Problem Set 5

Problem 1. (20 points)

What is the probability that a poker hand will have *n* kings in it? (Answer for each possible value of *n*: 0, 1, 2, 3, or 4.)

Suppose we play a game as follows: I deal you a poker hand, and then pay you $1 for each king in your hand. For instance, if your hand has 2 kings in it, you get $2. On average, how much money do you expect to win each time you play this game?

- Total amount of poker hands 52 choose 5 = **2,598,960**
  - Formula: $\mathbf{((_4C_n)*(_{48}C_{5-n}))/(_{52}C_5)}$
- For n = 0
  → ((1)*(1712304))/(2598960) = **0.6588**
- For n = 1
  → ((4)*(194580))/(2598960) = **0.2995**
- For n = 2
  → ((6)*(17296))/(2598960) = **0.0399**
- For n = 3
  → ((4)*(1128))/(2598960) = **0.0017**
- For n = 4
  → ((1)*(48))/(2,598,960) = **0.00002**

Problem 2. (20 points)

There are two urns, each with four ping-pong balls. In one urn, three of the balls are red, and one is white; in the other, three are white, and one is red. Without knowing which urn you are choosing, you reach inside and draw out, at random, a red ball. You put the ball back into the urn, mix up the contents, and repeat the experiment (remember there are four balls once more inside the urn). Again you draw out a red ball. Based on this two- part experiment, what is the probability that the urn you chose is the one with the three red balls in it?

- The probability of choosing two red balls from the urn that contains 3 red balls is 9/16, and therefore 56.25%. First we choose one ball from this urn at ¾ chance of choosing the red ball, then we choose again at ¾ and combine these two probabilities to get 9/16.
- The probability of choosing two red balls from the urn that contains only 1 red ball is 1/16, and therefore 6.25%. First we choose one ball from this urn at ¼ chance of choosing the red ball, then we choose again at ¼ and combine these two probabilities to get 1/16.

- $P(A|B_1) = (¾)$
- $P(A|B_2) = (¼)$
  - $P(B_1) = P(B_2) = (½)$, equal likelihood of each urn being chosen.
- **Therefore: $P(B_1 \mid A) = (⅜) / (½) = (¾)$, this is the probability that the urn we choose is the one with three red balls.**

Problem 3. (20 points)

Prove that it is impossible to have a graph in which there are an odd number of odd-degree vertices.

- **The degree sum formula implies:**
  The sum of all degrees is equal to twice the number of edges. Since the sum of the degree is even and the sum of the degrees of vertices with even degree is even, the sum of the degrees of vertices with odd degree must be even. If the sum of the degrees of vertices with odd degree is even, there must be an even number of these vertices.

Problem 4. (20 points)

Draw the adjacency matrix for the following graph, and use it to determine the number of four-step walks from vertex C to vertex D.

	A	B	C	D	E
A	0	0	1	1	0
B	0	0	0	1	1
C	1	0	0	1	0
D	1	1	1	0	1

E	0	1	0	1	0

Therefore:

C → D → B → E → D
C → D → A → C → D
C → D → E → B → D
C → A → D → C → D

Problem 5.

Consider the following graph:

Note that the graph has been constructed so that there are *no* cycles of length three (triangles) or four (quadrilaterals). Note also that the graph has a total of fifteen edges and ten vertices, and every vertex has degree 3.

Now, suppose this graph had a Hamiltonian cycle. That would mean that you could write out a list of ten edges, visiting each vertex exactly once. There would still be five edges in the graph to account for in the graph. Show that if you try to draw those remaining edges, you must end up with either a triangle or quadrilateral, and thus prove (by contradiction) that the graph does *not* have a Hamiltonian cycle.

Case 1. there are two connecting edges, four star edges and four external edges. all of the vertices need to be visited by the path of length 4 and all of the pentagon vertices need to be visited by a path of length 4. This is impossible because a path of length 4 on the pentagon must have ending vertices that are next to each other, A path of length 4 in the star vertices must have endpoints that are not next to each other.

Case 2. There are four connecting edges, for star edges and two edges to the pentagon. This is impossible because all the pentagon nodes cannot be reached.

Case 3. There are four connecting edges, two star edges and four pentagon edges. This is impossible because all five star noded cannot be reached.