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Decision Support

Dynamic pricing of multiple home delivery options

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ABSTRACT

Online grocers accept delivery bookings and have to deliver groceries to consumers' residences. Grocery stores operate on very thin margins. Therefore, a critical question that an online grocery store needs to address is the cost of home delivery operations. In this paper, we develop a Markov decision process-based pricing model that recognizes the need to balance utilization of delivery capacity by the grocer and the need to have the goods delivered at the most convenient time for the customer. The model dynamically adjusts delivery prices as customers arrive and make choices. The optimal prices have the following properties. First, the optimal prices are such that the online grocer gains the same expected payoff in the remaining booking horizon, regardless of the delivery option independently chosen by a consumer. Second, with unit order sizes, delivery prices can increase due to dynamic substitution effects as there is less time left in the booking horizon.

1. Introduction

An online grocery store allows customers to order products from virtual storefronts, and delivers products to customers' homes or workplaces. With over 750,000 regular customers and 200,000 orders per week, Tesco.com generated almost £1 billion of sales in 2005 (Tesco, 2006). Peapod.com increased its net sales by almost 30% in 2005 by increasing its number of costumers (Ahold, 2006). Forrester Research predicts that online grocery sales in Europe (USA) will increase from nearly 6.7 billion euros ($5.8 billion USD) in 2005 to 34 billion euros ($16.9 billion USD) in 2010 (Favier, 2006; Johnson, 2005). Most recently, online retailing giant Amazon.com entered the online grocery business by selling non-perishable goods.

In order to be successful in the online grocery business, online grocers must effectively address a number of issues, the most significant of which is complicated logistics (Punakivi and Saranen, 2001). Grocery stores operate on very thin margins. Therefore, a critical question that an online grocery store needs to address is the cost of home delivery operations. Yet, the strategy of acquiring ample delivery capacity will not resolve the delivery issue as the following quote from an interview with the Chairman of Tesco that appeared in the *Mckinsey Quarterly* explains: "Instead of spending lots of it, we put a lot of effort into getting the system and processes right: how to manage vehicle loading, for example. You can't be sending out half-empty vehicles if you want any financial returns. But this is exactly what some supermarkets ended up doing – and taking hours to make trips to and from the warehouse. Most of them have had to stop" (Child, 2002).

While home delivery generates increased operating costs and other operational challenges for an online grocer, it has its benefits. As with point of sale systems, the grocer has access to the purchase history of its customers but has greater flexibility in targeting promotions to specific customers during the purchase process and at the point of purchase – a unique feature of the online environment. Sometimes delivery cost is used only to recoup a portion of the delivery costs and occasionally it is used to cover some additional costs beyond just the delivery. However, the existence of low and high demand periods for delivery results in lost sales due to capacity constraints. The capacity constraints arise due to "rigid" warehouse capacity and the difficulty of temporarily hiring truck drivers who are trained in customer service (Agatz, 2005). We propose a dynamic pricing approach to alleviate this problem. Dynamic pricing of home delivery is appealing for two reasons. First, Punakivi and Saranen (2001) show that the more the customer can control the time of delivery, the higher the operating costs of the online grocer. Therefore, dynamic pricing can be used as a mechanism to decrease the cost of home delivery by providing the right incentives to customers. Second, it can increase revenue by charging higher delivery prices to customers who are willing to pay more for a convenient delivery time.

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Table 1
A hypothetical time-window availability grid (partly adapted from Peapod.com)

Wednesday, February 21 Delivery		
Morning (*Submit order by 8:00 p.m. on Tuesday February 20*)		
8:00–10:00 a.m.		Past cutoff
10:00–12:00 p.m.		Past cutoff
Afternoon (*Submit order by 11:00 p.m. on Tuesday February 20*)		
2:00–4:00 p.m.		Fully booked
4:00–6:00 p.m.	Save $2.00	Select

A few online grocers such as Peapod have adopted dynamic pricing policies. Table 1 shows a time-window availability grid for a specific zip code. Groceries can be delivered within different time-windows during the day. We call these time-windows *delivery options*. In Table 1, a customer cannot book the delivery option 8:00–10:00 a.m., since it is already past 8:00 p.m. on February 20 for the next morning delivery. That is to say, customers must book a desired delivery option before a cut-off time. Furthermore, a delivery option may not be available because of filled capacity. We also observe that this online grocer offers a discount for one of the delivery options.

Importantly, through its Web interface an online grocer can dynamically change the delivery fees by offering different levels of discounts for specific delivery options and charge different amounts to different types of customers such as customers making large purchases. As online grocers become more sophisticated in managing data and automating pricing decisions, they will adopt dynamic pricing policies similar to those of Peapod. Hence, an important question is how to model and structure these dynamic pricing policies. In this paper, we develop a Markov decision-process-based pricing model that recognizes the need to balance the grocer's utilization of delivery capacity and the customers' need to have the goods delivered at the most convenient time. This model dynamically adjusts delivery prices for different delivery options as customers arrive and make choices. To the best of our knowledge, this is the first study that incorporates these complex issues.

We model a setting in which customers choose from multiple delivery options. Online grocers can use dynamic pricing as a mechanism to control uncertainty created by consumers' choice of delivery options according to the utility they derive from each option. Particularly, the prices should be chosen such that the online grocer earns the same expected payoff with the selection of any of the delivery options. This makes the online grocer indifferent between delivering in any of the available delivery options. According to Agatz (2005), the control of consumers' choices can be achieved *coercively* by not displaying some of the available delivery options (e.g. Campbell and Savelsbergh, 2006's model with accept-and-allocate or reject decisions). However, the opportunity cost of a coercive method is lost sales and displeased customers. Therefore, a *persuasive* method that balances the trade-off between lost sales and expected payoffs from sales and delivery is more desirable for the long term success of an online grocer. Lost sales not only result in the loss of the delivery fee but also the grocery order, thus, optimal delivery pricing should also consider the monetary value of an order. Specifically, customers who make large purchases should be offered lower prices.

A natural question that arises is how prices should change over time and with decreasing capacity. The extant literature suggests that the price of a single perishable product should decrease, as there is less time and more capacity for additional bookings (Gallego and van Ryzin, 1994). When there are multiple delivery options, this does not necessarily hold. The price of a delivery option can be raised to increase demand for another delivery option. Specifically, an online grocer with a popular and a less popular delivery option should set prices so that the *popular* option may be filled *late* in the booking horizon, whereas the *less popular* option may be filled *early* in the booking horizon. The intuition behind this result is that it is more likely that the popular option will be chosen by an arriving consumer. Therefore, the online grocer can delay filling the popular option by keeping its price high hoping that consumers may pay more for this option or choose the other option. However, as the cut-off time gets closer, the online grocer would like to increase the likelihood of sales by decreasing the price of the popular option and increasing the price of the less popular option. This result highlights the importance of *dynamic substitution effects* in pricing multiple options.

The remainder of this paper is organized as follows. In the next section, we review the relevant literature and present our contribution to the literature. In Section 3, we present our model and in Section 4 we analyze the properties of the optimal pricing decisions. Section 5 concludes the paper with a discussion of extensions and future research.

2. Relevant literature

We identify two research streams relevant to our study: online grocery delivery research and revenue management. Each stream is discussed in turn.

Major motivation for our paper stems from the ECOMLOG (2002) research program at the University of Helsinki that produced a number of papers specific to online grocers and home delivery (Tanskanen et al., 2002). The first question tackled by these researchers was the determination of different home delivery concepts. Towards this end, Kämäräinen et al. (2001) propose a framework that identifies the various dimensions of a home delivery concept: product range offered, type of reception, delivery hours, length of time windows, cut-off times, and delivery pricing. Several papers study different delivery concepts with respect to their performance in terms of delivery costs (Saranen and Småros, 2001; Punakivi and Saranen, 2001; Punakivi et al., 2001; Yrjölä, 2001). The majority of the papers in this stream adopt a simulation approach. In this approach, a commercial vehicle routing software was used to simulate delivery concepts within an actual metropolitan area. Campbell and Savelsbergh (2005) contributed to this literature by developing specific vehicle routing algorithms for home delivery. A recurring theme in this literature is that operating costs are substantially increased by giving more control to customers in determining the delivery time, e.g. with shorter delivery options. Campbell and Savelsbergh (2006) resolves this issue with an accept/reject decision model and focus more on the performance of routing algorithms rather than on how to qualitatively set prices. Our study focuses on delivery pricing, which is not studied by the ECOMLOG program, and shows that delivery pricing can act as a mechanism to alleviate the lack of control over customers' delivery choices. Finally, Agatz et al. (2006) present a recent survey of the literature on Internet fulfillment.

