Labor

III 1. $f(0)$, $f(\frac{1}{2})$, $f(1)$

Lagrange interpolation, because we don't know any derivatives.

0	$f(0)$	$\frac{f(\frac{1}{2})-f(0)}{\frac{1}{2}-0} = 2(f(\frac{1}{2})-f(0))$	$\frac{2(f(1)-2f(\frac{1}{2})+f(0))}{1-0}$
$\frac{1}{2}$	$f(\frac{1}{2})$	$\frac{f(1)-f(\frac{1}{2})}{1-\frac{1}{2}} = 2(f(1)-f(\frac{1}{2}))$	0
1	$f(1)$	0	0

$L_1 f(x)$

$$L_2 f(x) = \underbrace{[f(0)+2(f(\tfrac{1}{2})-f(0))\cdot(x-0)]}_{L_1 f(x)} + 2(f(1)-2f(\tfrac{1}{2})+f(0))\cdot x(x-\tfrac{1}{2}) =$$

$$= f(0)(1-2x+2x^2-x) + f(\tfrac{1}{2})(2x-4x^2+2x) +$$

$$+ f(1)(2x^2-x)$$

$$= \boxed{f(0)(2x-1)(x-1) + f(\tfrac{1}{2})\cdot 4x(1-x) + f(1)\cdot x(2x-1)}$$

$$\frac{f'''(\xi)}{3!} = A$$

$$u(x) = (x-x_0)(x-x_1)\ ...\ (x-x_n)$$

$$R_2 f(x) = \frac{u(x)}{3!}\cdot f'''(\xi) = \frac{x(x-\frac{1}{2})(x-1)}{3!}\cdot f'''(\xi), \quad \xi \in (0,1)$$

n $\quad (n+1)!$

$$R = u(x)\cdot A$$

$$R_n f(x) = \frac{u(x)}{(n+1)!}\cdot f^{(n+1)}(\xi)$$

$|R_2 f(x)| \leq ?$ $\Rightarrow a\cdot A < R < b\cdot A$

$a < u < b$

$$u(x) = (x^2-x)(x-\tfrac{1}{2}) = x^3 - \frac{x^2}{2} - x^2 + \frac{x}{2} = x^3 - \frac{3x^2}{2} + \frac{x}{2}$$

$$u'(x) = 3x^2 - 3x + \frac{1}{2} = 0$$

$$\Delta = b^2 - 4ac$$

$$\Delta = 9 - 6 = 3$$

$$x_{1,2} = \frac{3 \pm \sqrt{3}}{6} \in (0,1)$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\boxed{(x^n)' = n x^{n-1}}$$

$u''(x) = 6x - 3$

$u''\left(\frac{3-\sqrt{3}}{6}\right) = -\sqrt{3} < 0 \Rightarrow$ local max. point

$u''\left(\frac{3+\sqrt{3}}{6}\right) = \sqrt{3} > 0 \Rightarrow$ local min. point

a, b

				max		min	
x	a		x_1		x_2		b
f		$\nearrow$	$f(x_1)$	$\searrow$	$f(x_2)$	$\nearrow$	
f'	semnul lui a	$+++$	0	$--$	0	$+++$	opusa
f''			$-$		$+$		

(vezi c/s. 12)

$\left|u\left(\frac{3\pm\sqrt{3}}{6}\right)\right| = \frac{\sqrt{3}}{36} \Rightarrow \max_{x\in[0,1]} |u(x)| \le \frac{\sqrt{3}}{36}$

$-\frac{\sqrt{3}}{36} < u < \frac{\sqrt{3}}{36}$

$\boxed{|R_2 f(x)| \le \frac{\sqrt{3}}{36 \cdot 3!} \cdot |f'''(\xi)| \le} \frac{\sqrt{3}}{216} \cdot \|f'''\|$

$a \cdot A < R < b \cdot A$

$-\frac{\sqrt{3}}{36} \cdot A < R < \frac{\sqrt{3}}{36} \cdot A \Rightarrow |R| < \frac{\sqrt{3}}{36} \cdot A$

2) $f(-1), f(0), f'(0), f(1)$

Hermite interpolation because at $x_1 = 0$ we have both the function and the derivative

