习题参考答案

第 1 章　函数、极限与连续

习　题　1-1

A 组

1. (1) 是;　(2) 不是.

2. (1) 偶函数;　(2) 既非奇函数也非偶函数;　(3) 奇函数;　(4) 奇函数.

3. (1) $y=u^{\frac{4}{5}}, u=\sin x$;　(2) $y=\mathrm{e}^{u}, u=-v^{2}, v=\cos w, w=\dfrac{1}{x}$;

(3) $y=u^{2}, u=\arctan v, v=\dfrac{x}{2}$;　(4) $y=\sqrt{u}, u=\ln v, v=\sin w, w=4^{x}$.

4. -1,3, 2.　5. (1) $[0,3]$;　(2) $[-5,5)$.

6. (1) $[-1,1]$;　(2) $\left[2k\pi, 2k\pi+\dfrac{\pi}{2}\right]$　$(k=0,\pm1,\pm2,\cdots)$;　(3)$[-1,0]$;　(4)$[1,\mathrm{e}]$.

7. 略.　8. $f[g(x)]=\begin{cases}\ln x^{2}, & x\leqslant 1\text{且}x\neq 0,\\ \ln x^{3}, & x>1,\\ 0, & x=0.\end{cases}$

9. $f[g(x)]=\begin{cases}1, & x<0,\\ 0, & x=0,\\ -1, & x>0;\end{cases}$　$g[f(x)]=\begin{cases}\mathrm{e}, & |x|<1,\\ 1, & |x|=1,\\ \dfrac{1}{\mathrm{e}}, & |x|>1.\end{cases}$　10. $1-\cos 2x$.

11. $V=\pi h\left(r^{2}-\dfrac{h^{2}}{4}\right), h\in(0,2r)$.　12. $s(t)=\begin{cases}0.25t^{2}, & 0\leqslant t\leqslant 2,\\ t-1, & 2<t\leqslant 9,\\ t-1-0.25(t-9)^{2}, & 9<t\leqslant 11.\end{cases}$

B 组

1. B.　2. $f(x)=-\left(x^{2}+x+1\right)$.

3. 略. (提示: 先求出函数 $f(x)$, 再证明其为奇函数)

习 题 1-2

A 组

1. (1) 0; (2) 3; (3) 1; (4) 1; (5) 不存在; (6) 不存在. 2. 略. 3. 略.

B 组

1. 略. 例如数列 $\{u_n=(-1)^n\}$, $\lim\limits_{n\to\infty}|u_n|=\lim\limits_{n\to\infty}1=1$, 而 $\lim\limits_{n\to\infty}u_n=\lim\limits_{n\to\infty}(-1)^n$ 不存在. 2. A. 3. 略.

习 题 1-3

A 组

1. 略. 2. 略. 3. δ 等于 0.1 或小于 0.1. 4. 不矛盾.

5–7. 略.

B 组

1. 略; 其逆命题不成立, 例如函数 $f(x)=\begin{cases}1, & x\in\mathbf{Q},\\ -1, & x\notin\mathbf{Q}.\end{cases}$ 2. 略.

习 题 1-4

A 组

1. 略. 2. 略. 3. 1.

4. (1) $x\to\infty$; (2) $x\to1$; (3) $x\to-\infty$; (4) $x\to0^-$.

5. (1) $x\to0$; (2) $x\to\infty$; (3) $x\to+\infty$; (4) $x\to0^+$.

6. (1) 0; (2) 0; (3) 0; (4) 0.

7. (1) 不一定, 例如 $x\to0$ 时 x^2 与 x^3 都是无穷小, 但它们的商 $\dfrac{1}{x}$ 当 $x\to0$ 时为无穷大 (不是无穷小); (2) 不一定, 例如 $x\to\infty$ 时 x^2 与 $-x^2+1$ 都是无穷大, 但它们的和等于 1, 故当 $x\to\infty$ 不是为无穷大.

B 组

1. (1) 0; (2) 0.

2. 证明提示: 分别取特殊序列: 如 $x=\dfrac{1}{2n\pi}\to0$ 与 $x=\dfrac{1}{2n\pi+\dfrac{\pi}{2}}\to0$ 时, 即可证得.

习 题 1-5

A 组

1. (1) $-\dfrac{1}{2}$; (2) 0; (3) $\dfrac{3}{2}$; (4) $-\dfrac{1}{3}$; (5) $-\dfrac{3}{2}$; (6) ∞; (7) $2x$; (8) -1.

2. (1) ∞; (2) $\dfrac{2}{5}$; (3) ∞; (4) 0; (5) $\dfrac{81}{2^{11}}$; (6) $\dfrac{1}{2}$; (7) 2; (8) $\dfrac{4}{3}$; (9) $\dfrac{1}{2}$; (10) $-\dfrac{1}{2}$; (11) $\dfrac{1}{5}$; (12) 1.

3. (1) 27; (2) $\dfrac{1}{6}$; (3) 2; (4) 5; (5) $\dfrac{2}{3}$; (6) $\dfrac{1}{2}$.

B 组

1. (1) $\sqrt{2}$; (2) 4. 2. $a=4, l=10$. 3. $k=1, m=\dfrac{1}{2}$. 4. $a=1, b=-1$.

习 题 1-6

A 组

1. (1) 2; (2) 0; (3) 0; (4) 2; (5) e^{-1}; (6) e^{-2}.

2. (1) $\dfrac{2}{3}$; (2) $\dfrac{3}{4}$; (3) 0; (4) 2; (5) $\dfrac{3}{2}$; (6) $-\sqrt{2}$; (7) 2; (8) 1.

3. (1) e^{-4}; (2) e^{-2}; (3) e^{-2}; (4) e^{2a}; (5) e^{-1}; (6) e^{-1}.

4. (1) 0; (2) 3; (3) $\dfrac{1}{3}$; (4) 1.

5. (1) $\dfrac{1+\sqrt{1+4a}}{2}$; (2) 2; (3) $\dfrac{1+\sqrt{5}}{2}$.

B 组

1. (1) $-\dfrac{2}{3}$; (2) $\dfrac{\sqrt{2}}{8}$; (3) e^{2}; (4) $a\mathrm{e}$; (5) $\mathrm{e}^{-\frac{1}{2}}$; (6) e^{-1}.

2. $\dfrac{1-\mathrm{e}^{2}}{\mathrm{e}\pi}$. 3. -1. 4. 略.

习 题 1-7

A 组

1. 是, 是. 2. (1) 三阶; (2) 二阶. 3. 略.

4. (1) 1; (2) $-\dfrac{2}{3}$; (3) 2; (4) $\dfrac{1}{2}$; (5) 2; (6) $\dfrac{n^2-m^2}{2}$; (7) $\dfrac{1}{2}$; (8) $\dfrac{\mathrm{e}}{2}$.

5. 略.

B 组

1. C. 2. B. 3. 0. 4. (1) 是, 2 阶; (2) 不是.

习 题 1-8

A 组

1. (1) 错; (2) 对; (3) 对; (4) 错.

2. (1) B; (2) $2k\pi+\dfrac{3}{2}\pi \quad (k\in\mathbf{Z})$. 3. (1) $\dfrac{\ln 2}{4}$; (2) -1. 4. 9.

5. (1) $x=1$ 是第一类跳跃型间断点; (2) $x=\pi$ 是连续点;

(3) $x=0$ 是第一类可去型间断点; (4) $x=2$ 是第二类无穷型间断点.

6. (1) $f(x)$ 在 $x=1, x=2$ 处间断, $x=1$ 为第一类可去型间断点, $x=2$ 为第二类无穷型间断点;

(2) $f(x)$ 在 $x=0, x=1$ 处间断, $x=1$ 为第二类无穷型间断点, $x=0$ 为第一类可去型间断点;

(3) $f(x)$ 在 $x=k\pi(k=0,\pm1,\pm2,\cdots)$ 处间断, $x=0$ 与 $x=\pi$ 均为第一类可去型间断点; $x=k\pi(k=-1,\pm2,\cdots)$ 为第二类无穷型间断点;

(4) $f(x)$ 在 $x=1$ 处间断, $x=1$ 为第二类振荡型间断点;

(5) $f(x)$ 在 $x=2k+1(k=0,\pm1,\pm2,\cdots)$ 处间断, $x=1$ 为第一类可去型间断点; $x=2k+1(k=\pm1,\pm2,\cdots)$ 为第二类无穷型间断点;

(6) $f(x)$ 在 $x=0$ 处间断, $x=0$ 是第一类跳跃型间断点.