Revenue management is a rich literature stream that includes the management of perishable assets (see surveys Weatherford and Bodily, 1992; McGill and van Ryzin, 1999). Delivery capacity for online grocers is a perishable asset as unutilized capacity on a given day is a lost revenue opportunity. An important component of revenue management is dynamic pricing. Elmaghraby and Keskinocak (2003), in a review of dynamic pricing literature, found that a few of the reviewed papers considered multiple products. We are aware of at least two multiple product dynamic pricing models. In Gallego and van Ryzin (1997), the demand for each of the products depends on the vector of prices for the products. The relationship between stochastic and deterministic formulations of the problem is analyzed. Using this relationship, heuristics are proposed which are asymptotically optimal. Maglaras and Meissner (2004) show that the multi-product dynamic pricing problem can be studied as a single product dynamic pricing problem by a demand aggregation technique and propose several heuristics. These papers, thus, study theoretical relationships of general dynamic pricing models and heuristics that can produce near optimal solutions. On the contrary, we are interested in the structure of the prices for the particular dynamic pricing problem facing online grocers. Apart from these models, Talluri and Van Ryzin (2004) address the issue of choosing a fare class for a single-flight leg according to a discrete choice model. From this perspective, customers' decisions in our model can be considered as choosing another flight on the same airline on the same leg. Gallego and Phillips (2004) provide a model where flexibility in managing capacity is provided not with pricing but a flexible product – a menu of multiple substitutes. The specific substitute is chosen by the airline for a customer who bought a flexible product. This model is a two-period capacity allocation model with exogenous prices and price independent aggregate demand functions.

3. Model

3.1. Delivery options

Delivery options are restricted to a limited geographical area such as a zip code (i.e. Peapod's delivery options). This ensures that the same capacity pool is available for arriving customers. In effect, we consider each geographical area independently and the model is solved for a single geographical area. There are $N+1$ options available for customers: they may either choose one of N delivery options ($n = 1,\ldots,N$) or the no-purchase option ($n = 0$). We consider a discrete booking horizon $[T,0]$, in which time represents the time remaining in the booking horizon, with time being in reverse chronological order. Delivery options become available at time T and the last delivery option is closed for booking at time 0. An order can be delivered if there is sufficient time and capacity for fulfillment and delivery. We represent the end of the booking or cut-off time of delivery option n with $\tau_n(\tau_1 \geqslant \tau_2 \geqslant \cdots \geqslant \tau_N = 0)$. The capacity vector at time t, $\mathbf{x}(t) = (x^1(t), x^2(t),\ldots,x^N(t))$ represents the N-vector of remaining capacities at time t. We assume that the capacity levels are committed *a priori* and stay constant over the booking horizon.

3.2. Customer model

Customer population within a particular geographical area is segmented into customer classes according to notable demand characteristics. We expect to have a small number of customer classes that show significant behavioral differences. For example, a *three-class* customer categorization may include business customers who prefer delivery on Mondays and who are less price sensitive, customers who strongly prefer delivery on Fridays, and customers who are price sensitive and are willing to switch to a delivery option with the lowest price (Agatz, 2005). In order to capture these characteristics, we develop the following customer model.

There are M customer classes all with unit-sized orders. Each customer class m is characterized by an arrival probability at each period t p_{mt}, $t = T, T-1,\ldots,1$; an expected profit r_m; a vector of predictable utilities $\mathbf{U}_m = (U_m^0, U_m^1, \ldots, U_m^N)$ for each delivery option, $n = 1,\ldots,N$, and an outside (or no-purchase) option $n = 0$; and a price sensitivity coefficient $\beta_m > 0$ for which a customer's utility decreases with a unit increase in delivery price. Let p_{0t} represent the no-arrival probability at period t, then the arrival probabilities satisfy the following equality:

$$p_{0t} + \sum_{m=1}^{M} p_{mt} = 1. \tag{1}$$

Interpretation of the customer model is as follows: the arrival probabilities can be generated from a non-homogeneous Poisson arrival process via uniformization (see Subramanian et al., 1999; Lee and Hersh, 1993) by allowing at most one arrival per period according to Eq. (1). The expected order profit r_m represents the average profit this customer class brings to the store after deduction of cost of goods sold and fulfillment costs and does not include delivery fees. We assume that the order size, which is the number of delivery boxes the order requires, is equal to one for all customer classes. This allows us to study the form of prices over time in the absence of multi-unit orders with different sizes. It is well known that for different order sizes, the prices are not necessarily monotonic in capacity, even for a single delivery option (see Lee and Hersh, 1993, for a discussion in the context of seat inventory control).

We believe that the online grocery store cannot predict each customer's utility with certainty; hence we adopt a random utility model (Ben-Akiva and Lerman, 1985). Specifically, we use a logit-based model of choice. Each customer's utility for delivery is composed of a predictable part and a random part. Each element of the utility vector, $\mathbf{U}_m, U_m^n$, represents the predictable part of utility of a customer class m for delivery option n when the option is available for a zero delivery fee (price), except that U_m^0 represents the utility of no purchase. A unit increase in the price of a delivery option n reduces the utility for that option by the price coefficient $\beta_m > 0$ for the customer class m. Let $\mathbf{a}_m = (a_m^1, \ldots, a_m^n, \ldots, a_m^N)$ be the vector of prices, where a_m^n is the price for option n for the customer class m. Thus, a customer of class m facing a price menu $\mathbf{a}_m$ has a utility of

$$u_m^n(\mathbf{a}_m) = U_m^n - \beta_m a_m^n + \tilde{\varepsilon}_m^n \quad \text{for } n = 1, \ldots, N, \quad \text{and} \quad u_m^0 = U_m^0 + \tilde{\varepsilon}_m^0, \tag{2}$$

where $\tilde{\varepsilon}_m^n$ for $n = 0, 1,\ldots,N$ are independent and identically distributed random variables distributed according to Gumbel distribution. A particular delivery option is chosen by a customer of class m, if the utility associated with that delivery option exceeds the utility from other options.

Before we can specify the choice probability (a customer of class m choosing option n), we need to define the set of all available delivery options. A delivery option is available at time t, if there is sufficient time and the remaining fulfillment and delivery capacity is at least one. The set of available delivery options $\mathbf{A}(\mathbf{x}(t))$ is defined as $\mathbf{A}(\mathbf{x}(t)) = \{n{:}t > \tau^n, x^n(t) \geqslant 1\}$. Let $\tilde{n}_m$ represent the random variable denoting a customer's choice; the choice probability for delivery option n is formally defined as $\Pr(\tilde{n}_m = n|\mathbf{x}), n = 0, 1, \ldots, N$ and satisfy[1]

$$\Pr(\tilde{n}_m = n|\mathbf{x}) = \begin{cases} \Pr(u_m^n(\mathbf{a}_m) > \max\limits_{i \neq n} u_m^i(\mathbf{a}_m) \geqslant u_m^0) & \text{for } n \neq 0,\ i \in \mathbf{A}, \\ \Pr(u_m^n > \max\limits_{i} u_m^i(\mathbf{a}_m)) & \text{for } n = 0,\ i \in \mathbf{A}, \\ 0 & \text{for } n \notin \mathbf{A}. \end{cases} \tag{3}$$

Each customer, thus, makes an incentive compatible choice by choosing the option that gives his/her the highest utility and reveals the most preferred option. Therefore, for each customer class m we have

$$\Pr(\tilde{n}_m = n) = \begin{cases} e^{U_m^n - \beta_m a_m^n} \Big/ \left(e^{U_m^0} + \sum\limits_{i \in \mathbf{A}} e^{U_m^i - \beta_m a_m^i} \right) & \text{for } n \in \mathbf{A}, \\ e^{U_m^0} \Big/ \left(e^{U_m^0} + \sum\limits_{i \in \mathbf{A}} e^{U_m^i - \beta_m a_m^i} \right) & \text{for } n = 0, \\ 0 & \text{for } n \notin \mathbf{A} \end{cases} \tag{4}$$

and

$$\sum_{n=0}^{N} \Pr(\tilde{n}_m = n) = 1. \tag{5}$$

Understanding how customers will react to price changes is important in understanding how prices change dynamically over time and with remaining capacity. Below, we present the properties of our customer demand model.

