Universidade de Brasília - UnB
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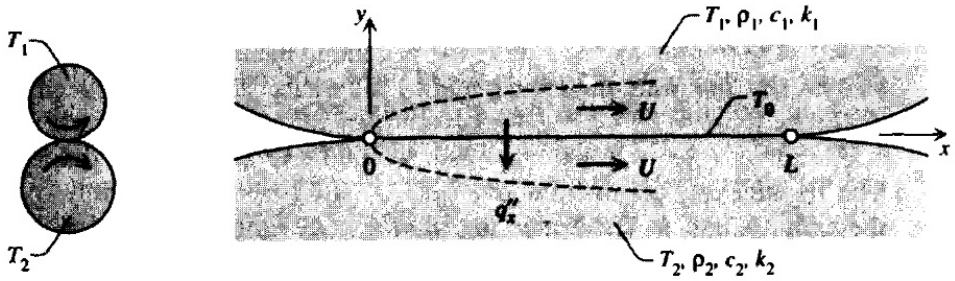
Programa de Pós-Graduação em Ciências Mecânicas
Transferência de Calor Avançada/Advanced Heat Transfer

Thermal contact and boundary layers

When two elastic cylinders are pressed against one another, they make contact over a strip of width L. This width is assumed known. In general, it depends on the elastic properties and radii of the two cylinders and on the force with one cylinder is pressed against the other. The radii of the cylinders are much larger than the contact width L. Two cylindrical bodies have different temperatures, T_1 and T_2, and roll past one another without slip. The peripheral velocity U with which both bodies pass through the frame of reference attached to the contact region is known. The objective of this exercise is to show how the "fluid" boundary layer method can be used to calculate the heat transfer rate between two *solids*.



1. Assume that the interface temperature T_0 (unknown) is uniform, that is, independent of x. Write the expression for the local heat flux q''_x by nothing that the "flow" of each solid through its respective thermal boundary layer region (with constant U) is similar to that of a fluid with extremely low Prandtl number.

2. Show that the interface temperature depends on the physical properties of the two solid in the following manner:

$$T_0 = \frac{(\rho_1 c_1 \kappa_1)^{1/2} T_1 + (\rho_2 c_2 \kappa_2)^{1/2} T_2}{(\rho_1 c_1 \kappa_1)^{1/2} + (\rho_2 c_2 \kappa_2)^{1/2}}. \tag{1}$$

3. Derive the following expression for the $L-$averaged heat flux between the two bodies:

$$\bar{q}'' = \frac{1.128}{1+r} \kappa_1 (T_1 - T_2) \left(\frac{U}{\alpha_1 L} \right)^{1/2}, \tag{2}$$

   where $r = \sqrt{(\rho_1 c_1 \kappa_1)/(\rho_2 c_2 \kappa_2)}$.

4. How fast must the cylinders roll for these analytical results to be valid?