7. $a=2,b=4$. 8. $a=-1$. 9. $a=-3$. 10. $k=\mathrm{e}^{-\frac{1}{2}}$.

B 组

1. B. 2. $x=1,x=-1$ 均为第一类跳跃型间断点.

3. $x=\dfrac{\pi}{4},\dfrac{3\pi}{4},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$ 为 $f(x)$ 的间断点, $x=\dfrac{\pi}{4},\dfrac{5\pi}{4}$ 都是第二类无穷型间断点; $x=\dfrac{3\pi}{4},\dfrac{7\pi}{4}$ 都是第一类可去型间断点.

习 题 1-9

A 组

1—8. 略.

B 组

1—3. 略.

总复习题 1

1. (1) 1; (2) $\dfrac{6}{5}$; (3) $-\dfrac{\sqrt{2}}{6}$; (4) 0; (5) 2.

2. (1) C; (2) C; (3) B; (4) A; (5) A .

3. (1) e^3; (2) $-\ln 2$; (3) $-\dfrac{1}{6}$; (4) $\dfrac{a^2}{b^2}$; (5) 1; (6) $\dfrac{\ln 3}{\ln 2}$; (7) $\sqrt{ab}$;

(8) $\ln a$; (9) 1; (10) $\mathrm{e}^{\frac{2}{\pi}}$; (11) $\mathrm{e}^{-\frac{a^2}{2}}$; (12) 1; (13) $\dfrac{1}{2}$; (14) 2.

4. $a=\ln 2$. 5. 6. 6. 略.

7. 略. (提示: 利用单调有界准则) 8. 证略 (提示: 利用单调有界准则), $\dfrac{3}{2}$.

9. $x=0$, $x=1$ 都为 $f(x)$ 的第一类可去型间断点, $x=-1$ 为 $f(x)$ 的第二类无穷型间断点.

10. $a=0$, $b=\mathrm{e}$. 11. 略. 12. 略.

第 2 章 导数与微分

习 题 2-1

A 组

1. $\overline{v}=14+3\Delta t, v\,|_{t=2}=14$.

2. (1) $[f(x_0)]'$ 表示常数 $f(x_0)$ 的导数; (2) $f'[\varphi(x_0)]$ 表示导函数 $f'(u)$ 在 $u=\varphi(x_0)$ 处的函数值.

3. (1) 2; (2) 50!. 4. 1. 5. (1) $-2f'(x_0)$; (2) $-\dfrac{1}{2}f'(1)$. 6. $(2,4)$.

7. $y-\dfrac{\sqrt{2}}{2}=-\dfrac{\sqrt{2}}{2}\left(x-\dfrac{\pi}{4}\right)$ 或 $y+\dfrac{\sqrt{2}}{2}=-\dfrac{\sqrt{2}}{2}\left(x-\dfrac{3\pi}{4}\right)$.

8. $f'(x)=\begin{cases}\mathrm{e}^x, & x<0,\\ \dfrac{1}{2\sqrt{x}}, & x>0.\end{cases}$ 9. $a=1,b=0$.

10. (1) 不连续也不可导; (2) 连续但不可导. 11. 略. 12. 2.

B 组

1. D. 2. D. 3. 间断且为第一类可去型间断点. 4. 略. 5. 略.

习 题 2-2

A 组

1. (1) x^2; (2) $\ln|x|$; (3) $\dfrac{1}{2}\tan x^2$; (4) $\dfrac{1}{5}\mathrm{e}^{5x}$; (5) $-2\sqrt{1-x^3}$; (6) $2\mathrm{e}^{\sqrt{x}}$.

2. (1) $\dfrac{7}{2}x^{\frac{5}{2}}+3x^{\frac{1}{2}}+x^{-\frac{3}{2}}+\dfrac{3}{2}x^{-\frac{5}{2}}$; (2) $30x^5-3^x\ln 3+\mathrm{e}^{x+1}$;

(3) $2\sec^2 x+\dfrac{1}{\sqrt{1-x^2}}$; (4) $2x\ln x+x$; (5) $\mathrm{e}^{-x}(-\sin x+\cos x)$; (6) $-\dfrac{4x^2}{(1+x^2)^2}$;

(7) $-2\tan 2x$; (8) $\sec x$; (9) $\dfrac{-2x}{\sqrt{x^2(2-x^2)}}$; (10) $y=\dfrac{1}{\sqrt{x^2+a^2}}$;

(11) $-x^{-\frac{1}{3}}(a^{\frac{2}{3}}-x^{\frac{2}{3}})^{\frac{1}{2}}$; (12) $\dfrac{2}{\mathrm{e}^x+\mathrm{e}^{-x}}$; (13) $-\dfrac{1}{x^2}\sin\dfrac{2}{x}\mathrm{e}^{\sin^2\frac{1}{x}}$;

(14) $-\dfrac{2}{(x-1)^2+(x+1)^2}$; (15) $\dfrac{x-a+b}{x}\mathrm{e}^x\cdot\left(\dfrac{b}{x}\right)^a\cdot\left(\dfrac{x}{a}\right)^b$; (16) $\dfrac{\mathrm{e}^x+2}{\mathrm{e}^{2x}+1}$;

(17) $x^{\frac{1}{x}}\dfrac{1-\ln x}{x^2}$; (18) $x^{\arccos x}\left(\dfrac{-\ln x}{\sqrt{1-x^2}}+\dfrac{\arccos x}{x}\right)$.

3. (1) 200; (2) $-\pi$.

4. (1) $\mathrm{e}^{f(x)}[\mathrm{e}^x f'(\mathrm{e}^x)+f(\mathrm{e}^x)\cdot f'(x)]$; (2) $-\dfrac{f'(x)g(x)-f(x)g'(x)}{f^2(x)+g^2(x)}$.

5. a^2. 6. (e,e), $x+2y-3\mathrm{e}=0$. 7. $f'(x)=\begin{cases}-5^{2-x}\ln 5, & x<2,\\ 5^{x-2}\ln 5, & x>2.\end{cases}$

8. (1) -4; (2) 5. 9. $-\dfrac{1}{(x+1)^2}$. 10. $y=\dfrac{3}{2}x$.

B 组

1. A. 2. -2. 3. $\dfrac{3\pi}{4}$. 4. 略.

习 题 2-3

A 组

1. (1) $(x+2)\mathrm{e}^x$; (2) $2\sin t\mathrm{e}^{-t}$; (3) $-\dfrac{x}{(1+x^2)^{\frac{3}{2}}}$; (4) $2\arctan x+\dfrac{2x}{1+x^2}$.

2. (1) $4x^2f''(x^2)+2f'(x^2)$; (2) $\dfrac{f''(x)f(x)-[f'(x)]^2}{f^2(x)}$.

3. 略.

4. (1) $(-1)^n n!\left[\dfrac{1}{(x-1)^{n+1}}-\dfrac{1}{(x+4)^{n+1}}\right]$;

(2) $-2^{n-1}\cos\left(2x+\dfrac{n\pi}{2}\right)$ 或 $2^{n-1}\sin\left[2x+\dfrac{(n-1)\pi}{2}\right]$;

(3) $y'=2x\ln x+x,\quad y''=2\ln x+3,\ y'''=\dfrac{2}{x},\quad y^{(n)}=\dfrac{(-1)^{n-3}2\,(n-3)!}{x^{n-2}}\ (n\geqslant 3)$;

(4) $(-1)^n\,\mathrm{e}^{-x}\left[x^3-3nx^2+3n(n-1)x-n(n-1)(n-2)\right]$.

5. $(\ln 2)^{n-2}n(n-1)$.

B 组

1. $-\dfrac{3}{2}$. 2. $4^{n-1}\cos\left(4x+\dfrac{n\pi}{2}\right)$.

习 题 2-4

A 组

1. (1) $\dfrac{\cos x-y\mathrm{e}^{xy}}{x\mathrm{e}^{xy}+1}$; (2) $\dfrac{ay-x^2}{y^2-ax}$; (3) $\dfrac{x+y}{x-y}(x\neq y)$; (4) $1-\dfrac{\pi}{2}$.

