial^2 w}{\partial x\partial z}=\frac{\partial}{\partial z}(f_1'+yzf_2')=\frac{\partial f_1'}{\partial z}+yf_2'+yz\frac{\partial f_2'}{\partial z};$$

例6 设 $w=f(x+y+z,xyz)$，f 具有二阶连续偏导数，求 $\dfrac{\partial w}{\partial x}$ 和 $\dfrac{\partial^2 w}{\partial x\partial z}$.

续解

$$\frac{\partial f_1'}{\partial z}=\frac{\partial f_1'}{\partial u}\cdot\frac{\partial u}{\partial z}+\frac{\partial f_1'}{\partial v}\cdot\frac{\partial v}{\partial z}=f_{11}''+xyf_{12}'';$$

$$\frac{\partial f_2'}{\partial z}=\frac{\partial f_2'}{\partial u}\cdot\frac{\partial u}{\partial z}+\frac{\partial f_2'}{\partial v}\cdot\frac{\partial v}{\partial z}=f_{21}''+xyf_{22}'';$$

于是
$$\frac{\partial^2 w}{\partial x\partial z}=f_{11}''+xyf_{12}''+yf_2'+yz(f_{21}''+xyf_{22}'')$$

$$=f_{11}''+y(x+z)f_{12}''+xy^2zf_{22}''+yf_2'.$$

例7 设 $z=f(u,x,y),u=xe^y$, f具有二阶连续偏导数, 求$\frac{\partial z}{\partial x},\frac{\partial^2 z}{\partial x\partial y}$

解 $$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial x}=f_1'\cdot e^y+f_2'$$

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial}{\partial y}\left(f_1'\cdot e^y+f_2'\right)=\left(\frac{\partial f_1'}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f_1'}{\partial y}\right)e^y+f_1'e^y+\frac{\partial f_2'}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f_2'}{\partial y}$$

$$=(f_{11}''\cdot xe^y+f_{13}'')e^y+f_1'e^y+f_{21}''\cdot xe^y+f_{23}''$$

$$=xf_{11}''\cdot e^{2y}+f_{13}''\cdot e^y+f_1'e^y+xf_{21}''\cdot e^y+f_{23}''$$

例8 设$u=f(x,y)$的所有二阶偏导数连续，把下式转换成极坐标系中的形式 (1) $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2$, (2) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$.

解 由直角坐标与极坐标间的关系式得

$$u=f(x,y)=f(\rho\cos\theta,\rho\sin\theta)=F(\rho,\theta),$$

其中 $x=\rho\cos\theta, y=\rho\sin\theta,$

$$\rho=\sqrt{x^2+y^2},\quad \theta=\arctan\frac{y}{x}.$$

应用复合函数求导法则，得

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial x}+\frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial x}=\frac{\partial u}{\partial \rho}\frac{x}{\rho}-\frac{\partial u}{\partial \theta}\frac{y}{\rho^2}=\frac{\partial u}{\partial \rho}\cos\theta-\frac{\partial u}{\partial \theta}\frac{y\sin\theta}{\rho}$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial y}+\frac{\partial u}{\partial \theta}\frac{\partial \theta}{\partial y}=\frac{\partial u}{\partial \rho}\frac{y}{\rho}+\frac{\partial u}{\partial \theta}\frac{x}{\rho^2}=\frac{\partial u}{\partial \rho}\sin\theta+\frac{\partial u}{\partial \theta}\frac{\cos\theta}{\rho}$$

得两式平方后相加，得

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial \rho}\right)^2+\frac{1}{\rho^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

再求二阶偏导数，得

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial \rho}\left(\frac{\partial u}{\partial x}\right)\cdot\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial \theta}\left(\frac{\partial u}{\partial x}\right)\cdot\frac{\partial \theta}{\partial x}$$

$$=\frac{\partial}{\partial \rho}\left(\frac{\partial u}{\partial \rho}\cos\theta-\frac{\partial u}{\partial \theta}\frac{\sin\theta}{\rho}\right)\cdot\cos\theta-\frac{\partial}{\partial \theta}\left(\frac{\partial u}{\partial \rho}\cos\theta-\frac{\partial u}{\partial \theta}\frac{\sin\theta}{\rho}\right)\cdot\frac{\sin\theta}{\rho}$$

$$=\frac{\partial^2 u}{\partial \rho^2}\cos^2\theta-2\frac{\partial^2 u}{\partial \rho\partial \theta}\frac{\sin\theta\cos\theta}{\rho}+\frac{\partial^2 u}{\partial \theta^2}\frac{\sin\theta^2}{\rho^2}+\frac{\partial u}{\partial \theta}\frac{2\sin\theta\cos\theta}{\rho^2}+\frac{\partial u}{\partial \rho}\frac{\sin^2\theta}{\rho}$$

