Vechicle Routing Problems with Time-Window Incentives

The incentive problem

This problem treats the home delivery service issue: the need for a company to create a routing plan for its vehicles so they could satisfy the highest number of customer orders keeping a good overall profit.

After buying what he needs, a customer chooses a particular time-window when he wants his purchase to be delivered. The company has to bias customers, with some sort of incentives on the listed time-windows, so there is a high probability that the customer chooses the time-window that the company wants him to pick.

The main and most important part of this procedure is to place the incentives, that is to choose in which time-window we should place them and how big every incentive has to be.

This problem could be described as follows, having:

C customers/consumers spread into a limited geographical area (e.g. ZIP-code division, districts)

V company vehicles, each one having Q^s storage and Q^f fuel capacity[1]

T delivery time window[2] options, each being L[3] wide (time windows are chosen by the company itself)

I_t possible incentives, one per time window

p_i^t the probability that the customer i chooses the time window t to get his items delivered

Being restricted by:

- Don't exceed vehicles capacity, both storage and fuel (or delivery costs)
- Make sure that every delivery will stay in the defined time-window, remembering also that every delivery has a service time[4]
- $\forall t \in T : I_t \geq 0$

Knowing that the final objective could be:

- $MAX\ Profit = MAX\ \sum revenue - \sum delivery\_costs - \sum incentives$
- $MAX\ Orders\_delivered$

[1] Every vehicle could have a different capacities.
[2] Time windows could be overlapped with each other or not.
[3] The larger the better: we have more flexibility. (Campbell and Savelsbergh, 2005)
[4] Time between vehicle arrival to the customer and its departure.

We also make some assumptions:

- Real-time delivery are not available, we want at least 1-day notice to create the best plan that we can
- Delivered products are not subject to any particular restriction: they are unbreakable and without expiry date (e.g. no food)
- We treat every geographical region by itself, there isn't any possible interaction between two or more regions
- We keep a low number of C customers because the problem is hard to solve even with small sets of input
- $\forall i \in V : Q_i^s = a, Q_i^f = b, a\, b \in \mathbb{N}$

Future extensions or possible changes:

- Everything that could vary from the assumptions
- Different time-windows: different L, overlap between windows
- Predict customers: study customers behaviour to predict their choices, knowing that they could be biased by incentives
- Negative incentives: make the delivery cost more high for those customers that demand a specific delivery that is unprofitable for the company, without a negative incentive