2. (1) $(x-5)\cdot\sqrt[3]{\dfrac{(x-1)^2}{x-3}}\left(\dfrac{1}{x-5}+\dfrac{2}{3}\cdot\dfrac{1}{x-1}-\dfrac{1}{3(x-3)}\right)$;

(2) $\dfrac{\mathrm{e}^{2x}(x+1)}{\sqrt{(x-2)(x+5)}}\left[2+\dfrac{1}{x+1}-\dfrac{1}{2(x-2)}-\dfrac{1}{2(x+5)}\right]$;

(3) $(\sin x)^x(\ln\sin x+x\cot x)$; (4) $-(1+\cos x)^{\frac{1}{x}}\dfrac{x\tan\frac{x}{2}+\ln(1+\cos x)}{x^2}$.

3. (1) $\dfrac{\cos t-\cos 2t}{\sin 2t-\sin t}$; (2) $\dfrac{3\,(1+t)}{2}$; (3) $\dfrac{\cos t-\sin t}{\sin t+\cos t}$; (4) 2.

4. $y=x+a$. 5. (1) $y''=-\dfrac{2}{y^3}\left(1+\dfrac{1}{y^2}\right)(y\neq 0)$; (2) $\dfrac{\mathrm{e}^{2y}\,(3-y)}{(2-y)^3}$.

6. $2\mathrm{e}^2$. 7. $\dfrac{y\,(x\ln y-y)}{x(y\ln x-x)}$, 0. 8. (1) $\dfrac{1}{3\cos^4 t\sin t}$; (2) $\dfrac{1}{4}\tan t$.

9. $2x+3y-2=0$. 10. $\sqrt{2}$. 11. 略. 12. $\dfrac{1}{10\pi}$cm/s.

B 组

1. $\dfrac{f''}{(1-f')^3}$. 2. $y=\dfrac{-2}{\pi}x+\dfrac{\pi}{2}$. 3. $\dfrac{\left(y^2-\mathrm{e}^t\right)\left(1+t^2\right)}{2\,(1-ty)}$.

4. 0. 5. $\dfrac{16}{25\pi}$cm/min.

习 题 2-5

A 组

1. (1) x^4+c; (2) $\ln|x|+c$; (3) $\sqrt{x}+c$; (4) $-\mathrm{e}^{-x}+c$;

(5) $-\dfrac{1}{2}\cos 2x+\arctan x+c$; (6) $x\mathrm{e}^x+c$; (7) $\sin\sqrt{x}+c$; (8) $\dfrac{1}{3}\ln^3 x+c$.

2. B.

3. (1) $\dfrac{\mathrm{d}x}{\sqrt{1+x^2}}$; (2) $\dfrac{2}{1+4x^2}\mathrm{d}x$; (3) $8x\tan\left(1+2x^2\right)\sec^2\left(1+2x^2\right)\mathrm{d}x$;

(4) $(\sin x)^x(\ln\sin x+x\cot x)\mathrm{d}x$; (5) $\mathrm{e}^{2x}(2x^2+8x+5)\mathrm{d}x$;

(6) $\dfrac{2}{x\sqrt{1+x^2}}\mathrm{d}x$; (7) $\dfrac{2x-\mathrm{e}^y}{x\mathrm{e}^y-\mathrm{e}^{-y}+y\mathrm{e}^{-y}}\mathrm{d}x$; (8) $\dfrac{\mathrm{e}^t+t\mathrm{e}^t}{1-2t}\mathrm{d}x$.

4. $\mathrm{e}^{-x}f'\left(1-\mathrm{e}^{-x}\right)\mathrm{d}x$. 5. $\mathrm{d}y|_{x=0}=\left(\mathrm{e}-\mathrm{e}^2\right)\mathrm{d}x$.

6. $\Delta y=-1.141,\mathrm{d}y=-1.2$; $\Delta y=0.1206,\mathrm{d}y=0.12$.

7. $-\dfrac{\sqrt{3}\pi}{45\left(\sqrt{3}+1\right)^3}\approx-0.0059$. 8. $\dfrac{36\pi}{1000}\approx0.113\mathrm{m}^3$. 9. 略.

10. (1) 1.034 9; (2) 2.745 5.

11. $-43.63\mathrm{cm}^2$; $104.72\mathrm{cm}^2$. 12. $5.76\mathrm{m}^2$, $0.24\mathrm{m}^2$.

B 组

1. $\dfrac{1}{x\left(1+\ln y\right)}\mathrm{d}x$. 2. $\mathrm{e}^{f(x)}\left[\dfrac{1}{x}f'\left(\ln x\right)+f'\left(x\right)f\left(\ln x\right)\right]\mathrm{d}x$.

3. (1)$\dfrac{x\cos x-\sin x}{2x^3}$; (2)$1-10x^3-3x^6$.

4. (1) $\mathrm{d}y|_{x=x_0}=2(\cos x_0)\mathrm{d}x, f'(x_0)=2\cos x_0$; (2) 略.

总复习题 2

1. (1) $y=-\dfrac{1}{2}x+\dfrac{3}{2}$; (2) $a=-\pi,\ b=-\pi+1$; (3) 1; (4) $-\dfrac{1}{8}$; (5)$2^{-100}100!$.

2. (1) D; (2)A; (3) B; (4)B ; (5) C.

3. (1) $\dfrac{\mathrm{e}^{\arctan\sqrt{x}}}{2\sqrt{x}(1+x)}$; (2) $-\dfrac{3}{2}$;

(3) $\left(1+x^2\right)^x\left[\ln\left(1+x^2\right)+\dfrac{2x^2}{1+x^2}\right]$; (4) $\dfrac{2x^2\ln x-\left(1+x^2\right)\ln\left(1+x^2\right)}{x\left(1+x^2\right)\ln^2 x}\mathrm{d}x$.

4. (1) $\dfrac{2^x\ln 2-y}{x+2^y\ln 2}$; (2)$\dfrac{y^3}{1-3xy^2-f'\left(y\right)}$.

5. (1) -2; (2)$\dfrac{1}{f''\left(t\right)}$, $-\dfrac{f'''(t)}{[f''(t)]^3}$.

6. (1) $\sin 2x\cdot\left[f'\left(\sin^2 x\right)-f'\left(\cos^2 x\right)\right]$; (2) $2x\mathrm{e}^{x^2}\left[f\left(x^2\right)+f'\left(x^2\right)\right]$.

7. $K>3$. 8. 略.

9. (1) $\dfrac{n!}{(1-x)^{n+1}}$;

(2) $x^2\sin\left(x+\dfrac{n\pi}{2}\right)+2nx\sin\left(x+\dfrac{(n-1)\pi}{2}\right)+n\left(n-1\right)\sin\left(x+\dfrac{(n-2)\pi}{2}\right)$.

10. (1) $\dfrac{g''\left(0\right)-1}{2}$; (2) 处处连续. 11. $2\sqrt{2}V_0$.

12. (1) 略;

(2) $f'(x)=u_1'(x)u_2(x)\cdots u_n(x)+u_1(x)u_2'(x)u_3(x)\cdots u_n(x)+\cdots+u_1(x)u_2(x)u_3(x)\cdots u_n'(x)$.

第 3 章　微分中值定理与导数的应用

习　题　3-1

A 组

1. (1) 满足, 成立;　(2) 不满足, 不成立.

2. 四个根分别位于区间 $(1,2)$, $(2,3)$, $(3,5)$ 与 $(5,6)$ 内.

3–15. 略.

B 组

1–3. 略.

习　题　3-2

A 组

1. (1) $-\pi$;　(2) $\dfrac{7}{11}$;　(3) $1-\ln 2$;　(4) $-\dfrac{1}{2}$;　(5) $\dfrac{1}{2}\left(\beta^2-\alpha^2\right)$;　(6) $\dfrac{1}{6}$;　(7) 2;

(8) ∞;　(9) 1;　(10) $\dfrac{1}{2}$;　(11) $\dfrac{1}{3}$;　(12) e;　(13) $\mathrm{e}^{-\frac{2}{\pi}}$;　(14) $\dfrac{1}{3}$;　(15) $\dfrac{1}{2}$;　(16) $\mathrm{e}^{\frac{1}{2}}$.

2. (1) 1;　(2) 1.　3. -2.　4. 1.　5. 略.　6. 略.