$f(1), f'(1)$

~~$f(1), f'(-2)$~~

$x_1 = 0$ double node

$x_0 = -1,\ x_1 = 1$ simple nodes

-1	$f(-1)$	$\frac{f(0)-f(-1)}{1}$	$\frac{f'(0)-f(0)+f(-1)}{1}$	$\frac{f(1)-2f'(0)-f(-1)}{2}$
0	$f(0)$	$f'(0)$	$\frac{f(1)-f(0)-f'(0)}{1}$	0
0	$f(0)$	$\frac{f(1)-f(0)}{1}$	0	0
1	$f(1)$	0	0	0

$H_3 f(x) = f(-1) + (f(0) - f(-1))(x+1) + (f'(0) - f(0) + f(-1))(x+1)(x-0) +$

$+ \frac{1}{2}(f(1) - 2f'(0) - f(-1))(x+1)x^2$

with fundamental polynomials

$f^{(j)}(x_i)$

b_{ij} — i index of node; j ~~index~~ order of the derivative

b_{ij}

J

$$f(\overset{x_0}{0}) \to b_{00} \qquad f'(\overset{x_0}{0}) \to b_{01} \qquad f'(\overset{x_1}{1}) \to b_{11}$$

— have the same degree as $\boxed{B_2 f = 2}$

$$\boxed{b_{00}(x) = ax^2 + bx + c} \qquad \boxed{b_{00}'(x) = 2ax + b}$$

$b_{ij}^{(\cancel{k})}(x_i) = 1$, $j = k$, otherwise 0

$$\begin{cases} b_{00}(0) = 1 \\ b_{00}'(0) = 0 \\ b_{00}'(1) = 0 \end{cases} \Rightarrow \begin{cases} c = 1 \\ b = 0 \\ 2a + b = 0 \end{cases} \Rightarrow \begin{cases} c = 1 \\ b = 0 \\ a = 0 \end{cases} \Rightarrow b_{00}(x) = \cancel{2x^2}\ 1$$

$$\begin{cases} b_{01}(0) = 0 \\ b_{01}'(0) = 1 \\ b_{01}'(1) = 0 \end{cases} \Rightarrow \begin{cases} c = 0 \\ b = 1 \\ 2a + b = 0 \end{cases} \Rightarrow \begin{cases} c = 0 \\ b = 1 \\ a = -\frac{1}{2} \end{cases} \Rightarrow b_{01}(x) = -\frac{x^2}{2} + x$$

$$\begin{cases} b_{11}(0) = 0 \\ b_{11}'(0) = 0 \\ b_{11}'(1) = 1 \end{cases} \Rightarrow \begin{cases} c = 0 \\ b = 0 \\ 2a + b = 1 \end{cases} \Rightarrow \begin{cases} c = 0 \\ b = 0 \\ a = \frac{1}{2} \end{cases} \Rightarrow b_{11}(x) = \frac{x^2}{2}$$

$$B_2 f(x) = b_{00}(x) \cdot f(0) + b_{01}(x) \cdot f'(0) + b_{11}(x) \cdot f'(1) =$$
$$= f(0) + \left(x - \frac{x^2}{2}\right) f'(0) + \frac{x^2}{2} f'(1)$$

~~$B_2 f(0) = f(0)$~~

~~$B_2 f$~~

$$R_3 f(x) = \frac{(x+1)x^2(x-1)}{4!} \cdot f^{(4)}(\xi), \quad \xi \in (-1,1)$$

$$\max_{x\in[-1,1]} |u(x)| = \left|u\left(\pm \frac{\sqrt{2}}{2}\right)\right| = \frac{1}{4}$$

$$|R_3 f(x)| \le \frac{1}{4\cdot 4!} |f^{(4)}(\xi)| \le \frac{1}{96} \|f^{(4)}\|$$

