Computational Colour and Spectral Imaging

"Advanced Colour and Spectral Imaging"

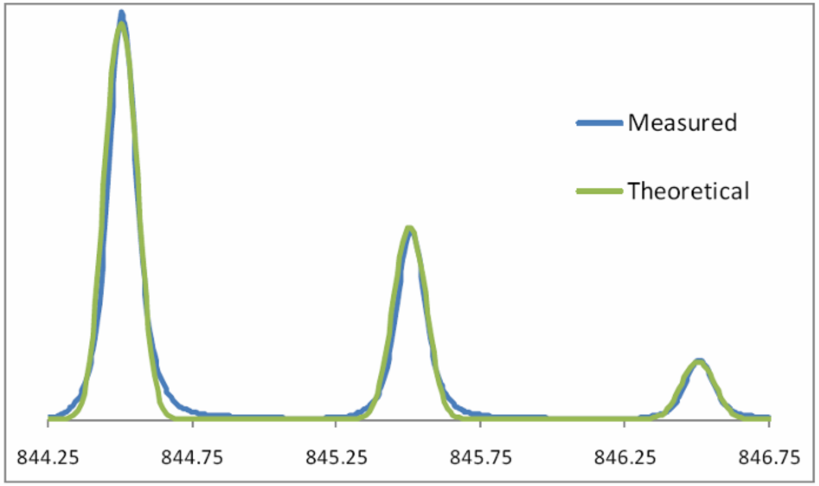
Chapter 4: Spectral metrics

Javier Hernández-Andrés and Eva M. Valero
javierha@ugr.es **valerob@ugr.es**
Colour Imaging Lab (*colorimaginglab.ugr.es*)

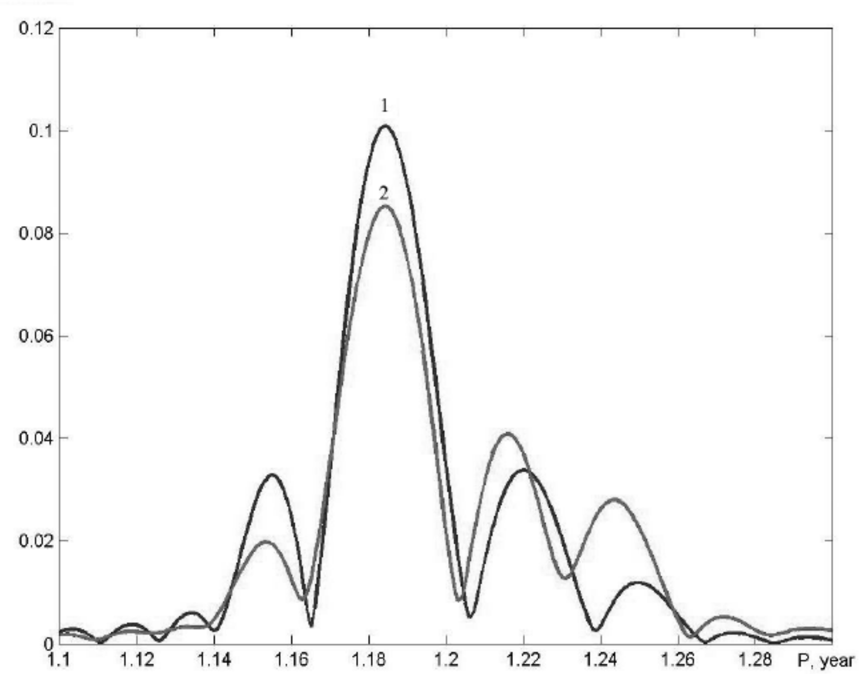
COLOUR IMAGING LAB
University of Granada
SPAIN

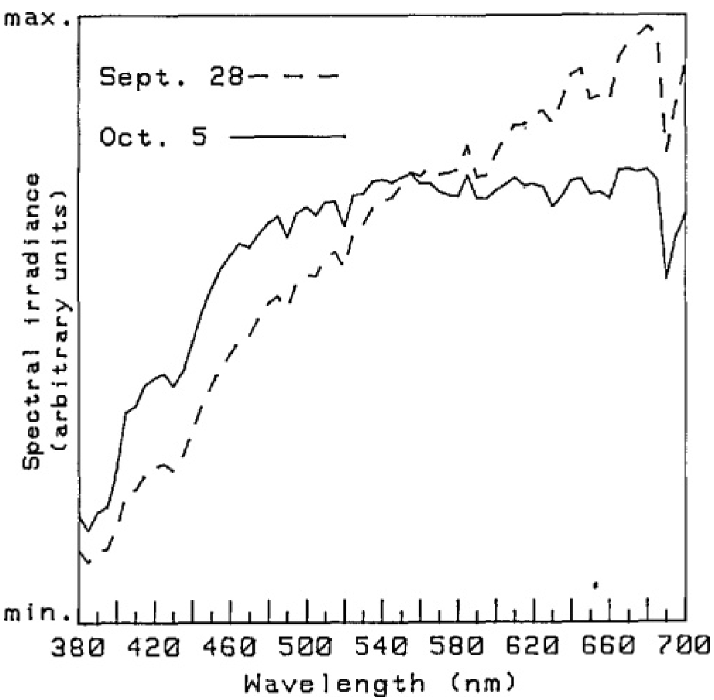
UNIVERSIDAD DE GRANADA

Departamento de Óptica, Facultad de Ciencias,
Universidad de Granada, 18071-Granada (SPAIN)



How similar? How different?
How to evaluate their similarity?





1. Metrics at a pixel
- 1.a. Colorimetric metrics
- 1.b. Spectral metrics
- 1.c. Metameric indexes
- 1.d. Weighted spectral metrics

2. What properties a metric should have?
- 2.a. Accuracy
- 2.b. Precision

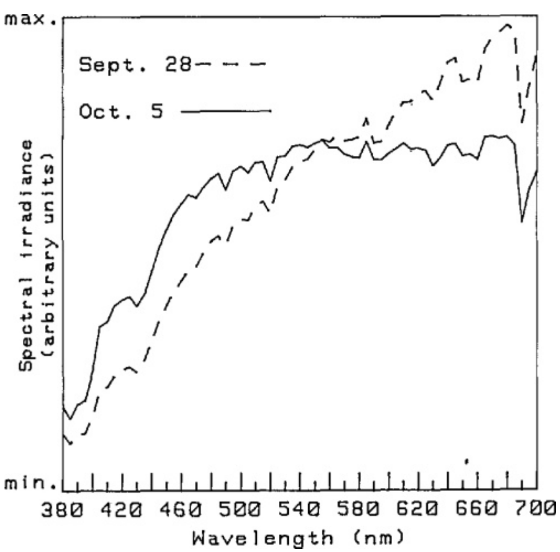
3. Combined metric at a pixel

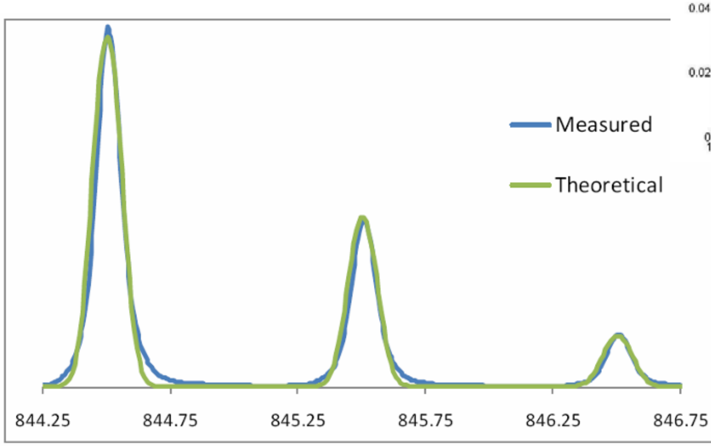
Annex: How to build a metric?

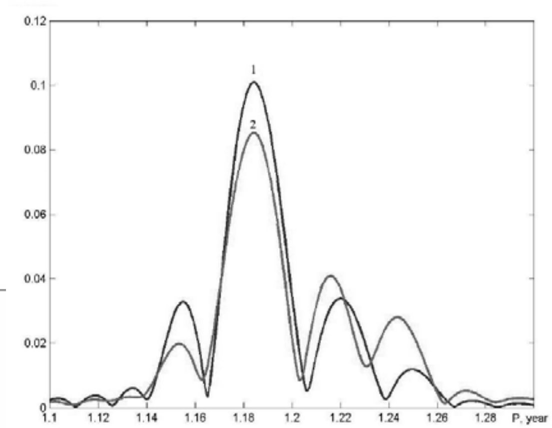
1. Metrics at a pixel
(or “cost” functions or “merit” functions)

Spectral Accuracy is a measure of the similarity between two spectra (normally between the measured spectrum (or estimated) against that of the theoretical or exact spectrum).

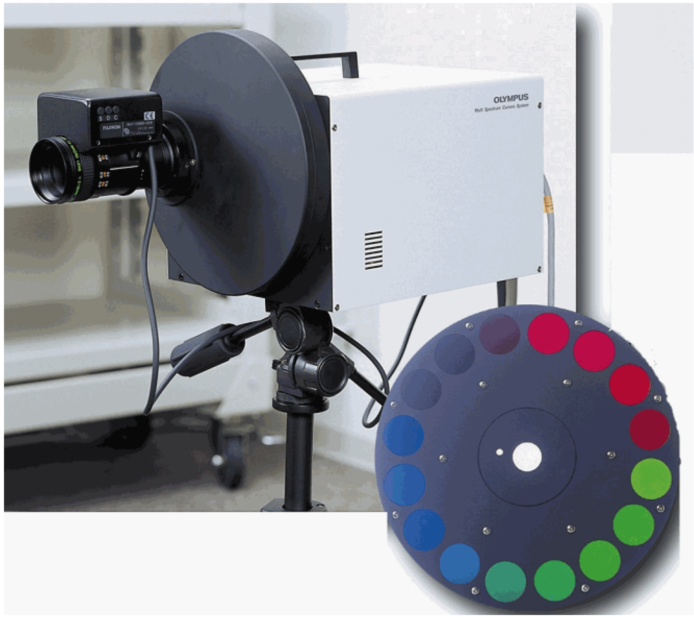
How to ***evaluate*** the spectral accuracy?







Our choice can impact everything: for example, the selection of the filters.



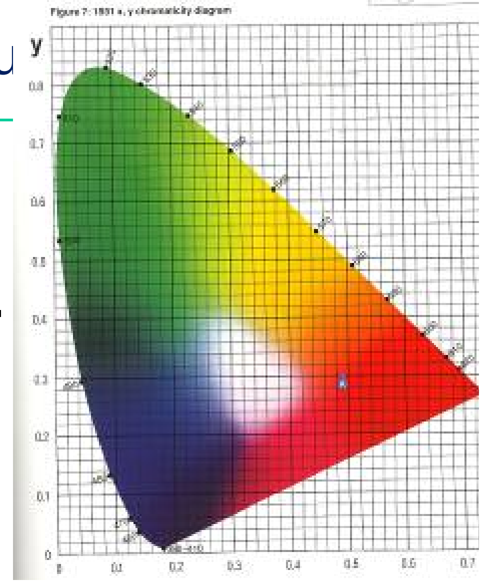
The same metric for artwork reproduction or to telemedicine?

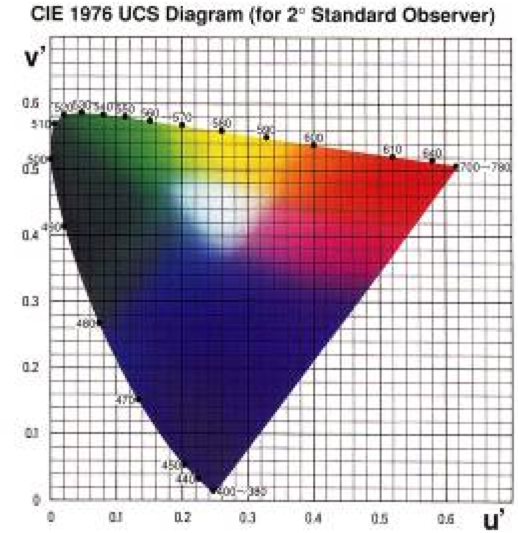
Depending on the shape and magnitude of spectral curves, one metric could perform better than another.

No consensus on which metric should be applied!
(from Imai 2002)

Four classes (each one with advantages and drawbacks):

a. **CIE color difference equations,**
b. **Spectral metrics,**
c. **Metameric indexes**
d. **Weighted spectral metrics.**





Which metric?

