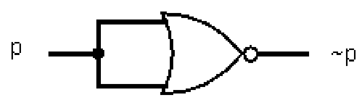
CPSC 121: Models of Computation
Assignment #1
SOLUTIONS

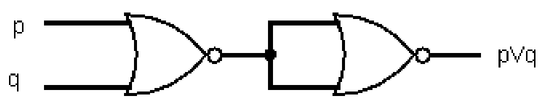
Note: For most problems, the solution shown here is not the only correct solution.

Question 1 [12]

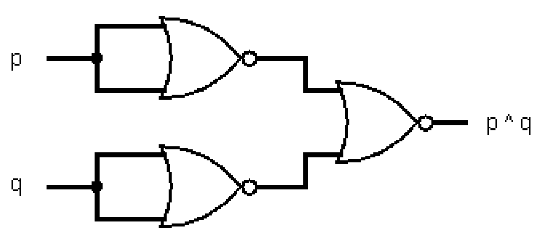
a. ~p ≡ ~ (p V p) . Therefore it is implemented by



b. p V q ≡ ~ (~(p V q) V ~(p V q)) and is implemented by



c. p Λ q ≡ ~ (~p V ~q) and is implemented by



d. The logic formula represented by a table with k propositions and n rows is of the form:

$$row_1 \ \mathrm{v} \ row_2 \ \mathrm{v} \ \dots \ \mathrm{v} \ row_n$$

where row_i has the form

$$p_1 \ \hat{} \ p_2 \ \hat{} \ \dots \ \hat{} \ p_k$$

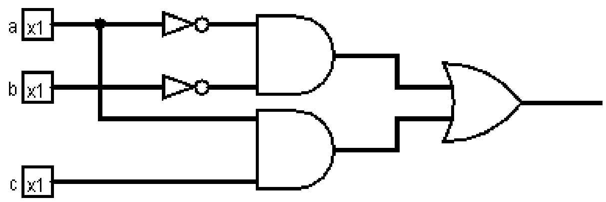
where p_j is either the jth proposition (if it has the value T in that row) or its negation (if it has the value F in that row).

Since negation and conjunction can be replaced by NOR gates, any row can be re-written using only NOR gates. Because a disjunction can also be replaced by NOR gates, the disjunction of the rows, and therefore the table itself, can be re-written using NOR gates only.

Question 2 [6]

a. The table has four rows with output T: row 1,2, 6 and 8. Rows 1 and 2 are the only rows in which a is false and b is false. Therefore, the formula for these two rows is **~a ^ ~b** . Rows 6 and 8 are the only rows in which a is true and c is true, and the formula for these rows is **a ^ c** . Therefore the formula for this table is (**~a ^ ~b) v (a ^ c) [or ~(a v b) v (a ^ c)]**.

b. The circuit for the given table is:



Question 3 [9]

- **(a) is equivalent to (c)**

  Proof:

  $(c \wedge a) \rightarrow (c \wedge b)$

$\equiv \sim(c \wedge a) \vee (c \wedge b)$	equivalence of $\rightarrow$
$\equiv (\sim c \vee \sim a) \vee (c \wedge b)$	DeMorgan's
$\equiv (c \wedge b) \vee (\sim c \vee \sim a)$	commutativity (may skip)
$\equiv ((c \wedge b) \vee \sim c) \vee \sim a$	associativity
$\equiv ((c \vee \sim c) \wedge (b \vee \sim c)) \vee \sim a$	distributive
$\equiv (T \wedge (b \vee \sim c)) \vee \sim a$	negation
$\equiv (b \vee \sim c) \vee \sim a$	identity
$\equiv \sim a \vee (b \vee \sim c)$	commutativity (may skip)
$\equiv \sim a \vee (\sim c \vee b)$	commutativity (may skip)
$\equiv \sim a \vee (c \rightarrow b)$	equivalence of $\rightarrow$

- **(b) is equivalent to (f)**

  The formula for (f) is : $\sim(a \wedge \sim c) \vee \sim(b \vee c)$. We'll show that (b) is equivalent to it.

