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GEANT4 X-ray transition radiation package

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Abstract

Implementation of particular C + + classes in GEANT4 X-ray transition radiation (XTR) library is discussed. Recent developments concerning the transparent regular XTR radiator and XTR generated in straw tube (consisting three media) are considered in details. Simulation results are compared with experimental data.

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GEANT4 models [1] of X-ray transition radiation (XTR) are based mainly on the approach of the radiation energy loss, i.e. the work done by the relativistic charge e crossing the boundary between two media against an additional electric fields induced by the charge in the vicinity of the boundary. This field is defined from the boundary conditions.

Using the methods developed in Refs. [2–4] one can derive the relation describing the mean number of XTR photons, $\bar{N}_{\rm in}$, generated per unit photon frequency ω and θ^2 (θ is the emitting angle), $\mathrm{d}^2\bar{N}_{\rm in}/\hbar\,\mathrm{d}\omega\,\mathrm{d}\theta^2$, *inside* radiator for the most general XTR radiator consisting of n different absorbing media with fluctuating gap thicknesses t_j ($j = 2, 3, \ldots, n-1$):

$$\frac{\mathrm{d}^2\bar{N}_{\rm in}}{\hbar\,\mathrm{d}\omega\,\mathrm{d}\theta^2} = \frac{\alpha}{\pi\hbar c^2}\omega\theta^2 \mathrm{Re}\left\{\sum_{i=1}^{n-1}(Z_i - Z_{i+1})^2 + 2\sum_{k=2}^{n-1}(Z_k - Z_{k+1}) \times\sum_{i=1}^{k-1}(Z_i - Z_{i+1})\prod_{j=i+1}^{k} H_j\right\}, \quad (1)$$

where α is the fine structure constant, and $\hbar$ is the Planck's constant. In the case of gamma distributed gap thicknesses t_j the correlation factors H_j are

$$H_j = \int_0^\infty \mathrm{d}t_j \left(\frac{v_j}{\bar{t}_j}\right)^{v_j} \frac{t_j^{v_j-1}}{\Gamma(v_j)} \exp\left[-\frac{v_j t_j}{\bar{t}_j} - i\frac{t_j}{2Z_j}\right] = \left[1 + i\frac{\bar{t}_j}{2Z_j v_j}\right]^{-v_j}, \quad (2)$$

where Γ is the Euler gamma function, $\bar{t}_j$ is the mean thickness of the jth medium in the radiator and $v_j > 0$ is the parameter roughly describing the relative fluctuations of t_j. In fact, the relative fluctuation of t_j is, $\delta_j = 1/\sqrt{v_j}$. The complex formation zone of XTR in the jth medium [2,5], Z_j ($j = 1, 2, \ldots, n$) is defined by

$$Z_j = \frac{L_j}{1 - i(L_j/l_j)}, \quad L_j = \frac{c}{\omega}\left[\gamma^{-2} + \frac{\omega_j^2}{\omega^2} + \theta^2\right]^{-1}, \quad (3)$$

where $i^2 = -1$, θ is the angle between the charged particle velocity and the direction of XTR photon, $\gamma^{-2} = 1 - \beta^{-2}$, $\beta = v/c$ is the ration of the particle velocity v and the speed of light c; ω_j and l_j are the plasma frequency and the photon absorption length in the medium, respectively. In the case of a transparent medium, $l_j \to \infty$, the complex formation zone is reduced to the *coherence length* L_j of XTR, which is real number.

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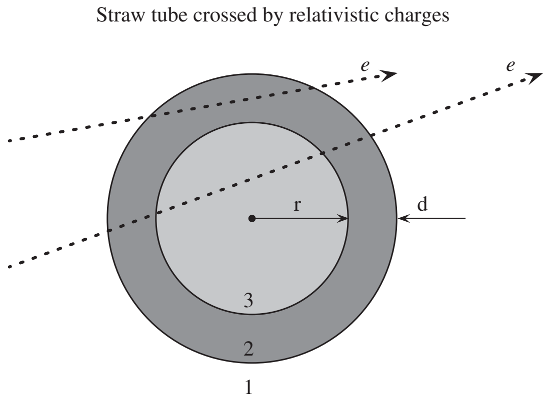

Fig. 1. The diagram of straw tube crossed by charged particles with the charge e generating XTR on the straw tube surface interfaces. The medium 1 is air, the medium 2 is the straw tube wall, and the medium 3 is the detector gas mixture.

