velocity[10] and the value we obtain from velocity measurements at the very low temperature is 210°K. This is lower than the value 260°K quoted by Mott and Jones,[11] but is slightly higher than the specific heat value 185°K computed by Keesom and Van Laer.[12]

We are indebted to Dr. W. D. Mason and Dr. H. J. McSkimin of The Bell Laboratories for calling our attention to the use of Dow Corning 200 Fluid in low-temperature work.

[10] C. F. Squire, *Low Temperature Physics* (McGraw-Hill Book Company, Inc., New York, 1953).
[11] N. F. Mott and H. Jones *Theory of the Properties of Metals and Alloys* (Oxford University Press, London, 1936).
[12] W. Keesom and P. Van Laer, Physica **5**, 193 (1938).

PHYSICAL REVIEW VOLUME 95, NUMBER 5 SEPTEMBER 1, 1954

# Bohr's Theory of Energy Losses of Moving Charged Particles

JACOB NEUFELD
*Oak Ridge National Laboratory, Oak Ridge, Tennessee*
(Received February 4, 1954; revised manuscript received April 5, 1954)

The stopping power of a particle having charge $Z_{\text{eff}}\epsilon$ and moving with velocity $v$ has been determined by Bohr for $\kappa=2Z_{\text{eff}}\epsilon^2/\hbar v$ and $\eta_s=2v/u_s>2$, where $u_s$ is the velocity of the orbital electron that is effective in stopping the particle. Bohr's results are formulated differently for $\kappa/\eta_s<1$ and $\kappa/\eta_s>1$ and are based on an orbital picture of the processes involved. An alternative method has been developed in which the space surrounding the particle track has been subdivided into three regions: (a) the region of validity of the Rutherford formula (adjacent to the track), (b) the intermediate region involving large perturbations for which no adequate theory exists at the present time, and (c) the region of validity of the quantum perturbation theory (the most remote from the track). By extrapolating the formulas valid in regions (a) and (c) into the region (b), the contribution to the stopping power due to the region (b) has been estimated, and adding to it the contribution due to the regions (a) and (c) an expression has been derived from the total stopping power that is applicable whenever $\kappa>1$ and $\eta_s>1$. This expression has a wider region of applicability than the corresponding Bohr's formula since the latter can be used only for $\kappa\ll\eta_s^3$ if $\kappa/\eta_s>1$. The physical picture used for deriving this expression is somewhat similar to the one used by Bohr for $\kappa/\eta_s<1$ but is different from the one used by Bohr for $\kappa/\eta_s>1$.

## I. INTRODUCTION

THE present investigation deals with Bohr's theory of energy losses due to excitation and ionization processes by moving charged particles. This theory is particularly applicable to heavy ions when

$$\kappa=2Z_{\text{eff}}\epsilon^2/\hbar v\gg 1, \tag{1}$$

where $Z_{\text{eff}}\epsilon$ is the effective charge of the ion and $v$ its velocity. A further requirement is that

$$\eta_s=2v/u_s>2, \tag{2}$$

where $u_s$ is the orbital velocity of the $s$th electron in one of the atoms of the stopping substance.

The stopping power of a moving charged particle can often be determined by separately evaluating the contribution $\sigma_s$ of the $s$th orbital electron and then summing to get the total effect as $\sum_s \sigma_s$. In these calculations the processes resulting from the loss and capture of orbital electrons of the moving particle are not considered and a term depending on these processes should be added to $\sum_s \sigma_s$ in order to determine the total stopping power.