1.a. Colorimetric metrics:

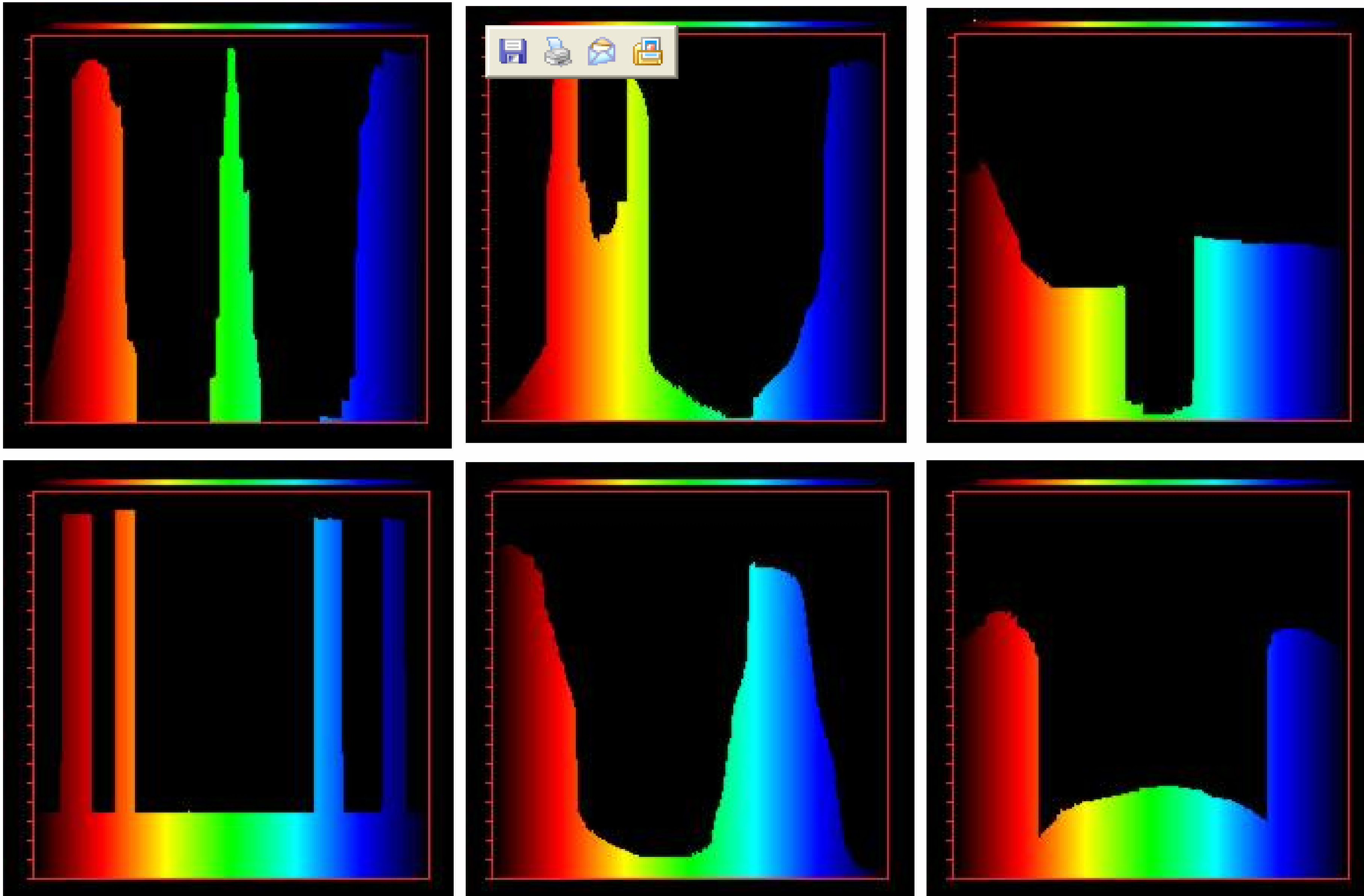
CIELUV
CIELAB
CIE94
CIEDE2000

$$\text{CIELAB} \quad \Delta E^*_{ab} = \left[\left(\Delta L^*\right)^2 + \left(\Delta a^*\right)^2 + \left(\Delta b^*\right)^2 \right]^{1/2}$$

All of them measure the Euclidean distance between two “colors” in a 3D color space.

Color difference equations produce bad correlation to spectral matches, particularly for metameric pairs.

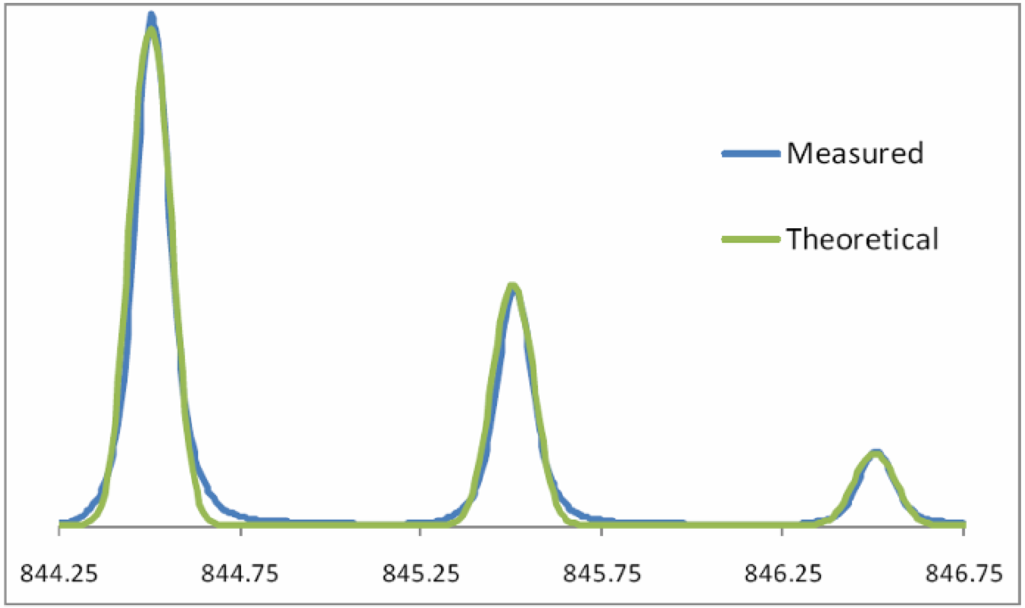
The six spectra below look the same purple to normal color-vision people =>



Spectra obtained using a simulation by Hughes, Bell and Doppelt (Brown University)

1.b. Spectral metrics:

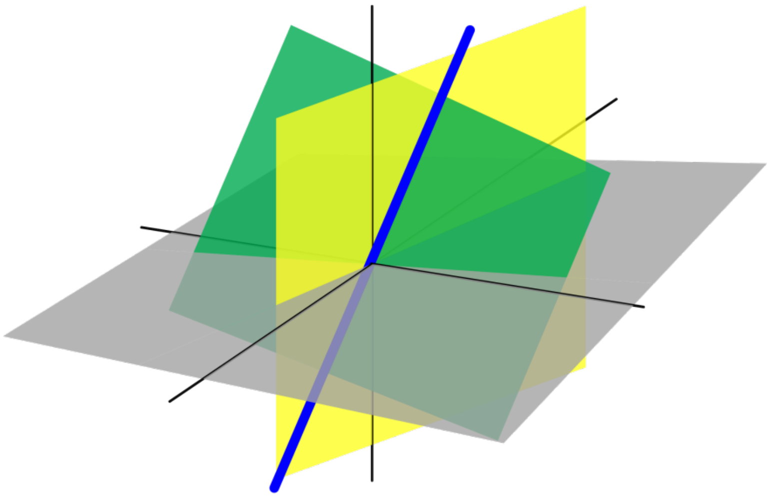
They distinguish between metamers **but do not take into account human vision.**

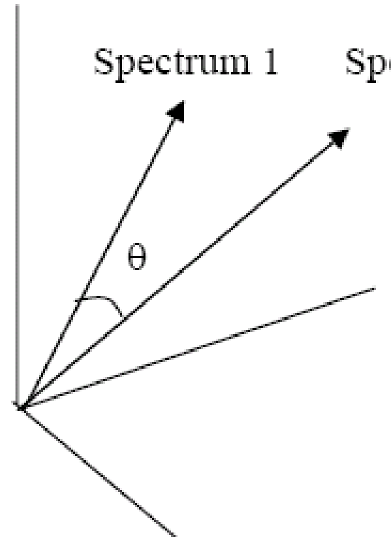


It is difficult to use multispectral/hyperspectral data without resorting at last to colorimetry. To which extent spectral metrics can be used to predict perceptual differences of colors whose spectra are known?

1.b. Spectral metrics:

GOAL: finding a spectral metric **with a clear meaning and known quantitative features**.





Such a metric should capture the **"geometry"** of the **"space of spectra",** and allow to quantitatively compare spectra while still having high confidence in the corresponding colorimetric results for given illuminant and observer.

Features of a metric?

Features of a metric?

Minimum value?

Maximum value?

Negative values?

Depends on the number of samples (wavelengths)?

Easy to compute?

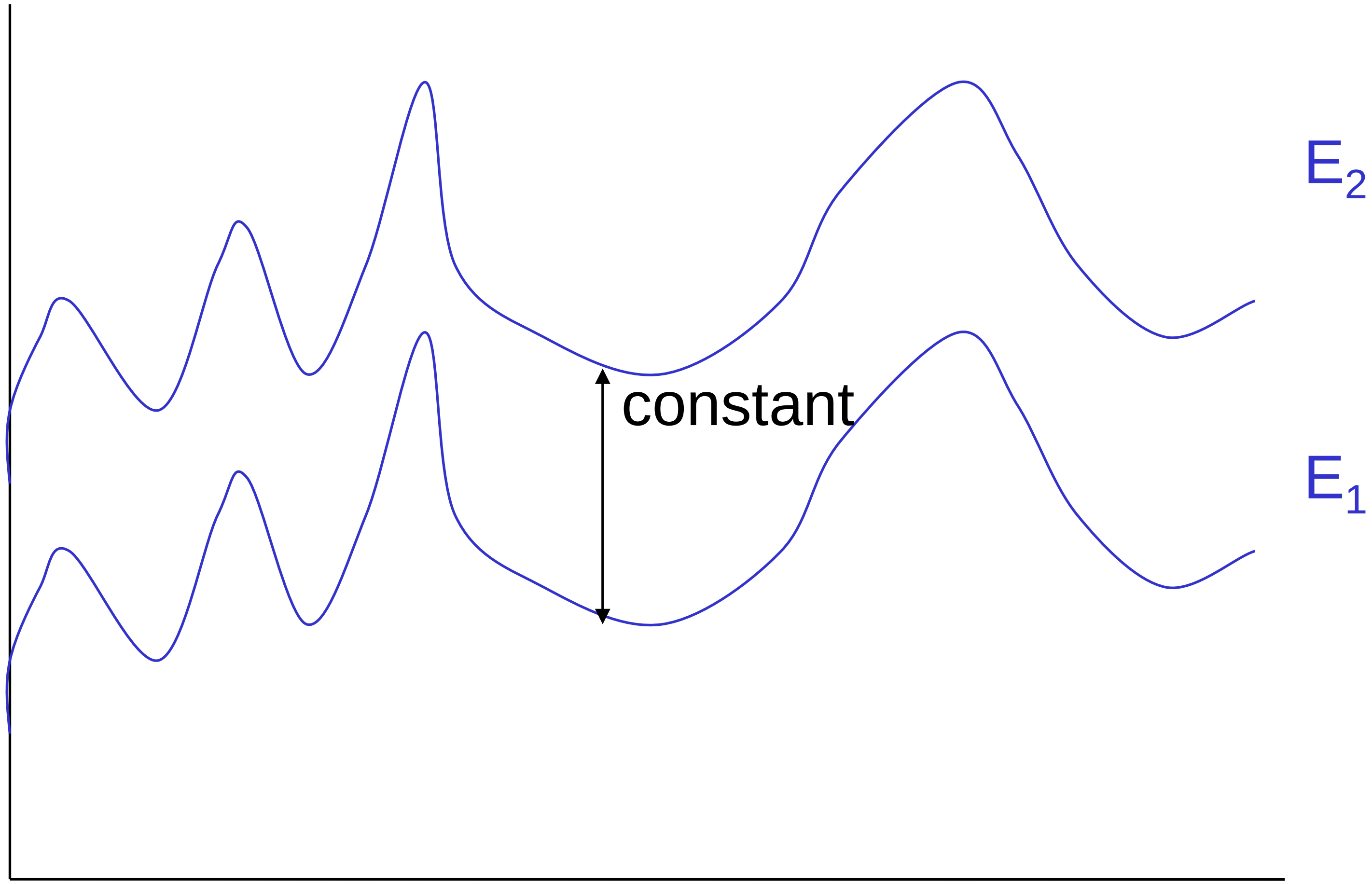
Units?

Thresholds?

Conmutative?

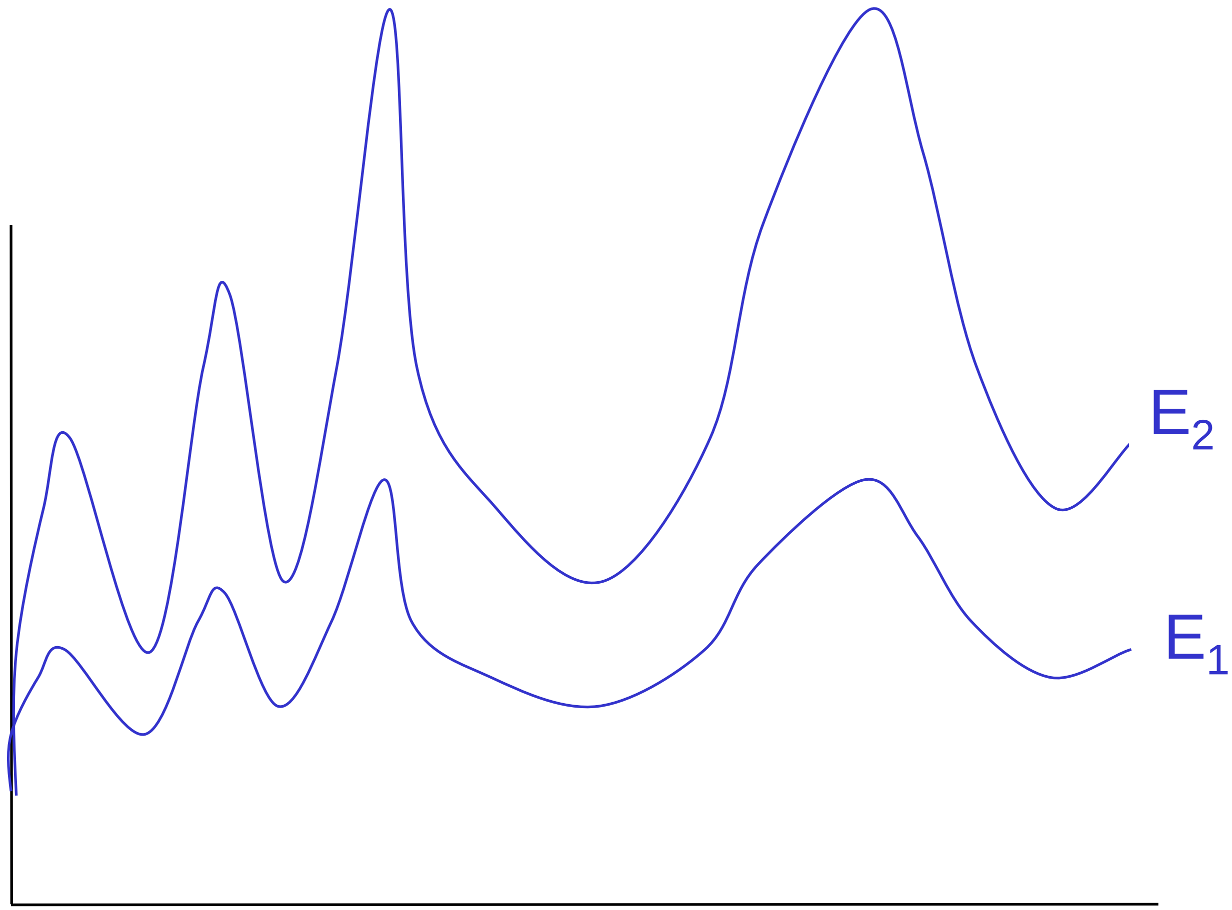
Relation with other metrics?

