

Bewijzen - Inleveropgave 1

B. H. J. van Boxtel

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(a) $I = \{[3 - \frac{1}{n}, 6]\}_{n \in \mathbb{N}}$

(b) $n \neq m \implies \frac{1}{n} \neq \frac{1}{m} \implies [3 - \frac{1}{n}, 6] \neq [3 - \frac{1}{m}] \implies A_n \neq A_m.$

(c) Stel $n, m \in \mathbb{N}$ met $n < m$ en $n \neq m$.

$$\text{Dan } n < m \implies \frac{1}{n} > \frac{1}{m} \implies 3 - \frac{1}{n} < 3 - \frac{1}{m}.$$

$$\text{Dus } A_m \subseteq A_n.$$

Omdat $1 \leq x$ voor alle $x \in \mathbb{N}$, $A_m \subseteq A_1$ voor elke $m \in \mathbb{N}$.

$$\text{Dus } \bigcup_{n \in \mathbb{N}} A_n \subseteq A_1 \iff \bigcup_{n \in \mathbb{N}} A_n \subseteq [2, 6].$$

$$n = 1 \implies A_n = [2, 6], \text{ dus } [2, 6] \subseteq \bigcup_{n \in \mathbb{N}} A_n.$$

$$\text{Dus } \bigcup_{n \in \mathbb{N}} A_n = [2, 6].$$

(d) Er bestaat geen getal $n \in \mathbb{N}$ waarvoor $3 - \frac{1}{n} > 3$, dus $[3, 6] \subseteq A_n$ voor elke $n \in \mathbb{N}$. Dus $[3, 6] \subseteq \bigcap_{n \in \mathbb{N}} A_n$.

(e) Omdat $|\mathbb{N}| = \infty$, geldt:

$$\bigcap_{n \in \mathbb{N}} A_n = \lim_{a \rightarrow \infty} \bigcap_{k=1}^a A_k \tag{1}$$

Neem nu $n, m \in \mathbb{N}$ met $n < m$ en $n \neq m$.

$$\begin{aligned} n < m &\implies 3 - \frac{1}{n} < 3 - \frac{1}{m} \implies [3 - \frac{1}{m}, 6] \subseteq [3 - \frac{1}{n}, 6] \text{ en } [3 - \frac{1}{n}, 6] \not\subseteq [3 - \frac{1}{m}, 6] \\ &\implies [3 - \frac{1}{m}, 6] \cap [3 - \frac{1}{n}, 6] = [3 - \frac{1}{m}, 6]. \end{aligned}$$

$$\text{Dus } n < m \implies A_m \cap A_n = A_m \implies \bigcap_{k=1}^a A_k = A_a.$$

Samen met (1) geeft dit:

$$\bigcap_{n \in \mathbb{N}} A_n = \lim_{a \rightarrow \infty} A_a = \lim_{a \rightarrow \infty} [3 - \frac{1}{a}, 6] = [3, 6] \tag{2}$$