The contribution $\sigma_s$ to the stopping power is often expressed as a function of three parameters, two of which characterize the moving particle; e.g., the effective charge $Z_{\text{eff}}\epsilon$ and velocity $v$, and one characterizing the orbital electron that is effective in stopping the particle. The latter parameter can be either the "cyclic frequency" $\omega_s$, "orbital velocity" $u_s$, "orbital radius" $a_s$, or the energy $I_s$ necessary to remove the $s$th electron from its orbit. These quantities are defined by Bohr as follows:[1]

$$I_s=\hbar\omega_s=\tfrac{1}{2}mu_s^2, \tag{3}$$

$$a_s=\hbar/mu_s. \tag{4}$$

In our calculations we shall replace the above three parameters by three other parameters that are nondimensional, i.e.,

$$\sigma_s=f(\kappa,\eta_s,Z_{\text{eff}}). \tag{5}$$

Bohr has expressed the stopping power by means of two formulas having different regions of applicability. The first of Bohr's formulas, derived in 1912, is as follows:[2]

$$\sigma_s=2B_\epsilon \ln\frac{kmv^3}{Z_{\text{eff}}\epsilon^2\omega_s}=2B_\epsilon \ln\frac{k\eta_s^2}{\kappa}, \tag{6}$$

where

$$B_\epsilon=2\pi Z_{\text{eff}}^2\epsilon^4/mv^2=\pi I_s\kappa^2a_s^2 \tag{7}$$

and

$$k=1.25. \tag{8}$$

[1] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, No. 8 (1948), formulas (3.1.7), (3.3.1), and (3.3.2).
[2] N. Bohr, Phil. Mag. **25**, 10 (1913).

This formula is applicable in the region where

$$\kappa/\eta_s<1. \tag{9}$$

The second of Bohr's formulas, derived in 1948, is as follows:[3]

$$\sigma_s=B_\epsilon \ln\frac{2^{\frac{1}{2}}m^{\frac{3}{2}}v^4}{\hbar^{\frac{1}{2}}\omega_s^{\frac{1}{2}}Z_{\text{eff}}\epsilon^2}=B_\epsilon \ln\frac{\eta_s^3}{\kappa}, \tag{10}$$

and is applicable in the region where

$$\kappa/\eta_s>1. \tag{11}$$

We shall now consider the range of values of $\kappa/\eta_s$ and the regions of applicability of formulas (6) and (10) if the velocity of a moving ion having an atomic number $Z$ satisfies the following relationship:

$$\epsilon^2/\hbar<v<Z^{\frac{2}{3}}\epsilon^2/\hbar. \tag{12}$$

Then the effective charge number of the ion is roughly equal to:[4]

$$Z_{\text{eff}}\sim Z^{\frac{1}{3}}\hbar v/\epsilon^2. \tag{13}$$

Substituting (13) in (1), we find that in the velocity region considered, $\kappa$ is substantially constant, i.e.,

$$\kappa\sim 2Z^{\frac{1}{3}}. \tag{14}$$

The minimum value of $\eta_s$ is given by (2). The maximum value of $\eta_s$ is determined by taking from (12) $v_{\max}=Z^2\epsilon^2/\hbar$ and assuming that the velocity of the most loosely bound electron in the stopping atom is roughly equal to $u_1=\epsilon^2/\hbar$. Consequently,

$$2<\eta_s<2Z^{\frac{2}{3}}, \tag{15}$$

and we obtain from (14) and (15)

$$Z^{-\frac{1}{3}}<\kappa/\eta_s<Z^{\frac{1}{3}}. \tag{16}$$

For a "representative" heavy fission fragment such as iodine ($Z=53$) we obtain $0.26<\kappa/\eta_s<3.75$. If we consider recoil heavy atoms released for instance by bombardment of solids by neutrons the range of $\kappa/\eta_s$ may be somewhat more extended, i.e., for lead ($Z=82$) we find $0.23<\kappa/\eta_s<4.34$.

