CPSC 121: Models of Computation
Assignment #1

Due: Friday September 26, 5:00 pm **Total: 36 Marks**

<u>Submission Instructions</u>-- read carefully

All assignments should be done in groups of 2. It is very important to work with another student and exchange ideas. **Each group should submit ONE assignment**. Type or write your assignment on clean sheets of paper with question numbers prominently labelled. Answers that are difficult to read or locate may lose marks. We recommend working problems on a draft copy then writing a separate final copy to submit.

Your submission must be **STAPLED** and include the **CPSC 121 assignment cover page** – located at the Assignments section of the course web page. Additionally, include your names at the top of each page. We are not responsible for lost pages from unstapled submissions.

Submit your assignment to the appropriately marked box (**box 21**) in room **ICCS X235** by the due date and time listed above. **Late submissions are not accepted**.

Note: the number of marks allocated to a question appears in square brackets after the question number.

<u>A Note on the Marking Scheme</u>

Most items (i.e., question or, for questions divided into parts, part of a question) will be worth 3 marks with the following general marking scheme:

- **3 marks:** correct, complete, legible solution.
- **2 marks**: legible solution contains some errors or is not quite complete but shows a clear grasp of how the concepts and techniques required apply to this problem.
- **1 mark**: legible solution contains errors or is not complete but shows a clear grasp of the concepts and techniques required, although not their application to this problem or the solution is somewhat difficult to read but otherwise correct.
- **0 marks**: the solution contains substantial errors, is far from complete, does not show a clear grasp of the concepts and techniques required, or is illegible.

This marking scheme reflects our intent for you to learn the key concepts and techniques underlying computation, determine where they apply, and apply them correctly to interesting problems. It also reflects a practical fact: we have insufficient time to decipher illegible answers. At the instructor's discretion, some items may be marked on a different scale. TAs may very occasionally award a bonus mark for exceptional answers.

Question 1 [12]

One way to better understand a computational system is to look at the minimum set of primitives (simple operations) that are sufficient to express all the tasks performed by the system. In this question, you will prove that every truth table can be implemented by a circuit that uses only 2-input NOR gates. To do that you need to show the following steps:

[3] a. Show that **~** can be simulated using NOR gates. That is, design a circuit of NOR gates, that takes as input a signal x, and whose output is ~x.

[3] b. Show that **V** can be simulated using NOR gates. That is, design a circuit whose only gates are NOR gates, that takes as inputs two signals x and y, and whose output is x V y. (Hint: take advantage of what you learned in the previous part!)

[3] c. Show that **Λ** can be simulated using NOR gates. That is, design a circuit whose only gates are NOR gates, that takes as inputs two signals x and y, and whose output is x Λ y. (Hint: take advantage of what you learned in the previous parts!)

[3] d. Now argue that any logic function that is represented by a truth table over k atomic propositions can be implemented with a circuit that uses only 2-input NOR gates.

Question 2 [6]

The truth table below defines the truth value of **f** for each combination of truth values of a, b and c.

a	**b**	**c**	**f**
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	T
T	T	F	F
T	T	T	T

a. [3] Find a logic formula for **f** that uses each variable name at most twice. Then verify the correctness of your formula by drawing a truth table corresponding to this formula, including the truth values of all the subformulas.

Hint: try to divide the rows of the truth table which contribute to the formula into two groups based on a*, find a logical formula for each group, and then combine them appropriately.*

b. [3] Draw a circuit with inputs **a**, **b** and **c** whose output is the value **f** described by the truth table.

Question 3 [9]

Consider the following logical expressions and circuits:

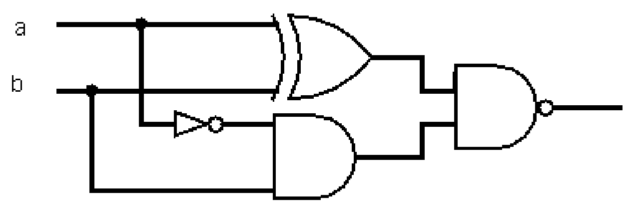
a) $(c \wedge a) \rightarrow (c \wedge b)$

b) $(a \wedge (b \vee c)) \rightarrow (a \wedge c)$

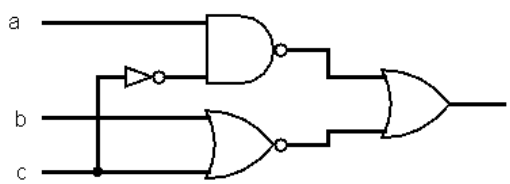
c) $\sim a \vee (c \rightarrow b)$

d) $a \vee \sim b$

e)



f)



Each proposition or circuit is logically equivalent to exactly one other proposition or circuit. For each of them, write down which one it is equivalent to and provide a proof for that equivalency. Each proof of the three logical equivalences must use a sequence of the logical equivalences that are listed in the “Logical Equvalence Laws”and “Implication” sections of the 121 “official” formula list shown at http://www.ugrad.cs.ubc.ca/~cs121/2014W1/handouts/formulasheet.pdf , not a truth table. (See Epp-4 theorem 2.1.1, Epp-3 theorem 1.1.1, Rosen-6 table 6 in section 1.2, Rosen-7 table 6 in section 1.3; you can also assume that $x \rightarrow y \equiv \sim x \vee y$). You may also use the abbreviations for the rule names which are shown on this handout.

Question 4 [6]

Design a circuit that takes three inputs a, b, and c and returns the value that the majority of inputs have (this is called a majority vote).

a. [3] Show the formula for this problem and explain how did you find it.
b. [3] Show the circuit.

Question 5 [3]

Consider the following small logic puzzle. Try to provide a reasonable argument demonstrating your thinking and reasoning. Even if you don't solve the puzzle perfectly, try to articulate your reasoning clearly. The puzzle is as following:

> The head of the CS department wanted to tease three logicians, let's call them A, B and C, who recently joined the department and asked them to participate in the following game. The department head asked A, B and C to sit one behind the other and put a hat on each of them. Then the head told them that each hat is either black or white, but not all their hats are white and that, if one of them guesses the color of his/her hat, all of them will get a raise. Note that nobody can see their own hat. A can see the hats of B and C. B can see the hat of C, and C cannot see any hat. After a short silence the logicians responded in the following order:
>
> A : I don't know.
> B: I don't know.
> C: Correctly predicted the color of her hat, and everyone was happy.

Explain what color was the hat of C and the reasoning that C used to derive her answer.

[Hint: Think about which hat arrangements are consistent with the facts and the answers of the three logicians .]