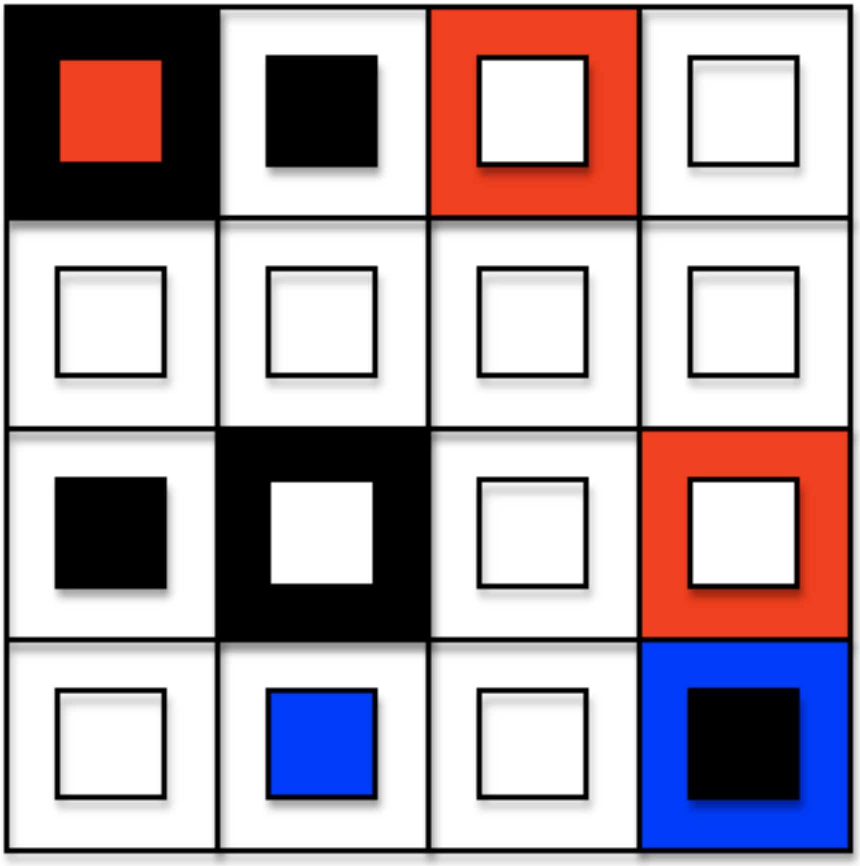
Discrete Structures, CSCI 2824, Fall 2015

Problem Set 1. Due Tuesday, Sept. 22 (hard copy, in class!)



Problem 1. The picture above shows a puzzle based on a four-by-four array of nested squares (each of the 16 locations has a "center" and a "border" square). The puzzle needs to be filled in with four colors: black, blue, gold, and red. (The "black and gold" part is for CU, of course.)

The puzzle rules are as follows:

Rule a. Each square is two-colored (the center and border are different colors).

Rule b. In each row of four squares (horizontal, vertical, or diagonal), all the centers have different colors and all the borders have different colors.

In the grid above, five black regions, three red, and two blue have been filled in. Anything shown in white needs to be filled with one of the four colors. Rows in the grid are numbered 1-4 from the top, and columns are numbered 1-4 from the left.

1a. (2 points) Solve the puzzle by filling in the remaining regions, and briefly explain your reasoning.

Now, suppose you have propositional logic statements of the form:

SQ11CENTRED("the center of the square in row 1, column 1 is red")

SQ32BORDBLACK ("the border of the square in row 3, column 2 is black")

As it happens, these propositions are both true in the given grid. Answer the following questions:

1b (2 points). How many distinct propositions of the form given in the previous two examples do we need to cover all combinations of square regions and colors?

1c (3 points) Show all the propositions you need to represent the idea that there is exactly one color in the center of the square in row 1, column 1.

1d. (3 points) Show all the propositions you need to represent the idea that all the borders in column four have distinct colors. (There are multiple ways of doing this, but regardless you'll need well more than a few propositions.)

Problem 2. (2 points) Negate each of the following statements: (a) All students live in the dormitories. (b) All mathematics majors are males. (c) Some students are 25 years old or older. Write both the initial statement and its negation in predicate logic.

Problem 3. (4 points) (***3.1***) You meet 3 inhabitants A, B and C of Smullyan's Island. Using p to represent "A is truthful", q to represent "B is truthful", and r to represent "C is truthful", how would you write each of these phrases?

a) A and B are lying

b) All three are lying

c) One of us is lying

d) Exactly one of the three is truthful

(***3.2***) On Smullyan's Island two strangers, A and B approach you. Stranger B is holding a rake. The two strangers say the following to you:

Stranger A: If I were a Truth-Teller, I would say that B is holding a rake.

Stranger B: A is a Truth-Teller.

What can you say about the nature of Strangers A and B?

Problem 4. (3 points) In class, we talked about the Fibonacci numbers, defined as follows:

F(0) = 0

F(1) = 1

F(N) = F(N-1) + F(N-2) for all N > 1

The Fibonacci series begins as follows:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Prove by induction the following statement:

$F(1)^2 + F(2)^2 + F(3)^2 + \dots F(N)^2 = F(N) * F(N+1)$

Problem 5. (3 points) Prove by induction the following representation for Fibonacci numbers:

$$F(n) = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n\sqrt{5}},$$

Problem 6. (4 points) The *Lucas numbers* are a series rather like the Fibonacci numbers, but defined as follows:

L(0) = 2
L(1) = 1
L(n) = L(n-1) + L(n-2) for all n>= 2

Thus, the Lucas series begins as follows:

2, 1, 3, 4, 7, 11, 18, 29, etc.

(a) Prove the following statement by induction for all n>=1:

L(n) = F(n-1) + F(n+1)

where F(n) is the nth Fibonacci number.

(b) Now you can also (easily) prove: F(n) + L(n) = 2F(n+1)