Substituting in (9) and (11) the values $\kappa$ and $\eta_s$ as given by (14) and (2), respectively, we find that the electronic orbital velocities for which the Bohr formula (6) is applicable are approximately given by

$$u_s<v/Z^{\frac{1}{3}}. \tag{17}$$

Similarly the Bohr formula (10) is applicable for

$$v>u_s>v/Z^{\frac{1}{3}}. \tag{18}$$

As an example of the applicability of the two Bohr formulas, consider a fission fragment moving in hydrogen. In the beginning of the range of the fission fragment, $Z_{\text{eff}}\sim 20$ and $v\sim 1.4\times 10^9$ cm/sec. Consequently, $\kappa\sim 6$ and $\eta_s\sim 10$ and the stopping power should be determined by means of the Bohr formula (6). However, close to the end of the range $Z_{\text{eff}}\sim 2$ and $v$ is somewhat above $2.18\times 10^8$ cm/sec. This gives $\kappa\sim 4$ and $\eta_s\sim 2$ and, consequently, the Bohr formula (10) is applicable.

Our purpose is to investigate the physical assumptions that led to the formulation of the first and the second Bohr formulas, and to discuss modification of the assumptions on which the second Bohr formula is based.

The evaluation of the stopping power involves considerations that are based partly on an orbital picture and partly on quantum mechanical arguments. We shall use an impact parameter and divide the space surrounding the track of the particle into three regions designated as (a), (b), and (c), and defined by concentric cylinders aligned along the particle track: (a) is the "region of the orbital representation" and is adjacent to the particle track; (b) is an "intermediate region" involving large perturbations, and (c) is the "region of small perturbations," the region most remote from the track. The stopping power can thus be represented as a sum of three terms:

$$\sigma_s=\sigma_s^{(a)}+\sigma_s^{(b)}+\sigma_s^{(c)}, \tag{19}$$

representing individually the contributions of each of the above three regions.

## II. REGION OF THE ORBITAL REPRESENTATION

In the region of the orbital representation we neglect the binding of electrons in the stopping atom and assume that the energy transfer from the particle to an electron expressed in terms of the impact parameter is[5]

$$T^{(a)}(p)=\frac{B_\epsilon}{2\pi}\frac{8}{(4p^2+b^2)}, \tag{20}$$

where $B_\epsilon$ is defined by (7) and the quantity $b$ is the collision diameter, i.e.,

$$b=2Z_{\text{eff}}\epsilon^2/mv^2=2\kappa a_s/\eta_s. \tag{21}$$

We assume that this region extends from $p=0$ to $p=li_s$, where $l\ll 1$ and $i_s$ is the value of the impact parameter corresponding to the energy transfer $I_s$, i.e., $T^{(a)}(i_s)=I_s$. We have, approximately,[6]

$$i_s\sim\kappa a_s. \tag{22}$$

The contribution to the stopping power due to collisions in the region (a) is

$$\sigma_s^{(a)}=2\pi\int_0^{li_s} pT^{(a)}(p)dp=B_\epsilon \ln(l^2\eta_s^2+1). \tag{23}$$

[3] See reference 1, formula (3.3.11).
[4] See reference 1, formula (4.4.1).
[5] See reference 1, formula (1.1.10).
[6] See reference 1, formula (3.3.3).

### III. REGION OF SMALL PERTURBATIONS

In this region the field exerted by the moving particle upon the stopping atom is sufficiently small so that the method of variation of constants can be applied. We consider the particle as moving along the $z$ axis and assume that an isotropic oscillator having a cyclic frequency $\omega_s$ is located on the $x$ axis at a distance $p$ from the origin. The value $p$ is the impact parameter and $t=0$ corresponds to the instant at which the particle passes through the origin. Since $v\ll c$, we consider the perturbation due to the scalar potential only, i.e.,

$$V_x=\epsilon(\partial\phi/\partial x)_{x=p}r_x \quad \text{and} \quad V_z=\epsilon(\partial\phi/\partial z)_{z=0}r_z. \tag{24}$$

In the above expression $\phi$ designates the perturbing potential, i.e.,

$$\phi=Z_{\text{eff}}\epsilon/(p^2+v^2t^2)^{\frac{1}{2}}; \tag{25}$$

$r_x$ and $r_z$ are the projections of the vector $\mathbf{r}$ on the $x$ and $z$ axes, respectively. The vector $\mathbf{r}$ represents the position of the electron with respect to the center of the oscillator.