Features of a metric?



Shift in amplitude $E_2=E_1$+constant

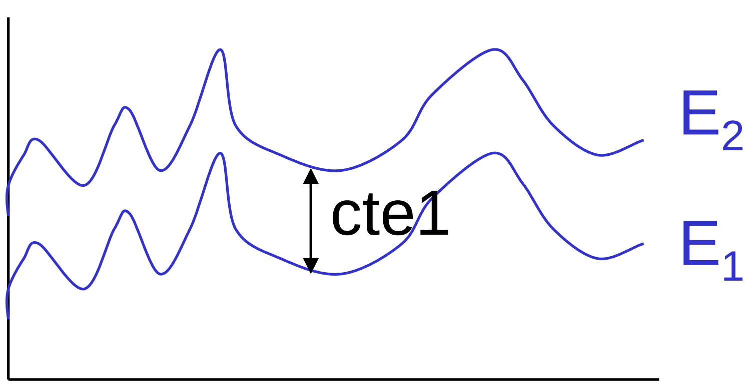
Features?



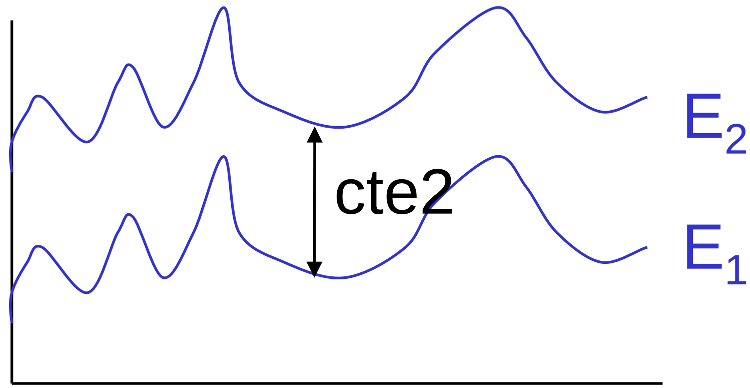
Shift in scale E_2= constant · E_1

Features?

- Metric values are scalable with magnitude change?



Shift in amplitude $E_2=E_1+cte1$
metric value1

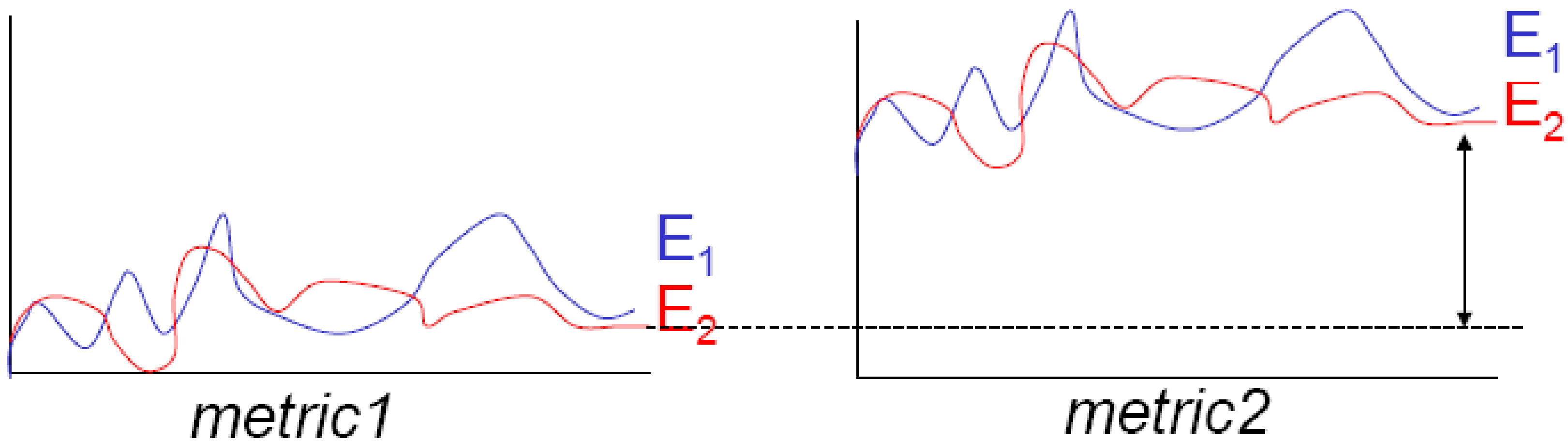


Shift in amplitude $E_2=E_1+cte2$
metric value2

What is the relation between **metric value1** and **metric value 2**?

Features?

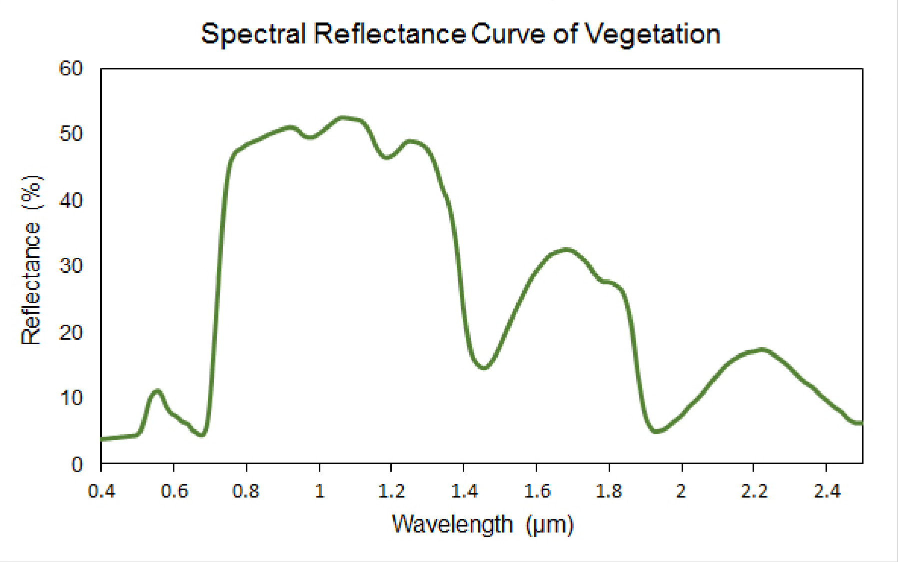
Affected by absolute values of the spectra (*light versus dark*)?

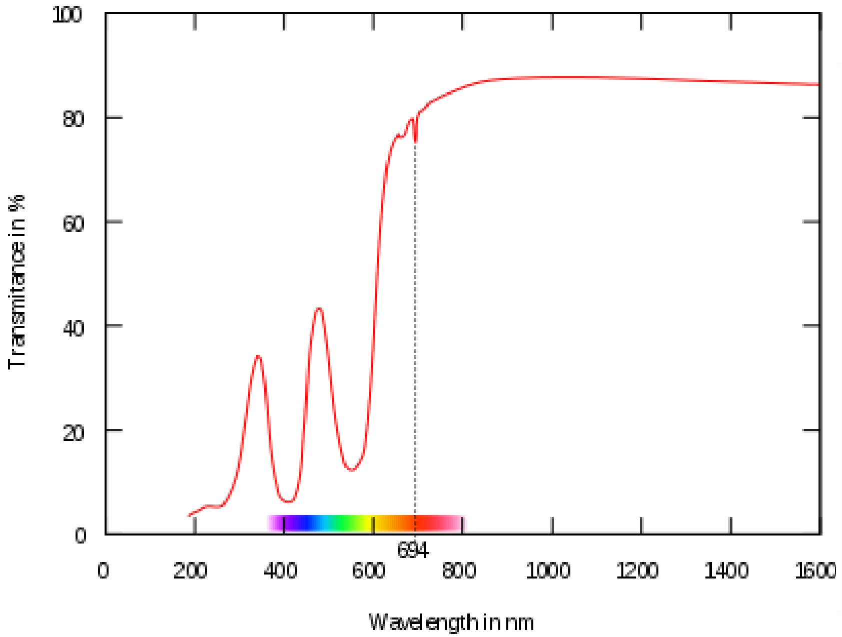


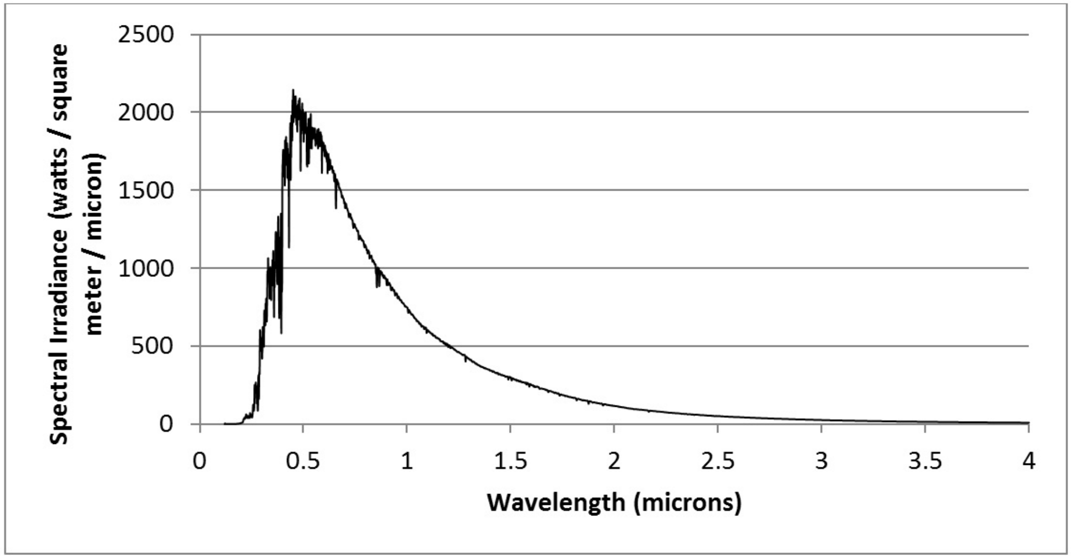
What is the relation between metric1 and metric 2?

Remember…

The two spectra we want to compare can be Color Signals, just illuminants or just reflectances (or transmittances…)







1.b. Spectral metrics:

Root Mean Square Error (RMSE o RMS):
(standard deviation)

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(E(\lambda_j) - E_R(\lambda_j)\right)^2}$$

n= number of samples (wavelengths)
E= measured
E_R= reconstructed

1.b. Spectral metrics:

Root Mean Square Error:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(E(\lambda_j) - E_R(\lambda_j)\right)^2}$$

Features

- Perfect match: RMSE?
- Worst match: RMSE?
- Identical shape but shift in amplitude ($E=E_R+0.1$) leads to RMSE?
- Shift in scale ($E=cteE_R$) leads to RMSE?
- Is RMSE scalable with magnitude change?

  RMSE1 for $E=E_R+0.1$
  RMSE2 for $E=E_R+0.2$
  RMSE2=2*RMSE1 ???
- Is RMSE affected by absolute values of the spectra (*light versus dark*)?

1.b. Spectral metrics:

Root Mean Square Error:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(E(\lambda_j) - E_R(\lambda_j)\right)^2}$$

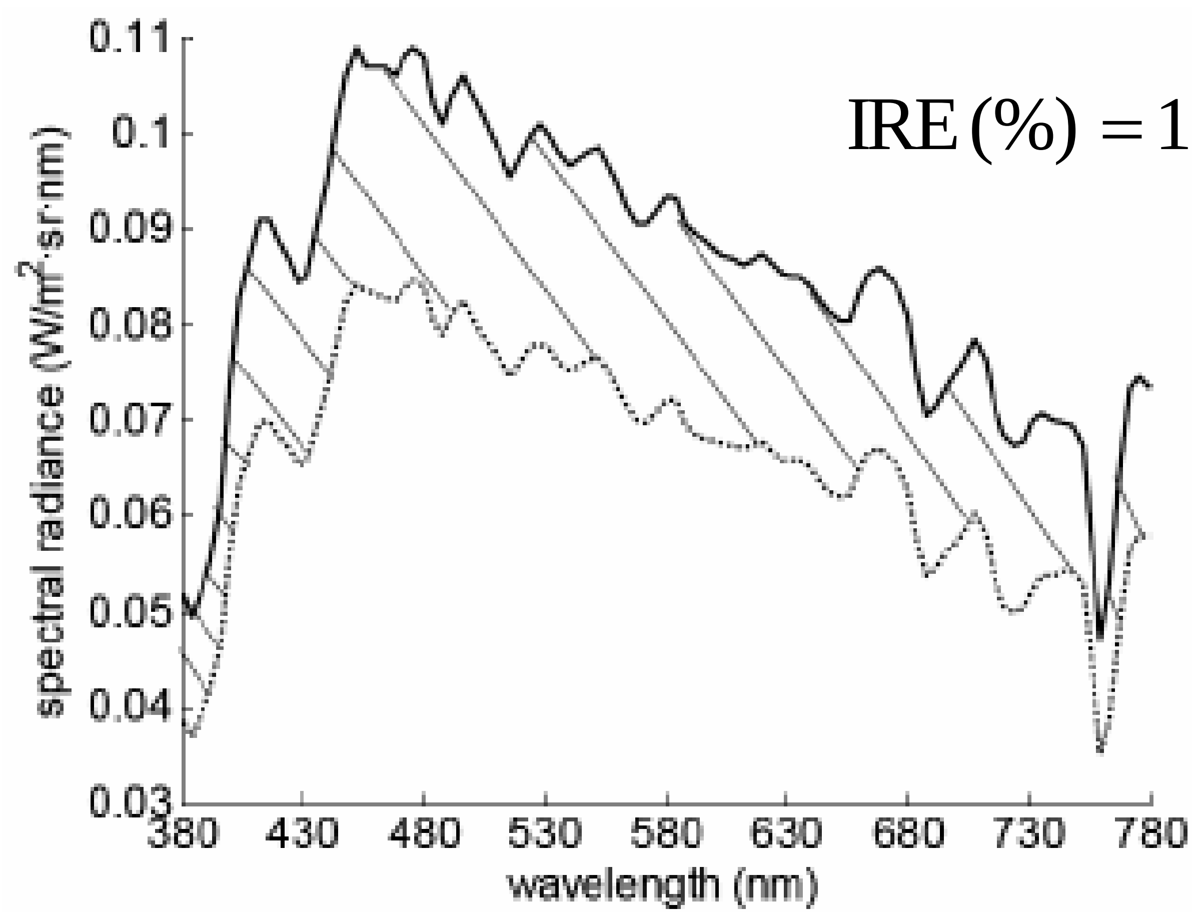
Features

- Perfect match RMSE=0
- Worst match RMSE= infinity
- Identical shape but shift in amplitude ($E=E_R+0.1$) leads to RMSE=0.1
- Shift in scale ($E=cteE_R$) leads to RMSE not equal to 0.
- Is RMSE scalable with magnitude change?