Lemma 1. *Customer choice model properties. For each customer class m facing a price menu* $\mathbf{a}$*:*

a. *when the price of option n increases, its demand changes by*

$$\frac{\partial \Pr(\tilde{n}_m = n)}{\partial a^n} = -\beta_m \Pr(\tilde{n}_m = n) \sum_{\substack{i \neq n \\ i \in \mathbf{A} \cup 0}}^{0} \Pr(\tilde{n}_m = i) = -\beta_m \Pr(\tilde{n}_m = n)(1 - \Pr(\tilde{n}_m = n)) < 0 \quad \textit{for } n \in \mathbf{A};$$

b. *when the price of option i increases, the demand for option n, $i \neq n$ increases by:*

$$\frac{\partial \Pr(\tilde{n}_m = n)}{\partial a^i} = \beta_m \Pr(\tilde{n}_m = n) \Pr(\tilde{n}_m = i) > 0 \quad \textit{for } i \neq n;\ i \in \mathbf{A},\ n \in \mathbf{A} \cup 0; \quad \textit{and}$$

c. *let $\mathbf{A}_1$ and $\mathbf{A}_2$ be two sets of available delivery options. If $\mathbf{A}_1 \subset \mathbf{A}_2$; then $\Pr(\tilde{n}_m = n|\mathbf{A}_1) \geqslant \Pr(\tilde{n}_m = n|\mathbf{A}_2)$ for $n \in \mathbf{A}_1 \cup 0$.*

Proof. All the proofs are included in the Appendix.

This form has intuitive characteristics we would like to capture in a multi-option demand model. Part (a) implies that as the price of delivery option n (a^n) increases, the demand for option n decreases proportional to the price sensitivity (β_m), n's existing demand ($\Pr(\tilde{n}_m = n)$), and the total demand for other options $(1 - \Pr(\tilde{n}_m = n))$. Part (b) implies that this reduction in demand for option n, generated by the increase in a^n, is transferred to each option i in proportion to its existing demand $\Pr(\tilde{n}_m = i)$. Consequently, popular options observe a higher increase in demand than unpopular options do when the price of another option rises. Secondly, when a price of delivery option n is decreased, the majority of the increased demand for this option will come from more price-sensitive customer classes. In Part (c), we note that as the delivery options become unavailable the probability of no purchase and the demand for delivery options that are still available increase. These characteristics capture significant effects on dynamic pricing policies. Increasing the price of a delivery option may be used to shift demand to a delivery option with high capacity. However, this also creates an adverse effect, due to elastic customer demand; it decreases the overall demand for delivery. We also observe this adverse effect when the number of delivery options is decreased.

3.3. System states and optimality equations

We use a discrete-time, discrete-state Markov Decision Process (Puterman, 1994) to model the pricing decisions of the online grocer. As mentioned above, there are T stages numbered in reverse chronological order $t = T, T-1, \ldots, 1, 0$. At each stage, we assume that one and only one of an arrival or a null event occurs. We define the reduced system state with the vector of remaining capacities $\mathbf{x}(t)$ at stage t. We define the full system state as $(\mathbf{x}(t), m)$ that includes the class of the customer who arrived to the system.

Our objective is to maximize the expected total net benefit of operating the system over the horizon from stage T to 0, starting from the empty state. Let $V_t(\mathbf{x})$ denote the maximal expected net benefit of operating the system over the period from t to 0. We call $V_t(\mathbf{x})$ the value function, which represents the maximum payoff that can be generated with available capacity $\mathbf{x}(t)$. $V_t(\mathbf{x}, m)$ is the value function at state $(\mathbf{x}(t), m)$. The optimal value functions $V_t(\mathbf{x})$ and $V_t(\mathbf{x}, m)$ are determined recursively using Bellman's Principle of Optimality:

[1] Note on notation: The choice probability is a function of both capacity vector $\mathbf{x}$ and price menu $\mathbf{a}$. Moreover price menu $\mathbf{a}$ is a function of time t and capacity vector $\mathbf{x}$. In order to avoid cumbersome notations, we have dropped $\mathbf{x}$, $\mathbf{a}$, and t from several expressions when it causes no confusion. Furthermore, we occasionally use $\mathbf{A}$ to represent the set of available delivery options.

$$V_t(\mathbf{x}) = p_{0t}V_{t-1}(\mathbf{x}) + \sum_{m=1}^{M} p_{mt}V_t(\mathbf{x}, m) \quad \text{for } T \geqslant t \geqslant 1, \tag{6}$$

$$V_t(\mathbf{x}, m) = \max_{\mathbf{a}_{mt}} \left[\Pr(\tilde{n}_{mt} = 0)V_{t-1}(\mathbf{x}) + \sum_{n=1}^{N} \Pr(\tilde{n}_{mt} = n)\{r_m + a_{mt}^n + V_{t-1}(\mathbf{x} - \mathbf{I}^n)\} \right] \quad \text{for } T \geqslant t \geqslant 1, \quad \text{and} \tag{7}$$

$$V_0(\mathbf{x}) = 0, \tag{8}$$

where $\mathbf{I}^n$ is the nth unit vector.

The optimality Eqs. (6)–(8) characterize how the delivery system moves from one state to another. In addition, they determine how optimal prices are determined. Eq. (6) shows that when no arrival occurs, the system moves to the next stage with the same capacity $\mathbf{x}$. If an arrival occurs, an optimal price vector $\mathbf{a}$ is chosen according to Eq. (7). Eq. (7) specifies that if the customer makes a no-purchase decision, the system state remains the same in the next stage. If the customer chooses a delivery option, the profits for the purchase order and the price for that delivery option are collected. As a result, the available capacity vector is modified to reflect the new remaining capacity. At time 0, the value of the remaining capacity is zero. Two characteristics of the value function $V_t(\mathbf{x})$ can be obtained from Eqs. (6) and (7):

Lemma 2. *Characteristics of the value function. $V_t(\mathbf{x})$ is non-decreasing in both $\mathbf{x}$ and t.*

The intuition behind this result is the following. The optimal solution of a home delivery problem with smaller capacity is a feasible solution to the same problem instance with higher capacity if the prices are set so that no order is allocated to the additional capacity of the latter system. Similarly, the optimal solution of a home delivery problem with shorter remaining time is also a feasible solution to the same problem instance with longer remaining time if pricing is such that no purchase occurs during the extra time. Finally, note that Lemma 2 follows since the optimal solution is weakly better than any feasible solution. With the optimality conditions in place, we are now ready to analyze the structure of optimal price menus.

4. Optimal price menus and dynamic pricing

4.1. Price menus

Before we can study how prices change over time and available capacity, we need to understand the nature of the price menu shown to a customer after his or her arrival. This requires us to study Eq. (7). By rearranging the terms in (7) and using Eq. (5), we can re-write Eq. (7) as

$$V_t(\mathbf{x}, m) = \max_{\mathbf{a}_{mt}} \left[V_{t-1}(\mathbf{x}) + \sum_{n=1}^{N} \Pr(\tilde{n}_{mt} = n)\{r_m + a_{mt}^n - v_{t-1}^n(\mathbf{x})\} \right], \tag{9}$$

where $v_{t-1}^n(\mathbf{x}) = V_{t-1}(\mathbf{x}) - V_{t-1}(\mathbf{x} - \mathbf{I}^n)$ is the opportunity cost of delivery option n. Theorem 1 characterizes the optimal solution to Eq. (9).

Theorem 1. *Optimal Price Menu. Let m be the class of the customer arriving at stage t, then for a price vector $\mathbf{a}_{mt}^*$ to be optimal, the following condition is necessary and sufficient*

$$a_{mt}^{i*} + r_m - v_{t-1}^i(\mathbf{x}) = a_{mt}^{j*} + r_m - v_{t-1}^j(\mathbf{x}) = (\beta_m \Pr(\tilde{n}_{mt} = 0))^{-1} \quad \textit{for } \forall i, j \in \mathbf{A}(\mathbf{x}), \tag{10}$$

where $v_{t-1}^n(\mathbf{x}) = V_{t-1}(\mathbf{x}) - V_{t-1}(\mathbf{x} - \mathbf{I}^n)$ is the opportunity cost of delivery option n.