Let $a_x^{(s)}(t)$ and $a_z^{(s)}(t)$ represent the transition probabilities for excitation of the oscillator along the $x$ and $z$ axes due to the moving particle at the instant $t$. We have[7]

$$a_x^{(s)}(t)=(i\hbar)^{-1}\int_{-\infty}^{t}\langle 1|V_x|0\rangle\exp(i\omega_s t)dt, \tag{26}$$

and a similar expression for $a_z^{(s)}(t)$.

We are concerned here with the transition probability after the particle has gone to infinity, i.e., at $t=\infty$. Substituting (24) in (26) and taking into account (1), (2), (3), and (4), we obtain

$$\begin{aligned}|a_x^{(s)}(\infty)|&=\frac{\kappa}{\eta_s}K_1\left(\frac{p}{\eta_s a_s}\right),\\ |a_y^{(s)}(\infty)|&=\frac{\kappa}{\eta_s}K_0\left(\frac{p}{\eta_s a_s}\right),\end{aligned} \tag{27}$$

where $K_0$ and $K_1$ are modified Bessel functions of the second kind, having the order 0 and 1, respectively.[8]

The method of variation of constants can be applied for those impact parameters for which the probability of transition is small, i.e.,

$$|a_x^{(s)}(\infty)|^2\ll 1 \quad \text{and} \quad |a_z^{(s)}(\infty)|^2\ll 1. \tag{28}$$

Taking into account that

$$K_0\left(\frac{p}{\eta_s a_s}\right)<\frac{\eta_s a_s}{p} \quad \text{and} \quad K_1\left(\frac{p}{\eta_s a_s}\right)<\frac{\eta_s a_s}{p}, \tag{29}$$

and substituting (27) in (28), we find that for

$$p\gg a_s\kappa=i_s \tag{30}$$

the method of variation of constants can be applied.

We shall, therefore, assume that the region of small perturbations extends from $p=l'i_s$ to $p=\infty$, where $l'\gg 1$.

The energy transfer at a distance $p$ from the particle track is[7]

$$T^{(c)}(p)=\frac{B_\epsilon}{2\pi}\frac{2}{\eta_s^2 a_s^2}\left[K_0^2\left(\frac{p}{\eta_s a_s}\right)+K_1^2\left(\frac{p}{\eta_s a_s}\right)\right], \tag{31}$$

where $K^2(a)\equiv[K(a)]^2$ and the contribution to the stopping power due to the region (c) can be represented as[7]

$$\begin{aligned}\sigma_s^{(c)}&=2\pi\int_{l'p}^{\infty}pT^{(c)}(p)dp\\ &=2B_\epsilon l'\frac{\kappa}{\eta_s}K_0\left(l'\frac{\kappa}{\eta_s}\right)K_1\left(l'\frac{\kappa}{\eta_s}\right).\end{aligned} \tag{32}$$

### IV. INTERMEDIATE REGION

We have determined by means of expressions (23) and (32) two of the terms entering into the stopping power $\sigma_s$. The term $\sigma_s^{(b)}$ representing the contribution of the intermediate region cannot be determined with a precision comparable to those of (23) and (32). We are dealing here with quantum mechanical systems subjected to large perturbations and no adequate theory exists for dealing with such cases.

In view of the above difficulty there is no exact theory for determining the stopping power when $\kappa>1$. Bohr has derived two approximate expressions for the stopping power which apply to cases when $\kappa/\eta_s<1$ and $\kappa/\eta_s>1$, respectively.

### V. BOHR'S EXPRESSIONS FOR $\sigma_s$ WHEN $\kappa/\eta_s<1$

We briefly sketch the derivation of formula (6).

