CSCI 2824: Discrete Structures Problem Set 5
Due Tuesday 12/8 in class (*hard copy*)

Problem 1. (20 points)

What is the probability that a poker hand will have *n* kings in it? (Answer for each possible value of *n*: 0, 1, 2, 3, or 4.)

Suppose we play a game as follows: I deal you a poker hand, and then pay you $1 for each king in your hand. For instance, if your hand has 2 kings in it, you get $2. On average, how much money do you expect to win each time you play this game?

Problem 2. (20 points)

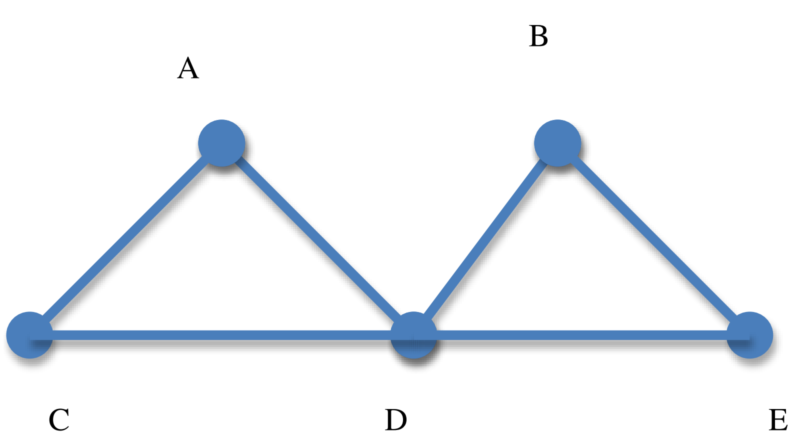
There are two urns, each with four ping-pong balls. In one urn, three of the balls are red, and one is white; in the other, three are white, and one is red. Without knowing which urn you are choosing, you reach inside and draw out, at random, a red ball. You put the ball back into the urn, mix up the contents, and repeat the experiment (remember there are four balls once more inside the urn). Again you draw out a red ball. Based on this two-part experiment, what is the probability that the urn you chose is the one with the three red balls in it?

Problem 3. (20 points)

Prove that it is impossible to have a graph in which there are an odd number of odd-degree vertices.

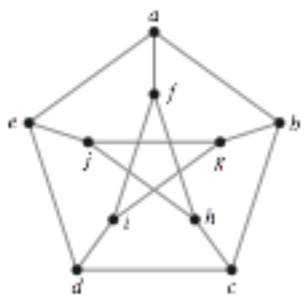
Problem 4. (20 points)

Draw the adjacency matrix for the following graph, and use it to determine the number of four-step walks from vertex C to vertex D.



Problem 5.

Consider the following graph:



Note that the graph has been constructed so that there are *no* cycles of length three (triangles) or four (quadrilaterals). Note also that the graph has a total of fifteen edges and ten vertices, and every vertex has degree 3.

Now, suppose this graph had a Hamiltonian cycle. That would mean that you could write out a list of ten edges, visiting each vertex exactly once. There would still be five edges in the graph to account for in the graph. Show that if you try to draw those remaining edges, you must end up with either a triangle or quadrilateral, and thus prove (by contradiction) that the graph does *not* have a Hamiltonian cycle.