  RMSE1 for $E=E_R+0.1$
  RMSE2 for $E=E_R+0.2$
  RMS2=2*RMS1
- Is RMSE affected by absolute values of the spectra (*light versus dark*)? NO

1.b. Spectral metrics:

Integrated radiance error (or irradiance)



$$\mathrm{IRE}(\%) = 100 \frac{\left| \sum_{j=1}^{n} \left(E(\lambda_j) - E_R(\lambda_j) \right) \right|}{\left| \sum_{j=1}^{n} E(\lambda_j) \right|}$$

1.b. Spectral metrics:

Integrated radiance error

$$\mathrm{IRE}(\%) = 100 \frac{\left| \sum_{j=1}^{n} \left(E(\lambda_j) - E_R(\lambda_j) \right) \right|}{\left| \sum_{j=1}^{n} E(\lambda_j) \right|}$$

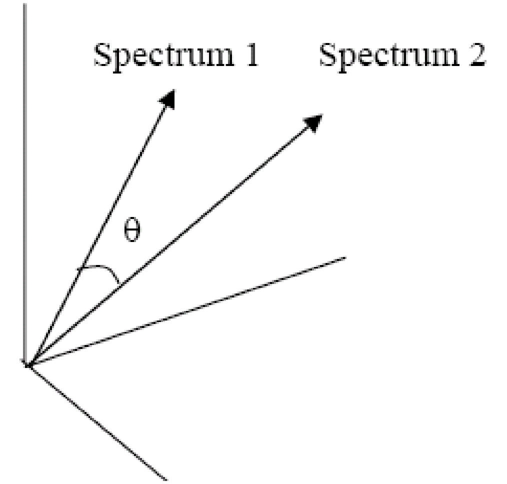
Features

- Perfect match IRE=0
- Worst match IRE=infinity
- Identical shape but shift in amplitude leads to IRE= **for students?**
- Shift in scale ($E=cteE_R$) leads to IRE not equal to 0.
- Is IRE scalable with magnitude change? **for students?**
- Is IRE affected by absolute values of the spectra (*light versus dark*)? **for students?**

1.b. Spectral metrics:

Goodness of Fit Coefficient: GFC is based on the inequality of Schwartz and it is described by the equation.

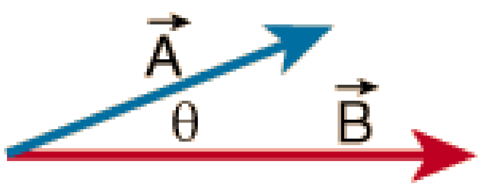
$$GFC = \frac{\left| \sum_{j=1}^{n} E(\lambda_j) E_R(\lambda_j) \right|}{\left| \sum_{j=1}^{n} \left[E(\lambda_j) \right]^2 \right|^{1/2} \left| \sum_{j=1}^{n} \left[E_R(\lambda_j) \right]^2 \right|^{1/2}}$$



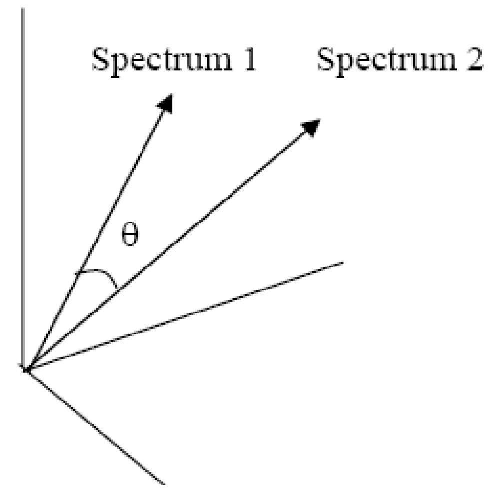
which is the cosine of the angle between two spectra if these are intended to be vectors in a Hilbert space.

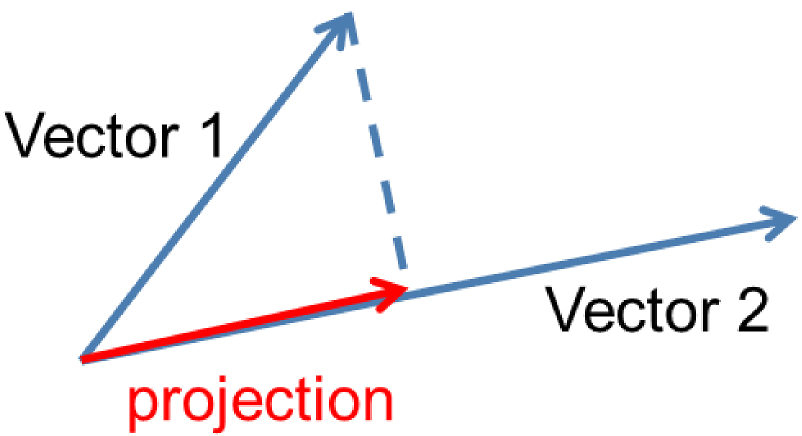
$$\vec{A} \cdot \vec{B} = A\,B \cos\theta$$

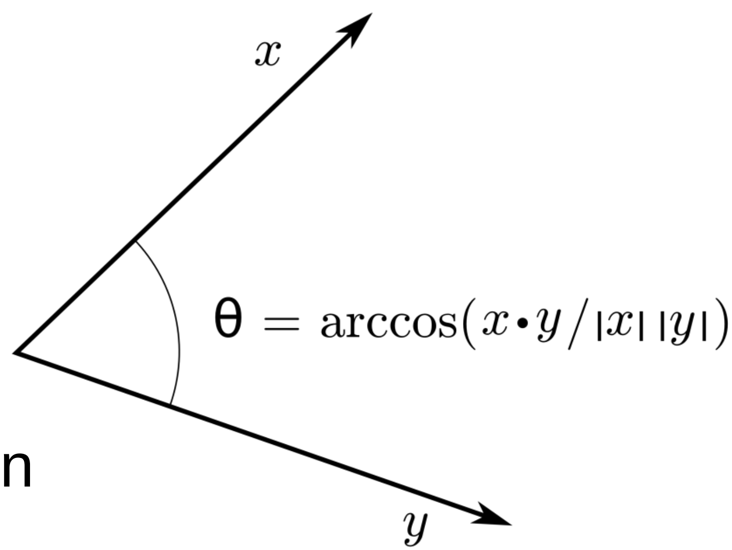
Calculation



$$\mathrm{GFC} = \frac{\left|\sum_{j=1}^{n} E(\lambda_j) E_R(\lambda_j)\right|}{\left|\sum_{j=1}^{n} [E(\lambda_j)]^2\right|^{1/2} \left|\sum_{j=1}^{n} [E_R(\lambda_j)]^2\right|^{1/2}}$$

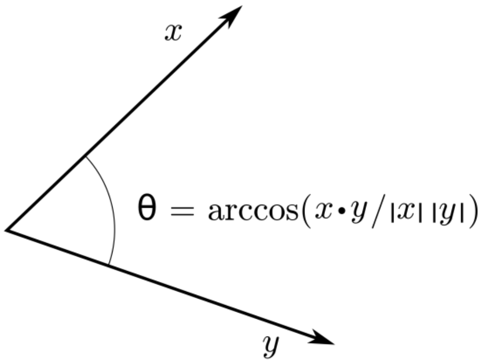






GFC is the cosine of the angle between two spectra if these are intended to be vectors in a Hilbert space.

Talking about projections….



$$R = k \int_{\lambda_{\min}}^{\lambda_{\max}} Light(\lambda)\ SensorR(\lambda)\ d\lambda$$

$$R = k \sum_{n=1}^{N} Light(\lambda_n)\ SensorR(\lambda_n)$$

$$R = k\ Light \bullet SensorR$$

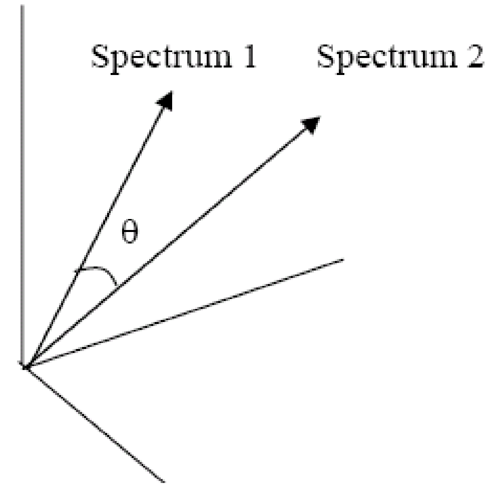
inner product
(or scalar product
or dot product)

$$X = k \int_{\lambda_{\min}}^{\lambda_{\max}} Light(\lambda)\ \overline{x}(\lambda)\ d\lambda \qquad X = k\ Light \bullet \overline{x}$$

1.b. Spectral metrics:

Goodness of Fit Coefficient: GFC is based on the inequality of Schwartz and it is described by the equation.

$$\mathrm{GFC} = \frac{\left|\sum_{j=1}^{n} E(\lambda_j) E_R(\lambda_j)\right|}{\left|\sum_{j=1}^{n} [E(\lambda_j)]^2\right|^{1/2} \left|\sum_{j=1}^{n} [E_R(\lambda_j)]^2\right|^{1/2}}$$



GFC is just the multiple correlation coefficient *R* and the square root of the variance accounted- for coefficient (VAF).

GFC ranges from 0 to 1, where 1 indicates a perfect reconstruction.

1.b. Spectral metrics:

Good of Fit Coefficient:

$$\mathrm{GFC} = \frac{\left|\sum_{j=1}^{n} E(\lambda_j)E_R(\lambda_j)\right|}{\left|\sum_{j=1}^{n}\left[E(\lambda_j)\right]^2\right|^{1/2}\left|\sum_{j=1}^{n}\left[E_R(\lambda_j)\right]^2\right|^{1/2}}$$

Features:

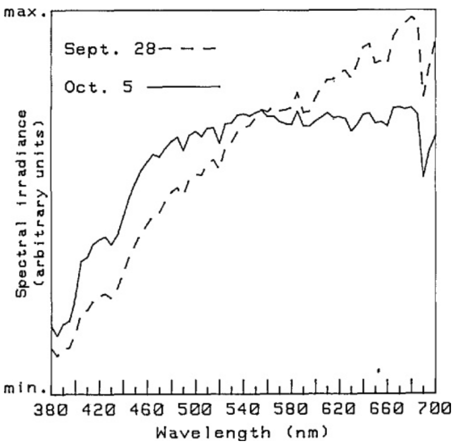
- Perfect match GFC=1
- Worst match GFC=0
- Identical shape but shift in amplitude ($E=E_R+0.1$) leads to GFC= **for students?**
- Shift in scale ($E=cteE_R$) leads to GFC=1. This metric is insensitive to shifts in magnitude (two spectra which differ by a multiplicative factor have the same score as a perfect match). ***Not necessarily a problem.***
- GFC values are not scalable with magnitude change.

  GFC1 for $E=E_R+0.1$

  GFC2 for $E=E_R+0.2$

  GFC2<GFC1
- Is GFC **affected** by absolute values of the spectra (*light versus dark*)? **for students???**

1.b. Spectral metrics:



$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(E(\lambda_j) - E_R(\lambda_j)\right)^2}$$

$$\mathrm{IRE}\,(\%) = 100\,\frac{\left|\sum_{j=1}^{n}\left(E(\lambda_j) - E_R(\lambda_j)\right)\right|}{\left|\sum_{j=1}^{n}E(\lambda_j)\right|}$$

$$\mathrm{GFC} = \frac{\left|\sum_{j=1}^{n}E(\lambda_j)E_R(\lambda_j)\right|}{\left|\sum_{j=1}^{n}\left[E(\lambda_j)\right]^2\right|^{1/2}\left|\sum_{j=1}^{n}\left[E_R(\lambda_j)\right]^2\right|^{1/2}}$$

Others?...

1.c. Metameric indexes:

A metamerism index compares the extent to which two spectra are different between a reference condition and a test condition under different illuminants and **observers**.