Bohr treated the perturbation caused by the field of the moving particle upon a classical oscillator and obtained for the energy loss an expression somewhat different in form but equivalent to (31). He assumed that (31) is valid not only in the region of small perturbations but also in the portion of the intermediate region extending from $p=l'i_s$ to $p=i_s$. He obtained thus as a contribution to the stopping power due to (31):

$$\sigma_s^{(A)}=\frac{2\omega_s i_s}{v}K_0\left(\frac{\omega_s i_s}{v}\right)K_1\left(\frac{\omega_s i_s}{v}\right). \tag{33}$$

Furthermore, Bohr assumed that (20) is valid not only in the region of orbital representation but also in the portion of the intermediate region extending from $p=li_s$ to $p=i_s$. He obtained thus as a contribution to the stopping power due to (20):

$$\sigma_s^{(B)}=2B_\epsilon\ln(2i_s/b). \tag{34}$$

[7] J. Neufeld, Proc. Phys. Soc. (London) **A66**, 590 (1953).

[8] For tables of $K_0$ and $K_1$ functions, see for instance: British Association for the Advancement of Science, *Mathematical Tables* (Cambridge University Press, Cambridge, 1937), Vol. 6, pp. 264–271.

By adding the expressions (33) and (34), we obtain (6) provided that

$$k=\frac{\kappa}{\eta_s}\exp\left[\frac{\kappa}{\eta_s}K_0\left(\frac{\kappa}{\eta_s}\right)K_1\left(\frac{\kappa}{\eta_s}\right)\right]. \tag{35}$$

As shown in Fig. 1, $k$ is a slowly varying function of $\kappa/\eta_s$ confined within a relatively narrow range from 1.055 to 1.288 when $\kappa/\eta_s$ varies from 0.01 to 1. Consequently, the constant value attributed to $k$ in the expression (8) is within the above range.

Bohr's method is based essentially on two assumptions; first, an extrapolation of the expressions (20) and (31) from the respective regions in which they are valid into the intermediate region, and second, that the point $p=i_s$ is the boundary separating the regions of applicability of (20) and (31).

The feasibility of the above assumptions can be verified by applying the formulas (20) and (31) to the point $p=i_s$ at which they are presumably valid and comparing the values obtained. These two values of the energy transfer are $T^{(a)}(i_s)$ and $T^{(c)}(i_s)$ and should be approximately equal one to another.

We have from (20) and (31)

$$\frac{T^{(c)}(i_s)}{T^{(a)}(i_s)}=\left(\frac{\kappa}{\eta_s}\right)^2\left[K_0^2\left(\frac{\kappa}{\eta_s}\right)+K_1^2\left(\frac{\kappa}{\eta_s}\right)\right]. \tag{36}$$

As shown in Fig. 2, the expression (36) is confined within the range from 1.033 to 0.539 when $\kappa/\eta_s$ varies from 0.02 to 1. This range is within acceptable limits and, therefore, $T^{(c)}(i_s)$ and $T^{(a)}(i_s)$ are of the same order of magnitude.

The expression (6) has also been derived by Bohr using an alternative method based on the use of an adiabatic limit. This limit corresponds to an impact parameter

$$d_s=v/\omega_s=a_s\eta_s. \tag{37}$$

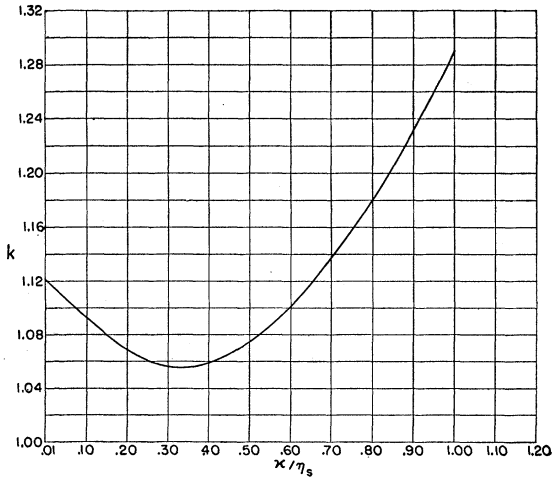


FIG. 1. Plot of $k=\left(\frac{\kappa}{\eta_s}\right)\exp\left[\left(\frac{\kappa}{\eta_s}\right)K_0\left(\frac{\kappa}{\eta_s}\right)K_1\left(\frac{\kappa}{\eta_s}\right)\right]$.