B 组

1. D.　2. 略.　3. 略.

习　题　3-3

A 组

1. $\dfrac{1}{2}+\dfrac{1}{2^2}(x-1)+\dfrac{1}{2^3}(x-1)^2+\dfrac{1}{2^4}(x-1)^3+o\left((x-1)^3\right)$.

2. $\ln 2-\dfrac{3}{2}x-\dfrac{5}{8}x^2-\dfrac{3}{8}x^3+o\left(x^3\right)$.

3. $-2(x+1)^3+11(x+1)^2-13(x+1)-5$.

4. $1+3x+9x^2+(3x)^3+\cdots+(3x)^n+\dfrac{3^n n!}{(1-3\theta x)^{n+1}}x^{n+1}(0<\theta<1)$.

5. $x+x^2+\dfrac{1}{2!}x^3+\cdots+\dfrac{1}{(n-1)!}x^n+o(x^n)$.

6. (1) 0.309017;　(2) 0.1823215.　7. 4 阶.

8. (1) $\dfrac{1}{6}$;　(2) $-\dfrac{1}{32}$.

B 组

1. $a=-1, b=-\dfrac{1}{2}, k=-\dfrac{1}{3}$.

2. 提示: 利用麦克劳林公式及介值定理.

3. 提示: 利用泰勒公式.

习 题 3-4

A 组

1. (1) 单调增加区间为 $\left[\frac{1}{2},+\infty\right)$, 单调减少区间为 $\left(0,\frac{1}{2}\right]$;

(2) 单调增加区间为 $(-\infty,-1],[3,+\infty)$, 单调减少区间为 $[-1,3]$;

(3) 单调增加区间为 $(-\infty,0]$, 单调减少区间为 $[0,+\infty)$;

(4) 单调增加区间为 $[0,+\infty)$, 单调减少区间为 $(-1,0]$;

(5) 单调增加区间为 $(-\infty,-3]$ 与 $[-1,+\infty)$, 单调减少区间为 $[-3,-1]$;

(6) 单调增加区间为 $[0,n]$, 单调减少区间为 $[n,+\infty)$.

2. 略. 3. 小于. 4. 略.

5. (1) 凹区间为 $(-\infty,0)$ 与 $(1,+\infty)$, 凸区间为 $(0,1)$, 拐点为 $(0,0)$ 及 $(1,-1)$;

(2) 凹区间为 $(-1,1)$, 凸区间为 $(-\infty,-1)$ 与 $(1,+\infty)$, 拐点为 $(-1,\ln 2)$ 及 $(1,\ln 2)$;

(3) 凹区间为 $\left(-\infty,\frac{1}{2}\right)$, 凸区间为 $\left(\frac{1}{2},+\infty\right)$, 拐点为 $\left(\frac{1}{2},\mathrm{e}^{\arctan\frac{1}{2}}\right)$;

(4) 凹区间为 $(b,+\infty)$; 凸区间为 $(-\infty,b)$, 拐点为 (b,a).

6. 略. 7. $a=-1,b=2$. 8. $k=\pm\dfrac{\sqrt{2}}{8}$.

9. $f(x)$ 在 $(-\infty,0)$ 与 $(0,+\infty)$ 内都是凸的, 不存在拐点.

10. $(1,4),(1,-4)$. 11. 略.

B 组

1. C. 2. B. 3. 略. 4. 略.

习 题 3-5

A 组

1. (1) 当 $x=2$ 时取得极小值为 3; (2) 在 $x=0$ 处取极小值 0;

(3) 在 $x=\dfrac{\pi}{4}+2k\pi(k\in\mathbf{Z})$ 处取极大值 $\dfrac{\sqrt{2}}{2}\mathrm{e}^{\frac{\pi}{4}+2k\pi}$, 在 $x=\dfrac{5\pi}{4}+2k\pi(k\in\mathbf{Z})$ 处取极小值 $-\dfrac{\sqrt{2}}{2}\mathrm{e}^{\frac{5\pi}{4}+2k\pi}$;

(4) 在 $x=0$ 处取极大值 0, 在 $x=1$ 处取极小值 -3;

(5) 在 $x=\dfrac{1}{3}$ 时取极大值为 $\dfrac{\sqrt[3]{4}}{3}$; 在 $x=1$ 时取极小值为 0;

(6) 在 $x=\mathrm{e}$ 处取极大值 $\mathrm{e}^{\frac{1}{\mathrm{e}}}$.

2. $a=-\dfrac{2}{3},b=-\dfrac{1}{6}$, 极小值 $f(1)=\dfrac{5}{6}$, 极大值 $f(2)=\dfrac{4}{3}-\dfrac{2}{3}\ln 2$.

3. $a=1,b=-3,c=-24,d=16$. 4. 当 $x=1$ 时取极小值 $y(1)=1$.

5. (1) 最小值 $y(1)=-\dfrac{5}{2}$, 最大值 $y(4)=65$; (2) 最小值 $y(\pm2)=\sqrt[3]{4}-\sqrt[3]{3}$, 最大值 $y\left(\pm\dfrac{\sqrt{2}}{2}\right)=\sqrt[3]{4}$; (3) 无最小值, 最大值 $y\left(\dfrac{\pi}{4}\right)=1$.

6. 极大值 $f\left(\arcsin\dfrac{1}{a}\right)=\pi$. 7. x^3-6x^2+9x+2. 8. $x+y-2=0$.

9. 当圆柱形锅炉的高与底直径都等于 $\sqrt[3]{20}$ 米时, 用料最省.

10. 圆形的周长为 $\dfrac{\pi a}{4+\pi}$, 正方形的周长为 $\dfrac{4a}{4+\pi}$. 11. 略. 12. 略.

B 组

1. D. 2. 略. 3. 略.

4. $f'(x)=\begin{cases}2x^{2x}(\ln x+1), & x>0,\\ \mathrm{e}^x(x+1), & x<0,\end{cases}$

$f(x)$ 极大值有 $f(0)=1$, 极小值有 $f(-1)=1-\dfrac{1}{\mathrm{e}}$ 与 $f\left(\dfrac{1}{\mathrm{e}}\right)=\mathrm{e}^{-\frac{2}{\mathrm{e}}}$.

习 题 3-6

A 组

1. (1) $y=x-\dfrac{2}{3}$ 为曲线的斜渐近线; (2) $y=0$ 为曲线的水平渐近线, $x=2$ 为曲线的垂直渐近线; (3) 水平渐近线为 $y=1$; (4) 水平渐近线为 $y=\dfrac{\pi}{4}$, 垂直渐近线为 $x=0$.

2. (1)—(4) 略.

B 组

1. $y=x+\dfrac{\pi}{2}$. 2. C.

3. 水平渐近线为 $y=1$, 垂直渐近线为 $x=0$.

习 题 3-7

A 组

1. (1) $|\csc x|\,\mathrm{d}x$; (2) $\dfrac{\mathrm{e}^{\frac{x}{a}}+\mathrm{e}^{-\frac{x}{a}}}{2}\mathrm{d}x$ 或 $\mathrm{ch}\dfrac{x}{a}\mathrm{d}x$; (3) $\sqrt{1+9x^4}\mathrm{d}x$.

2. (1) $K=\dfrac{2}{\sqrt[3]{25}},\rho=\dfrac{\sqrt[3]{25}}{2}$; (2) $K=\dfrac{1}{2\sqrt{2}},\rho=2\sqrt{2}$; (3) $K=2,\rho=\dfrac{1}{2}$.

3. $k=\dfrac{2\sqrt{2}ab}{(a^2+b^2)^{\frac{3}{2}}}$. 4. $x^2+\left(y-\dfrac{1}{2}\right)^2=\dfrac{1}{4}$. 5. $\left(\dfrac{9}{8},-3\right)$ 和 $\left(\dfrac{9}{8},3\right)$. 6. $\dfrac{2}{3}$.

B 组

1. $\left(\dfrac{\sqrt{2}}{2},-\dfrac{1}{2}\ln 2\right),R=\dfrac{3\sqrt{3}}{2}$. 2. $\dfrac{(1.2)^{3/2}}{2\sqrt[4]{0.2}}$.

总复习题 3

1. (1) $[0,+\infty)$; (2) 3; (3) -14; (4) $\dfrac{1}{2}$; (5) $y=x+2$.

2. (1) D; (2) D; (3) D; (4) C; (5)D.

