Tarea 7

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Tabla de Contenidos

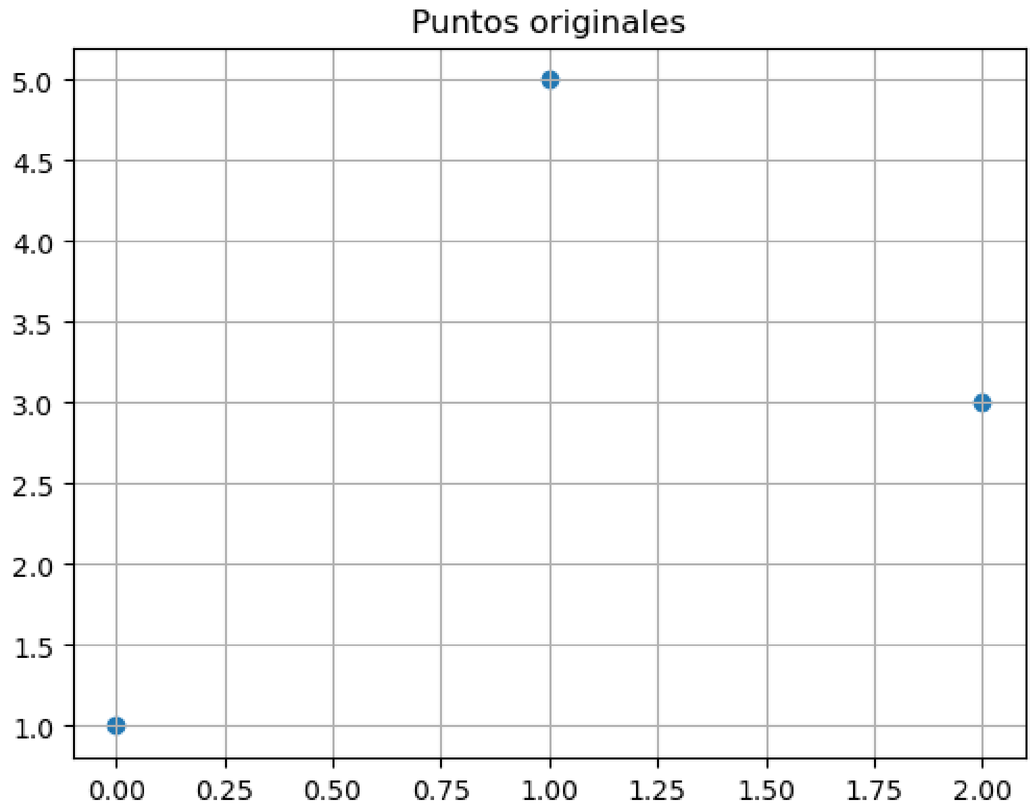
Splines cúbicos.

Conjunto de ejercicios

1. Dados los puntos (0,1),(1,5),(2,3), determine el spline cúbico

A continuación realizaremos la gráfica de los puntos para entender su comportamiento

```
import matplotlib.pyplot as plt
import numpy as np
x_puntos=[0,1,2]
y_puntos=[1,5,3]
plt.scatter(x_puntos,y_puntos)
plt.title('Puntos originales')
plt.grid(True)
plt.show()
```



A continuación se determinarán los dos spines cúbicos S_0 y S_1 necesarios para realizar la interpolación de la función

A partir de S_0 podemos obtener las siguientes ecuaciones:

- $S_0(x_0) = y_0$ $a_0 = 1$
- $S_0(x_1) = y_1$ $b_0 + c_0 + d_0 = 5$
- $s'_0(x_1) = s'_1(x_1)$ $b_0 + 2c_0 + 3d_0 = 1$
- $s''_0(x_0) = 0$ $c_0 = 0$

A partir de S_1 se obtiene: * $S_1(x_1) = y_1$ $a_1 = 5$

- $S_1(x_2) = y_2$ $b_1 + c_1 + d_1 = -2$
- $S''_1(x_2) = 0$ $c_1 = -3d_1$
- $S''_1(x_1) = S''_0(x_1)$ $c_1 = c_0 + 3d_0$

Resolviendo el sistema de ecuaciones:

$a_0 = 1$

$b_0 = \frac{11}{2}$

$c_0 = 0$

$d_0 = -3/2$

$a_1 = 5$

$b_1 = 1$

$c_1 = -9/2$

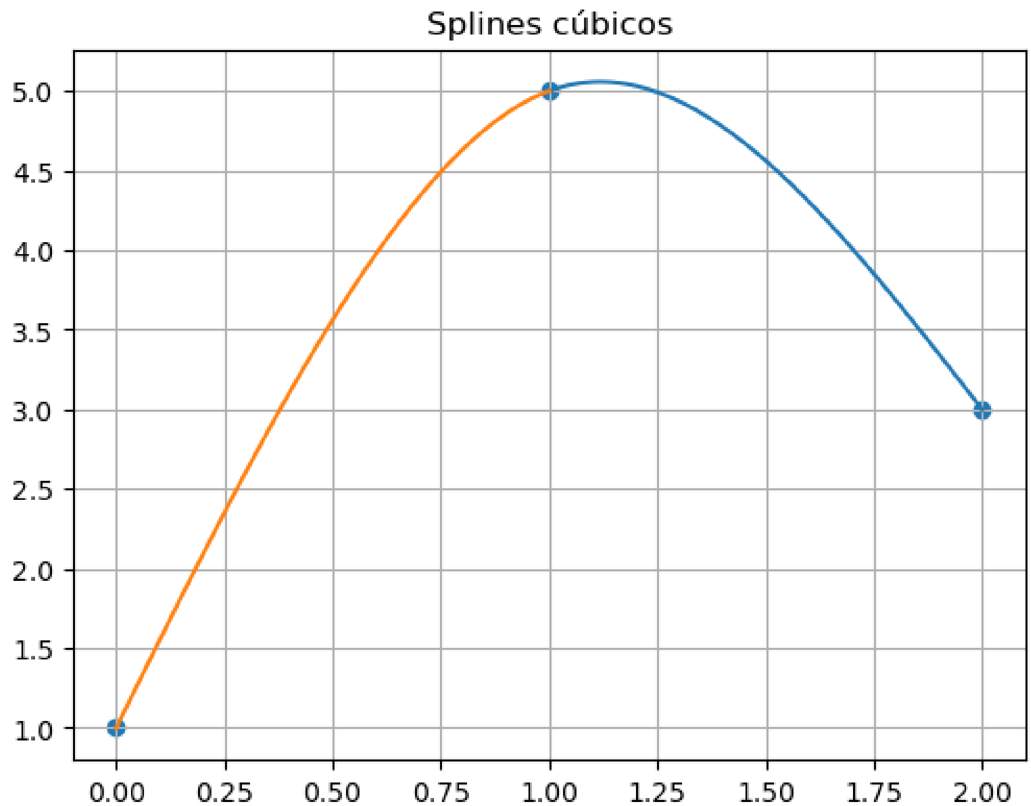
$d_1 = 3/2$

Por lo tanto las funciones para los splines son:

$S_0 = 1 + \frac{11}{2} * x - \frac{3}{2} * x^3$. $S_1 = 5 + x - 1 - \frac{9}{2} * (x-1)^2 + \frac{3}{2} * (x-1)^3$

A continuación graficaremos la función resultante junto con los puntos por los que debe pasar.