$(f''(0) \Rightarrow I_0 = \{0,1,2\})$

3) $f(0)$, $f'(0)$ $\boxed{f'(1)}$

$$x_0 = 0 \qquad I_0 = \{0, 1\}$$
$$x_1 = 1 \qquad I_1 = \{1\}$$

$f \quad f'$

Birkhoff, because at $x_1 = 1$ we only know the derivative without the function value

$$|I_0| + |I_1| - 1 = 2$$

$I_0 = \{0, 1\}$, x_0

$\Rightarrow$ degree $= n = |I_0| + |I_1| - 1 = 2 + 1 - 1 = 2$

$I_1 = \{1\}$, x_1

Direct way

$$B_2 f(x) = ax^2 + bx + c$$

$$(B_2 f)'(x) = 2ax + b$$

$$2a + b = f'(1)$$

$$a = \frac{f'(1) - b}{2}$$

$B_2 f(0) = \boxed{c = f(0)}$

$(B_2 f)'(0) = \boxed{b = f'(0)}$

$(B_2 f)'(1) = 2a + b \Rightarrow a = \dfrac{f'(1) - f'(0)}{2}$

$$\Rightarrow \boxed{B_2 f(x) = \frac{f'(1) - f'(0)}{2} x^2 + f'(0) x + f(0)}$$

$B_2 f(0) = f(0)$

$(B_2 f)'(x) = (1-x) f'(0) + x f'(1)$

$(B_2 f)'(0) = f'(0)$

$(B_2 f)'(1) = f'(1)$

Lab 09

3.

$f(0),\ f'(0),\ f'(1)$

$B_2 f(x) = f(0) + \frac{1}{2}x(2-x)f'(0) + \frac{1}{2}x^2 f'(1)$

$f(x) = B_2 f(x) + R_2 f(x) \Rightarrow R_2 f(x) = f(x) - B_2 f(x)$

degree of precision $=2$

$e_0 = 1;\ e_1 = x;\ e_2 = x^2;\ e_3 = x^3;\ \ldots$

$(R_2 e_0)(x) = e_0(x) - \left(e_0(0) + \frac{1}{2}x(2-x)\underbrace{e_0'(0)}_{0} + \frac{1}{2}x^2 \underbrace{e_0'(1)}_{0}\right) =$

$= 1 - (1 + 0 + 0) = 0$

$(R_2 e_1)(x) = e_1(x) - \left(e_1(0) + \frac{1}{2}x(2-x)\underbrace{e_1'(0)}_{1} + \frac{1}{2}x^2 \underbrace{e_1'(1)}_{1}\right) =$

$= x - x + \frac{x^2}{2} - \frac{x^2}{2} = 0$

$(R_2 e_2)(x) = e_2(x) - \left(\underbrace{e_2(0)}_{0} + \frac{1}{2}(2-x)\underbrace{e_2'(0)}_{0} + \frac{1}{2}x^2 \underbrace{e_2'(1)}_{2}\right) =$

$= x^2 - (0 + 0 + x^2) = 0$

$(R_2 e_3)(x) = e_3(x) - \left(\underbrace{e_3(0)}_{0} + \frac{1}{2}(2-x)\underbrace{e_3'(0)}_{0} + \frac{1}{2}x^2 \underbrace{e_3'(1)}\right) =$

$= x^3 - \left(0 + 0 + \frac{3x^2}{2}\right) \neq 0$

The remainder of Peano's theoreme is:

$R_2 f(x) = \int_0^1 k_2(x,t) f^{'''}(t)\,dt$ ($n+1$)

$x_+ = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$k(x,t) = R_2\left(\frac{(x-t)_+^2}{2!}\right)$

we check the sign of $k_2(x,t)$ on $[0,1]$

$t \in [0,1]$ $\quad K_2(x,t) = \frac{1}{2} R_2((x-t)_+^2)$

$$R_2((x-t)_+^2) = (x-t)_+^2 - \left((0-t)_+^2 + \frac{1}{2}x(2-x)\cdot 2(0-t)_+ + \frac{1}{2}x^2(1-t)_+\right)$$