Color rendering index (CRI)

General index of metamerism *(from Berns 2000)*

Fairman special index of metamerism *(from Fairman et al. 1997)*

1.c. Metameric indexes:

Viggiano's perception-reference method compares radiance ratio spectra. *(from Viggiano 2004).* Abbreviated as **SCI**.

$$M_V = \sum_{\lambda=1}^{n} w(\lambda) \left\| \Delta\beta(\lambda) \right\|$$

where $w(\lambda)$ is the weight, n is the number of the wavelengths, $\Delta\beta(\lambda)$ is the difference between the two spectra.

$$w(\lambda) = \sqrt{\left(\frac{dL^*}{d\beta(\lambda)}\right)^2 + \left(\frac{da^*}{d\beta(\lambda)}\right)^2 + \left(\frac{db^*}{d\beta(\lambda)}\right)^2}$$

It is a refinement of a spectral-based metameric index based on a weighted sum of the absolute differences between two spectra proposed by Nimeroff and Yukov.

$$\frac{dL^*}{d\beta(\lambda_i)} = 116\, k\, S(\lambda_i)\, \bar{y}(\lambda_i)\, \frac{d}{dY} f\left(\frac{Y}{Y_n}\right)$$

$$\frac{da^*}{d\beta(\lambda_i)} = 500\, k\, S(\lambda_i) \cdot \left[\bar{x}(\lambda_i) \frac{d}{dX} f\left(\frac{X}{X_n}\right) - \bar{y}(\lambda_i)\, \frac{d}{dY} f\left(\frac{Y}{Y_n}\right)\right]$$

$$\frac{db^*}{d\beta(\lambda_i)} = 200\, k\, S(\lambda_i) \cdot \left[\bar{y}(\lambda_i)\, \frac{d}{dY} f\left(\frac{Y}{Y_n}\right) - \bar{z}(\lambda_i) \frac{d}{dZ} f\left(\frac{Z}{Z_n}\right)\right]$$

$$\frac{d}{du} f\left(\frac{u}{u_n}\right) = \begin{cases} \dfrac{7.787}{u_n} & si \;\; \dfrac{u}{u_n} \leq 0.008856 \\[2ex] \dfrac{1}{3u} f\left(\dfrac{u}{u_n}\right) & si \;\; \dfrac{u}{u_n} > 0.008856 \end{cases}$$

u tristimulus values X, Y and Z

u_n triestimulus values for the white reference

1.d. Weighted spectral metrics:

To **weight spectral reflectance** **factor rms error** between reference and test curves in a way that consider some properties of human visual system.

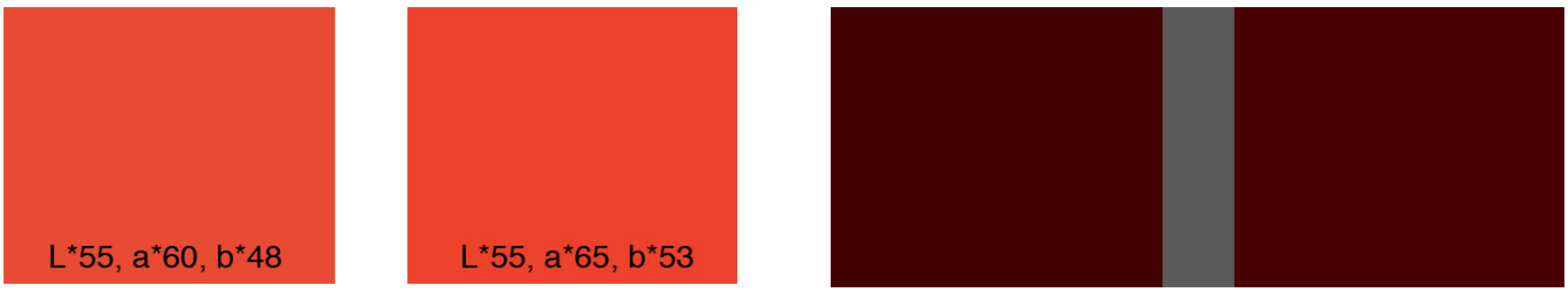
$$wrms = \sqrt{\frac{\sum_{\lambda=1}^{n}\left(\sqrt{w(\lambda)}\Delta\beta(\lambda)\right)^2}{n}}$$

where $w(\lambda)$ is the weight, n is the number of the wavelengths, $\Delta\beta(\lambda)$ is the difference between the two spectra.

1.d. Weighted spectral metrics:

Inverse of the reference spectra:

it is more important to weight spectral data with small magnitude than the ones with larger magnitude because human visual system is more sensitive to mismatches in dark colors than light colors. (R_m is the reference spectrum)



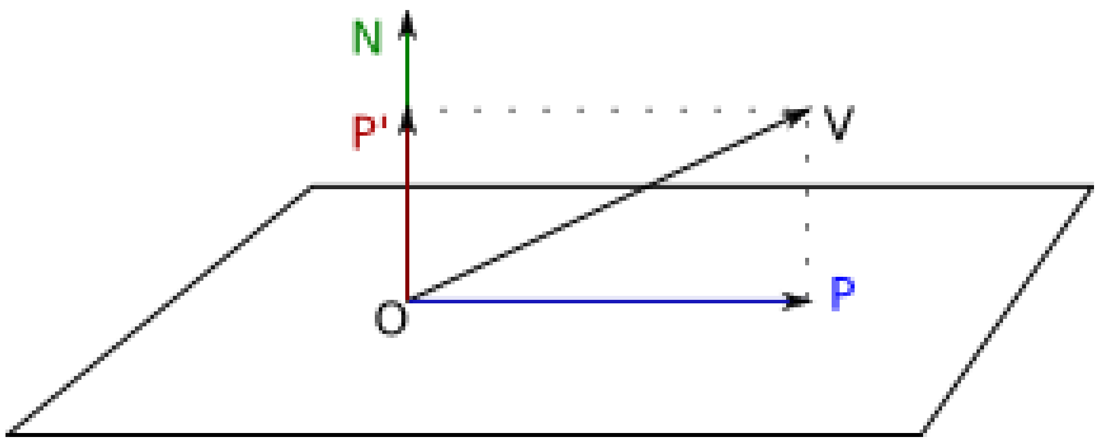
$$w_{invR}(\lambda) = \frac{1}{R_m(\lambda)}$$

1.d. Weighted spectral metrics:

Diagonal of matrix [***R***] Cohen developed a mathematical technique, known as Matrix [***R***] based on Wyszecki's hypothesis that **any stimulus can be decomposed into a fundamental stimulus with identical tristimulus values and a residual metameric black.**

any spectrum | fundamental spectrum | metameric black spectrum

$$\beta = \beta_F + \beta_B$$



1.d. Weighted spectral metrics:

Diagonal of matrix [***R***]

The matrix [***R***] can be easily calculated from the matrix ***A*** of weights for the reference illuminant and observer as

$$[R] = A\,(A^t A)^{-1}\,A^t$$

What matrix R represents?

A= color matching functions matrix (observer and illuminant). For 5nm resolution, A is a 61x3 matrix.

The diagonal of the matrix [***R***] (a 61x61 matrix) can be used as the weighting function for the RMS calculation:

$$w_{diagR}(\lambda) = diag([R])$$

1.d. Weighted spectral metrics:

Diagonal of matrix [***R***]

$$[R] = A\,(A^t A)^{-1}\,A^t$$

$$w_{diagR}(\lambda) = diag([R])$$

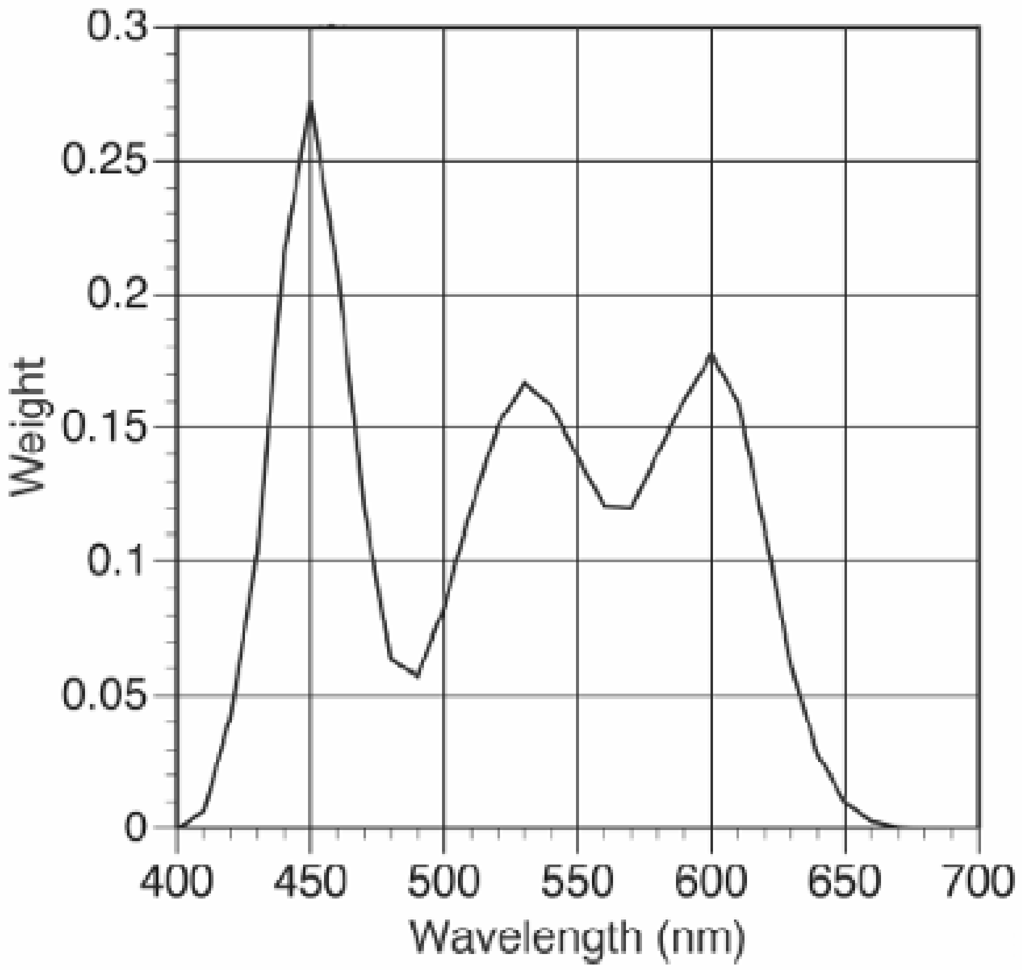


Figure 1. Weighting functions calculated from matrix [R] for D65 illuminant and 10 degree observer.

Figure from Imai (2002)

1. **Metrics at a pixel**
    1.a. Colorimetric metrics
    1.b. Spectral metrics
    1.c. Metameric indexes
    1.d. Weighted spectral metrics

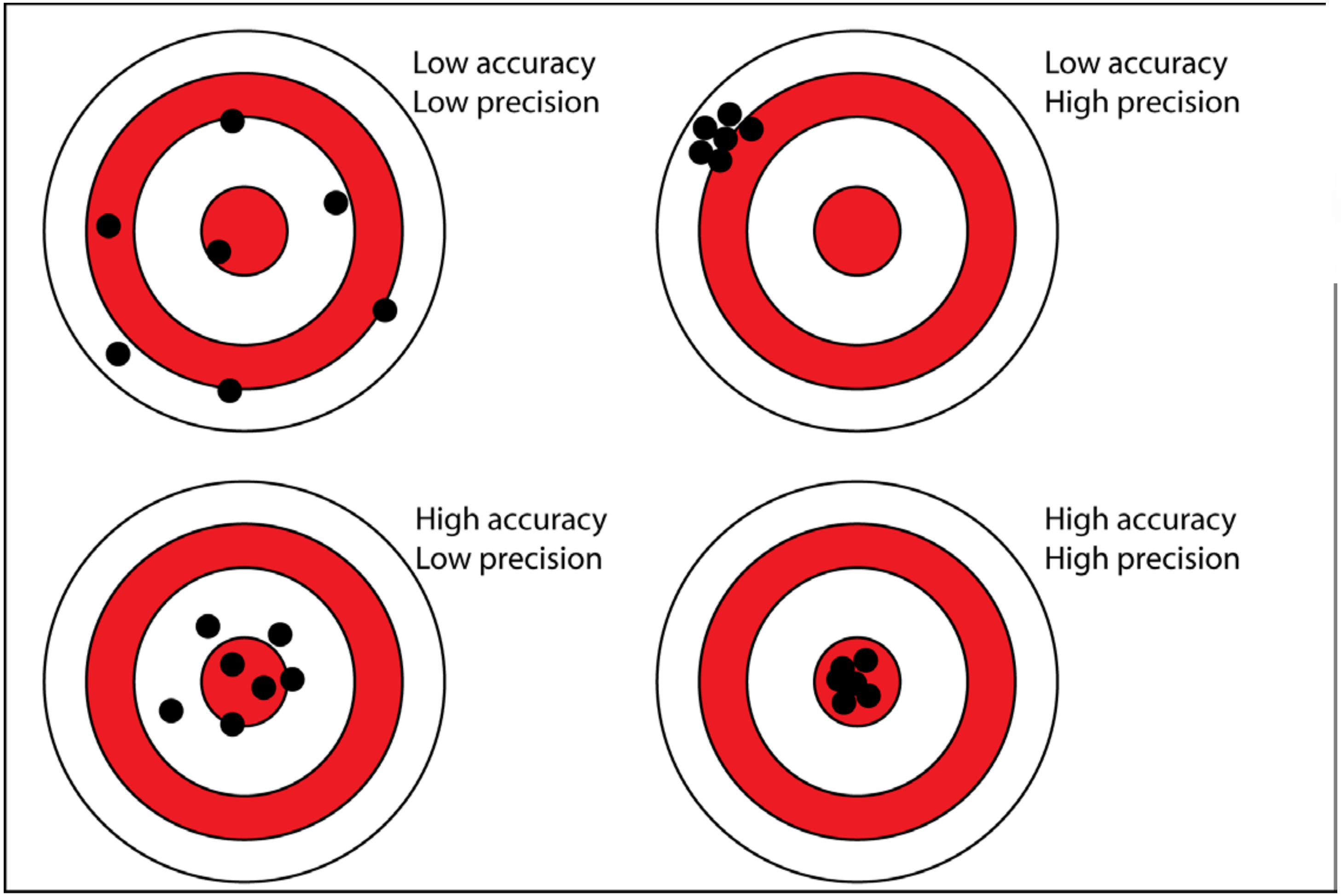
2. What properties a metric should have?
    2.a. Accuracy
    2.b. Precision

3. Combined metric at a pixel

Annex: How to build a metric?

2. What **properties** a metric should have if **color** is important?

Accuracy and Precision?