3. 略. 4. 略. 5. 略.

6. (1) $-\dfrac{1}{3}$; (2) $\dfrac{2}{3}$; (3) $\dfrac{1}{6}$; (4) $\dfrac{1}{2}$; (5) a; (6) $\mathrm{e}^{-\frac{2}{\pi}}$.

7. 1.　8. 略.

9. 当 $a>0$ 时, 极大值 $y\left(-\sqrt{a}\right)=2\left(a\sqrt{a}+1\right)$, 极小值 $y\left(\sqrt{a}\right)=2\left(1-a\sqrt{a}\right)$; 当 $a>1$ 时, 方程有三个不同的实根; $0<a<1$ 时方程有唯一实根.

10. (1) 略;　(2) $f(0)$ 为 $f(x)$ 的极小值.　11. $(1,2)$ 和 $(-1,-2)$.

12. (1) $y-\dfrac{1}{x_0^2}=-\dfrac{2}{x_0^3}(x-x_0)$;　(2) $\dfrac{3\sqrt{3}}{2}$.　13. $h=\dfrac{\sqrt{2}r}{2}$.　14. $\sqrt[3]{3}$.

15. (1) 最小值 $f(1)=1$;　(2) 利用单调有界定理证明, $\lim\limits_{n\to\infty}x_n=1$.　16. 略.

第 4 章　不 定 积 分

习　题　4-1

A 组

1. $\arccos\sqrt{x}+C$.　2. $f(x)=x+\dfrac{1}{3}x^3+1$.

3. (1) $-\dfrac{2}{\sqrt{x}}-\ln x+\mathrm{e}^x+C$;　(2) $\dfrac{2^x}{\ln 2}+\dfrac{1}{3}x^3+C$;

(3) $\ln|x|+3\arctan x-2\arcsin x+C$;　(4) $\mathrm{e}^x-3\ln|x|+C$;

(5) $\sin x-\cos x+C$;　(6) $-\cot x-x+C$;　(7) $\dfrac{1}{3}x^3-x+\arctan x+C$;

(8) $-\dfrac{2}{5^x\ln 5}+\dfrac{1}{5\ln 2\cdot 2^x}+C$;　(9) $x-\ln x+C$;　(10) $\mathrm{e}^x-2\sqrt{x}+C$;

(11) $-4\cot x+C$;　(12) $-\dfrac{1}{x}+\arctan x+C$;　(13) $\dfrac{90^x}{\ln 90}+C$;

(14) $\tan x-\sec x+C$;　(15) $\dfrac{1}{2}x+\dfrac{1}{2}\sin x+C$;　(16) $\dfrac{1}{2}\tan x+C$.

4. $y=\ln x+\dfrac{1}{x}-1$.　5. $y(t)=\dfrac{2}{3}kt^{\frac{3}{2}}$.

B 组

1. (1) $\dfrac{1}{x\sqrt{1-x^2}}$;　(2) $-\sin x+C_1x+C_2$;　(3) $4\mathrm{e}^{2x}$.

2. $f(x)=x-\dfrac{1}{2}x^2$.　3. $\begin{cases}\mathrm{e}^x, & x\geqslant 0,\\ x+\dfrac{1}{2}x^2+1, & x<0.\end{cases}$

4. $\begin{cases}-\dfrac{1}{2}x^2-\dfrac{1}{2}+C, & x<-1,\\ x+C, & -1\leqslant x\leqslant 1,\\ \dfrac{1}{2}x^2+\dfrac{1}{2}+C, & x>1.\end{cases}$

习　题　4-2

A 组

1. (1) $\dfrac{1}{7}$;　(2) $\dfrac{1}{12}$;　(3) $-\dfrac{1}{2}$;　(4) $\dfrac{1}{3}$;　(5) $\dfrac{\sin(\omega t+\varphi)}{\omega}$;

(6) $\dfrac{1}{k}\mathrm{e}^{kx}$;　(7) $\sqrt{x^2+a^2}$;　(8) $\dfrac{1}{2}\sin^2 x$.

2. (1) $-\frac{1}{7}(1-x)^7+C$; (2) $\frac{2}{9}(2+3x)^{\frac{3}{2}}+C$; (3) $\frac{1}{12}\left(1+2x^2\right)^3+C$;

(4) $\frac{1}{a}\ln|ax+b|+C$; (5) $\ln(1+\mathrm{e}^x)+C$; (6) $\mathrm{e}^{\mathrm{e}^x}+C$;

(7) $-\frac{1}{3}\cos(3x+2)+C$; (8) $\arcsin(x-1)+C$; (9) $-\frac{4}{3}(1-\sqrt{x})^{\frac{3}{2}}+C$;

(10) $\ln|\ln\ln x|+C$; (11) $-\ln\left|\cos\sqrt{1+x^2}\right|+C$; (12) $\arctan\mathrm{e}^x+C$;

(13) $-\frac{3}{4}\ln\left|1-x^4\right|+C$; (14) $-\frac{1}{3\omega}\cos^3(\omega t+\varphi)+C$;

(15) $\frac{3}{2}\sqrt[3]{(\sin x-\cos x)^2}+C$; (16) $\sin x-\frac{\sin^3 x}{3}+C$;

(17) $\frac{t}{2}+\frac{1}{4\omega}\sin 2(\omega t+\varphi)+C$; (18) $\frac{1}{3}\sin\frac{3x}{2}+\sin\frac{x}{2}+C$;

(19) $\ln|\tan x|+C$; (20) $-\frac{1}{10}\cos 5x+\frac{1}{2}\cos x+C$;

(21) $\frac{1}{3}\sec^3 x-\sec x+C$; (22) $-\frac{10^{2\arccos x}}{2\ln 10}+C$; (23) $\arctan^2\sqrt{x}+C$;

(24) $-\frac{1}{x\ln x}+C$; (25) $\frac{1}{2}\ln^2\tan x+C$; (26) $\arccos\frac{1}{|x|}+C$.

3. $-\frac{1}{2}\left(1-x^2\right)^2+C$. 4. $-\frac{1}{x-2}-\frac{1}{3}(x-2)^3+C$.

B 组

(1) $\sqrt{x^2-9}-3\arccos\frac{3}{|x|}+C$; (2) $\frac{a^2}{2}\left(\arcsin\frac{x}{a}-\frac{x}{a^2}\sqrt{a^2-x^2}\right)+C$;

(3) $\sqrt{2x}-\ln(1+\sqrt{2x})+C$; (4) $\frac{1}{3}\ln\left|\frac{x-2}{x+1}\right|+C$;

(5) $\frac{1}{2}\arctan\frac{x+1}{2}+C$; (6) $\frac{1}{\sqrt{1+x^2}}+\sqrt{1+x^2}+C$;

(7) $\frac{1}{2}\arctan x-\frac{x}{2(1+x^2)}+C$; (8) $-\frac{\sqrt{x^2+a^2}}{a^2x}+C$;

(9) $-\frac{1}{14}\ln\left|x^7+2\right|+\frac{1}{2}\ln|x|+C$; (10) $\ln\frac{\sqrt{1+\mathrm{e}^x}-1}{\sqrt{1+\mathrm{e}^x}+1}+C$;

(11) $\sqrt{1+x^2}-\ln\left(1+\sqrt{1+x^2}\right)+C$; (12) $-\frac{1}{\sqrt{x^2+2x}}+C$.

习 题 4-3

A 组

1. (1) $-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x+C$; (2) $x(\ln x)^2-2x\ln x+2x+C$;

(3) $-\frac{1}{2}\cot x\csc x+\frac{1}{2}\ln|\csc x-\cot x|+C$; (4) $\frac{x}{2}[\sin(\ln x)-\cos(\ln x)]+C$;

(5) $-\frac{1}{2}x^2\mathrm{e}^{-x^2}-\frac{1}{2}\mathrm{e}^{-x^2}+C$; (6) $2\sqrt{x}\mathrm{e}^{\sqrt{x}}-2\mathrm{e}^{\sqrt{x}}+C$;

(7) $\ln x(\ln\ln x-1)+C$; (8) $x\tan x+\ln|\cos x|-\frac{1}{2}x^2+C$;

(9) $-\dfrac{2}{17}\mathrm{e}^{-2x}\left(\cos\dfrac{x}{2}+4\sin\dfrac{x}{2}\right)+C$; (10) $\dfrac{1}{3}x^3\arctan x-\dfrac{1}{6}x^2+\dfrac{1}{6}\ln\left(1+x^2\right)+C$;

(11) $\dfrac{1}{(1-n)x^{n-1}}\left(\ln x-\dfrac{1}{1-n}\right)+C$; (12) $-x\cot x+\ln|\sin x|+C$;

(13) $-\dfrac{1}{4}x\cos 2x+\dfrac{1}{8}\sin 2x+C$; (14) $x\ln\left(1+x^2\right)-2x+2\arctan x+C$;

(15) $\dfrac{x^2}{2}\mathrm{e}^{x^2}+C$; (16) $-\dfrac{1}{2}x^4\cos x^2+x^2\sin x^2+\cos x^2+C$;

(17) $\sqrt{1+x^2}\arctan x-\ln\left(x+\sqrt{1+x^2}\right)+C$; (18) $x\tan x+\ln|\cos x|+C$.

