Tutorial Week 2

0. **EXAMPLE:** Show $\sim(\sim(p \vee q \vee p) \wedge p) \equiv T$

$$
\begin{aligned}
& \sim(\sim(p \vee q \vee p) \wedge p) & \\
\equiv\ & ((p \vee q \vee p) \vee \sim p) & \text{by DeMorgan's Law} \\
\equiv\ & p \vee q \vee p \vee \sim p & \text{by Associative Law} \\
\equiv\ & p \vee p \vee \sim p \vee q & \text{by Commutative Law} \\
\equiv\ & p \vee \sim p \vee q & \text{by Idempotent Law} \\
\equiv\ & T \vee q & \text{by Negation Law} \\
\equiv\ & T & \text{by Universal Bound Law}
\end{aligned}
$$

1. Show $p \vee q \equiv \sim(\sim p \wedge ((\sim q \wedge \sim p) \vee (\sim q \wedge p)))$

$$
\begin{aligned}
& \sim(\sim p \wedge ((\sim q \wedge \sim p) \vee (\sim q \wedge p))) & \\
\equiv & (p \vee \sim((\sim q \wedge \sim p) \vee (\sim q \wedge p))) & \text{by DeMorgan's Law} \\
\equiv & (p \vee (\sim(\sim q \wedge \sim p) \wedge \sim(\sim q \wedge p))) & \text{by DeMorgan's Law} \\
\equiv & (p \vee ((q \vee p) \wedge (q \vee \sim p))) & \text{by DeMorgan's Law} \\
\equiv & p \vee (q \vee (p \wedge \sim p)) & \text{by Distributive Law} \\
\equiv & p \vee q \vee F & \text{by Negation Law} \\
\equiv & p \vee q & \text{by Identity Law}
\end{aligned}
$$

2. Translate the following Truth Table to a propositional logic statment: Can you simplify it?

   First approach maybe:

$$(\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r)$$

   (**hints** : the brute force approach needs three variables to describe each individual time the output is 1. is there a way to use less than three variables to cover more than 1 case where the output is 1?)

Truth Table			
p	q	r	output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

A simplified answer: observe that the rule $(\sim p \wedge \sim r)$ covers the 1st and 3rd rows, and the rule $(\sim q \wedge r)$ covers the other two rows where the circuit is true (2nd and 6th)

$$\begin{gathered}(\sim p \wedge \sim r) \vee (\sim q \wedge r) \\ \text{or, with deMorgan's law} \\ \sim(p \vee r) \vee (\sim q \wedge r)\end{gathered}$$

Another simplified answer: Observe that the first three rows are true, which is $\sim p$ plus the opposite of $(q \wedge r)$, then covering the last row the brute force way

$$(\sim p \wedge \sim(q \wedge r)) \vee (p \wedge \sim q \wedge r)$$

3. Prove the following equivalence: $(\sim p \vee s) \wedge (p \to (s \to \sim p)) \equiv \sim p$

$(\sim p \vee s) \wedge (p \to (s \to \sim p))$	
$\equiv (\sim p \vee s) \wedge (\sim p \vee (\sim s \vee \sim p))$	by definition of conditional
$\equiv (\sim p \vee s) \wedge ((\sim p \vee \sim p) \vee \sim s)$	by associative law
$\equiv (\sim p \vee s) \wedge (\sim p \vee \sim s)$	by idempotent law
$\equiv \sim p \vee (s \wedge \sim s)$	by distributive law
$\equiv \sim p \vee F$	by negation law
$\equiv \sim p$	by identity law

4. Prove that $(p \vee q) \equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee ((p \wedge \sim p) \wedge (q \vee \sim q))$

$(\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee ((p \wedge \sim p) \wedge (q \vee \sim q))$	
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee (F \wedge T)$	by negation
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q) \vee F$	by universal bound
$\equiv (\sim q \to p) \vee \sim(\sim p \wedge \sim q)$	by identity
$\equiv (\sim q \to p) \vee (p \vee q)$	by De Morgan's
$\equiv (\sim p \to q) \vee (p \vee q)$	by contrapositive
$\equiv (p \vee q) \vee (p \vee q)$	by definition of conditional
$\equiv p \vee q$	by Idempotent