In many high energy physics experiments the XTR is detected by the arrays of straw tube gas proportional counters inserted in radiator. It is interesting therefore to estimate the contribution of straw tube to the generation of XTR, since charged particles passing through the radiator will cross many straw tubes. Fig. 1 shows the geometry of a straw tube with the radius r and the wall thickness d, when the charge serially crosses mainly the boundaries 1-2-3-2-1 and rarely 1-2-1. Simplifying (1) one gets

$$\frac{d^2\bar{N}_{in}}{\hbar\, d\omega\, d\theta^2} = \frac{\alpha\omega\theta^2}{\pi\hbar c^2}\mathrm{Re}\{pR^{(12321)} + (1-p)R^{(121)}\},$$

$$\begin{aligned} R^{(12321)} = 2\{&(Z_1 - Z_2)^2[1 - H_2^2 H_3] \\ &+ (Z_2 - Z_3)^2[1 - H_3] \\ &+ 2(Z_1 - Z_2)(Z_2 - Z_3)H_2[1 - H_3]\}. \end{aligned} \tag{4}$$

$$R^{(121)} = 2(Z_1 - Z_2)^2[1 - H_2]. \tag{5}$$

Here p is the probability to cross the interfaces in the sequence 1-2-3-2-1. The correlation factors depend on the mode of straw tube intersection and can be evaluated using the mean, $\bar{t}$ and variance, $v = \bar{t}^2/(\bar{t^2} - \bar{t}^2)$ of the thickness distributions. In the case of uniform shooting perpendicular to the straw tube, assuming $d \ll r$, they read

$$\bar{t}_3 = \frac{\pi r}{2}, \quad v_3 = \frac{3\pi^2}{32 - 3\pi^2} \simeq 12.4,$$

$$\bar{t}_2 = \sqrt{2rd} + (\pi - 1)d \simeq \sqrt{2rd}, \quad v_2 \simeq \tfrac{1}{3}.$$

The G4StrawTubeXTRadiator class describes the generation of XTR in straw tube under approximation, $d \ll r$ and $p = 0$. Fig. 2 shows the plot of XTR photon number emitted inside straw tube versus the Lorentz factor of incident charged particle. The straw tube diameter is 4 mm and its wall thickness is 0.07 mm. The gas mixture is 0.7 $Xe + 0.27CO_2 + 0.03O_2$ at 20° and 1 atm. For the intersection of 35 to 40 straw tubes by relativistic charged

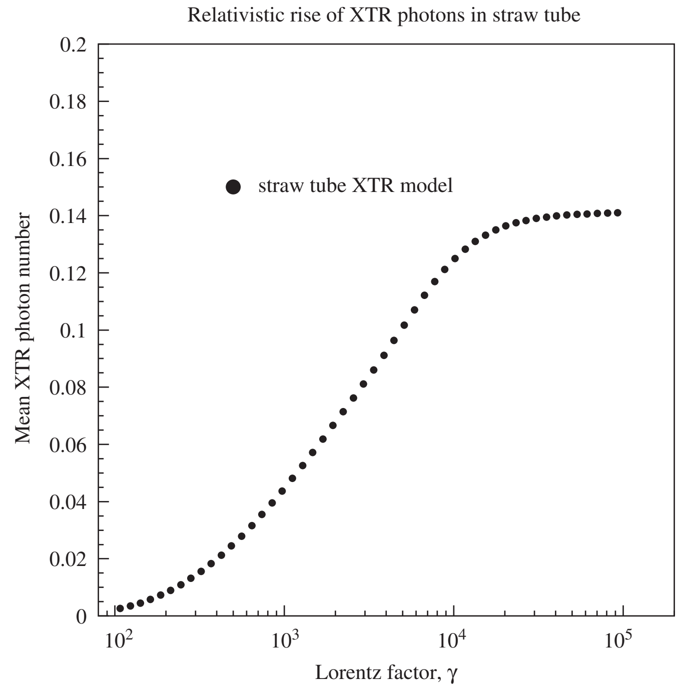

Fig. 2. The plot of XTR photon number emitted inside straw tube versus the Lorentz factor of incident charged particle. The straw tube diameter is 4 mm and its wall thickness is 0.07 mm. The gas mixture is 0.7 $Xe +$ $0.27\ CO_2 + 0.03\ O_2$ at 20° and 1 atm.

particle with Lorentz factor $\gtrsim 10^4$ they will produce about 5 XTR photons in addition to layered XTR radiator.

