Bewijzen - Inleveropgave 1

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21 september 2022 - Week 38

(a). $I = \{[\,3 - \frac{1}{n}, 6\,]\}_{n\in\mathbb{N}}$

(b). $n \neq m \implies \frac{1}{n} \neq \frac{1}{m} \implies [3 - \frac{1}{n}, 6] \neq [3 - \frac{1}{m}] \implies A_n \neq A_m$.

(c). Stel $n, m \in \mathbb{N}$ met $n < m$ en $n \neq m$.

Dan $n < m \implies \frac{1}{n} > \frac{1}{m} \implies 3 - \frac{1}{n} < 3 - \frac{1}{m}$.

Dus $A_m \subseteq A_n$.

Omdat $1 \leq x$ voor alle $x \in \mathbb{N}$, $A_m \subseteq A_1$ voor elke $m \in \mathbb{N}$.

Dus $\bigcup_{n\in\mathbb{N}} A_n \subseteq A_1 \iff \bigcup_{n\in\mathbb{N}} A_n \subseteq [2, 6]$.

$n = 1 \implies A_n = [2, 6]$, dus $[2, 6] \subseteq \bigcup_{n\in\mathbb{N}} A_n$.

Dus $\bigcup_{n\in\mathbb{N}} A_n = [2, 6]$.

(d). Er bestaat geen getal $n \in \mathbb{N}$ waarvoor $3 - \frac{1}{n} > 3$, dus $[3, 6] \subseteq A_n$ voor elke $n \in \mathbb{N}$. Dus $[3, 6] \subseteq \bigcap_{n\in\mathbb{N}} A_n$.

(e). Omdat $|\mathbb{N}| = \infty$, geldt:

$$\bigcap_{n\in\mathbb{N}} A_n = \lim_{a\to\infty} \bigcap_{k=1}^{a} A_k \tag{1}$$

Neem nu $n, m \in \mathbb{N}$ met $n < m$ en $n \neq m$.

$n < m \implies 3 - \frac{1}{n} < 3 - \frac{1}{m} \implies [3 - \frac{1}{m}, 6] \subseteq [3 - \frac{1}{n}, 6]$ en $[3 - \frac{1}{n}, 6] \nsubseteq [3 - \frac{1}{m}, 6]$

$\implies [3 - \frac{1}{m}, 6] \bigcap [3 - \frac{1}{n}, 6] = [3 - \frac{1}{m}, 6]$.

Dus $n < m \implies A_m \bigcap A_n = A_m \implies \bigcap_{k=1}^{a} A_k = A_a$.

Samen met (1) geeft dit:

$$\bigcap_{n\in\mathbb{N}} A_n = \lim_{a\to\infty} A_a = \lim_{a\to\infty} [3 - \frac{1}{a}, 6] = [3, 6] \tag{2}$$