  Proof:

  $(a \wedge (b \vee c)) \rightarrow (a \wedge c)$

$\equiv \sim(a \wedge (b \vee c)) \vee (a \wedge c)$	equivalence of $\rightarrow$
$\equiv (\sim a \vee \sim(b \vee c)) \vee (a \wedge c)$	DeMorgan's

$\equiv (a \wedge c) \vee (\sim a \vee \sim(b \vee c))$	commutative (may skip)
$\equiv ((a \wedge c) \vee \sim a) \vee \sim(b \vee c)$	associative
$\equiv (\sim a \vee (a \wedge c)) \vee \sim(b \vee c)$	commutative (may skip)
$\equiv ((\sim a \vee a) \wedge (\sim a \vee c)) \vee \sim(b \vee c)$	distributive
$\equiv (T \wedge (\sim a \vee c)) \vee \sim(b \vee c)$	negation
$\equiv (\sim a \vee c) \vee \sim(b \vee c)$	identity
$\equiv (\sim a \vee \sim\sim c) \vee \sim(b \vee c)$	double negation (may skip)
$\equiv \sim(a \wedge \sim c) \vee \sim(b \vee c)$	DeMorgan's

Another way to do this proof is to reduce BOTH formulas to ~a v ~b v c and then stitch the two proofs together.

- **(d) is equivalent to (e)**

  The formula for (e) is : $\sim((a \oplus b) \wedge (\sim a \wedge b))$ which is equivalent to (d).

  Proof:

  $\sim((a \oplus b) \wedge (\sim a \wedge b))$

$\equiv \sim(a \oplus b) \vee \sim(\sim a \wedge b)$	DeMorgan's
$\equiv \sim[(a \vee b) \wedge \sim(a \wedge b)] \vee \sim(\sim a \wedge b)$	definition of $\oplus$
$\equiv [\sim(a \vee b) \vee \sim\sim(a \wedge b)] \vee \sim(\sim a \wedge b)$	DeMorgan's
$\equiv [\sim(a \vee b) \vee (a \wedge b)] \vee \sim(\sim a \wedge b)$	double negation (may skip)
$\equiv [(\sim a \wedge \sim b) \vee (a \wedge b)] \vee \sim(\sim a \wedge b)$	DeMorgan's
$\equiv [((\sim a \wedge \sim b) \vee a) \wedge ((\sim a \wedge \sim b) \vee b)] \vee \sim(\sim a \wedge b)$	distributive
$\equiv [((\sim a \vee a) \wedge (\sim b \vee a)) \wedge ((\sim a \wedge \sim b) \vee b)] \vee \sim(\sim a \wedge b)$	distributive
$\equiv [(T \wedge (\sim b \vee a)) \wedge ((\sim a \wedge \sim b) \vee b)] \vee \sim(\sim a \wedge b)$	negation
$\equiv [(\sim b \vee a) \wedge ((\sim a \wedge \sim b) \vee b)] \vee \sim(\sim a \wedge b)$	identity
$\equiv [(\sim b \vee a) \wedge ((\sim a \vee b) \wedge (\sim b \vee b))] \vee \sim(\sim a \wedge b)$	distributive
$\equiv [(\sim b \vee a) \wedge ((\sim a \vee b) \wedge T)] \vee \sim(\sim a \wedge b)$	negation
$\equiv [(\sim b \vee a) \wedge (\sim a \vee b)] \vee \sim(\sim a \wedge b)$	identity
$\equiv [(\sim b \vee a) \wedge (\sim a \vee b)] \vee (a \vee \sim b)$	DeMorgan's
$\equiv (a \vee \sim b) \vee [(\sim b \vee a) \wedge (\sim a \vee b)]$	commutative (may skip)
$\equiv (a \vee \sim b) \vee [(a \vee \sim b) \wedge (\sim a \vee b)]$	commutative (may skip)
$\equiv a \vee \sim b$	absorption