2. $-\dfrac{x\sin x+\cos x}{x}-\dfrac{\cos x}{x}+C$; $\dfrac{-x^2\cos x+2x\sin x+2\cos x}{x^2}+\dfrac{x\sin x+\cos x}{x^2}+C$.

3. $-\dfrac{\ln(1+\mathrm{e}^x)}{\mathrm{e}^x}-\ln(1+\mathrm{e}^x)+x+C$. 4. $-2\sqrt{1-x}\arcsin\sqrt{x}+2\sqrt{x}+C$.

B 组

1. (1) $-\dfrac{x^2\mathrm{e}^x}{x+2}+x\mathrm{e}^x-\mathrm{e}^x+C$; (2) $-\dfrac{\arctan\mathrm{e}^x}{2\mathrm{e}^{2x}}-\dfrac{1}{2\mathrm{e}^x}-\dfrac{1}{2}\arctan\mathrm{e}^x+C$;

(3) $\mathrm{e}^{\arctan x}+C$; (4) $x\left[\sin^2(\ln x)-\dfrac{1}{5}\sin(2\ln x)+\dfrac{2}{5}\cos(2\ln x)\right]+C$.

2. $xf^{-1}(x)-F[f^{-1}(x)]+C$. 3. $\dfrac{1}{2}\mathrm{e}^{2x}\arctan\sqrt{\mathrm{e}^x-1}-\dfrac{1}{6}(\mathrm{e}^x+2)\sqrt{\mathrm{e}^x-1}+C$.

4. 略.

习 题 4-4

A 组

(1) $-\dfrac{1}{33(x-1)^{99}}-\dfrac{3}{49(x-1)^{98}}-\dfrac{6}{97(x-1)^{97}}-\dfrac{1}{48(x-1)^{96}}+C$;

(2) $\dfrac{1}{3}x^3+\dfrac{1}{2}x^2+x+8\ln|x|-4\ln|x+1|-3\ln|x-1|+C$;

(3) $\ln|x+1|-\dfrac{1}{2}\ln\left|x^2-x+1\right|+\sqrt{3}\arctan\dfrac{2x-1}{\sqrt{3}}+C$;

(4) $\dfrac{1}{x+1}+\dfrac{1}{2}\ln\left|x^2-1\right|+C$;

(5) $-\dfrac{1}{2}\ln\dfrac{x^2+1}{x^2+x+1}+\dfrac{\sqrt{3}}{3}\arctan\dfrac{2x+1}{\sqrt{3}}+C$;

(6) $\dfrac{x^3}{3}-\dfrac{x^2}{2}+3x-\dfrac{16}{3}\ln(x+2)+\dfrac{1}{3}\ln|x-1|+C$;

(7) $\dfrac{u}{16\left(u^2+4\right)^2}+\dfrac{3u}{128\left(u^2+4\right)}+\dfrac{3}{256}\arctan\dfrac{u}{2}+C$;

(8) $x+3\ln|x-3|-3\ln|x-2|+C$; (9) $\dfrac{1}{8}\cot^2\dfrac{x}{2}-\dfrac{1}{4}\ln\left|\tan\dfrac{x}{2}\right|+C$;

(10) $\dfrac{1}{2}\ln|\tan x|+\dfrac{1}{2}\tan x+C$; (11) $\dfrac{1}{4}\tan^2\dfrac{x}{2}+\tan\dfrac{x}{2}+\dfrac{1}{2}\ln\left|\tan\dfrac{x}{2}\right|+C$;

(12) $-\dfrac{1}{2}\ln\dfrac{1+\cos x}{1-\cos x}+\dfrac{1}{2\sqrt{2}}\ln\dfrac{\sqrt{2}+\cos x}{\sqrt{2}-\cos x}+C$;

(13) $\frac{1}{2}(\sin x-\cos x)-\frac{1}{2\sqrt{2}}\ln\left|\csc\left(x+\frac{\pi}{4}\right)-\cot\left(x+\frac{\pi}{4}\right)\right|+C$;

(14) $2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ln(1+\sqrt[6]{x})+C$; (15) $4\ln\left(\frac{\sqrt{x+3}+\sqrt{x-1}}{\sqrt{x+3}-\sqrt{x-1}}\right)+C$;

(16) $2\sqrt{3-4x}-\sqrt{3}\ln\left|\frac{\sqrt{3-4x}+\sqrt{3}}{\sqrt{3-4x}-\sqrt{3}}\right|+C$;

(17) $\ln\left|\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\right|-2\arctan\sqrt{\frac{1-x}{1+x}}+C$;

(18) $x-4\sqrt{1+x}+\ln(\sqrt{1+x}+1)^4+C$.

B 组

(1) $\frac{12}{7}(1+x^{\frac{1}{4}})^{\frac{7}{3}}-3(1+x^{\frac{1}{4}})^{\frac{4}{3}}+C$;

(2) $-\frac{1}{4}\ln\frac{\sqrt[4]{1+x^4}+x}{\sqrt[4]{1+x^4}-x}+\frac{1}{2}\arctan\frac{\sqrt[4]{1+x^4}}{x}+C$;

(3) $\ln\left|\cos^2 x-2\cos x-1\right|+\frac{1}{\sqrt{2}}\ln\left|\frac{\cos x-1-\sqrt{2}}{\cos x-1+\sqrt{2}}\right|+C$;

(4) $\frac{1}{2}(x+\ln|\cos x|+x\tan x)+C$;

(5) $3(1+x)^{\frac{1}{3}}-6(1+x)^{\frac{1}{6}}+6\ln\left[1+(1+x)^{\frac{1}{6}}\right]+C$;

(6) $\frac{1}{\sqrt[3]{3x+2}+1}+\frac{5}{3}\ln|\sqrt[3]{3x+2}+1|+\frac{4}{3}\ln|\sqrt[3]{3x+2}-2|+C$;

(7) $2\ln\left|x+\sqrt{x^2-x+1}\right|-\frac{3}{2}\ln\left|2x+2\sqrt{x^2-x+1}-1\right|-\frac{3}{2\left(2x+2\sqrt{x^2-x+1}-1\right)}+C$;

(8) $-2\ln|x-1|-\frac{3}{x-1}+\ln(x^2+x+1)+C$.

习 题 4-5

(1) $\frac{1}{2}\ln|2x+\sqrt{4x^2-9}|+C$; (2) $\frac{1}{2}\arctan\frac{x+1}{2}+C$;

(3) $\ln\left[(x-2)+\sqrt{5-4x+x^2}\right]+C$; (4) $\frac{x}{2(1+x^2)}+\frac{1}{2}\arctan x+C$;

(5) $\left(\frac{x^2}{2}-1\right)\arcsin\frac{x}{2}+\frac{x}{4}\sqrt{4-x^2}+C$;

(6) $-\sqrt{1+x-x^2}+\frac{1}{2}\arcsin\frac{2x-1}{\sqrt{5}}+C$;

(7) $\frac{1}{6}\cos^5 x\sin x+\frac{5}{24}\cos^3 x\sin x+\frac{15}{24}\left(\frac{x}{2}+\frac{\sin 2x}{4}\right)+C$;

(8) $\frac{1}{\sqrt{21}}\ln\left|\frac{\sqrt{3}\tan\frac{x}{2}+\sqrt{7}}{\sqrt{3}\tan\frac{x}{2}-\sqrt{7}}\right|+C$; (9) $\frac{e^{2x}}{5}(\sin x+2\cos x)+C$;

(10) $\frac{1}{2}\ln|x^2-2x-1|+\frac{3}{\sqrt{2}}\ln\left|\frac{x-(\sqrt{2}+1)}{x+(\sqrt{2}+1)}\right|+C$;

(11) $\dfrac{1}{12}x^3-\dfrac{25}{16}x+\dfrac{125}{32}\arctan\dfrac{2x}{5}+C$;

(12) $x\ln^3 x-3x\ln^2 x+6x\ln x-6x+C$.