同理可得

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial \rho^2}\sin^2\theta+2\frac{\partial^2 u}{\partial\rho\partial\theta}\frac{\sin\theta\cos\theta}{\rho}+\frac{\partial^2 u}{\partial\theta^2}\frac{\cos\theta^2}{\rho^2}$$

$$-\frac{\partial u}{\partial\theta}\frac{2\sin\theta\cos\theta}{\rho^2}+\frac{\partial u}{\partial\rho}\frac{\cos^2\theta}{\rho}$$

两式相加，得

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial \rho^2}+\frac{1}{\rho}\rho+\frac{1}{\rho^2}\frac{\partial^2 u}{\partial\theta^2}=\frac{1}{\rho^2}\left[\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right)+\frac{\partial^2 u}{\partial\theta^2}\right]$$

二、全微分不变性

全微分形式不变性： 设函数 $z=f(u,v)$ 具有连续偏导数，则有全微分 $\mathrm{d}z=\frac{\partial z}{\partial u}\mathrm{d}u+\frac{\partial z}{\partial v}\mathrm{d}v$. 当 $u=\phi(x,y)$、$v=\psi(x,y)$ 时，有

$$\boxed{\mathrm{d}z=\frac{\partial z}{\partial x}\mathrm{d}x+\frac{\partial z}{\partial y}\mathrm{d}y}=\left(\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\right)\mathrm{d}x+\left(\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\right)\mathrm{d}y$$

$$=\frac{\partial z}{\partial u}\left(\frac{\partial u}{\partial x}\mathrm{d}x+\frac{\partial u}{\partial y}\mathrm{d}y\right)+\frac{\partial z}{\partial v}\left(\frac{\partial v}{\partial x}\mathrm{d}x+\frac{\partial v}{\partial y}\mathrm{d}y\right)\boxed{=\frac{\partial z}{\partial u}\mathrm{d}u+\frac{\partial z}{\partial v}\mathrm{d}v.}$$

实质： 无论u和v是自变量还是中间变量，函数$z=f(u,v)$的**全微分形式是一样的**.

例9 求函数$u=f(x+y,x-y)$的全微分.

解 令$s=x+y, t=x-y$，则 $u=f(s,t)$，

则 $\mathrm{d}u = f_1'\,\mathrm{d}s + f_2'\,\mathrm{d}t = f_1'(\mathrm{d}x+\mathrm{d}y)+f_2'(\mathrm{d}x-\mathrm{d}y)$

$$=(f_1'+f_2')\mathrm{d}x+(f_1'-f_2')\mathrm{d}y$$

例10 已知 $e^{-xy}-2z+e^{z}=0$，求 $\dfrac{\partial z}{\partial x}$和 $\dfrac{\partial z}{\partial y}$.

解 $\because d(e^{-xy}-2z+e^{z})=0$

$\therefore e^{-xy}d(-xy)-2dz+e^{z}dz=0$ ， $(e^{z}-2)dz=e^{-xy}(xdy+ydx)$

故 $dz=\dfrac{ye^{-xy}}{e^{z}-2}dx+\dfrac{xe^{-xy}}{e^{z}-2}dy$

得 $\boxed{\dfrac{\partial z}{\partial x}=\dfrac{ye^{-xy}}{e^{z}-2},\dfrac{\partial z}{\partial y}=\dfrac{xe^{-xy}}{e^{z}-2}}$

三、内容小结

1、链式法则

（分三种情况，特别要注意课中所讲的特殊情况）

2、全微分形式不变性（理解其实质）

思考题 设$z=f(u,v,x)$，而$u=\varphi(x),v=\psi(x)$,

则 $\dfrac{\mathrm{d}z}{\mathrm{d}x}=\dfrac{\partial f}{\partial u}\dfrac{du}{dx}+\dfrac{\partial f}{\partial v}\dfrac{dv}{dx}+\dfrac{\partial f}{\partial x}$, 试问$\dfrac{\mathrm{d}z}{\mathrm{d}x}$与$\dfrac{\partial f}{\partial x}$是否相同？

解答 不相同.

等式左端的z是作为一个自变量x 的函数，

而等式右端最后一项f 是关于u,v,x 的三元函数.

$$\left.\frac{\mathrm{d}z}{\mathrm{d}x}\right|_x=\left.\frac{\partial f}{\partial u}\right|_{(u,v,x)}\cdot\left.\frac{\mathrm{d}u}{\mathrm{d}x}\right|_x+\left.\frac{\partial f}{\partial v}\right|_{(u,v,x)}\cdot\left.\frac{\mathrm{d}v}{\mathrm{d}x}\right|_x+\left.\frac{\partial f}{\partial x}\right|_{(u,v,x)}.$$

为明天做准备的最好方法就是集中你所有智慧，所有的热忱，把今天的工作做得尽善尽美，这就是你能应付未来的唯一方法。