$t \in [0,1] \Rightarrow (0-t)_+ = 0 = (0-t)_+^2$

$(1-t)_+ = 1-t$

$$R_2((x-t)_+^2) = (x-t)_+^2 - x^2(1-t)$$

Case I: $0 \le x \le t \le 1$

$$R_2((x-t)_+^2) = -x^2(1-t) < 0$$

Case II: $0 \le t < x \le 1$

$$R_2((x-t)_+^2) = (x-t)^2 - x^2(1-t) = x^2 - 2xt + t^2 - x^2 + x^2 t =$$

$$= t(x^2 - 2x + t)$$

$g(x) = x^2 - 2x + t$

$D = 4 - 4t = 4(1-t)$

$x_{1,2} = \frac{2 \pm 2\sqrt{1-t}}{2} = 1 \pm \sqrt{1-t}$

x		x_1		x_2	
$g(x)$	+ + +	0	− − −	0	+ + +

$g(x)$ is negative on $(1-\sqrt{1-t};\ 1+\sqrt{1-t})$

$(1-\sqrt{1-t} \quad (t;1] \quad 1+\sqrt{1-t})$

$0 \le 1-\sqrt{1-t} \le t \le 1 \le 1+\sqrt{1-t}$

$1-t \le \sqrt{1-t} \Rightarrow 1-\sqrt{1-t} \le t$

for $x \in (t, 1] \subseteq \underbrace{(1-\sqrt{1-t}, 1+\sqrt{1-t})}_{\text{on this interval } g(x)<0} \Rightarrow g(x)$ negative $\Rightarrow$

$\Rightarrow R_2((x-t)_+^2)$ is negative

In both cases $R_2((x-t)_+^2)$ is negative $\Rightarrow k_2(x,t)$ has constant sign on $[0,1]$ $\xRightarrow[\text{Th}]{\text{Peano's}}$ $R_2 f(x) = \frac{f'''(\xi)}{3!} \cdot (R_2 e_3)(x) = \frac{x^3 - \frac{3x^2}{2}}{6} \cdot f'''(\xi)$, $\xi \in (0,1)$

$h(x) = x^3 - \frac{3x^2}{2}$

$h' = 3x^2 - 3x \leq 0 \Rightarrow h$ is decreasing $\Rightarrow h(1) \leq h(x) \leq h(0)$

$-\frac{1}{2} \leq h(x) \leq 0 \Rightarrow$

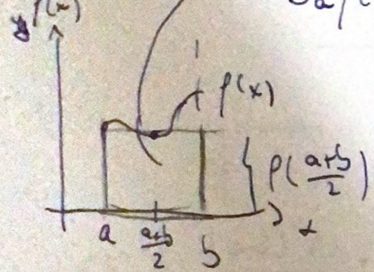
$\Rightarrow |h(x)| \leq \frac{1}{2}$

$|\cancel{R_2 f(x)}|$

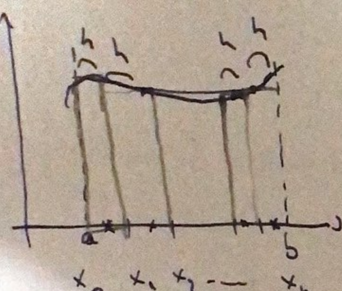
$|R_2 f(x)| \leq \frac{\frac{1}{2}}{6} |f'''(\xi)| \leq \frac{1}{12} \| f''' \|$

Lab 10

$$\int_a^b f(x)\,dx \text{ - area}$$



$$\text{area} \simeq (b-a) f\left(\frac{a+b}{2}\right)$$



$$h = \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx = h \cdot \sum_{i=0}^{n-1} f\left(a+\left(i+\frac{1}{2}\right)h\right) + \frac{h^2(b-a)}{2h} \cdot f''(\xi);$$

Composite rectangle $\quad \xi \in (a,b)$

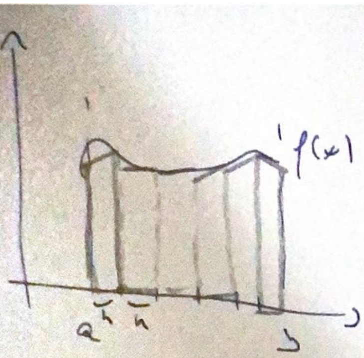
Octave

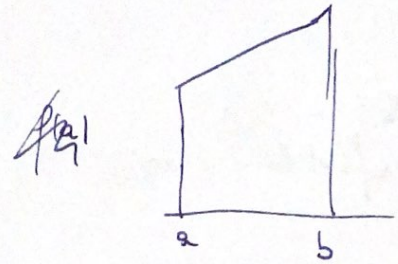
ex_01

```
function I = comp-rectangle (f, a,b, n)
    h=(b-a)/n;
    I = h* sum ( f(a+([0:n-1]+1/2)*h))
end;
```