总复习题 4

1. (1) $2x\mathrm{e}^{2x}(1+x)$; (2) $F(\ln x)+C$; (3) $\dfrac{1}{2}x^2+\dfrac{1}{4}x^4+C$;
(4) $-\ln(1-x)-x^2+C$; (5) $\dfrac{1}{2}f\left[\cos\left(1-x^2\right)\right]+C$.

2. (1) C; (2) B; (3) A; (4) B; (5) D.

3. (1) $\mathrm{e}^x+\tan x+C$; (2) $2\sqrt{\mathrm{e}^x-1}-2\arctan\sqrt{\mathrm{e}^x-1}+C$;
(3) $\dfrac{1}{\ln 2}\arcsin 2^x+C$; (4) $-\ln\left|\dfrac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|+C$;
(5) $-\ln\left(1+\sqrt{4-x^2}\right)+C$; (6) $-\dfrac{1}{1+\tan x}+C$;
(7) $\dfrac{2-x}{4\left(x^2+2\right)}+\ln\sqrt{x^2+2}-\dfrac{\sqrt{2}}{8}\arctan\dfrac{x}{\sqrt{2}}+C$;
(8) $2\left[\sqrt{x}-\ln(1+\sqrt{x})\right]+C$; (9) $\dfrac{1}{20}\ln\dfrac{x^{10}}{2+x^{10}}+C$;
(10) $x+\ln|5\cos x+2\sin x|+C$; (11) $\dfrac{2}{5}(x\ln x)^{\frac{5}{2}}+C$; (12) $\mathrm{e}^x\tan\dfrac{x}{2}+C$;
(13) $x\tan\dfrac{x}{2}+2\ln|\cos x|+C$; (14) $-\dfrac{x\mathrm{e}^x}{x+1}+\mathrm{e}^x+C$;
(15) $(x+1)\arctan\sqrt{x}-\sqrt{x}+C$; (16) $x\tan x+\ln|\cos x|+C$;
(17) $\ln\left|\dfrac{\mathrm{e}^x+1}{\mathrm{e}^x+2}\right|+C$; (18) $\ln\left|\tan\dfrac{x}{2}\right|-\cos x\ln\tan x+C$;
(19) $\dfrac{1}{2}\arctan(\sin^2 x)+C$; (20) $\ln|\csc 2x-\cot 2x|-\dfrac{1}{2}\csc 2x+C$;
(21) $-\dfrac{\sqrt{2}}{16}x-\dfrac{3\sqrt{2}}{10}\ln|2\cos x+\sin x|+C$; (22) $2\ln\left|\cos\dfrac{x}{2}\right|+\tan\dfrac{x}{2}+C$;
(23) $\dfrac{1}{2}\arctan(2\sin^2 x-1)+C$; (24) $-\dfrac{1}{6}\sin 3x+\dfrac{1}{2}\sin x+C$;
(25) $-2\sqrt{1-x}\arccos\sqrt{x}+2\sqrt{x}+C$; (26) $\dfrac{1}{2}(x+\ln|\cos x|+x\tan x)+C$.

4. $\dfrac{f(x)}{x\mathrm{e}^x}+C$. 5. $I_n=x\ln^n x-nI_{n-1},\quad n\geqslant 1$.

6. $\displaystyle\int\max\left\{1,x^2,x^3\right\}\mathrm{d}x=\begin{cases}\dfrac{1}{3}(x^2-2)+C, & x\leqslant -1,\\ x+C, & -1<x<1,\\ \dfrac{1}{4}(x^4+3)+C, & x\geqslant 1.\end{cases}$

7. $\dfrac{x\mathrm{e}^{\frac{x}{2}}}{2(1+x)^{\frac{3}{2}}}$. 8. $\tan x+\sec x+C$.

第5章 定 积 分

习 题 5-1

A 组

1. (1) $\frac{1}{2}$; (2) $\mathrm{e}-1$. 2. (1) $\frac{\pi}{4}$; (2) 0.

3. $M=\int_a^b (1+x^2)\mathrm{d}x$. 4. $Q=\int_{T_1}^{T_2} \sin(\omega t)\mathrm{d}t$. 5. $\int_{-1}^{1}(2-2x^2)\mathrm{d}x$.

6. (1) $\int_0^1 x^2\mathrm{d}x \geqslant \int_0^1 x^3\mathrm{d}x$; (2) $\int_3^4 \ln^2 x\mathrm{d}x \leqslant \int_3^4 \ln^3 x\mathrm{d}x$;

(3) $\int_{-\frac{\pi}{2}}^{0} \sin x\mathrm{d}x \leqslant \int_0^{\frac{\pi}{2}} \sin x\mathrm{d}x$; (4) $\int_1^0 \ln(1+x)\mathrm{d}x < \int_1^0 \frac{x}{1+x}\mathrm{d}x$.

7. (1) $6 \leqslant I \leqslant 51$; (2) $1 \leqslant I \leqslant \mathrm{e}$; (3) $\int_0^{\frac{\pi}{2}} \sin x\mathrm{d}x$; (4) $-2\mathrm{e}^2 \leqslant I \leqslant -2\mathrm{e}^{-\frac{1}{4}}$.

8. 略.

B 组

1. 0. 2. $a=0,\ b=1$. 3. 略. 4. $2\int_0^1 \ln(1+x)\mathrm{d}x$.

习 题 5-2

A 组

1. (1) $\sqrt{1+x^2}$; (2) $\sin x^2$; (3) $\frac{2\cos t}{\sqrt{t}}$; (4) $2\ln(1+4x^2)-\ln(1+x^2)$.

2. (1) 0; (2) $\frac{7}{3}$.

3. (1) $\frac{29}{6}$; (2) $\sqrt{2}-1$; (3) $\frac{\pi}{6}$; (4) $\frac{3}{2}$; (5) $1+\frac{\pi}{4}$; (6) $1-\frac{\pi}{4}$;

(7) $\frac{\pi}{4}+\frac{1}{2}$; (8) $\frac{\pi}{8}$; (9) $\frac{\pi}{6}$; (10) $\frac{10}{3}$; (11) $1-\frac{1}{\sqrt{3}}+\frac{\pi}{12}$; (12) $2\sqrt{2}$.

4. (1) $\frac{4}{5}$; (2) $2(\sqrt{2}-1)$; (3) $\frac{10}{3}$; (4) $\frac{23}{6}$.

5. $\Phi(x)=\begin{cases} 0, & x<0, \\ \frac{1}{2}(1-\cos x), & 0\leqslant x\leqslant \pi, \\ 1, & x>\pi, \end{cases}$ 6. 略.

B 组

1. (1) $\sqrt[3]{36}$; (2) $\frac{3\sqrt{2}+2}{2}$. 2. $x=0$ 时, 取极小值 $y=0$. 3. 略.

4. $f(x)=4x+5,\ a=\frac{1}{2}$ 或 -3. 5. 略.

习 题 5-3

A 组

1. (1) $\frac{7}{72}$; (2) $\frac{\pi}{2}$; (3) $-\ln(\sqrt{2}-1)$; (4) $\frac{\pi}{2}-\frac{4}{3}$; (5) 12;
(6) $\arctan e$; (7) $2(\sqrt{3}-1)$; (8) 2; (9) 1; (10) $\frac{2}{7}$;
(11) $e^{-\frac{1}{2}}-e^{-1}$; (12) $\frac{4}{5}$; (13) $\frac{2}{3}$; (14) $2\sqrt{2}$; (15) $\frac{1}{2}-\ln\sqrt{2}$;
(16) $\frac{\pi}{4}-\frac{2}{3}$; (17) $\frac{5}{3}$; (18) $7+2\ln 2$; (19) $2+\ln\frac{3}{2}$; (20) $\frac{1}{4}-\frac{1}{2(e^2+1)}$.