2. What **properties** a metric should have if **color** is important?

(from Viggiano 2004 and Pellegri et al. 2005)

Accuracy: How the associated colorimetric errors varies given the spectral error. That is, the extent to which the metric will exhibit a ***proportional relationship*** with CIELAB total color difference

spectral metric

RMSE
IRE(%)
GFC

CIELAB ΔE

*How to test the **accuracy** of some used metrics?*

2. What **properties** a metric should have if **color** is important?

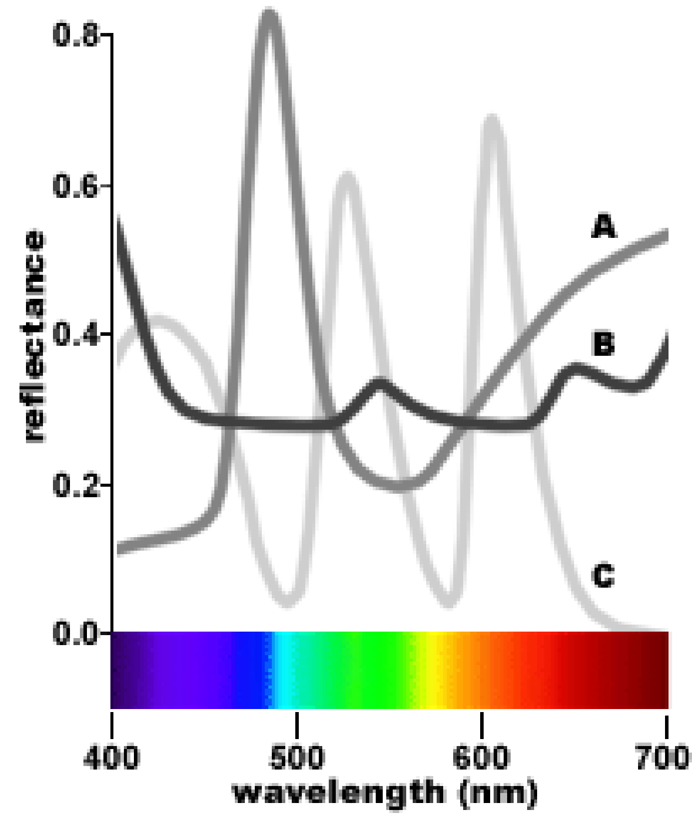
(from Viggiano 2004 and Pellegri et al. 2005)

Precision: the relative ***compactness*** of the distribution of a metric for spectral pairs which differ by a given level of CIELAB total color difference.

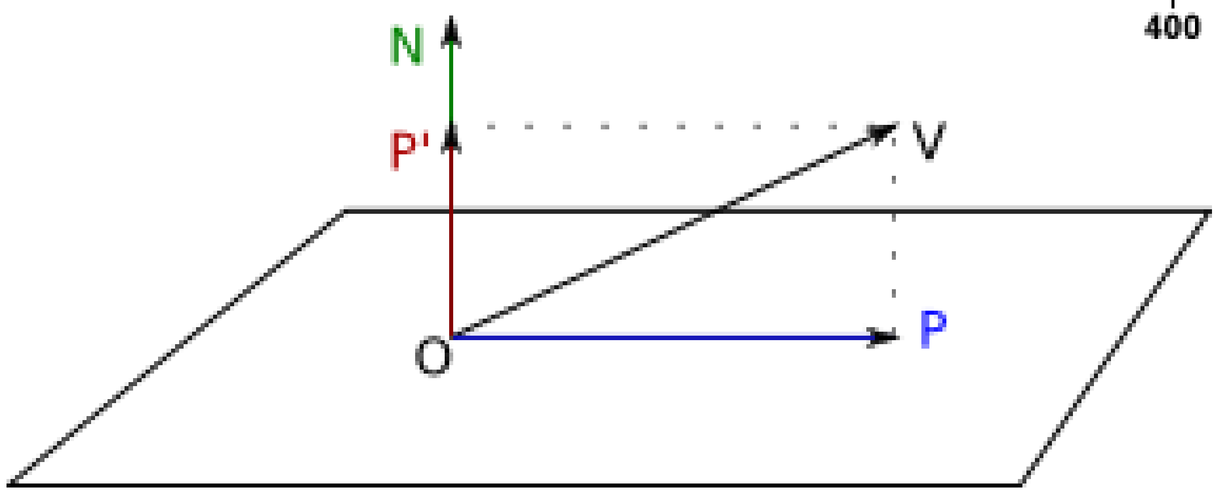
That is, given a spectral error, the range of associated colorimetric errors is as small as possible and as "concentrated" as possible around the most likely value, which can be determined by statistical means over a convenient test set.

*How to test the **precision** of some used metrics?*

Metameric spectra: two spectra which differ in the visible portion of the spectrum, but produce identical tristimulus values for a given combination of observer and illuminant.

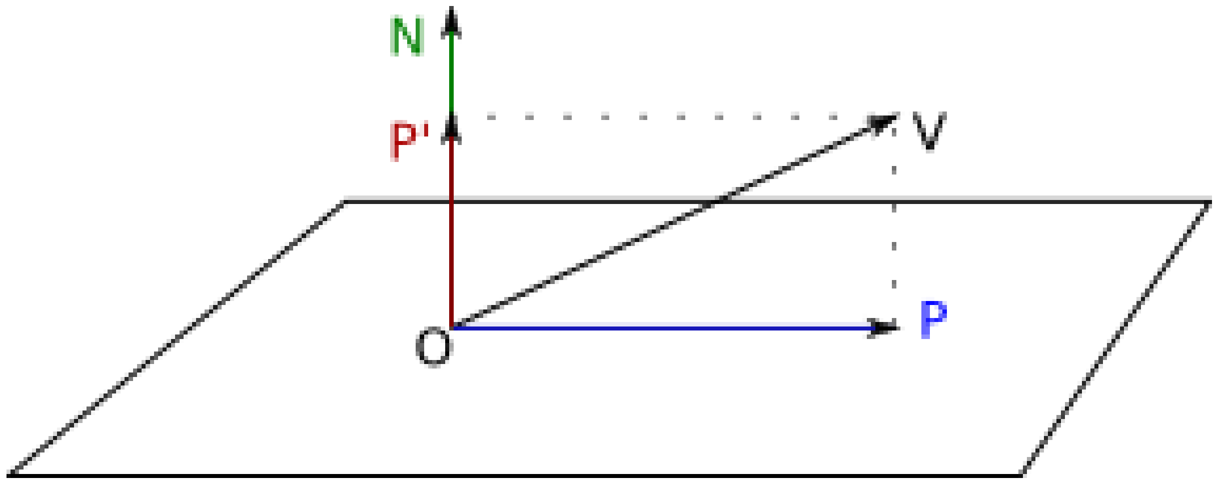


Mathematically ***Metameric*** spectra are spectra which have identical fundamental spectra, but different metameric black spectra.



any spectrum | fundamental spectrum | metameric black spectrum

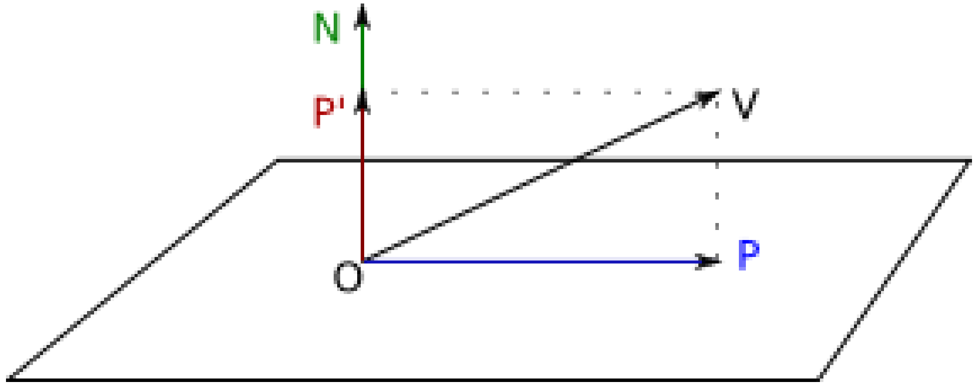
$$\beta = \beta_F + \beta_B$$



Non-metameric spectra are two spectra which have identical metameric black spectra, but possibly different fundamental spectra.

The ***fundamental of a spectrum*** is its projection onto observer/illuminant space.

$$\beta = \beta_F + \beta_B$$



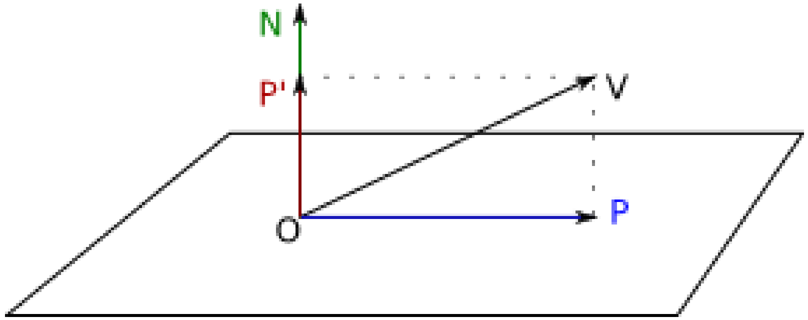
$$\boldsymbol{\beta}_F = \boldsymbol{W}^t \cdot (\boldsymbol{W} \cdot \boldsymbol{W}^t)^{-1} \cdot \boldsymbol{W} \cdot \boldsymbol{\beta}$$

where $\boldsymbol{\beta}$ is the column vector containing the spectral radiance ratios (*e.g.*, reflectances);

W is a 3-rowed matrix containing weights for tristimulus integration (the combined effect of observer and illuminant); and

$\boldsymbol{\beta}_F$ is the column vector containing the fundamental spectrum.

$$\beta = \beta_F + \beta_B$$



β given spectrum

Triestimulus values of the spectrum

$$\beta_F = W^t \cdot (W \cdot W^t)^{-1} \cdot W \cdot \beta$$ fundamental spectrum

$$\beta_B = \beta - \beta_F$$ metameric black spectrum

Non metameric spectra: if they have the same metameric black spectrum.

How to generate a trial spectrum which is non-metameric to a given standard spectrum? *(from Viggiano 2004)*

1. Given standard spectrum β

2. Given triestimulus values for the trial spectrum ***x***

3. Compute the metameric black spectrum of the standard β

$$\beta_F = W^t \cdot (W \cdot W^t)^{-1} \cdot W \cdot \beta \longrightarrow \boxed{\beta_B} = \beta - \beta_F \quad (1)$$

4. Compute the fundamental spectrum of the trial spectrum

$$\boxed{\beta_F} = F \cdot x \quad (2) \qquad F = W^t \cdot (W \cdot W^t)^{-1}$$

5. Add the metameric black to the fundamental **(1)+(2)**

$$\beta_{TRIAL} = \beta_F + \beta_B$$

Figure from Viggiano (2004)

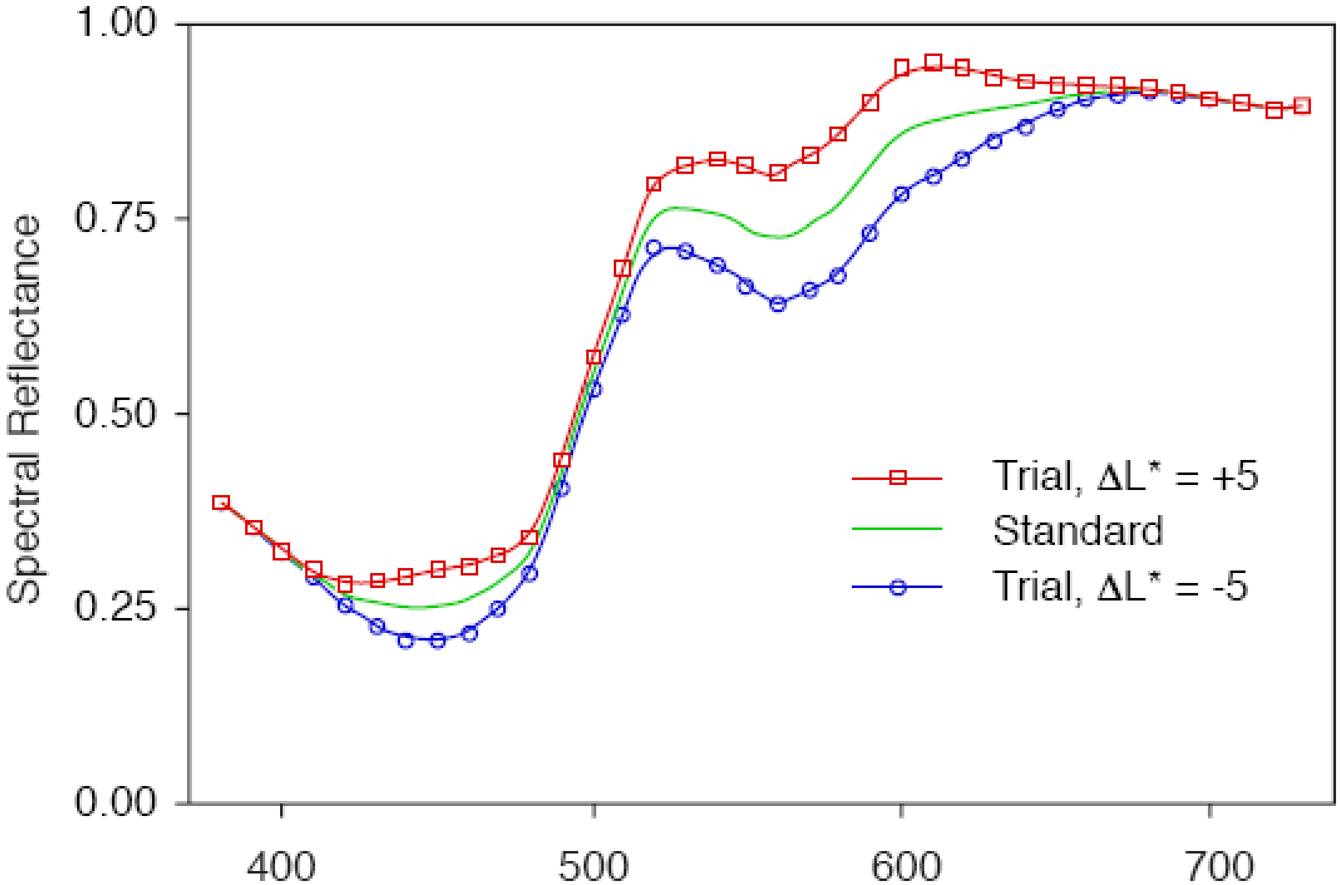


Figure 2: A Standard spectrum (middle curve) is flanked by two non-metameric Trial spectra which differ from the Standard by 3 units in L (upper and lower curves). All three spectra share a common Metameric Black spectrum, and, under the definition advanced in this paper, are regarded as "non-metameric."*

Accuracy: proportional relationship with a color difference?