Question 4 [6]

The following is the truth table for this problem :

a	b	c	out
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	T
T	T	T	T

If we apply the direct process of constructing the formula from the table we get:

(a ^ b ^ c) v (a ^ b ^ ~c) v (a ^ ~b ^ c) v (~a ^ b ^ c)

If we look at the true rows closer we can see that each of these rows, except the last one, is the only row that the two variables are true. Therefore each of these rows can be defined by the two true variables and the false variable can be omitted. We then can get a simpler formula like the following:

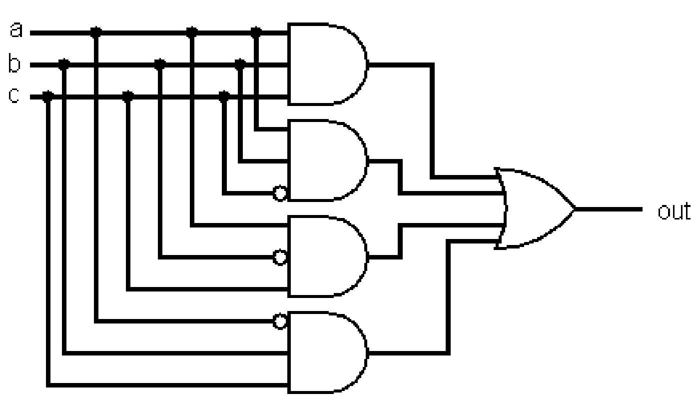
(a ^ b ^ c) v (a ^ b) v (a ^ c) v (b ^ c)

Now you can see that the first term, (a ^ b ^ c), is not needed. When this term is true the rest of the formula is also true. When this term is false, the final value of the formula is determined by the other three terms. Therefore, we can remove the first term and our final formula is:

(a ^ b) v (a ^ c) v (b ^ c)

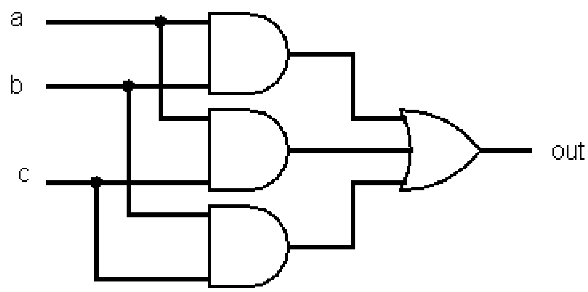
which is quite intuitive. It says that the output is true if and only if any two of the three variables is true. Otherwise the output is false.

b. If we use the full formula, the circuit is



Note that a bubble on an input to a gate is short-hand for negating that input.

If we use the short formula the circuit is:



Question 5 [3]

C's had was **black** and here is how we can formally show it. Let

- a = A's hat is white
- b = B's hat is white
- c = C's has is white

and create a truth table with all the possible combinations:

Row #	a	b	c
1	F	F	F
2	F	F	T
3	F	T	F
4	F	T	T
5	T	F	F
6	T	F	T
7	T	T	F
8	T	T	T

We can observe the following:

- Row 8 is not possible as not all hats are white.
- Row 4 is excluded because in not consistent with A's answer. If that were the case, A would know that he had a black had, as he can see the other two white hats.
- Rows 2 and 6 are excluded because are not consistent with B's answer. In both cases B knows that B and C cannot both have white hats and therefore B would know that she has a black hat.

Therefore the only possible rows are rows 1, 3, 5, and 7. In all of them C has a black hat.

We can describe C's reasoning in a less formal way as following:

Since A answered “Idon’t know”, not both of B and C have white hats (if they had, then A would had known his own hat was black, since not all hats are white). Now, if C’s hat were white, then B, knowing that she and C cannot both have white hats, would have known she had a black hat. Since B does not know, this must mean that C’s hat is black.