2. (1) 1; (2) $\frac{\pi}{4}-\frac{1}{2}$; (3) $\frac{e}{2}(\sin 1-\cos 1)+\frac{1}{2}$; (4) $\frac{\pi^3}{6}-\frac{\pi}{4}$;
(5) $8\ln 2-4$; (6) $\frac{\pi}{4}-\frac{\sqrt{3}}{9}\pi+\ln\frac{\sqrt{6}}{2}$; (7) $2-\frac{2}{e}$; (8) $\frac{1}{5}(e^{\pi}-2)$.

3. (1) 0; (2) $4\sqrt{2}$; (3) $\ln 3$; (4) $\frac{\pi}{12}$. 4. $\frac{62}{3}$. 5. 20.

6. 提示: 令 $t=a+b-x$. 7. 提示: 令 $t=\frac{1}{u}$. 8. $-\ln\pi-\sin 1$. 9. 略.

B 组

1. (1) $-\frac{4}{3}$; (2) $\ln\frac{(1+\sqrt{2})e}{1+\sqrt{1+e^2}}$; (3) $\frac{\pi}{6}$; (4) $\frac{\sqrt{2}}{2}$; (5) $\frac{1}{3}$; (6) $\frac{\pi}{2}$;
(7) $\frac{\pi^2}{64}+\frac{\pi}{16}-\frac{1}{8}$; (8) $\frac{\pi}{8}-\frac{1}{4}$; (9) $\frac{\ln 2}{3}$; (10) $\frac{\pi}{4}-\ln\sqrt{2}$.

2. 1. 3. $f(x)=x-1$.

4. $I_m=\begin{cases}\frac{m-1}{m}\cdot\frac{m-3}{m-2}\cdot\frac{m-5}{m-4}\cdots\cdots\frac{4}{5}\cdot\frac{2}{3}m, & m\text{为正奇数},\\ \frac{m-1}{m}\cdot\frac{m-3}{m-2}\cdot\frac{m-5}{m-4}\cdots\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi^2}{2}, & m\text{为正偶数}.\end{cases}$ 5. 略.

习 题 5-4

A 组

1. (1) $\frac{1}{3}$; (2) $\frac{\pi}{4}+\frac{1}{2}\ln 2$; (3) $1-\ln 2$; (4) π; (5) $\frac{1}{a^2+4}$;
(6) $2(1-\ln 2)$; (7) 发散; (8) $\frac{\pi}{2}$; (9) 发散; (10) $\frac{1}{8}\sqrt{\frac{\pi}{2}}$.

2. $e^{-\frac{1}{4}}\pi^{-\frac{1}{2}}$.

3. 当 $k>1$ 时, 收敛于 $\frac{1}{(k-1)(\ln 2)^{k-1}}$; 当 $k\leqslant 1$ 时, 发散; 当 $k=1-\frac{1}{\ln\ln 2}$ 时取最小值.

4. 4000 人.

5. (1) 收敛; (2) 收敛; (3) 发散; (4) 收敛; (5) 收敛; (6) 收敛; (7) 发散;
(8) 发散; (9) 收敛; (10) $m<3$ 时, 收敛; $m\geqslant 3$ 时, 发散.

6. 略.

B 组

1. $-\dfrac{1}{2}$. 2. 3. 3. (1) 条件收敛; (2) 条件收敛. 4. 略.
5. $0<\alpha<1$ 时, 收敛; $\alpha\geqslant 1$ 时, 发散.

总复习题 5

1. (1) 2; (2) 7; (3) $\dfrac{1}{4}(\cos 1-1)$; (4) $\mathrm{e}^{x}+6x$; (5) $\dfrac{\sqrt{3}+1}{12}\pi$.
2. (1) C; (2) C; (3) A; (4) D; (5) D.
3. (1) $\dfrac{1}{3}$; (2) $\dfrac{3}{2}$; (3) 1; (4) $\dfrac{2}{3}(2\sqrt{2}-1)$; (5) e^{-1}; (6) 0; (7) 0.
4. 提示: $1-x^{p}<\dfrac{1}{1+x^{p}}<1$.
5. (1) 提示: $\forall t\in\mathbf{R}$, $\displaystyle\int_a^b f^2(x)\mathrm{d}x+2t\int_a^b f(x)g(x)\mathrm{d}x+t^2\int_a^b g^2(x)\mathrm{d}x\geqslant 0$. (2) 略.
6. (1) $\dfrac{\pi}{2}$; (2) $\dfrac{\pi}{8}\ln 2$; (3) $-\dfrac{\pi}{2}\ln 2$; (4) $\dfrac{5\pi}{64}$; (5) $\dfrac{\pi}{2\sqrt{2}}$; (6) $\pi+\ln(2+\sqrt{3})$;
(7) $\dfrac{\pi}{4\mathrm{e}^2}$; (8) 0.
7. $\dfrac{1}{2}$. 8. 0. 9. (1) 略; (2) $\dfrac{\pi}{2}$. 10. 略. 11. 略. 12. 略.

第 6 章 定积分的应用

习 题 6-2

A 组

1. (1) $\dfrac{3}{2}-\ln 2$; (2) $\mathrm{e}+\dfrac{1}{\mathrm{e}}-2$; (3) $\dfrac{16}{3}p^2$; (4) $\dfrac{2\pi\sqrt{\pi}}{3}-2$.
2. $\dfrac{3}{8}\pi a^2$. 3. $3\pi a^2$. 4. (1) $\dfrac{3}{2}\pi a^2$; (2) $\dfrac{\pi}{6}+\dfrac{1-\sqrt{3}}{2}$.
5. (1) $\dfrac{18}{35}\pi$; (2) $\dfrac{128\pi}{7}, \dfrac{64\pi}{5}$. 6. $a=7\sqrt{7}$. 7. $2\pi\left(1-\dfrac{\mathrm{e}}{3}\right)$.
8. $5\pi^2a^3, 6\pi^3a^3, 7\pi^2a^3$. 9. $\dfrac{16}{3}R^3$.
10. (1) $2\sqrt{3}-\dfrac{4}{3}$; (2) 4; (3) $\dfrac{\sqrt{1+a^2}}{a}(\mathrm{e}^{a\varphi}-1)$; (4) $8a$. 11. $\dfrac{\pi^2a}{2}$.

B 组

1. 略. 2. πa^2.

习 题 6-3

A 组

1. $\dfrac{1}{4}\pi gR^4\cdot 10^3(\mathrm{J})$

2. $9.72 \times 10^5\,(\mathrm{kJ})$. 3. $\dfrac{4}{3} \times 10^3 \pi g R^4$. 4. $\dfrac{2}{3}\rho g R^3$. 5. $17.3\,(\mathrm{kN})$.

6. $F = (F_x, F_y)$, 其中 $F_x = km\mu\left(\dfrac{1}{a} - \dfrac{1}{\sqrt{a^2+l^2}}\right)$, $F_y = -\dfrac{km\mu l}{a\sqrt{a^2+l^2}}$.

B 组

1. $\sqrt{2}-1\,(\mathrm{cm})$. 2. 引力的大小为 $\dfrac{2Gm\mu}{R}\sin\dfrac{\varphi}{2}$, 方向为 M 指向圆弧的中点.

总复习题 6

1. (1) πa^2; (2) $\dfrac{3}{2}$; (3) $\dfrac{\pi}{12}$; (4) $\dfrac{\ln 3}{2}$; (5) $\dfrac{1}{3}\rho g a^3$.

2. (1) B; (2) C; (3) D; (4) D; (5) A.

3. $\dfrac{\pi}{3} + 2 - \sqrt{3}$. 4. $t = \dfrac{1}{2}$. 5. (1) $A\,(1,1)$; (2) $\dfrac{\pi}{30}$.

6. (1) $\dfrac{\pi a}{2}$; (2) $2\pi a^2$; (3) $\pi a\left(2\ln 2 - \dfrac{1}{2}\right)$.

7. (1) $f(x) = \dfrac{x}{\sqrt{1+x^2}}, x \in (0, +\infty)$; (2) $V = \dfrac{\pi^2}{6}$.

8. $\dfrac{8}{9}\left[\left(\dfrac{5}{2}\right)^{\frac{3}{2}} - 1\right]$. 9. 4. 10. $s = \dfrac{22}{3}$, $A = \dfrac{425}{9}\pi$.