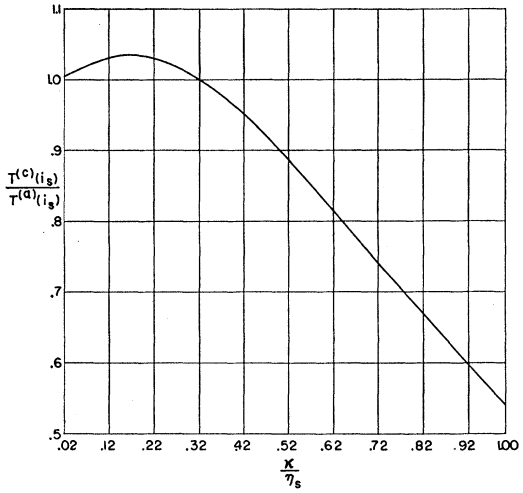


FIG. 2. Plot of $\frac{T^{(c)}(i_s)}{T^{(a)}(i_s)}=\left(\frac{\kappa}{\eta_s}\right)^2\left[K_0^2\left(\frac{\kappa}{\eta_s}\right)+K_1^2\left(\frac{\kappa}{\eta_s}\right)\right]$.

The space surrounding the particle track and comprised within the adiabatic limit (i.e., corresponding to $p\leqslant d_s$) was divided into two regions: first, the region of "free collisions" extending from $p=0$ to $p\sim i_s$, in which the energy transfer was represented by (20) and, second, the region of resonance effects extending from $p\sim i_s$ to $p=d_s$ in which the formula (31) was used in a somewhat modified form. In the region of "free collisions," the binding energy of the electrons is neglected and the energy transferred exceeds $I_s$. In the region of "resonance effects" the moving field interacts with the oscillator and the energy transferred is smaller than $I_s$.

This alternative method is based on an obvious assumption that

$$li_s<i_s<d_s. \tag{38}$$

By taking into account (22) and (37) we obtain from (38)

$$\frac{i_s}{d_s}=\frac{\kappa}{\eta_s}<1. \tag{39}$$

The inequality (39) expresses the condition of validity of (6).

### VI. BOHR'S EXPRESSION FOR $\sigma_s$ WHEN $\kappa/\eta_s>1$

Since $\kappa/\eta_s>1$ we obtain from (39) that $i_s>d_s$. We face here an unusual situation in which the adiabatic limit is within the region of free collisions and, therefore, the reasoning leading to (6) cannot be applied. In treating this case, Bohr did not consider resonance collisions since their aggregate effect upon stopping power is small, and assumed that the region of free collisions has been reduced in size. In the previous case (for $\kappa/\eta_s<1$) it was assumed that this region extended from $p=0$ to $p=i_s$. In this case, however, Bohr assumed that this region extended from $p=0$ to $p=d_s{}^*$. The new

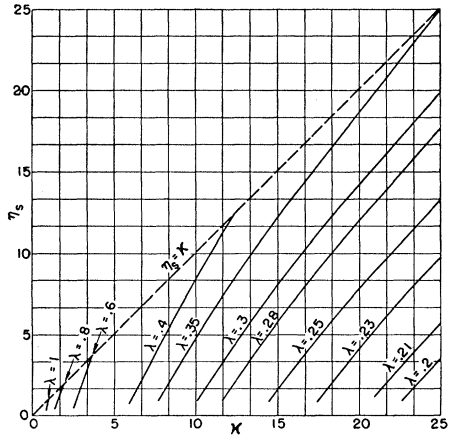


FIG. 3. Plot of $\lambda$ as a function of $\kappa$ and $\eta_s$ (for $\kappa>\eta_s$).

limit $d_s^*$ is smaller than $i_s$ and was expressed as[9]

$$d_s^*=i_s^{\frac{1}{2}}d_s^{\frac{1}{2}}. \tag{40}$$

Bohr obtained the expression (10) for the stopping power by taking into account energy transfer as expressed by (20) and the maximum impact parameter as expressed by (40).