Notable structural properties of optimal price menus emerge in Theorem 1. Before explaining these properties, we focus on the meaning of the opportunity cost, which plays a major role in the formation of price menus. The opportunity cost for a delivery option is the value lost by allocating capacity for an order using that option. Alternatively, $v_{t-1}^n(\mathbf{x})$ is the value of one unit of additional capacity when the capacity vector is $\mathbf{x} - \mathbf{I}^n$. The additional value comes from the future arrival of customers and the purchase decisions they make. Because the right-hand side of (10) is positive, the total value collected from each arriving customer $(a_{mt}^{i*} + r_m)$ has to exceed the opportunity cost. The result that an order must have a positive gain generalizes earlier results in the revenue management literature (e.g. see Van Slyke and Young, 2000). Moreover, we find that the gains from choosing any available option should be the same for each customer class. Put differently, the arriving customer's delivery option choice (if he or she chooses one) does not affect the expected payoffs in the remaining horizon (i.e. $a_{mt}^{i*} + r_m + V_{t-1}(\mathbf{x} - \mathbf{I}^i) = a_{mt}^{j*} + r_m + V_{t-1}(\mathbf{x} - \mathbf{I}^j)$). Thus, *the optimal price menu has the appealing property in that it manages choice uncertainty by making the online grocer indifferent between delivering in any available delivery window*. This balancing property has two consequences. First, the shape of the discrete price functions each customer class is facing over time is the same because the way prices change over time is determined by the opportunity costs. However, the level of prices is determined by customer class characteristics. Second, the form of (10) has a useful computational benefit. Because the price of each delivery option can be written in terms of the price of the first delivery option, it converts the multi-dimensional problem of finding prices for multiple delivery options into finding only the price of the first option. Not only does (10) balance the payoffs from different delivery options, it also adjusts prices depending on a customer's price sensitivity and the utility received from delivery. Corollary 1 deals with how price changes according to customer classes.

Corollary 1. *Pricing behavior according to customer class. Let m be the class of the customer arriving at stage t. For an optimal price vector $\mathbf{a}_{mt}^*$, the optimal price of any available delivery option i decreases with the price sensitivity of the customer and the expected order profit. Formally, $\frac{\partial a_{mt}^{i*}}{\partial \beta_m} < 0$ and $\frac{\partial a_{mt}^{i*}}{\partial r_m} < 0$ for $i \in \mathbf{A}(\mathbf{x})$.*

Less price-sensitive customer classes, such as businesses, face higher delivery prices in the optimal price menu. The customer classes that bring higher profits to the online grocer are offered lower prices. This form has been used by Peapod and HomeGrocer.

Corollary 2. *Pricing behavior according to opportunity cost. Let m be the class of the customer arriving at stage t. For an optimal price vector $\mathbf{a}_{mt}^*$, the optimal price of option i increases with its opportunity cost and decreases with the opportunity cost of an alternative. Formally*

Table 2
Example parameters

T	r_1	U_1^0	U_1^1	U_1^2	β_1	p_{1t}	$x^1(60)$	$x^2(60)$
60	10	0.5	1	0.5	0.1	0.3 for all t	10	10

$$1 > \frac{\partial a_{mt}^{i*}}{\partial v_{t-1}^{i}(\mathbf{x})} = 1 - \Pr(\tilde{n}_{mt} = i) > 0 \quad \textit{and} \quad -1 < \frac{\partial a_{mt}^{i*}}{\partial v_{t-1}^{j}(\mathbf{x})} = -\Pr(\tilde{n}_{mt} = j) < 0 \quad \textit{and for } i \neq j,\ i,j \in \mathbf{A} \quad \textit{and} \quad m = 1,\ldots,M.$$

As expected, the price of a delivery option increases with its opportunity cost. However, the price of an option decreases when the opportunity cost of an alternative option increases. Corollary 2 implies that prices increase at a slower rate than opportunity costs do. Theorem 1 suggests that the nature of dynamic prices at different states over time mainly depend on the opportunity costs. Proposition 1 provides additional insight by analyzing how opportunity costs behave according to time and capacity.

Proposition 1. *Characteristics of opportunity costs.*

a. $v_t^n(\mathbf{x})$ *is non-increasing in* $x^n(>0)$ *for every* $n(>0)$ *and* $\mathbf{x}$.
b. $v_t^n(\mathbf{x})$ *is non-decreasing in t.*

An immediate implication of Proposition 1 is that optimal price is monotonic in both capacity (price decreases as there is more capacity) and time (price increases as there is more time before the end of the booking horizon) for a single delivery option problem instance. This result is expected and has been shown earlier by Gallego and van Ryzin, (1994). Surprisingly, Proposition 1 does not imply that prices are always monotonic when there are multiple delivery options. To understand this result, we consider an example with two delivery options. Consider the difference between the optimal price of the second delivery option at time $t(a_{mt}^{2*})$ and at time $t-1(a_{m,t-1}^{2*})$. As opposed to a single-delivery-option problem instance, the difference between the opportunity costs of both options affect how the price changes. If this difference is not too large, we can approximate the price difference as follows

$$a_{mt}^{2*} - a_{m,t-1}^{2*} \cong \frac{\partial a_{mt}^{2*}}{\partial v_{t-1}^{1}(\mathbf{x})}\{v_{t-1}^{1}(\mathbf{x}) - v_{t-2}^{1}(\mathbf{x})\} + \frac{\partial a_{mt}^{2*}}{\partial v_{t-1}^{2}(\mathbf{x})}\{v_{t-1}^{2}(\mathbf{x}) - v_{t-2}^{2}(\mathbf{x})\}. \tag{11}$$

Applying Corollary 2, we get

$$a_{mt}^{2*} - a_{m,t-1}^{2*} \cong -\Pr(\tilde{n}_{mt} = 1)\{v_{t-1}^{1}(\mathbf{x}) - v_{t-2}^{1}(\mathbf{x})\} + \{1 - \Pr(\tilde{n}_{mt} = 2)\}\{v_{t-1}^{2}(\mathbf{x}) - v_{t-2}^{2}(\mathbf{x})\}. \tag{12}$$

Proposition 1b implies that $v_{t-1}^n(\mathbf{x}) - v_{t-2}^n(\mathbf{x}) \geqslant 0, n = 1,2$. An inspection of (12) reveals why prices are not necessarily monotonic in time. There is a negative effect from the increase in the opportunity cost of Option 1 and a positive effect from the increase in the opportunity cost of Option 2. Therefore, moving from time $t-1$ to t, if the opportunity cost of Option 1 increases much faster than the opportunity cost of Option 2 does, time t price a_{mt}^{2*} is less than time $t-1$ price $a_{m,t-1}^{2*}$ and vice versa. We investigate these interdependency results further with a numerical example to make the intuition clearer.

4.2. An example

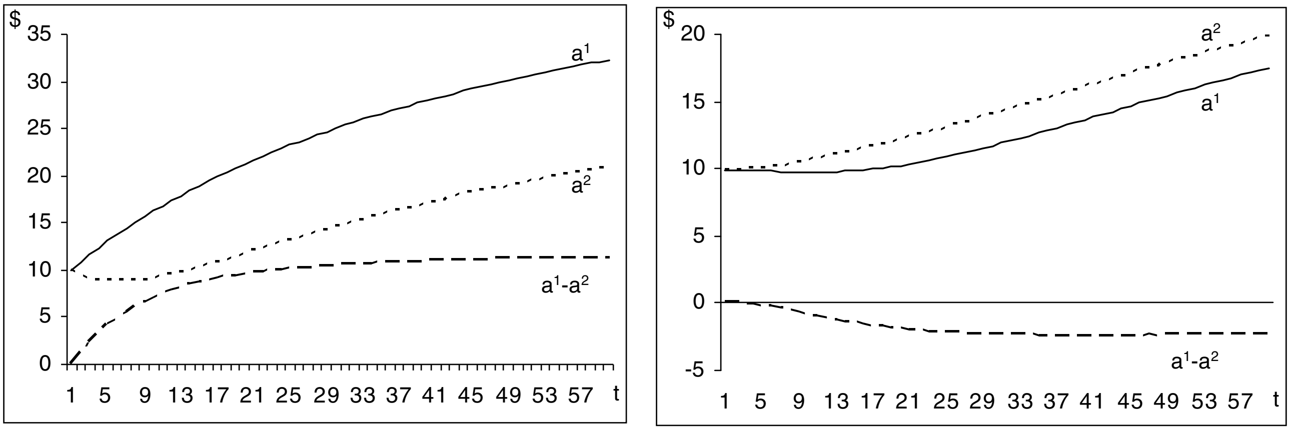
A complicated example would take us away from our objective of delving into the inner workings of our model. We therefore use a simple example with only one customer class arriving with a constant rate and two delivery options with Option 1 more popular than Option 2. Table 2 lists the parameters of the example.

