2024

1.

$$x_{n+1} = 2 - (1+c)\,x_n + c \cdot x_n^2 \qquad \alpha = 1$$

$$g(x) = 2 - (1+c)x + cx^2$$

$$g(1) = 2 - (1+c) + c = 2 - 1 - c + c = 1 \quad \checkmark$$

$$g'(x) = -1 - c + 2cx$$

$$g'(1) = -1 - c + 2c = c - 1$$

$$|g'(1)| < 1$$

$$|c-1| < 1$$

$$-1 < c-1 < 1 \quad | +1$$

$$0 < c < 2 \Rightarrow c \in (0;2)$$

$$g''(x) = 2c$$

$$g''(1) = 2c \neq 0$$

$\Rightarrow p = 2$, for $c = 1$

$$err = \lim_{n \to \infty} \frac{x_{n+1} - \alpha}{(x_n - \alpha)^p} = \frac{1}{p!} \cdot g^{(p)}(\alpha) = \frac{1}{2} \cdot 2c = 1, \text{ for } c = 1$$

3. $A=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ $\quad b=\begin{pmatrix}-1\\ -2\end{pmatrix}$ $\quad x^0=\begin{pmatrix}0\\ 0\end{pmatrix}$

$D=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$ $\quad x^{k+1}=(D-L)^{-1}Ux^{(k)}+(D-L)^{-1}b$

$-U=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$

$-L=\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$

I $k=1$

$$x^1=\underbrace{\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}}_{B}^{-1}\begin{pmatrix}0 & -1\\ 0 & 0\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\end{pmatrix}+\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}^{-1}\begin{pmatrix}-1\\ -2\end{pmatrix}$$

$\det B=4$

$B^t=\begin{pmatrix}2 & 1\\ 0 & 2\end{pmatrix}$ $\quad B^*=\begin{pmatrix}2 & 0\\ -1 & 2\end{pmatrix}$ $\quad B^{-1}=\begin{pmatrix}\frac{1}{2} & 0\\ -\frac{1}{4} & \frac{1}{2}\end{pmatrix}$

$$x^1=\begin{pmatrix}\frac{1}{2} & 0\\ -\frac{1}{4} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}0 & -1\\ 0 & 0\end{pmatrix}\begin{pmatrix}0\\ 0\end{pmatrix}+\begin{pmatrix}\frac{1}{2} & 0\\ -\frac{1}{4} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}-1\\ -2\end{pmatrix}$$

$$x^1=\begin{pmatrix}0\\ 0\end{pmatrix}+\begin{pmatrix}-\frac{1}{2}\\ -\frac{3}{4}\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}\\ -\frac{3}{4}\end{pmatrix}$$

$\frac{1}{4}-1=-\frac{3}{4}$

II $k=2$

$$x^2 = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$x^2 = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$x^2 = \begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix}$$

$$x^2 = \begin{pmatrix} \frac{3}{8} \\ -\frac{3}{16} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix} = \begin{pmatrix} \frac{-1}{8} \\ \frac{-15}{16} \end{pmatrix}$$

III $k=3$

$$x^3 = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{8} \\ -\frac{15}{16} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix}$$

$$x^3 = \begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} -\frac{1}{8} \\ -\frac{15}{16} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix}$$

$$x^3 = \begin{pmatrix} \frac{15}{32} \\ -\frac{15}{64} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{4} \end{pmatrix} = \begin{pmatrix} -\frac{1}{32} \\ \frac{-63}{64} \end{pmatrix}$$

4.

x	0	1	2
y	1	-3	7

$Ax+B$

$$\begin{cases} a\sum_{i=1}^{3} x_i^2 + b\sum_{i=1}^{3} x_i = \sum_{i=1}^{3} x_i y_i \\ a\sum_{i=1}^{3} x_i + 3b = \sum_{i=1}^{3} y_i \end{cases}$$

$$\begin{cases} 5a + 3b = 11 \\ 3a + 3b = 5 \end{cases}$$

$$2a = 6 \Rightarrow \boxed{a=3} \Rightarrow \boxed{b = \frac{-4}{3}} \Rightarrow \boxed{P(x) = 3x - \frac{4}{3}}$$