## VII. PROPOSED FORMULA FOR $\kappa/\eta_s>1$

The proposed expression utilizes an extrapolation of (20) and (31) into the intermediate region: namely, we assume that (20) is valid for $0<p<\lambda i_s$ and (31) is valid for $\lambda i_s<p<\infty$. The value $\lambda$ is such that

$$T^{(a)}(\lambda i_s)=T^{(c)}(\lambda i_s). \tag{41}$$

Substituting $p=\lambda i_s$ in (20) and (31), the equality (41) can be expressed as

$$K_0^2\left(\lambda\frac{\kappa}{\eta_s}\right)+K_1^2\left(\lambda\frac{\kappa}{\eta_s}\right)=\frac{{\eta_s}^4}{\lambda^2\eta_s^2\kappa^2+{\eta_s}^2}. \tag{42}$$

The relation (42) is illustrated diagrammatically in Fig. 3 by means of which we can determine $\lambda$ corresponding to any pair of values $(\kappa,\eta_s)$ for which $\kappa/\eta_s>1$.

Our expression for the stopping power becomes

$$\sigma_s=2\pi\left[\int_0^{\lambda i_s}pT^{(a)}(p)dp+\int_{\lambda i_s}^{\infty}pT^{(c)}(p)dp\right]$$

$$=B_\epsilon\left[\ln(\lambda^2\eta_s^2+1)+2\lambda\frac{\kappa}{\eta_s}K_0\left(\lambda\frac{\kappa}{\eta_s}\right)K_1\left(\lambda\frac{\kappa}{\eta_s}\right)\right]. \tag{43}$$

In view of the absence of an exact theory of energy losses in the intermediate region, it may be feasible to use (43) as an approximate value for the stopping power when $\kappa/\eta_s>1$. This expression is based on an approach similar to that which lead to (6).

[9] See reference 1, formula (3.3.10).

## VIII. DISCUSSION OF THE RESULTS

The general considerations leading to (43) are apparently in agreement with those stated by Bohr. This can be illustrated by means of Fig. 4 and 5 which correspond, respectively, to the diagrams of Fig. 7-II and Fig. 7-I of Bohr.[10] The ordinates represent the ratio between the actual differential cross section for energy transfers $T$ and the cross section corresponding to collisions with a free electron. The abscissas represent $\log(T_m/T)$, where $T_m$ is the maximum value of the energy transfer.

It should be mentioned that the diagrams made by Bohr are simplified; i.e., in order to avoid complications in the drawing, Bohr has eliminated in his representation the discrete resonance lines corresponding to excitation processes with $T<I_s$.[11] In order to show more clearly the analogy between the Bohr representation and the one used in this discussion the diagrams of Bohr have been completed so as to show the excitation processes.

Figure 4 illustrates the case when $\kappa/\eta_s<1$. The region represented by vertical hatchings is the region (a) of

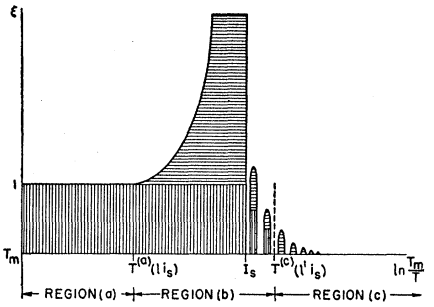


FIG. 4. Statistical distribution of energy losses for $\kappa/\eta_s<1$.

the orbital representation and it corresponds to energy transfers $T_m>T>T^{(a)}(li_s)$. The region represented partly by vertical and partly by horizontal and vertical hatchings is the intermediate region (b) corresponding to $T^{(a)}(li_s)>T>T^{(c)}(l'i_s)$ and the region (c) of small perturbations for $T<T^{(c)}(l'i_s)$ is represented by horizontal hatchings. The horizontal hatchings indicated by Bohr in the region (b) show that the problem is essentially quantum-mechanical and that the energy transfers cannot be *accurately* determined in this region by means of an orbital representation.