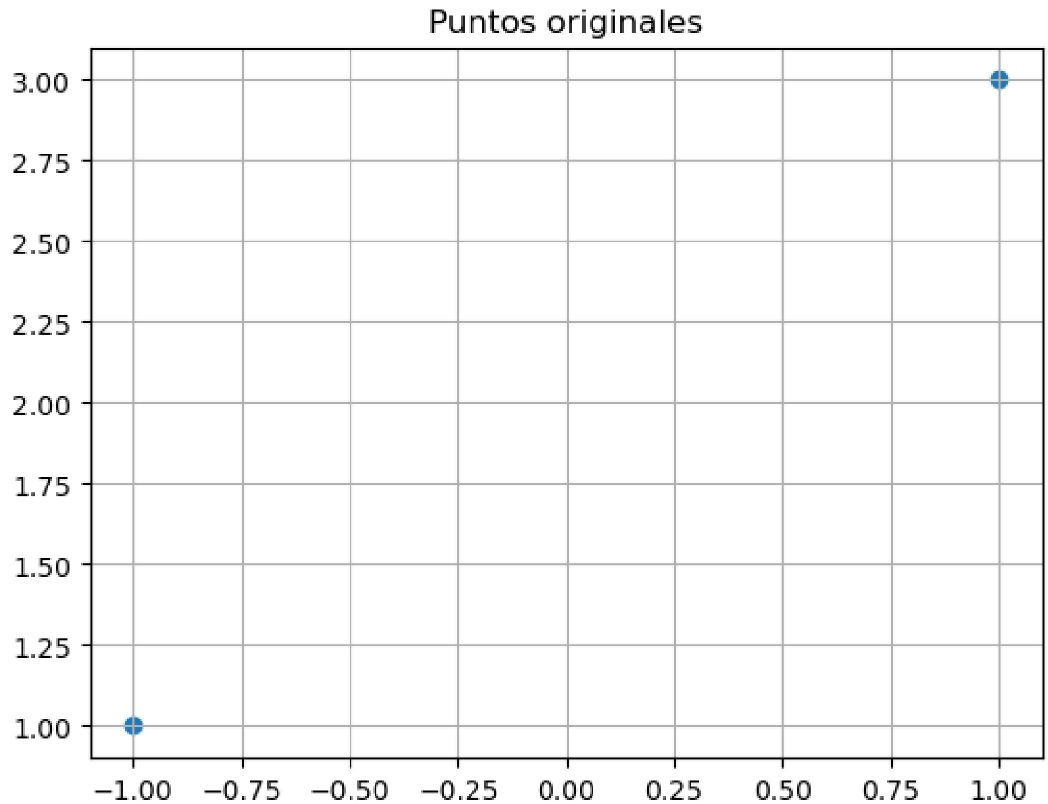
```python
def so(x):
    return 1+ 11/2 * x - 3/2 * (x**3)
def s1(x):
    return 5 + (x-1) - 9/2 * (x-1)**2 + 3/2 * (x-1)**3
plt.scatter(x_puntos,y_puntos)
plt.title('Splines cúbicos')
plt.grid(True)
x_s0=np.linspace(0,1,100)
y_s0=so(x_s0)
x_s1=np.linspace(1,2,100)
y_s1=s1(x_s1)
plt.plot(x_s1,y_s1,label='spline 1')
plt.plot(x_s0,y_s0,label='spline 0')
plt.show()
```



2. Dados los puntos(-1,1),(1,3),determine el spline cúbico sabiendo que $f'(x_0) = 1$,$f'(x_n) = 2$

A continuación graficaremos los puntos dados

```
x_p=[-1,1]
y_p=[1,3]
plt.scatter(x_p,y_p)
plt.title('Puntos originales')
plt.grid(True)
plt.show()
```



Para obtener la función obtenemos las siguientes ecuaciones: * $S(x_0) = y_0$ $a_0 = 1$

- $S(x_1) = y_1$ $b_0 + 2c_0 + 4d_0 = 1$
- $S'(x_0) = 1$ $b_0 = 1$
- $S'(x_1) = 2$ $4c_0 + 12d_0 = 1$