Fig. 1 shows how optimal prices change over time for two states. The graphs in this figure depict prices as the capacity of the more popular Option 1 is increased from 1 to 4 and the capacity of Option 2 remains at 2 (i.e. (1,2) → (4,2)). A careful examination of these graphs illustrates the results of the previous section. A comparison of graphs 1a and 1b reveals that Option 1 price (a^1) decreases compared to Option 2 price (a^2) as the capacity of Option 1 increases.[2] In graph 1a, the price of Option 1 is much higher than the price of Option 2; in contrast, in graph 1b, Option 2 price is always larger than option 1 price. This effect exhibited in graphs 1a and 1b is due to the decrease in the opportunity cost of Option 1 as its capacity increases (Proposition 1a). Hence, this simple example shows that a popular option may have a lower price in situations in which prices are adjusted according to time and capacity.

Fig. 1a provides clues about the rationale behind increasing a delivery option's price (a^2), even though it is less popular. Mathematical intuition, presented in Eqs. (11),(12) in the previous section, can explain this behavior: since Option 1 is popular and has less available capacity ($\mathbf{x}(\bullet) = (1,2)$), its opportunity cost decreases much faster than the opportunity cost of Option 2 does, thereby pushing up the price of delivery Option 2 as t decreases. The prices are eventually equal at $t = 1$, because the opportunity costs are zero at the end of the booking horizon ($t = 0$). We can better understand this pricing dynamic by looking at the choice probabilities depicted in Fig. 2. When there is more time for booking possible customer orders, the online grocer posts higher prices for Option 1; customers have higher utilities for this option and thus are willing to pay more. However, as time passes, the expected number of customer arrivals decreases (opportunity costs decrease). To ensure that the capacity of Option 1 – which is more likely to create an order – is utilized, the online grocer decreases the price of Option 1 and increases the price of Option 2. Hence, optimal dynamic pricing creates an intriguing substitution effect. *Popular delivery option 1 will be filled late in the booking horizon, whereas less popular delivery option 2 will be filled early in the booking horizon*. As can be seen from Fig. 1, the popularity of an option (customer utilities) becomes less important as there is less time in the booking horizon.

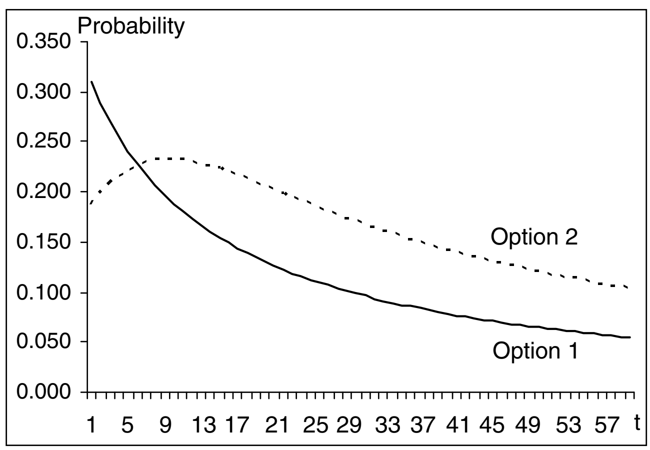
Fig. 3 shows the results of a single simulation run of this example. As evident from Figs. 1 and 2, the optimal dynamic pricing policy first allocates orders into the less popular Option 2 and then into Option 1 ($\mathbf{x}(20) = (8,5)$; $\mathbf{x}(10) = (6,5)$). Interestingly, Option 1 brings a payoff of

[2] Customer class is dropped because there is only one customer class in this example.



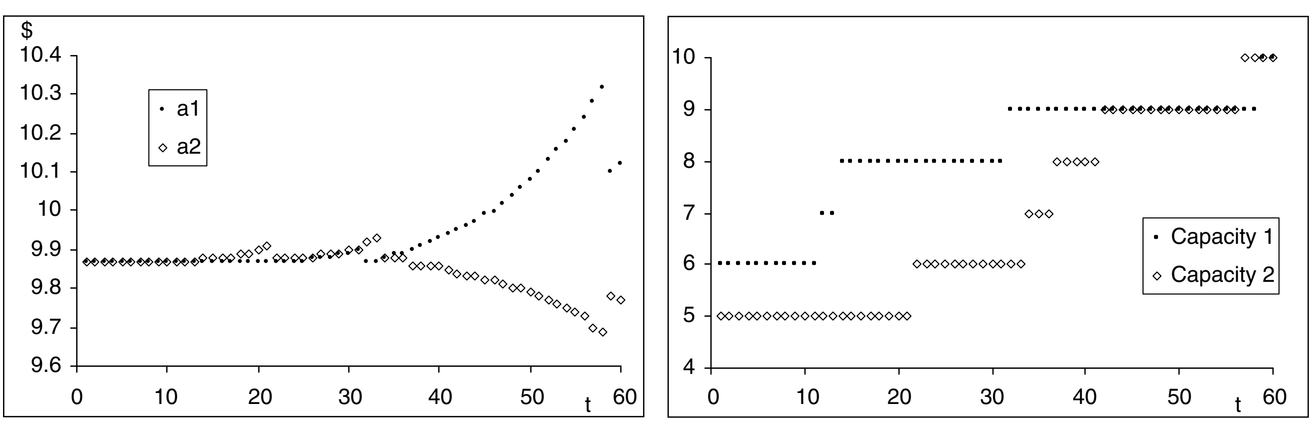
a. $\mathbf{x}(\bullet)=(1,2)$ **b.** $\mathbf{x}(\bullet)=(4,2)$

Fig. 1. Prices over time for different capacity levels, with delivery Option 1 being more popular.



$\mathbf{x}(\bullet)=(1,2)$

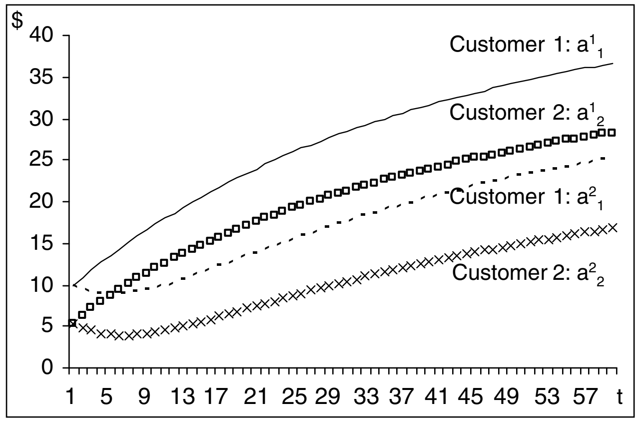
Fig. 2. Choice probabilities generated by the prices in Fig. 1a.



$\mathbf{x}(60)=(10,10)$ $\mathbf{x}(60)=(10,10)$

Fig. 3. Sample price and capacity path generated by simulation.

$79.71, which is somewhat lower than the $99.16 payoff generated by Option 2. Because there is much greater capacity than expected demand, the prices are in the neighborhood of the zero opportunity cost price of $9.87. This finding highlights the possibility of using fixed prices under certain situations instead of dynamic pricing policies.



x(•)=(1,2)

Fig. 4. Prices with two customer classes; class 2 is more profitable and has an order size of 1.

To illustrate how prices change across customer classes, we modify our example by introducing a new customer class with all the same parameters (except that $r_2 = 20$; $p_{2t} = 0.15$) and reducing the arrival rate of customer class 1 to $p_{1t} = 0.15$. Thus, the total arrival rate of customers does not change, and a more profitable customer class is introduced. Fig. 4 shows the prices corresponding to state $\mathbf{x}(\bullet) = (1,2)$. We observe that the shape of the prices is not altered; however the level of prices for the more profitable class 2 is much lower than for class 1.

