The opening chapter deals with Schur's lemma, the Jacobson radical and artin rings. These are used to prove Maschke's theorem on the semi-simplicity of a finite group algebra and there is a note about the present stage of development of the problem for infinite groups. This material has of course been given many times before, but here it is rather neatly carried out and there is an interesting application to proving that an artin ring with zero characteristic has a unit. Next follows the density theorem and matrix representations of rings, more properties of group algebras and a solution of the Burnside problem for matrix groups, solved by Burnside and revised by Kaplansky.

There follows a chapter discussing conditions under which a ring is commutative, the classical result due to Wedderburn is that a finite division ring is a field, and this has been widely generalized by Jacobson and the author. Moving back to simple algebras, we are given the Brauer group, maximal subfields of algebras and a study of crossed products. This chapter is of some depth, the reader has to be familiar with tensor products and the author is certainly exceeding his brief. The risk is well worthwhile, the chapter is essential to the book and nicely done. Next we move to group representations; this can still be thought of as ring theory thanks to E. Noether, and proceed to a point where the Burnside theorem, that groups of order $p^\alpha q^\beta$ are soluble, is a natural application. He also proves the beautiful theorem of Hurwitz, which states that

$$\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right) = \sum_{i=1}^{n} z_i^2,$$

where the $z_i$ are bilinear functions of the $x$'s and $y$'s, can occur only when $n = 1, 2, 4, 8$.

The remaining chapters move away from the classical treasures to modern developments, beginning with rings having polynomial identities and including a proof for P.I. algebras of the Kurosch conjecture that algebraic algebras are locally finite. There follows a discussion of quotient rings with an application to the important theorem of Posner on prime rings with polynomial identity. The book closes with a brief chapter on the work of Golod–Schafarevitch which yielded counter-examples to the Kurosch problem and the general Burnside problem. This book covers a good deal of ground and presents its information in a clear, down-to-earth and interesting style. It has developed from lectures given at Chicago by the author, many have already studied these in duplicated form. Now the whole can be slipped into a jacket pocket and every algebraist should do this with it sometime.

A. W. GOLDIE

## INTRODUCTION TO DIFFERENTIAL GEOMETRY AND RIEMANNIAN GEOMETRY

*By* ERWIN KREYSZIG: pp. xii, 370. £5.5s. (University of Toronto Press, 1968).

The first part of this book is concerned with the local differential geometry of curves and surfaces in three-dimensional Euclidean space and it is based on the author's

*Differential Geometry* (University of Toronto Press, 1959) which is now out of print. Attractively written and very well illustrated, it deals with such fundamental topics as the first and second fundamental forms of a surface, geodesics, mappings of surfaces, affine connexions and curvature. It includes results from the global differentiable geometry of surfaces and some special surfaces, such as minimal surfaces and modular surfaces, are also described. All this is a branch of classical mathematics and the historical background is not neglected.

Some of the previous ideas are generalized in the last four chapters to give a short introduction to Riemannian geometry. It is here that the author is not quite so successful. Although differentiable manifolds and Riemannian manifolds are defined in the modern way, the point of view then changes rather sharply and the rest of the work continues on the same lines as the earlier part of the book. The topics discussed include affine connexions and curvature on a differentiable manifold and some geometrical properties of submanifolds of a Riemannian manifold with particular reference to hypersurfaces. While this gives an idea of the historical development of Riemannian geometry (and this should never be ignored), it probably does not provide an effective introduction to a subject which has changed so dramatically in the last 30 years.

R. S. CLARK

## THEORIE AXIOMATIQUE DES ENSEMBLES

*By* JEAN-LOUIS KRIVINE: pp. 118; 10F. (Presses Universitaires de France, Paris, 1969).

This little book in the SUP series gives an account of the Zermelo-Fraenkel axioms for set theory, and studies the question of the relative consistency of the axiom of choice and the continuum hypothesis by means of the Fraenkel–Mostowski model (set theory with atomic elements) and the Gödel model (constructible sets). The book is intended as an introduction to Paul Cohen's recent work on the independence of the axiom of choice and of the continuum hypothesis but does not include an account of these results.

R. L. GOODSTEIN

## ADVANCED CALCULUS

*By* LYNN H. LOOMIS and SHLOMO STERNBERG: pp. ix, 580; 89*s*. (Addison–Wesley Publishing Co., Inc., London, 1968).

This book is devoted to a fairly sophisticated account of the calculus of several real variables, treated in the setting of normed vector spaces, and of the calculus of differentiable manifolds. It is intended for undergraduates who already have a good grounding in elementary analysis and some acquaintance with linear algebra, although it is in fact logically self-contained.