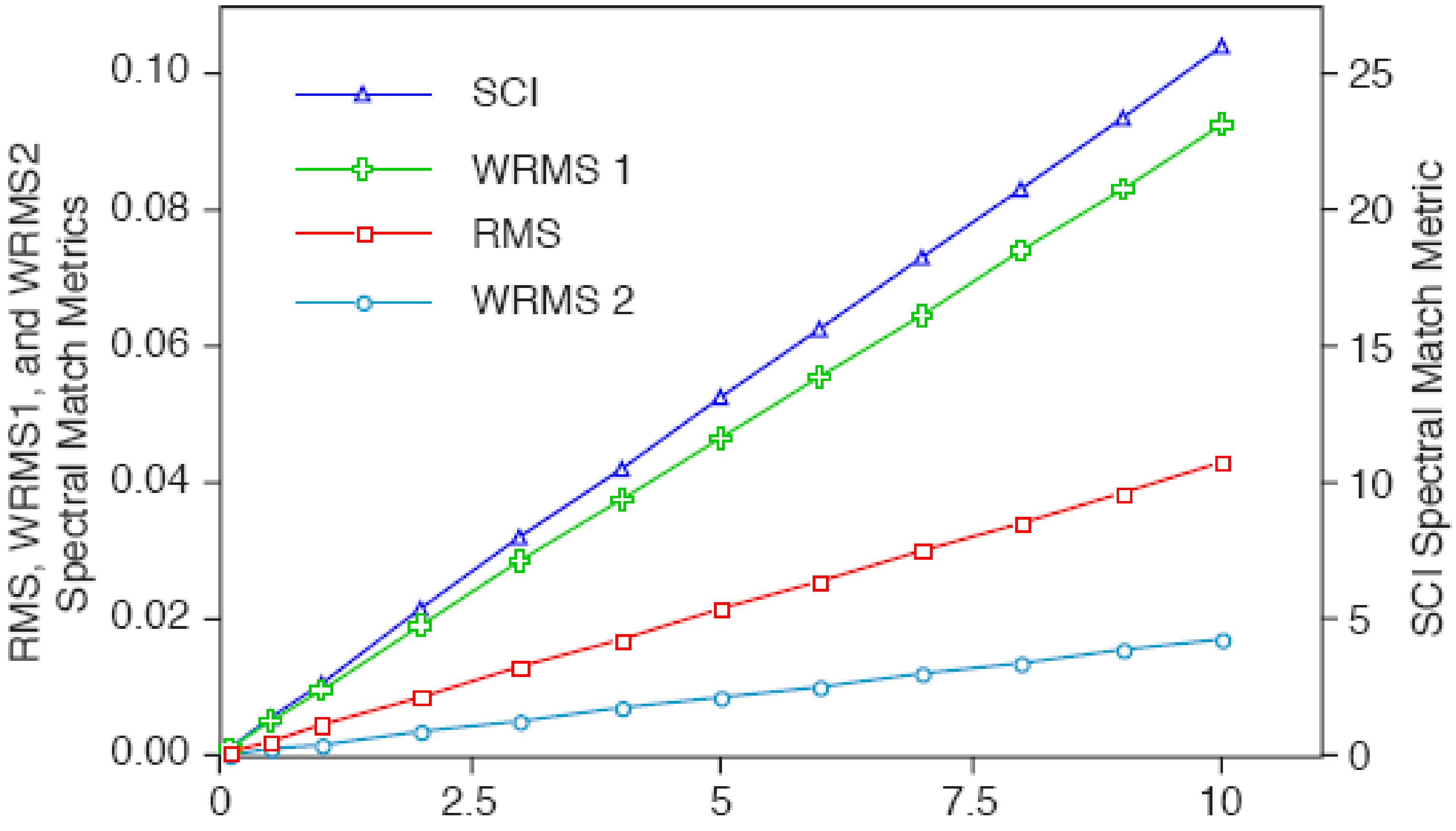
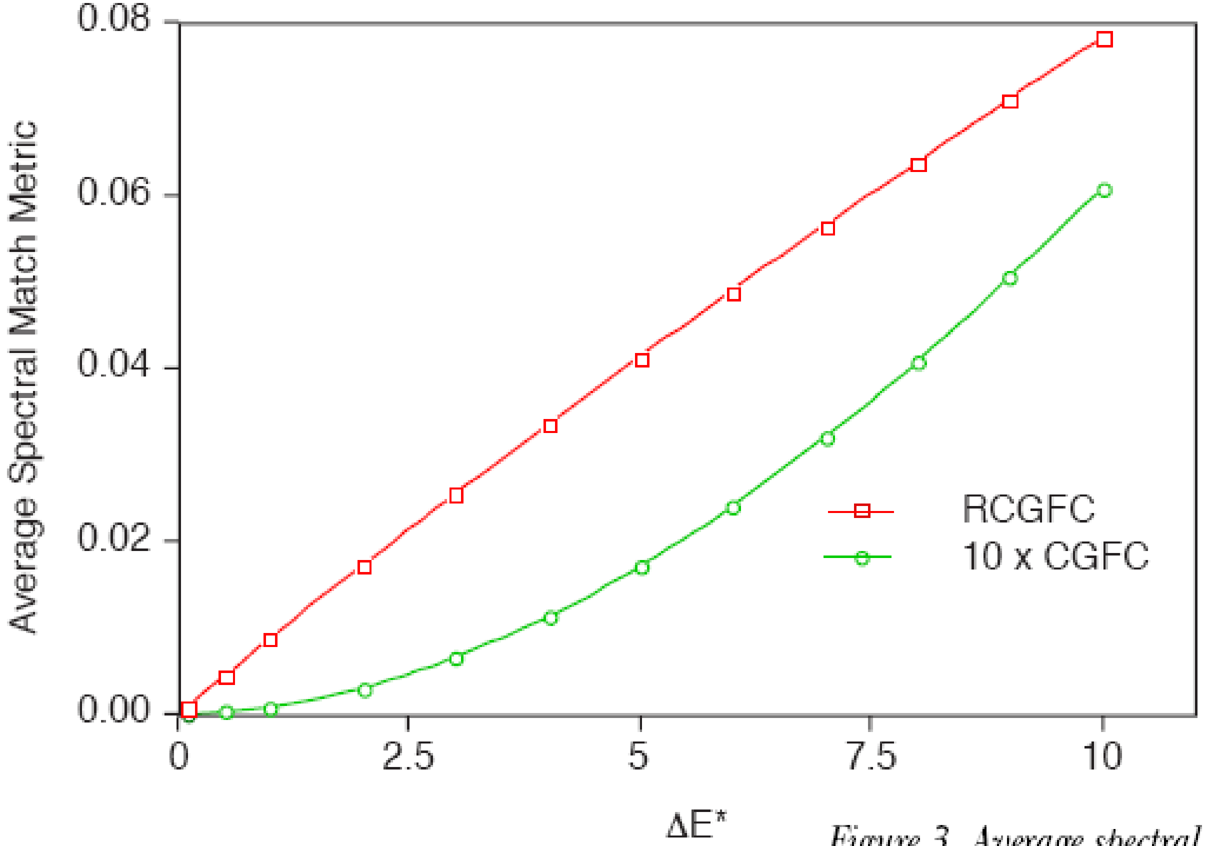


Figure 4. Average spectral match metrics as functions of ΔE. These metrics all exhibit excellent linear tracking of ΔE*. Because of its very different scale, the Viggiano Spectral Comparison Index uses the vertical axis to the right. The other metrics use the axis at the left. WRMS1: RMS Reflectance Difference weighted by reciprocal of spectral reflectance of Standard. WRMS2: RMS Reflectance Difference weighted by diagonal of Matrix R.*

Figure from Viggiano (2004)

Accuracy: proportional relationship with a color difference?



GFC

CGFC=1-GFC

RCGFC=(CGFC)$^{1/2}$

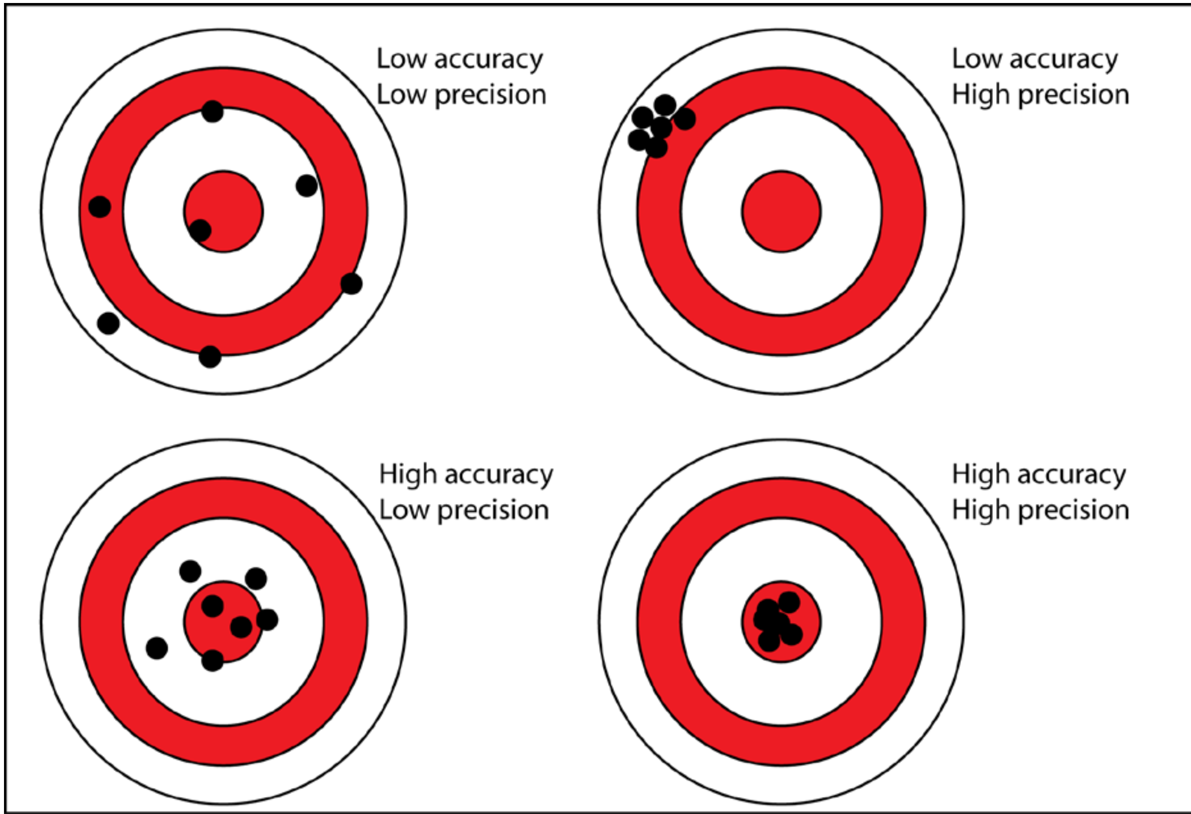
Figure 3. Average spectral match metrics as functions of ΔE for the Complemented Hernández-Andrés, Romero metric (CGFC, bottom curve) and its square root (RCGFC, upper curve). Because of a difference in scale, the latter is plotted at 10 times its actual value. This plot illustrates the non-linear manner in which the CGFC metric tracks ΔE*. The RCGFC enjoys a reasonably proportional relationship with ΔE*.*

Figure from Viggiano (2004)

Cha

Precision: the relative compactness of the distribution of a metric for spectral pairs which differ by a given level of CIELAB total color difference.

How to evaluate precision? *Viggiano 2004*: its ability to produce a tight cluster of **values for non-metameric** standard/test pairs which differ by a certain, constant, amount in human perception.



To evaluate the dispersion of metric values for a given ΔE*

metric $\quad \Delta E^* \quad \dfrac{metric}{\Delta E^*} \quad std\left(\dfrac{metric}{\Delta E^*}\right)$

Given a proportional relationship, if a metric value is divided by ΔE*, a constant value (the constant of proportionality) should result. Because the different metrics shall have different scalings, it is **not appropriate to compare the standard deviations** of their proportionality constants in order to assess precision and accuracy.

However, the **Coefficient of Variation (CV), which is the standard deviation divided by the mean**, is dimensionless and shall remove the non-uniformity.

$$CV = \frac{std\left(\dfrac{metric}{\Delta E^*}\right)}{mean\left(\dfrac{metric}{\Delta E^*}\right)}$$

CV of Spectral Match Metric

ΔE*	RMS	RCGFC	SCI	WRMS1	WRMS2
0,1	0,82	10,58	0,60	11,15	54,79
0,5	0,82	2,19	0,60	2,30	10,99
1	0,82	1,20	0,60	1,26	5,54
2	0,82	0,78	0,60	0,81	2,86
3	0,83	0,67	0,60	0,70	2,01
4	0,83	0,63	0,61	0,66	1,61
5	0,84	0,61	0,62	0,64	1,39
6	0,84	0,59	0,63	0,64	1,26
7	0,85	0,58	0,64	0,64	1,17
8	0,85	0,57	0,65	0,64	1,11
9	0,86	0,56	0,66	0,65	1,07
10	0,86	0,56	0,67	0,65	1,04

Lower CVs imply greater precision

Results of CV from 4096 reflectance spectra

Table from Viggiano (2004)

1. **Metrics at a pixel**
    1.a. Colorimetric metrics
    1.b. Spectral metrics
    1.c. Metameric indexes
    1.d. Weighted spectral metrics

2. **What properties a metric should have?**
    2.a. Accuracy
    2.b. Precision

3. **Combined metric at a pixel**

Annex: How to build a metric?