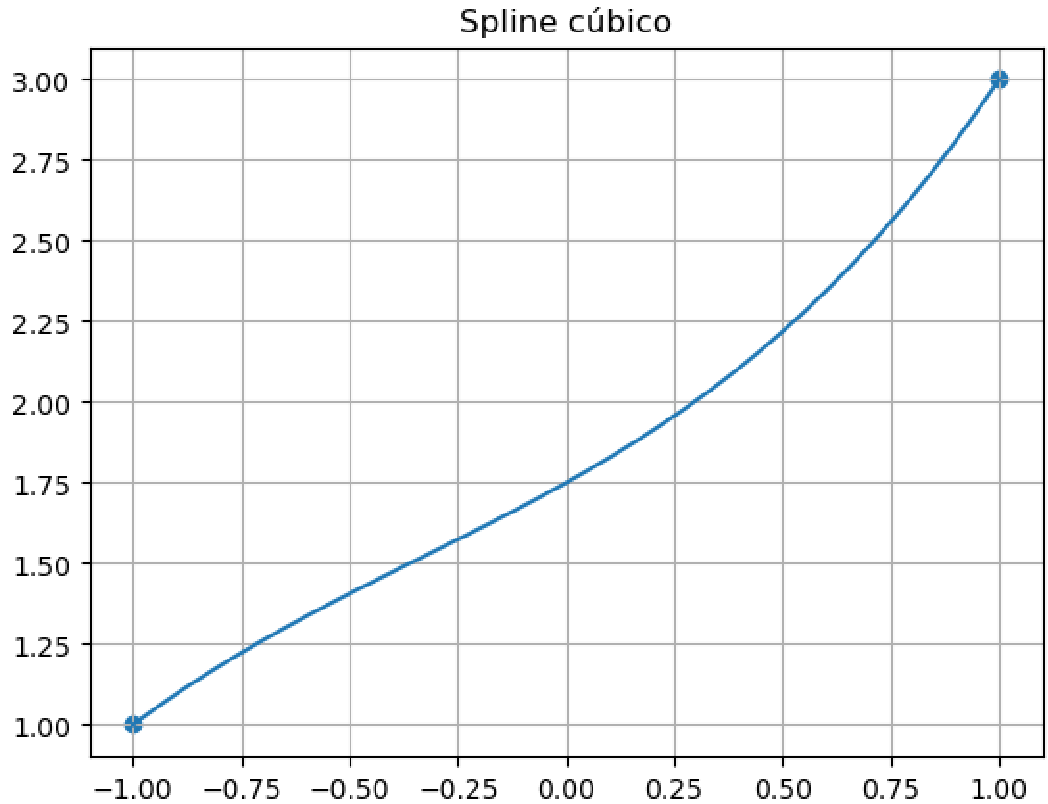
Obteniendo la siguiente función:

$1 + 1 * (x + 1) - \frac{1}{2} * (x + 1)^2 + \frac{1}{4} * (x + 1)^3$

la solución graficada es la siguiente:

```
def s(x):
    return 1+ 1 * (x+1) -1/2 * (x+1)**2 +1/4 * (x+1)**3
plt.scatter(x_p,y_p)
plt.title('Spline cúbico')
plt.grid(True)
x_s=np.linspace(-1,1,100)
y_s=s(x_s)
```

```
plt.plot(x_s,y_s,label='spline ')
plt.show()
```



3. Diríjase al pseudocódigo del spline cúbico con frontera natural provisto en clase, en base a ese pseudocódigo complete la función.

A continuación se muestra la función completa

```
import sympy as sym
from IPython.display import display



# ####################################################################
def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    """
    Cubic spline interpolation ``S``. Every two points are interpolated by a cubic polynomial
    ``S_j`` of the form ``S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3.``
```

```
xs must be different but not necessarily ordered nor equally spaced.

## Parameters
- xs, ys: points to be interpolated

## Return
- List of symbolic expressions for the cubic spline interpolation.
"""

points = sorted(zip(xs, ys), key=lambda x: x[0])  # sort points by x

xs = [x for x, _ in points]
ys = [y for _, y in points]

n = len(points) - 1  # number of splines

h = [xs[i + 1] - xs[i] for i in range(n)]  # distances between contiguous xs

alpha = [0] * (n + 1)
for i in range(1, n):
    alpha[i] = (
        3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])
    )

l = [1]
u = [0]
z = [0]

for i in range(1, n):
    l.append(2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1])
    u.append(h[i] / l[i])
    z.append((alpha[i] - h[i - 1] * z[i - 1]) / l[i])

l.append(1)
z.append(0)
c = [0] * (n + 1)

b = [0] * n
d = [0] * n
a = [ys[i] for i in range(n)]

x = sym.Symbol("x")
```

```
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b[j] = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d[j] = (c[j + 1] - c[j]) / (3 * h[j])
        a_j = a[j]
        b_j = b[j]
        c_j = c[j]
        d_j = d[j]

        S_j = a_j + b_j * (x - xs[j]) + c_j * (x - xs[j])**2 + d_j * (x - xs[j])**3
        splines.append(S_j)

    splines.reverse()
    return splines
```

4. Usando la función anterior, encuentre el spline cúbico para: xs=[1,2,3]

ys=[2,3,5]

```
xs = [1, 2, 3]
ys = [2, 3, 5]
splines = cubic_spline(xs, ys)
for i, spline in enumerate(splines):
    print(f"Spline {i}: ")
    display(spline)
```

```
Spline 0:
```

$$0.75x + 0.25\left(x - 1\right)^3 + 1.25$$

```
Spline 1:
```

$$1.5x - 0.25\left(x - 2\right)^3 + 0.75\left(x - 2\right)^2$$