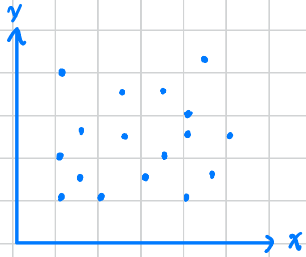
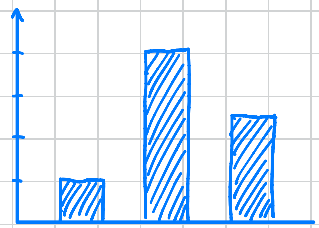
MCR3U : Functions 1.1 – What is a function?

Relations → scatter plots, bar graphs, value pairs (grade 9 and lower, i think)

Examples:

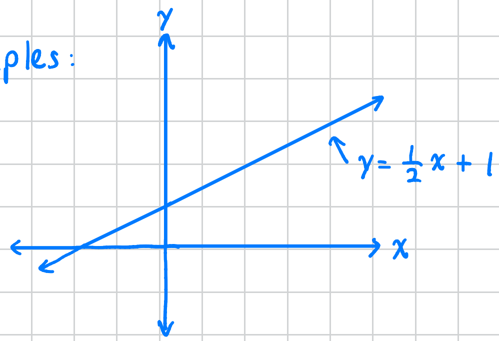
x	y
1	2
3	4
5	6
7	8

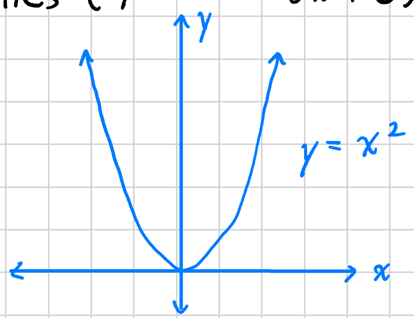




Equations → lines $(y = mx + b)$, quadratics $(y = ax^2 + bx + c)$ (grade 9+10)

Examples:





In grade 11 onwards we will focus on functions, instead of relations and equations for lines and quadratics. Functions are nothing to be scared of, we can think of functions as a new way to write things we already know about. Let's formally define how functions should be written.

Function Notation

→ notation just means the system of how we write things.

If we have an equation for a line like $y = 2x + 3$, we can write this as a function like so: $f(x) = 2x + 3$. Here are some things to note:

- We use f because f stands for function (duh!)
- f describes an operation on the input, which is x.
- $f(x)$ represents the output of the function after the operations are applied to x.
- x is still the independent variable, and y is still the dependent variable, we say that $y = f(x) \rightarrow y$ is a function of x.
- $f(3)$ represents the result after replacing x with 3 in the function.

Another thing to know about functions is that they all have a domain, codomain, and range/image. These things are sets, and you need to understand what a set is before you understand these other terms.

Set

A set is a collection of things. Sets could contain numbers, places, names, really anything, but we'll focus on numbers. To define a set you can do so explicitly like so $A = \{2, 4, 6, 8, 10\}$ or you can use set builder notation. An example of set builder notation is $B = \{x : x \geq 2\}$. The set B is defined to contain any number that is greater than or equal to 2.

Examples:

- Set with all non-negative numbers → $\{ x : x \geq 0 \}$
- set of all integers → $\{ \dots -2, -1, 0, 1, 2, \dots \}$ → usually written as $\mathbb{Z}$
- set of all numbers on the number line → written as $\mathbb{R}$ (real numbers)

Now that we know what sets are, we can define domain, codomain, and range/image.

Domain (of a function)

The set of values that can be used as an input to a function. If the function is $f(x) = 2x+3$ then the domain is the set of all possible values of x. The domain is defined by whoever defines the function.

Codomain (of a function)

The set of values that the function could output. A broad set encompassing values that the function might not output. The codomain is dependent on the domain and the range/image, the codomain should be equal to or contain the range/image.

Range/Image (of a function)

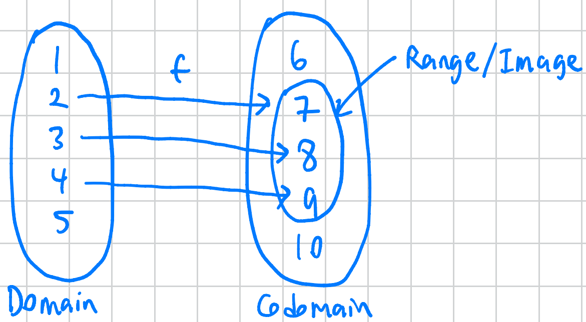
The set of values that the function actually outputs when given inputs in the domain. So all the values of $f(x)$ for every possible value of x. The range/image is entirely dependent on the domain.

Example:

$$\underset{\text{domain}}{f : \mathbb{R}} \to \underset{\text{codomain}}{\mathbb{R}}, \quad f(x) = \frac{1}{x} \qquad \underbrace{f(\mathbb{R})}_{\text{range/image}} = \mathbb{R} \text{ without } 0.$$

Notice how the codomain contains the range/image, and how if the domain was different then the range/image would also be different.
For example if the domain was $\{1, 2\}$ then the range/image would be $\{1, \frac{1}{2}\}$.

Example:

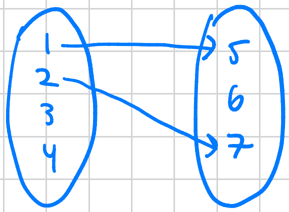


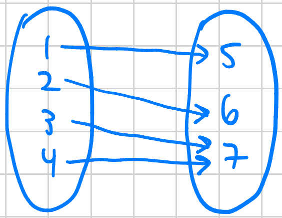
You should note that not every value in the domain needs an output, and not every value in the codomain needs to be the output of some input. But every value in the range/image must have a corresponding input.

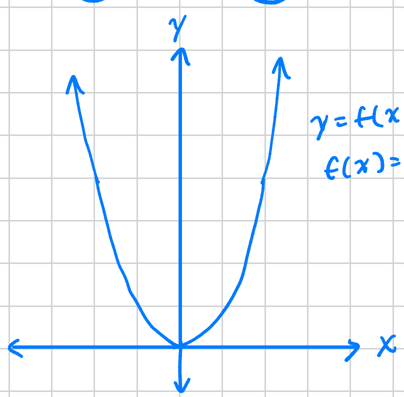
There is one more thing to know about functions. Every input of a function can only have one output. So if we have $f(4) = 2$ and $f(4) = -2$ then f is not a function because $f(4)$ has two different outputs.

Example:

FUNCTIONS



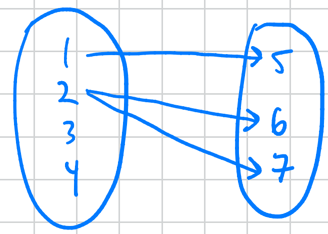


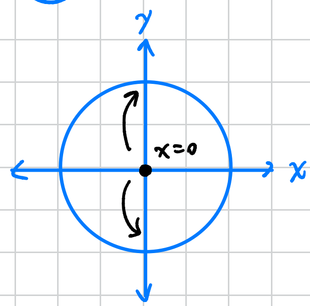


$y = f(x)$

$f(x) = x^2$

NOT FUNCTIONS





$f(0) = 2$

$f(0) = -2$

If you are looking at a graph and are trying to determine if what you are looking at is a function or not, try using the vertical line test.
Draw a vertical line, parallel with the y-axis, somewhere on the graph, if the line you drew hits more than one point on the graph, then it is not a function. Verify that this test works by doing it on the graphs above.