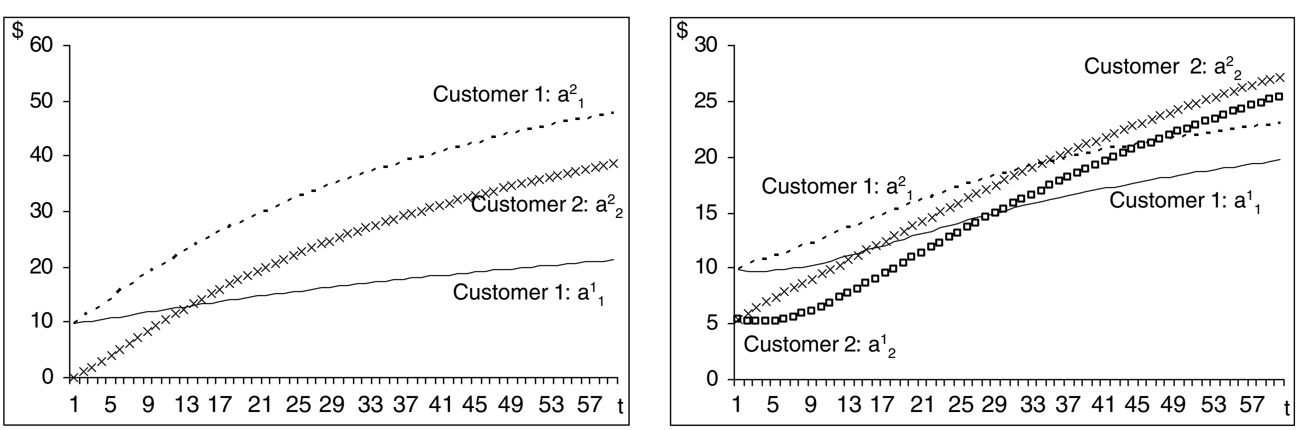
5. Extensions and future research

The basic model can be extended in several directions to include a number of features of the online grocery business. We first relax the unit-sized orders assumption in the previous section's example. Suppose that class 2 is more profitable because of a large order size. Formally, we now assume that the order size for class 2 is two instead of one. The resulting prices in Fig. 5a for the state $\mathbf{x}(\bullet) = (1,2)$ are dramatically different than the ones in Fig. 4. Surprisingly, the popular Option 1 is priced lower than the less popular Option 2. As class 2 orders can only be delivered with Option 2, Option 2 becomes the scarce resource that can generate more revenue than Option 1. Accordingly, the price shown to class 2 customers for Option 2 is further lower in Fig. 5a. Fig. 5b shows the prices when $\mathbf{x}(\bullet) = (4,2)$. Increasing the capacity of Option 1 significantly reduces all prices while Option 1 remains cheaper than Option 2 as expected. In contrast to Figs. 4 and 5a, class 2 prices for both options are higher than class 1 prices early in the booking horizon and lower closer to the end. Since a class 2 order requires a larger capacity commitment which reduces the flexibility in meeting different demand realizations later, the system charges class 2 higher prices at early stages. However, as time passes the possibility of unfilled capacity increases and this reverses the trade-off such that class 2 orders are allocated with higher probabilities as a result of lower class 2 prices.

Discounts on delivery fees may induce some customers to increase their purchase amounts. Equivalently, we can state this problem as customers decreasing their purchases by a proportion $\gamma(\gamma < 1)$ of the delivery price from their intended purchases when delivery is free. With this setup we obtain the first-order conditions as:

$$(1-\gamma)a_{mt}^{i*} + r_m - v_{t-1}^{i}(\mathbf{x}) = (1-\gamma)a_{mt}^{j*} + r_m - v_{t-1}^{j}(\mathbf{x}) = (1-\gamma)/\beta_m \Pr(\tilde{n}_{mt} = 0) \quad \text{for } \forall i,j \in \mathbf{A}(\mathbf{x}), \tag{13}$$

With (13), we show that the price of delivery option decreases with γ using the analysis of Corollary 1. Therefore, this extension reveals that if delivery discounts have a positive effect on the amount purchased, the delivery prices should be adjusted downward to account for this effect.



a. x(•)=(1,2) **b. x**(•)=(4,2)

Fig. 5. Prices with two customer classes; class 2 is more profitable and has an order size of 2.

In the future, the model can be extended to include other complexities of the environment. One such complexity is the resource interdependencies between delivery options that are overlapping. To incorporate this new feature, we can treat capacity as a resource pool such as fulfillment capacity – the pickers in the warehouse – and the delivery truck for the same geographical area. In this extended model we would, thus, have resource i with capacity x_i with delivery option n using y^i_{mn} units of resource i. Fortunately, Theorem 1 applies to this extended setting as well. This will enable us to model situations where some delivery options use common or separate resources.

The strategic design of delivery options with a longer planning horizon has implications on our dynamic pricing model. Investing in more capacity may change the cutoff times and shorten the duration of delivery options while leaving some capacity unutilized. However, this has proved not viable, as companies such as Webvan could not sustain their business models. Therefore, it is a critical success factor to improve the efficiency of delivery operations. One way of creating efficiency is to design vertically differentiated delivery options. Longer delivery options with more waiting time may be used to create lower quality options, since people do not like waiting in general. This can be handled in the model by properly setting the utility parameters U^n_m so that the more customers have to wait the less utility they have. Moreover, it is plausible that a longer delivery option may decrease the operational cost due to the flexibility in routing delivery trucks. This too can be handled within the model. Specifically, by extending the expected profit parameter r_m to $r^i_m = r_m - c^i$, where c^i is the cost of delivery with option i.

To sum up, we have developed a multi-product dynamic pricing model that adjusts the prices according to time, capacity, and demand characteristics of customers. We study the structure and properties of the optimal pricing policy and consider possible extensions.

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Appendix

Proof of Lemma 1. Let the denominator in (4) be $D = e^{U^0_m} + \sum_{i\in\mathbf{A}} e^{U^i_m - \beta_m a^i_m}$. It is straightforward to show that Part (a) holds

$$\frac{\partial \Pr(\tilde{n}_m = n)}{\partial a^n} = -\frac{\beta_m e^{U^n_m - \beta_m a^n_m}(D - e^{U^n_m - \beta_m a^n_m})}{D^2} = -\beta_m \Pr(\tilde{n}_m = n)(1 - \Pr(\tilde{n}_m = n)), \tag{A1}$$

and Part (b) follows from:

$$\frac{\partial \Pr(\tilde{n}_m = n)}{\partial a^i} = \frac{\beta_m e^{U^n_m - \beta_m a^n_m + U^i_m - \beta_m a^i_m}}{D^2} = \beta_m \Pr(\tilde{n}_m = n)\Pr(\tilde{n}_m = i). \tag{A2}$$

Part (c) follows from the fact that $e^{U^i_m - \beta_m a^i_m}$ is always positive. Hence, D increases with the number of options which, in turn, decreases the choice probabilities of delivery options. □

Proof of Lemma 2. A feasible solution is to (9) is to set all the prices to infinity, i.e. $a^1_{mt} = a^2_{mt} = \ldots = \infty$. In this solution, $V_t(\mathbf{x},m) = V_{t-1}(\mathbf{x})$. Thus, $V_t(\mathbf{x})$ is non-decreasing in t since the optimal solution is weakly better than any feasible solution. With the same argument, the expected payoff in each period is weakly increasing in the capacity of each delivery option. Furthermore, Part (c) of Lemma 1 guarantees that $\sum_{n=1}^{N} \Pr(\tilde{n}_{mt} = n)$ is weakly increasing with the available number of delivery options and, thus, weakly increasing in available capacity. Hence, the capacity result follows from Lemma 3.9.4 of Topkis (1998). □

Proof of Theorem 1. We present the objective function (9) in (A3) for exposition purposes

$$V_t(\mathbf{x}, m) = \max_{\mathbf{a}_{mt}} \left[V_{t-1}(\mathbf{x}) + \sum_{n=1}^{N} \Pr(\tilde{n}_{mt} = n)\{r_m + a^n_{mt} - v^n_{t-1}(\mathbf{x})\} \right], \tag{A3}$$

where $v^n_{t-1}(\mathbf{x}) = V_{t-1}(\mathbf{x}) - V_{t-1}(\mathbf{x} - \mathbf{I}^n)$ is the opportunity cost of delivery option n. Let F represent the objective function. That is, $F = V_{t-1}(\mathbf{x}) + \sum_{n=1}^{N} \Pr(\tilde{n}_{mt} = n)\{r_m + a^n_{mt} - v^n_{t-1}(\mathbf{x})\}$. The first order conditions of the objective function F in (A3) are:

$$\frac{\partial F}{\partial a^1_{mt}} = \Pr(\tilde{n}_{mt} = 1) + \sum_{n=1}^{N} \frac{\partial \Pr(\tilde{n}_{mt} = n)}{\partial a^1_{mt}}\{r_m + a^n_{mt} - v^n_{t-1}(\mathbf{x})\} = 0, \tag{A4}$$

$$\frac{\partial F}{\partial a^2_{mt}} = \Pr(\tilde{n}_{mt} = 2) + \sum_{n=1}^{N} \frac{\partial \Pr(\tilde{n}_{mt} = n)}{\partial a^2_{mt}}\{r_m + a^n_{mt} - v^n_{t-1}(\mathbf{x})\} = 0, \tag{A5}$$

$$\vdots$$

$$\frac{\partial F}{\partial a^N_{mt}} = \Pr(\tilde{n}_{mt} = N) + \sum_{n=1}^{N} \frac{\partial \Pr(\tilde{n}_{mt} = n)}{\partial a^N_{mt}}\{r_m + a^n_{mt} - v^n_{t-1}(\mathbf{x})\} = 0. \tag{A6}$$