In the particular case of n foils of the first medium interspersed with gas gaps of the second medium, the summation of Eq. (1) results in

$$\frac{d^2\bar{N}_{in}}{\hbar\, d\omega\, d\theta^2} = \frac{2\alpha}{\pi\hbar c^2}\omega\theta^2\mathrm{Re}\{\langle R^{(n)}\rangle\}, \quad H = H_1H_2,$$

$$\langle R^{(n)}\rangle = (Z_1 - Z_2)^2\left\{n\frac{(1-H_1)(1-H_2)}{1-H} + \frac{(1-H_1)^2H_2(1-H^n)}{(1-H)^2}\right\}. \tag{6}$$

This approach allows us to implement XTR as GEANT4 parametrisation [5], or as standard electro-magnetic process [2,3].

The integration of Eq. (6) in respect to θ^2 can be simplified for the case of regular radiator ($v_{1,2} \to \infty$) with transparent in terms of XTR generation media, and $n \gg 1$ [6]. The frequency spectrum of emitted XTR photons reads

$$\begin{aligned} \frac{d\bar{N}_{in}}{\hbar\, d\omega} = \int_0^{\sim 10\gamma^{-2}} d\theta^2 \frac{d^2\bar{N}_{in}}{\hbar d\omega\, d\theta^2} = \frac{4\alpha n}{\pi\hbar\omega}(C_1 + C_2)^2 \\ \times \sum_{k=k_{min}}^{k_{max}} \frac{(k - C_{min})}{(k - C_1)^2(k + C_2)^2} \\ \times \sin^2\left[\frac{\pi t_1}{t_1 + t_2}(k + C_2)\right], \end{aligned} \tag{7}$$

$$C_{1,2} = \frac{t_{1,2}(\omega_1^2 - \omega_2^2)}{4\pi c\omega},$$

$$C_{\min} = \frac{1}{4\pi c}\left[\frac{\omega(t_1+t_2)}{\gamma^2} + \frac{t_1\omega_1^2 + t_2\omega_2^2}{\omega}\right].$$

The sum in Eq. (7) is defined by terms with $k \gtrsim k_{\min}$ corresponding to the region of $\theta \gtrsim 0$. Therefore, $k_{\min}$ should be the nearest to $C_{\min}$ integer $k_{\min} \geq C_{\min}$. The value of $k_{\max}$ is defined by the maximum emitting angle $\theta_{\max}^2 \sim 10\gamma^{-2}$. It can be evaluated as the integer part of

$$C_{\max} = C_{\min} + \frac{\omega(t_1+t_2)}{4\pi c}\frac{10}{\gamma^2}.$$

This value usually results in

$$k_{\max} - k_{\min} \sim 10^2 - 10^3 \gg 1,$$

however numerically, only few tens of terms contribute substantially to the sum, i.e. one can choose $k_{\max} \sim k_{\min} + 20$. Eq. (7) corresponds to the spectrum of total number of photons emitted inside regular transparent radiator. Therefore, the mean interaction length, λ_{XTR}, of XTR process in this kind of radiators can be introduced as

$$\lambda_{\mathrm{XTR}} = n(t_1+t_2)\left[\int_{\hbar\omega_{\min}}^{\hbar\omega_{\max}} \hbar\,\mathrm{d}\omega \frac{\mathrm{d}\bar{N}_{\mathrm{in}}}{\hbar\,\mathrm{d}\omega}\right]^{-1},$$

where $\hbar\omega_{\min} \sim 1$ keV, and $\hbar\omega_{\max} \sim 100$ keV for the majority of high energy physics experiments. The spectrum of total number of XTR photons *after* regular transparent radiator is defined by Eq. (7) with

$$n \to n_{\mathrm{eff}} = \frac{1 - \exp[-n(\sigma_1 t_1 + \sigma_2 t_2)]}{1 - \exp[-(\sigma_1 t_1 + \sigma_2 t_2)]},$$

where σ_1 and σ_2 are photo-absorption cross-sections corresponding to the photon frequency ω in the first and the second medium, respectively. With this correction taking into account the XTR photon absorption in the radiator (7) corresponds to the results of Ref. [7].