3. Combined metrics at a pixel

Imai *et al.* suggest that "**mononumerosis**" should be avoided when evaluating the quality of spectral matches. That is: *several* metrics should be used to assess color reconstruction from both colorimetric and spectral standpoints.

RMSE IRE(%) GFC SCI

Others…

3. Combined metrics at a pixel

"**mononumerosis**" should be avoided

BUT sometimes (*simulated annealing algorithm*) we need to use only a single cost function.

$$CSCM = Ln(1+1000(1-GFC)) + \Delta E^{*}_{ab} + IRE(\%)$$

Combined metric that is zero for perfect matches (good candidate for developing an annealing search algorithm).

Its chief advantage is that it quantifies spectral mismatches among metamers, perceptual differences in color matches and differences in such integrated radiometric quantities as irradiance or radiance.
Combines the properties of various metrics relevant to skylight spectra.

Table 3. Means and standard deviations for the metrics used in different skylight spectral matches.

Match	GFC ±SD	CIELAB ΔE_{ab} ±SD	IIE(%) ±SD	CSCM ±SD	SCI ±SD	RMSE ±SD	wRMSE ±SD
2% shift.	0.99993 ±0.00004	0.46 ±0.04	3.99 ±0.14	4.55 ±0.21	0.238 ±0.002	0.036 ±0	0.007 ±0
5% shift.	0.99941 ±0.00021	1.12 ±0.09	9.97 ±0.36	11.57 ±0.58	0.595 ±0.006	0.09 ±0	0.017 ±0
3 eigenvct.	0.99952 ±0.00105	0.76 ±0.55	0.24 ±0.22	1.35 ±1.05	0.158 ±0.092	0.028 ±0.017	0.006 ±0.003
metamers	0.85044 ±0.02181	0 ±0	33.79 ±2.09	38.79 ±2.20	1.866 ±0.092	0.525 ±0.031	0.053 ±0.003

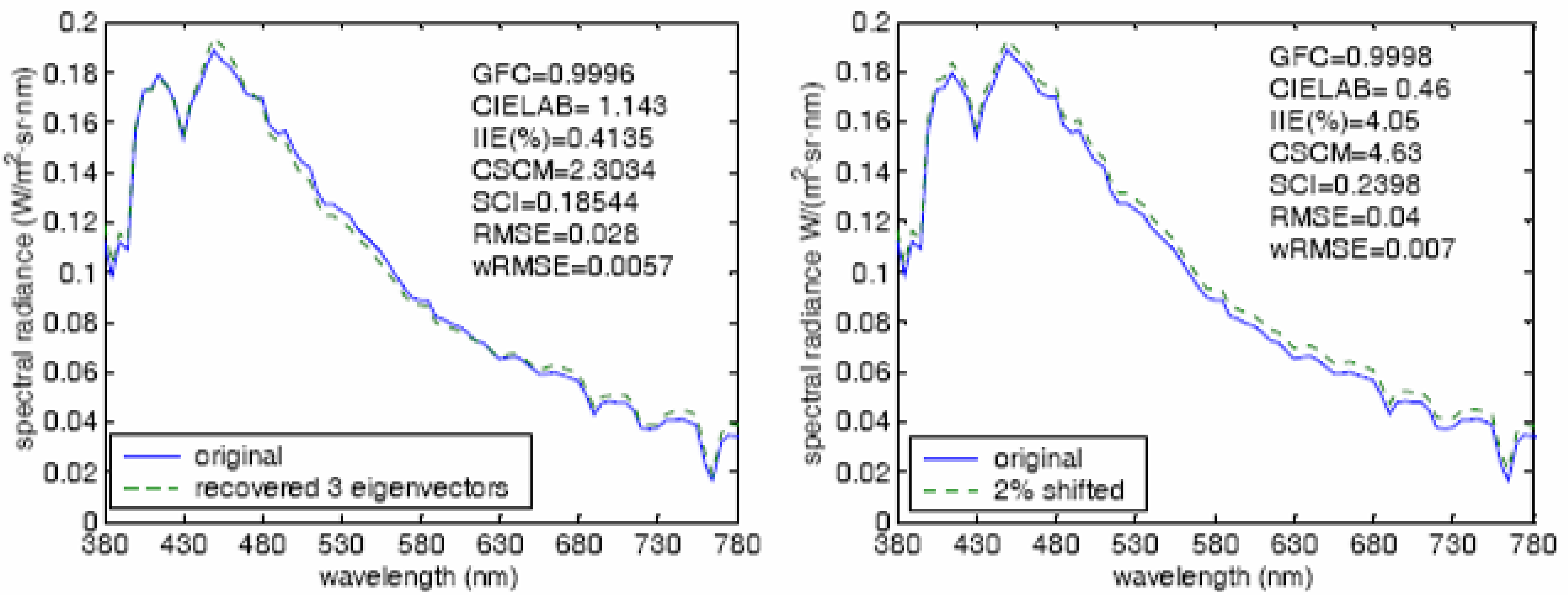
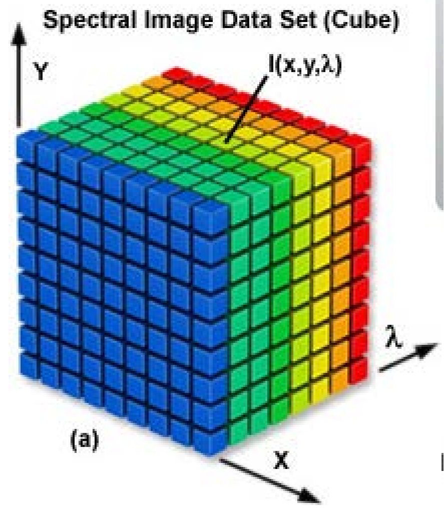


Figure 1. Samples of skylight spectral matches and their values for the metrics studied.

In a single pixel: a single value for just a metric

Avoiding mononumerosis: several values for just a pixel

In a spectral image: many many values for different metrics



How to present the data in a paper or in a report?

average

standard deviation

Percentiles (25%, 99%, 50%, 1%, 75%, 10%)

Quartiles (25%, 50%, 75%)

maximum

minimum

Interquartile range: difference between the upper and lower quartile

1. Metrics at a pixel
- 1.a. Colorimetric metrics
- 1.b. Spectral metrics
- 1.c. Metameric indexes
- 1.d. Weighted spectral metrics

2. What properties a metric should have?
- 2.a. Accuracy
- 2.b. Precision

3. Combined metric at a pixel

Annex: How to build a metric?

4. Annex: How to build a metric?

Pure spectral metrics: take as data only the spectral curves.
Should quantify the "diversity" of two spectra. The operation should be ***independent from the number of components*** in the spectra considered.

Mixed spectral metrics: take as data also the observer and illuminant.

From *Pellegri et al. 2005*

4. Annex: How to build a metric?

Problems: Difficulty in establishing a relationship between the spectral and colorimetric metrics lie in the effects of the observer and illuminant considered. Both act by enhancing the impact of the differences associated to certain wavelengths to the others: the observer also enhances differences on dark colors compared to brighter ones, resulting in a ***sort of non-linear correction***.

Any pure spectral metric can likely become a mixed metric with the proper changes!!!

From *Pellegri et al. 2005*

How to build a spectral metric?

Pellegri et al. 2005

Using different ingredients (quantifying spectral difference, correcting observer non-linearity, accounting for illuminant and observer wavelength biasing, etc.)

Two reflectance vectors $r_1 \quad r_2$

Standard difference vector $d = r_1 - r_2$

Component-wise ratio centered around 0 $d = \left[\dfrac{r_{1i}}{r_{2i}}\right]_i - 1$

Norm (i.e. rms) $N(d) = \dfrac{\|d\|}{\sqrt{L}}$

Pseudo-metric induced by the cosine of the angle between r_1 and r_2 $N(d) = \dfrac{1}{C} - 1 \qquad C = \dfrac{\langle r_1, r_2 \rangle}{\|r_1\| \|r_2\|}$

CGFC???

Pellegri et al. 2005

Absolute mean $N(d)=\frac{1}{L}\left|\sum_{i=1}^{L} d_i\right|$ Mean of absolute values $N(d)=\frac{1}{L}\sum_{i=1}^{L}\left|d_i\right|$

A correction for the observer non-linear response will try to enhance differences for dark colors compared to brighter ones! Can be done through a proper "weighting" of the vectors r_1 and r_2 before the difference vector d is calculated, or by multiplying N(d) for a convenient factor that depends on the vectors.

Weighting with the inverse of the average of the two spectra

$$w_1=\left[\frac{r_{1i}}{a_i}\right]_i \qquad a_i=(r_{i1}+r_{2i})/2$$

or more generally $w_1=\left[r_{i1}/a_i^{\,p}\right]_i$ Idem for w_2

where *p* is a convenient exponent

Pellegri et al. 2005

A correction?

Through a multiplicative factor using the inverse of the product of the vector norms:

$$D(r_1, r_2) = K\ N(d) \qquad K = \left(\|r_1\|^p \|r_2\|^p \right)^{-1}$$

as well as using the inverse of the product of vector means

$$K = \left(\left(\frac{1}{L} \sum_{i=1}^{L} r_{1i} \right)^p \left(\frac{1}{L} \sum_{i=1}^{L} r_{2i} \right)^p \right)^{-1}$$

One way to account for the illuminant: weighting the spectra before taking the vector difference

$$w_1 = \left[r_{1i} e_i^p \right]_i \qquad e = \left[e_i \right]$$

Represents the illuminant SPD

Pellegri et al. 2005

How to include the wavelength biasing effect of the observer?

By weighting: observer is represented by three sensitivity functions.

For example, using the diagonal of Cohen's matrix R

$$w_1 = \left[r_{1i}\rho_i^p\right]_i \qquad \left[\rho_i\right]_i = diag(R) \qquad R = \left(W^T\left(WW^T\right)^{-1}\right)W$$

W is the weight matrix for tristimulus integration made from the chosen observer and an equi-distribution illuminant.

Alternative for the weighting: the sum of the observer sensitivity functions

$$w_1 = \left[r_{1i}w_i^p\right]_i \qquad \left[w_i\right]_i = \left[\left(x_i + y_i + z_i\right)\right]_i$$

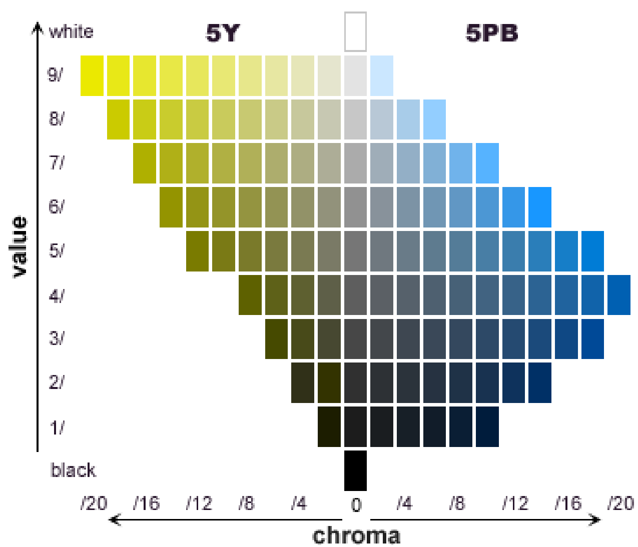
Choice of the test set used to evaluate a metric

Is a very important issue as it may greatly affect the results obtained.

In general a ***test set*** will consist of pairs of colors whose spectra and colorimetric coordinates are both known.

Test set should be homogeneously distributed throughout the color space as possible (unless the whole analysis is performed for a specific application).

Test set: two possibilities: (physical samples or mathematical samples)



References

1. "Colorimetric and Spectral Combined Metric for the Optimization of Multispectral Systems", *M.A. López-Álvarez, J. Hernández-Andrés, E.M. Valero and J.L. Nieves, AIC Colour 05 - 10th Congress of the International Colour Association, pp.*1685-1688 (2005)
2. "Metrics for Evaluating Spectral Matches: A Quantitative Comparison", *J A Stephen Viggiano, Proceedings of the Second European Conference on Colour in Graphics, Imaging and Vision* (Society for Imaging Science and Technology, Springfield, Va., 2004), pp. 286-291.
3. "Comparative study of metrics for spectral match quality," F. H. Imai, M. R. Rosen and R. S. Berns, in *Proceedings of the First European Conference on Colour in Graphics, Imaging and Vision* (Society for Imaging Science and Technology, Springfield, Va., 2002), pp. 492-496.
4. "A comparison between illuminants and light-source simulators", R. Roa, R. Huertas, M.A. López-Álvarez, L. Gómez-Robledo and M. Melgosa, Optica Pura y Aplicada, vol. 41, pp. 291-300.
5. "Metameric correction using parametric decomposition", H.S. Fairman, *Color Res. Appl*. **12,** 261-265 (1997)
6. "Billmeyer and Saltzman's Principles of Color Technology", R.S. Berns, John Wiley & Sons, 2000.
7. "Image-Based Spectral Reflectance Reconstruction Using the Matrix R Method", Y. Zhao, R.S. Berns, Color Research and Application, vol. 32, 343-351 (2007)

References (more)

8. “Metamer sets without spectral calibration”, A.Alsam, G.Finlayson, JOSA A, **24**, 2505-2512 (2007)
9. “Visual color and color mixture”, J.B. Cohen, Univ.of Illinois Press (2001)
10. “Spectral metrics and their ability to predict perceptual color differences: an evaluation”, P. Pellegri, G. Novati, R. Schettini, Colore e Colorimetria: contributi multidisciplinari, Collana Quaderni di Ottica e Fotonica 13, pp. 71-78, 2005.
11. “Similarity of color images”, M. Stricker and M. Orengo, Proc. SPIE Vol. 2420, p. 381-392, Storage and Retrieval for Image and Video Databases III, (1995)
12. “Quality Evaluation in Spectral Imaging – Quality Factors and Metrics” Raju Shrestha, Ruven Pillay, Sony George and Jon Y Hardeberg, Journal of the International Colour Association (2014): 12, 22-35