Let $\Lambda^n = r_m + a^{n*}_{mt} - v^n_{t-1}(\mathbf{x}, m)$; $P^n = \Pr(\tilde{n}_{mt} = n)$ and $P^n_i = \frac{\partial \Pr(\tilde{n}_{mt} = n)}{\partial a^i_{mt}}$. We can re-write the first-order conditions as

$$\begin{bmatrix} P^1_1 & P^2_1 & \cdots & P^N_1 \\ P^1_2 & P^2_2 & \cdots & P^N_2 \\ \vdots & \ldots & \ddots & \vdots \\ P^1_N & P^2_N & \cdots & P^N_N \end{bmatrix} \begin{bmatrix} \Lambda^1 \\ \Lambda^2 \\ \vdots \\ \Lambda^N \end{bmatrix} = \begin{bmatrix} -P^1 \\ -P^2 \\ \vdots \\ -P^N \end{bmatrix}. \tag{A7}$$

Using Cramer's Rule, the solution to (A7) is

$$\Lambda^n = \det\begin{bmatrix} P_1^1 & \cdots & P_1^{n-1} & -P^1 & P_1^{n+1} & \cdots & P_1^N \\ P_2^1 & \cdots & P_2^{n-1} & -P^2 & P_2^{n+1} & \cdots & P_2^N \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_N^1 & \cdots & P_N^{n-1} & -P^N & P_N^{n+1} & \cdots & P_N^N \end{bmatrix} \Bigg/ \det\begin{bmatrix} P_1^1 & P_1^2 & \cdots & P_1^N \\ P_2^1 & P_2^2 & \cdots & P_2^N \\ \vdots & \cdots & \ddots & \vdots \\ P_N^1 & P_N^2 & \cdots & P_N^N \end{bmatrix}. \tag{A8}$$

We calculate the denominator of (A8) using the properties of determinants of a matrix and the results from the properties (a) and (b) of the customer choice model in Lemma 1.

$$\det\begin{bmatrix} P_1^1 & P_1^2 & \cdots & P_1^N \\ P_2^1 & P_2^2 & \cdots & P_2^N \\ \vdots & \cdots & \ddots & \vdots \\ P_N^1 & P_N^2 & \cdots & P_N^N \end{bmatrix} = (\beta_m)^N P^1 P^2 \ldots P^N \det\begin{bmatrix} -(1-P^1) & P^2 & \cdots & P^N \\ P^1 & -(1-P^2) & \cdots & P^N \\ \vdots & \cdots & \ddots & \vdots \\ P^1 & P^2 & \cdots & -(1-P^N) \end{bmatrix}. \tag{A9}$$

The determinant on the right-hand side of (A9) equals

$$\det\begin{bmatrix} -(1-P^1) & P^2 & \cdots & P^N \\ P^1 & -(1-P^2) & \cdots & P^N \\ \vdots & \cdots & \ddots & \vdots \\ P^1 & P^2 & \cdots & -(1-P^N) \end{bmatrix} = \det\begin{bmatrix} -(1-P^1) & P^2 & \cdots & P^N \\ 1 & -1 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{bmatrix}$$
$$= \det\begin{bmatrix} -P^0 & 0 & \cdots & 0 \\ 1 & -1 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{bmatrix} = (-1)^N P^0. \tag{A10}$$

Thus (A9) equals

$$\det\begin{bmatrix} P_1^1 & P_1^2 & \cdots & P_1^N \\ P_2^1 & P_2^2 & \cdots & P_2^N \\ \vdots & \cdots & \ddots & \vdots \\ P_N^1 & P_N^2 & \cdots & P_N^N \end{bmatrix} = (-\beta_m)^N P^1 P^2 \ldots P^N P^0. \tag{A11}$$

The next step is to evaluate the numerator in (A8).

$$\det\begin{bmatrix} P_1^1 & \cdots & P_1^{n-1} & -P^1 & P_1^{n+1} & \cdots & P_1^N \\ P_2^1 & \cdots & P_2^{n-1} & -P^2 & P_2^{n+1} & \cdots & P_2^N \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_N^1 & \cdots & P_N^{n-1} & -P^N & P_N^{n+1} & \cdots & P_N^N \end{bmatrix}$$
$$= (\beta_m)^{N-1} P^1 P^2 \ldots P^N \det\begin{bmatrix} -(1-P^1) & \cdots & P^{n-1} & -1 & P^{n+1} & \cdots & P^N \\ P^1 & \cdots & P^{n-1} & -1 & P^{n+1} & \cdots & P^N \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P^1 & \cdots & P^{n-1} & -1 & P^{n+1} & \cdots & -(1-P^N) \end{bmatrix} \tag{A12}$$
$$= (\beta_m)^{N-1} P^1 P^2 \ldots P^N \det\begin{bmatrix} -1 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 0 & -1 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & -1 & 0 & \cdots & -1 \end{bmatrix} = (-1)^N (\beta_m)^{N-1} P^1 P^2 \ldots P^N. \tag{A13}$$

Therefore, (A8) with Eqs. (A11) and (A13), implies that

$$\Lambda^n = \Lambda = (\beta_m P^0)^{-1} \quad \text{for } n = 1, \ldots, N. \tag{A14}$$

Next, we check the second-order condition for the local maximum using the condition that the Hessian matrix is negative-definite at the point at which the first-order conditions (A14) hold. First, it is easy to verify that the cross partial derivatives of P^i have the following forms: $P_{nn}^n = (\beta_m)^2 P^n(1-P^n)(1-2P^n), P_{ni}^n = P_{in}^n = -(\beta_m)^2 P^n P^i(1-2P^n), P_{ii}^n = -(\beta_m)^2 P^n P^i(1-2P^i)$, and $P_{ij}^n = 2(\beta_m)^2 P^n P^i P^j$ where $n \neq i \neq j$. We will show that

$$\frac{\partial F}{(\partial a_{mt}^n)^2} = -\beta_m P^n < 0 \quad \text{and} \quad \frac{\partial F}{\partial a_{mt}^n \partial a_{mt}^i} = 0 \tag{A15}$$

for $n \neq i$. We prove (A15) by showing that these conditions are satisfied for $n = 1$ and $i = 2$, then (A15) follows by symmetry. The second-order partial derivative of F with respect to a_1 is:

$$\frac{\partial F}{(\partial a_{mt}^1)^2} = -2\beta_m P^1(1-P^1) + \sum_{n=1}^{N} P_{11}^n \{r_m + a_{mt}^n - v_{t-1}^n(\mathbf{x})\}. \tag{A16}$$

We evaluate (A16) when the first-order condition (A14) holds

$$\frac{\partial F}{(\partial a^1_{mt})^2} = -2\beta_m P^1(1-P^1) + \Lambda\sum_{n=1}^{N} P^n_{11} = -2\beta_m P^1(1-P^1) + \frac{1}{\beta_m P^0}(\beta_m)^2 P^1(1-2P^1)P^0 = -\beta_m P^1. \tag{A17}$$

The cross partial derivative of F with respect to a^1_{mt} and a^2_{mt} is

$$\frac{\partial F}{\partial a^1_{mt}\partial a^2_{mt}} = 2\beta_m P^1 P^2 + \sum_{n=1}^{N} P^n_{12}\{r_m + a^n_{mt} - v^n_{t-1}(\mathbf{x})\}. \tag{A18}$$

Similarly, we evaluate (A18) when the first-order condition (A14) holds.

