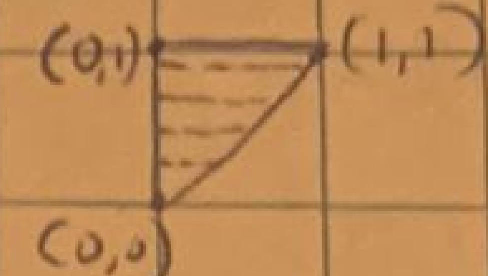
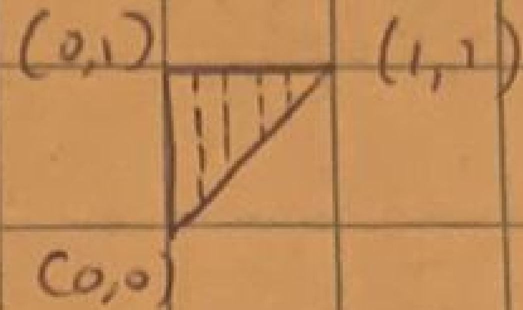
1. $\int_0^1 \left(\int_0^y \frac{y^3}{x^2+y^2} \, dx \right) dy$



$\int_0^1 \left(\int_x^1 \frac{y^3}{x^2+y^2} \, dy \right) dx$



2. we kiezen de 1 ste.

$$\int_0^1 \left(\int_0^y \frac{y^3}{x^2+y^2} \, dx \right) dy = \int_0^1 \left(y \int_0^y \frac{1}{\frac{x^2}{y^2}+1} \, dx \right) dy$$

we bekijken eerst de binnenste integraal.

$$\int_0^y \frac{1}{\left(\frac{x}{y}\right)^2+1} \, dx \qquad u = \frac{x}{y}, \quad \frac{du}{dx} = \frac{1}{y}, \quad dx = y\,du.$$

$$\Rightarrow y \int_0^1 \frac{1}{(u)^2+1} \, du$$

$$= y \arctan(1) - y \arctan(0) = y\frac{\pi}{4}$$

x	$u = \frac{x}{y}$
y	1
0	0

dus $= \int_0^1 y^2 \frac{\pi}{4} \, dy = \frac{1}{3}\frac{\pi}{4} = \frac{\pi}{12}$

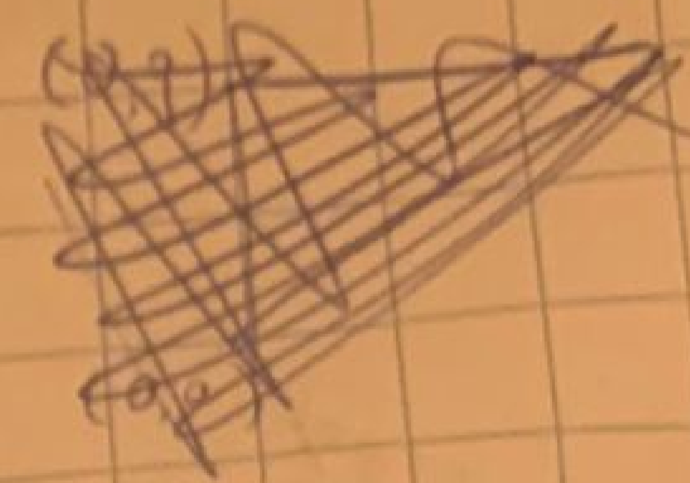
3. $$\int_0^q \int_0^{\frac{p}{q}y} \frac{y^3}{y^2+x^2}\,dx\,dy = I(p,q)$$

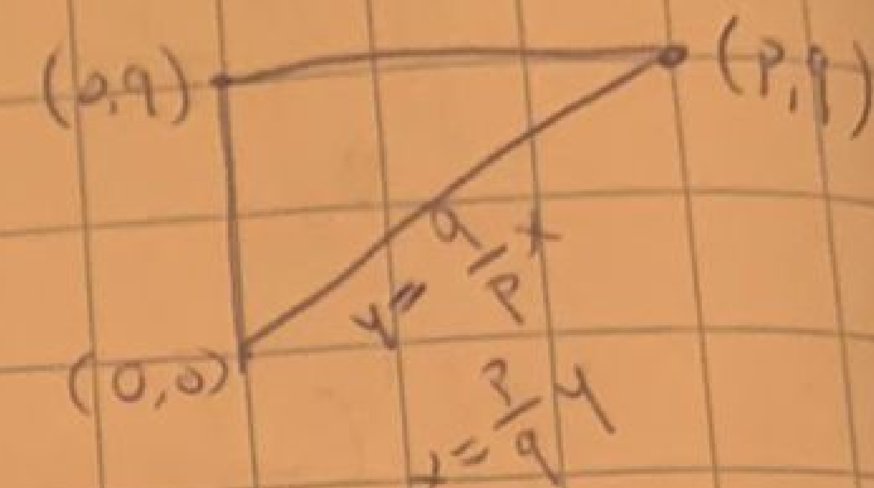
$$= \int_0^q y^2 \left(\arctan\left(\frac{x}{y}\right)\Big|_{x=0}^{x=\frac{p}{q}y}\right) dy$$

$$= \int_0^q y^2 \left(\arctan\left(\frac{p}{q}\right)\right) dy$$

$$= \arctan\left(\frac{p}{q}\right) \cdot \left(\frac{1}{3} y^3 \Big|_{y=0}^{y=q}\right)$$

$$= \frac{1}{3} q^3 \arctan\left(\frac{p}{q}\right) = I(p,q).$$





4. $$I(p,q) = I(1,1)$$

$$\Rightarrow \frac{1}{3} q^3 \arctan\left(\frac{p}{q}\right) = \frac{\pi}{12}$$

$$\arctan\left(\frac{p}{q}\right) = \frac{1}{4}\pi q^{-3}$$

$$\frac{p}{q} = \tan\left(\frac{1}{4}\pi q^{-3}\right)$$

$$p = q\tan\left(\frac{\pi}{4q^3}\right).$$

5. stel $q = t$, en $p = t\tan\left(\frac{\pi}{4t^3}\right)$.

Check of $(1,1)$ hierop ligt.

$q = 1 \Rightarrow t = 1$.

$t = 1 \Rightarrow p = 1 \cdot \tan\left(\frac{\pi}{4}\right) = 1$.

dus $(1,1)$ ligt op de kromme c geparametriseerd door $\left(t, t \cdot \tan\left(\frac{\pi}{4t^3}\right)\right)$.