Figure 5 illustrates the case when $\kappa/\eta_s>1$. The solid line represents the distribution of energy losses which qualitatively is the same in the representation of Bohr as in the one considered in the present investigation. We shall refer to this distribution as the "actual distribution." Bohr was not concerned in this particular case with the problem of statistical evaluation of energy losses; i.e., with the "actual distribution." His main

[10] See reference 1, p. 89.
[11] See reference 1, p. 90.

problem was to calculate the aggregate effect of all losses, and with this purpose in mind he replaced the "actual distribution" by an equivalent distribution represented by dotted lines. His distribution comprises only energy transfers in the range $T_m > T > D_s^*$, where $D_s^* = T^{(a)}(d_s^*)$ and is based on a collision with a free electron. We shall call it, therefore, the "free collision approximation."

Bohr's expression (10) contains a logarithmic term which does not have physical meaning for $\kappa \gtrsim \eta_s^3$. According to Bohr,[12] it is actually applicable if $\kappa \ll \eta_s^3$. Consequently, the expression (10) has a relatively limited region of applicability since it can be used only when

$$\eta_s < \kappa \ll \eta_s^3. \tag{44}$$

On the other hand, the expression (43) does not have this limitation since it is applicable to all those cases in which $\kappa > \eta_s$.

The physical picture used in this investigation may be helpful in evaluating the energy loss as a function of the impact parameter, and Fig. 6 illustrates two specific examples illustrating the "actual distribution."

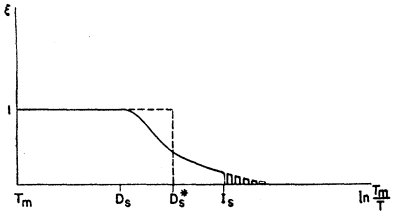


FIG. 5. Statistical distribution of energy losses for $\kappa/\eta_s > 1$.

Since Bohr was not concerned with the actual statistical distribution of the energy losses but with their aggregate effect, the "free collision approximation" was not intended to represent the behavior of the "actual distribution."

[12] See reference 1, p. 85.

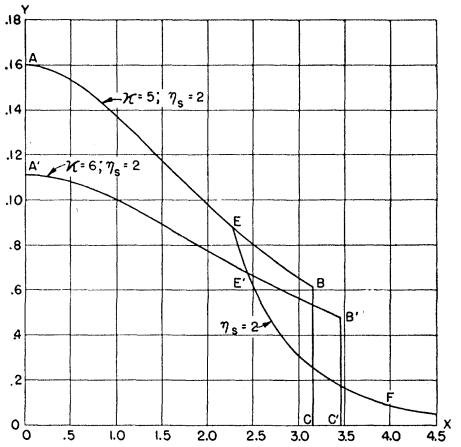


FIG. 6. Plot of the energy transfer as a function of the impact parameter.

Figure 6 shows the energy transfer at various impact parameters corresponding to the two distributions. The abscissa $X = p/a_s$ represents the impact parameter and the ordinate $Y = (\pi a_s^2/B_\epsilon)T(p)$ represents the energy transfer. The expression $T(p)$ can be represented either by (20) or (31). Two examples have been chosen:

Example 1: $\kappa = 5$; $\eta_s = 2$. For this case the stopping number $\sigma_s/B_\epsilon$, according to the Bohr formula (10), is 0.47 and according to the proposed formula (43) is 0.99. Curve $ABC$ represents the "free collision approximation," and the curve $AEF$ represents the distribution which is the basis of (43). More particularly, $AE$ represents the distribution (20) and $EF$ represents the distribution (31).

Example 2: $\kappa = 6$; $\eta_s = 2$. The stopping number, $\sigma_s/B_\epsilon$ according to (10) is equal to 0.29 and according to (43) is equal to 0.83. Curve $A'B'C'$ represents the "free collision approximation" and the curve $A'E'F'$ represents the distribution which is the basis of (43).