$$\frac{\partial F}{\partial a^1_{mt}\partial a^2_{mt}} = 2\beta_m P^1 P^2 + \Lambda\sum_{n=1}^{N} P^n_{12} = 2\beta_m P^1 P^2 - \frac{1}{\beta_m P^0}(\beta_m)^2 P^1 P^2\left(1-2P^1+1-2P^2-\sum_{n=3}^{N} P^n\right) = 0. \tag{A19}$$

This implies that the Hessian matrix is diagonal with all negative elements. Hence, the Hessian matrix is negative definite, which is a sufficient condition for a local maximum point. □

Proof of Corollary 1. We can re-write the optimality condition in (10) as follows

$$a^{i*}_{mt} + r_m - v^i_{t-1}(\mathbf{x}) - (\beta_m \Pr(\tilde{n}_{mt}=0))^{-1} = 0 \quad \text{for} \quad \forall i \in \mathbf{A}(\mathbf{x}) \tag{A20}$$

We utilize the implicit function theorem to derive the results using (A20):

$$\frac{\partial a^{i*}_{mt}}{\partial \beta_m} = -\frac{\frac{\partial}{\partial \beta_m}(a^{i*}_{mt} + r_m - v^i_{t-1}(\mathbf{x}) - \beta_m \Pr(\tilde{n}_{mt}=0))^{-1}}{\frac{\partial}{\partial a^{i*}_{mt}}(a^{i*}_{mt} + r_m - v^i_{t-1}(\mathbf{x}) - (\beta_m \Pr(\tilde{n}_{mt}=0))^{-1})} \tag{A21}$$

$$= -\frac{\partial}{\partial \beta_m}(-\beta_m \Pr(\tilde{n}_{mt}=0))^{-1} \Big/ \frac{\partial}{\partial a^{i*}_{mt}}(a^{i*}_{mt} - (\beta_m \Pr(\tilde{n}_{mt}=0))^{-1}) < 0. \tag{A22}$$

$$\text{Since,} \frac{\partial}{\partial \beta_m}(-\beta_m \Pr(\tilde{n}_{mt}=0))^{-1} > 0 \quad \text{and} \quad \frac{\partial}{\partial a^{i*}_{mt}}(a^{i*}_{mt} - (\beta_m \Pr(\tilde{n}_{mt}=0))^{-1}) > 0. \tag{A23}$$

Similarly

$$\frac{\partial a^{i*}_{mt}}{\partial r_m} = -1 \Big/ \frac{\partial}{\partial a^{i*}_{mt}}(a^{i*}_{mt} - (\beta_m \Pr(\tilde{n}_{mt}=0))^{-1}) < 0. \quad \square \tag{A24}$$

Proof of Corollary 2. The proof is similar to the proof of Corollary 1. Straightforward calculations produce the following results:

$$\frac{\partial a^{i*}_{mt}}{\partial v^i_{t-1}(\mathbf{x})} = 1 - \Pr(\tilde{n}_{mt}=i) > 0 \quad \text{and} \quad \frac{\partial a^{i*}_{mt}}{\partial v^j_{t-1}(\mathbf{x})} = -\Pr(\tilde{n}_{mt}=j) < 0. \tag{A25}$$

Therefore

$$\frac{\partial}{\partial v^i_{t-1}(\mathbf{x})}(r_m + a^{i*}_{mt} - v^i_{t-1}(\mathbf{x})) = 1 - \Pr(\tilde{n}_{mt}=i) - 1 = -\Pr(\tilde{n}_{mt}=i) < 0. \quad \square \tag{A26}$$

Proof of Proposition 1. The proof is by induction.

a. The proposition is trivially true for $t = 0$ and $t = 1$. Suppose that Part (a) is true for $t - 1$. That is, the induction hypothesis is

$$v^n_{t-1}(\mathbf{x} - \mathbf{I}^n) - v^n_{t-1}(\mathbf{x}) \geqslant 0. \tag{A27}$$

We will show that Part (a) also holds for t. To begin, we derive an expression for $v^n_t(\mathbf{x})$. From (6) and (9), we have

$$V_t(\mathbf{x}) = V_{t-1}(\mathbf{x}) + \sum_{m=1}^{M} p_{mt}\sum_{n=1}^{N}\Pr(\tilde{n}_{mt}=n|\mathbf{a}^*_{mt}(\mathbf{x}))\{r_m + a^{n*}_{mt}(\mathbf{x}) - v^n_{t-1}(\mathbf{x})\}. \tag{A28}$$

Similarly,

$$V_t(\mathbf{x}-\mathbf{I}^n) = V_{t-1}(\mathbf{x}-\mathbf{I}^n) + \sum_{m=1}^{M} p_{mt}\sum_{n=1}^{N}\Pr(\tilde{n}_{mt}=n|\mathbf{a}^*_{mt}(\mathbf{x}-\mathbf{I}^n))\{r_m + a^{n*}_{mt}(\mathbf{x}-\mathbf{I}^n) - v^n_{t-1}(\mathbf{x}-\mathbf{I}^n)\}. \tag{A29}$$

Therefore, using Theorem 1 repeatedly, we have

$$v^n_t(\mathbf{x}) = v^n_{t-1}(\mathbf{x}) + \sum_{m=1}^{M} p_{mt}\left\{\frac{1}{\beta_m}\left(\frac{1}{\Pr(\tilde{n}_{mt}=0|\mathbf{a}^*_{mt}(\mathbf{x}))}-1\right) - \frac{1}{\beta_m}\left(\frac{1}{\Pr(\tilde{n}_{mt}=0|\mathbf{a}^*_{mt}(\mathbf{x}-\mathbf{I}^n))}-1\right)\right\}, \tag{A30}$$

$$v^n_t(\mathbf{x}) = v^n_{t-1}(\mathbf{x}) + \sum_{m=1}^{M} p_{mt}\left\{\frac{1}{\beta_m \Pr(\tilde{n}_{mt}=0|\mathbf{a}^*_{mt}(\mathbf{x}))} - \frac{1}{\beta_m \Pr(\tilde{n}_{mt}=0|\mathbf{a}^*_{mt}(\mathbf{x}-\mathbf{I}^n))}\right\}, \tag{A31}$$

$$v^n_t(\mathbf{x}) = v^n_{t-1}(\mathbf{x}) + \sum_{m=1}^{M} p_{mt}\{a^{n*}_{mt}(\mathbf{x}) - v^n_{t-1}(\mathbf{x}) - (a^{n*}_{mt}(\mathbf{x}-\mathbf{I}^n) - v^n_{t-1}(\mathbf{x}-\mathbf{I}^n))\} \tag{A32}$$

and

$$v_t^n(\mathbf{x}-\mathbf{I}^n) = v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) + \sum_{m=1}^{M} p_{mt}\{a_{mt}^{n*}(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - (a_{mt}^{n*}(\mathbf{x}-2\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}-2\mathbf{I}^n))\}. \tag{A33}$$

The difference is

$$\begin{aligned} v_t^n(\mathbf{x}-\mathbf{I}^n) - v_t^n(\mathbf{x}) &= v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}) \\ &\quad + \sum_{m=1}^{M} p_{mt}\{2a_{mt}^{n*}(\mathbf{x}-\mathbf{I}^n) - 2v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - (a_{mt}^{n*}(\mathbf{x}-2\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}-2\mathbf{I}^n)) - (a_{mt}^{n*}(\mathbf{x}) - v_{t-1}^n(\mathbf{x}))\} \end{aligned} \tag{A34}$$

From Corollary 1 and the induction hypothesis, we know that

$$a_{mt}^{n*}(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - (a_{mt}^{n*}(\mathbf{x}-2\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}-2\mathbf{I}^n)) \geqslant 0. \tag{A35}$$

With Corollary 1 and the induction hypothesis, we can show that

$$\begin{aligned} \sum_{m=1}^{M} p_{mt}\{a_{mt}^{n*}(\mathbf{x}) - v_{t-1}^n(\mathbf{x}) - (a_{mt}^{n*}(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}-\mathbf{I}^n))\} &= \sum_{m=1}^{M} p_{mt}\{v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}) + a_{mt}^{n*}(\mathbf{x}) - a_{mt}^{n*}(\mathbf{x}-\mathbf{I}^n)\} \\ &\leqslant \sum_{m=1}^{M} p_{mt}\{v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x})\} \leqslant v_{t-1}^n(\mathbf{x}-\mathbf{I}^n) - v_{t-1}^n(\mathbf{x}). \end{aligned} \tag{A36}$$

Hence, the difference in (A34) is non-negative.

b. Part (b) follows from Corollary 1, the induction hypothesis, and (A32). □

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