1) $f = @(x)\ 1./x;$

comp-rectangle $(f, 1, 2, 15) \approx \log(2) \simeq 0.693$





$$\frac{f(a)+f(b)}{2}(b-a)$$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left(f(a)+f(b)+2^*\left(f_1+\dots+f_{n-1}\right)\right) - \frac{h^2(b-a)}{12}\cdot f''(\xi)$$

$$f_i = f(a+ih)$$

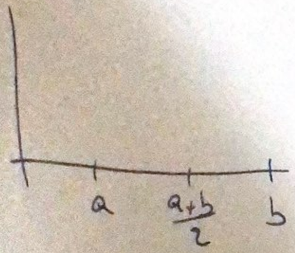
ex_02

```
function J = comp_trapezoid(f,a,b,n)
    h=(b-a)/n;
    J = h/2*(f(a)+f(b)+2*sum(f(a+[1:n-1]*h)));
end
```

2)

```
f=@(x) 1./x;
comp-trapezoid(f,1,2,15)
```

Simpson's Rule



a, b simple

$\frac{a+b}{2}$ double

$$h = \frac{b-a}{2m}$$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4\sum_{i=1}^{m} f_{2i-1} + 2\sum_{i=1}^{m} f_{2i} + f(b) \right] - \frac{h^4(b-a)}{180} \cdot f^{(4)}(\xi)$$

```
function J = simpsons(f, a, b, m)
    h = (b-a)/(2*m);
    J = h/3 * ( f(a) + f(b) + 4* ~~sum(a+([1:m]*2-1)~~
        sum(f(a+([1:m]*2-1)*h)) +
        +2*sum(f(a+([1:m-1]*2*h)) ~~*f(b))~~
end
```

```
f = @(x) 1./x;
simpsons(f,1,2,15)
```

```
function J = adaptquad(f, a, b, eps, met, n)
    J1 = met(f, a, b, n);
    J2 = met(f, a, b, 2*n);
    if |J1-J2| < eps
        J = J2
    else
        J = adaptquad(f, a, (a+b)/2, eps, met, n) + adaptquad(f, (a+b)/2, b, eps, met, n)
    endif
end
```

$$\int_0^1 \sqrt{1+(\pi\cos(\pi x))^2}\,dx$$

adaptquad(f, 0, 1, 10^{-6}, @simpsons, 4)

LUP and Cholesky

LU decomposition

$A = L \cdot U$, L - lower triangular matrix

U - upper —— " ——

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} = \left(\begin{array}{c|c} a_{11} & w^* \\ \hline v & A' \end{array}\right) =$$

$$= \left(\begin{array}{c|c} 1 & 0 \\ \hline \frac{v}{a_{11}} & I_{n-1} \end{array}\right) \cdot \left(\begin{array}{c|c} a_{11} & w^* \\ \hline 0 & A' - \frac{v}{a_{11}} \cdot w^* \end{array}\right)$$

Solve the system of eq using LUP decomposition

$$\begin{cases} 7x_1 - 2x_2 + x_3 = 12 \\ 14x_1 - 7x_2 - 3x_3 = 17 \\ -7x_1 + 11x_2 + 18x_3 = 5 \end{cases}$$

$$A = \begin{pmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{pmatrix} \qquad b = \begin{pmatrix} 12 \\ 17 \\ 5 \end{pmatrix}$$

$$Ax = b$$

$$A \sim \begin{pmatrix} 14 & -7 & -3 \\ 7 & -2 & 1 \\ -7 & 11 & 18 \end{pmatrix} \sim \begin{pmatrix} 14 & -7 & -3 \\ \frac{1}{2} & \frac{3}{2} & \frac{5}{2} \\ -\frac{1}{2} & \frac{15}{2} & \frac{33}{2} \end{pmatrix} \sim \begin{pmatrix} 14 & -7 & -3 \\ -\frac{1}{2} & \frac{15}{2} & \frac{33}{2} \\ \frac{1}{2} & \frac{3}{2} & \frac{5}{2} \end{pmatrix} \sim, \ P = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 1 \\ 11 & 18 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} -7 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 11 & 18 \end{pmatrix} - \begin{pmatrix} -\frac{7}{2} & -\frac{3}{2} \\ \frac{7}{2} & \frac{3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{2} & \frac{5}{2} \\ \frac{15}{2} & \frac{33}{2} \end{pmatrix}$$

