Assignment 4

Due: Nov 30, 2020

Question 1: (50 points)

The Black-Scholes formula for put option is given by

$$p(S_0, K, T, \sigma, r) = e^{-rT} E^Q[(K - S(T))_+] = -S_0 N(-d_1) + e^{-rT} K N(-d_2)$$

where d_1 and d_2 are the same as for call option, and Vega for put option is also the same as call option.

- Write a function to calculate option price using Black-Scholes formula for both calls of puts. Add one variable "type". The function returns call option price when "type" equals to "call", and returns put option price when "type" equals to "put". Calculate *both* call and put option prices using this function with $S0 = 100, K = 100, T = 1, \sigma = 0.2, r = 0.05$

- Write a function to calculate implied volatility for both calls and puts. Add one variable "type". The function returns implied volatility for call option when "type" equals to "call", and returns implied volatility for put option when type equals to "put". Calculate implied volatility using the function with $S0 = 100, K = 100, T = 1, r = 0.05$, and $P = 10$ for call option, and $P = 5$ for put option. Method is not limited.

Question 2: (50 points)

Since we can approximate put option price by replacing the mean with sample mean, then the put option can be approximated by:

$$p(S_0, K, T, \sigma, r) = e^{-rT} E^Q[(K - S(T))_+] \approx e^{-rT} \frac{1}{m} \sum_{j=1}^{m} (K - S^{(j)}(T))_+$$

- Using Monte-Carlo Simulation to estimate the put option price using the parameters from previous problems, you can use number of steps n = 252 and number of paths m = 10000, and check if it is close to 5.573526.