$$\sim \begin{pmatrix} 14 & -7 & -3 \\ -\frac{1}{2} & \frac{15}{2} & \frac{33}{2} \\ \frac{1}{2} & \frac{1}{5} & -\frac{4}{5} \end{pmatrix}$$

$$\cancel{P = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}} \quad P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{5} & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 14 & -7 & -3 \\ 0 & \frac{15}{2} & \frac{33}{2} \\ 0 & 0 & -\frac{4}{5} \end{pmatrix}$$

$$P \cdot A = L \cdot U \qquad P \cdot | \ Ax = b \Rightarrow L \cdot U \cdot x = P \cdot b \Rightarrow \begin{cases} L \cdot y = P \cdot b \\ U \cdot x = y \end{cases}$$

$$\cancel{P \underbrace{Ax}_{y} = P \cdot b \Rightarrow Py}$$

$$\begin{cases} y_1 = 17 \\ -\frac{1}{2} y_1 + y_2 = 5 \\ \frac{1}{2} y_1 + \frac{1}{5} y_2 + y_3 = 12 \end{cases} \Rightarrow y_1 = 17 \Rightarrow y_2 = \frac{27}{2} \Rightarrow y_3 = 12 - \frac{17}{2} - \frac{27}{10}$$

$$\begin{cases} 14x_1 - 7x_2 - 3x_3 = 17 \\ \frac{15}{2}x_2 - \frac{33}{2}x_3 = \frac{27}{2} \\ -\frac{4}{5}x_3 = y_3 \end{cases}$$

Cholesky

$$A = \begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix} \qquad A \text{ symetric \& positive definite}$$

All eigenvalues > 0

$$x \cdot A \cdot x^t > 0 \quad , \forall\, x \in \mathbb{R}^n \setminus \{0_n\}$$

$$25 > 0 \qquad \begin{vmatrix} 25 & 15 \\ 15 & 18 \end{vmatrix} > 0 \qquad \det(A) > 0 \Rightarrow A \text{ is positive definite}$$

$$\Rightarrow \exists\, R : A = R^t \cdot R \quad , \quad R = \text{upper triang}$$

$$\begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix} \sim \left(\begin{array}{c|c} \sqrt{25} & \\ \hline \frac{15}{5} & \\ -\frac{5}{5} & \end{array}\right) \sim \left(\begin{array}{c|cc} \sqrt{25} & & \\ \hline 3 & 9 & \\ -1 & 3 & 10 \end{array}\right) \sim$$

$$\begin{pmatrix} 18 & \cancel{0} \\ 0 & 11 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}\begin{pmatrix} 3 & -1 \end{pmatrix} = \begin{pmatrix} 18 & \cancel{0} \\ 0 & 11 \end{pmatrix} - \begin{pmatrix} 9 & \\ -3 & 1 \end{pmatrix}$$

$$=$$

$$\sim \underbrace{\left(\begin{array}{c|cc} 5 & & \\ \hline 3 & 3 & \cancel{4} \\ -1 & 1 & 3 \end{array}\right)}_{= R^t}$$

$$R = \begin{pmatrix} 5 & 3 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

Lab 12

Does this iteration

$$x_{n+1} = x_n^5 - 10x_n^3 - 20x_n^2 - 15x_n - 5$$

converges to $x=-1$ if x_0 is close enough to x. Det the order of convergence.

$g(x) = x^5 - 10x^3 - 20x^2 - 15x - 5$

$g(-1) = -1 \Rightarrow a = -1$ is a fixed point of g

$g'(x) = 5x^4 - 30x^2 - 40x - 15$

$g'(-1) = 5 - 30 + 40 - 15 = 0$ ✓ $\quad |g'(a)| < 1$

$g''(x) = 20x^3 - 60x - 40$

$g''(-1) = 0$

$g'''(x) = 60x^2 - 60 \qquad g'''(-1) = 0$

$g^{(4)}(x) = 120x \qquad g^{(4)}(-1) = -120 \neq 0 \Rightarrow p = 4$ order of convergence

$$\lim_{n\to\infty} \frac{x_{n+1} - \alpha}{(x_n - \alpha)^p} = \frac{1}{p!} g^{(p)}(x)$$

$$\lim_{n\to\infty} \frac{x_{n+1} + 1}{(x_n + 1)^4} = \frac{1}{4!} \cdot g^{(4)}(-1) = \frac{-120}{24} = -5$$