Fig. 3 shows the comparison between simulation and experimental data [8] for the total (ionisation and XTR absorption) energy loss distributions. Here the ionisation energy loss distribution produced by electrons with a momentum of 2 GeV/c in a gas mixture $0.9\mathrm{Xe} + 0.1\mathrm{CH_4}$ with a thickness of 3 cm at pressure 1 atm. Closed circles are the experimental data [8], the histogram is the simulation according to the photo-absorption ionisation (PAI) model [9], and the XTR model. The XTR radiator consists of 300 lithium foils with the thickness 0.04 mm separated by helium gas gaps 0.126 mm thick. Vertical and horizontal errors correspond to statistical and energy bin uncertainties, respectively. The distributions are normalised on the same integral statistics in the comparison region. The experimental and simulated mean energy losses are 29.74 and 29.97 keV, respectively.

Fig. 4 shows the same comparison based on experimental data published in Ref. [10]. The total ($\mathrm{d}E/\mathrm{d}x$ + XTR) energy loss distribution produced by electrons with a

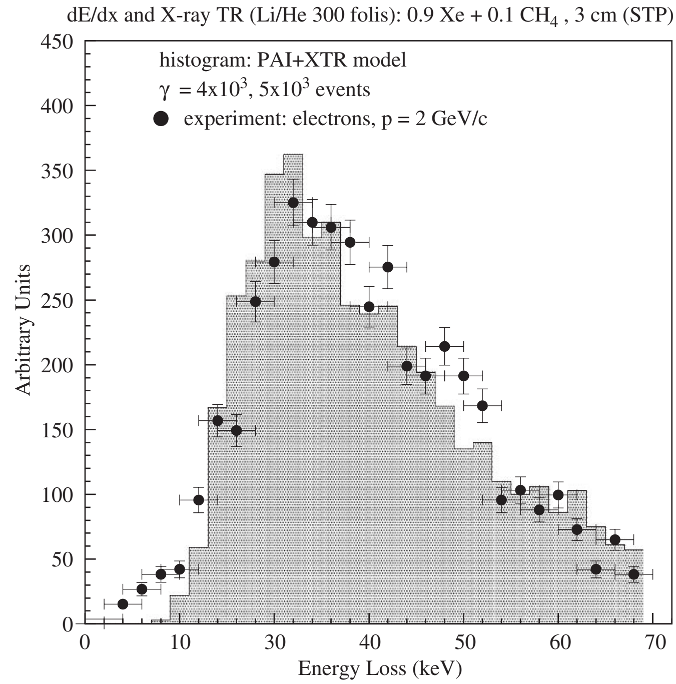

Fig. 3. The ionisation energy loss distribution produced by electrons with a momentum of 2 GeV/c in a gas mixture $0.9\mathrm{Xe} + 0.1\mathrm{CH_4}$ with a thickness of 3 cm at pressure 1 atm. Closed circles are the experimental data [8], the histogram is simulation according to the PAI model and the XTR model. XTR radiator: $300 \cdot (\mathrm{Li/He} = 0.04/0.126)$ mm.

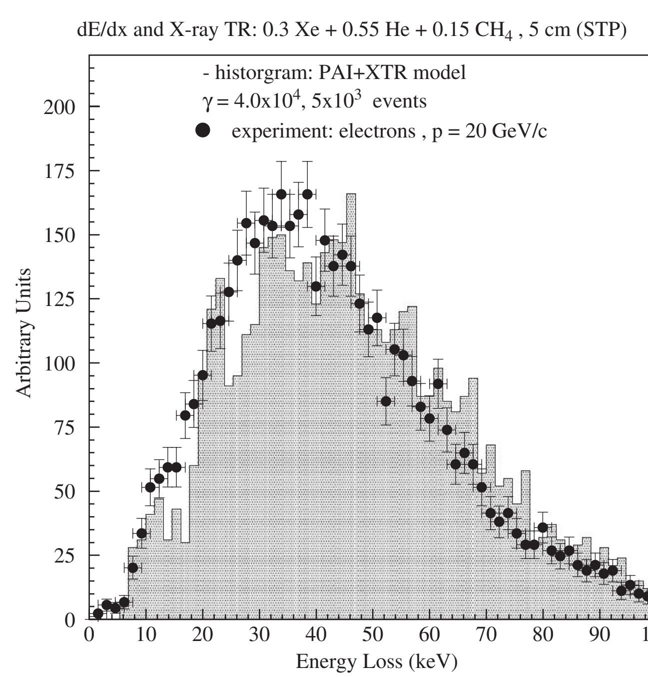

Fig. 4. The total ($\mathrm{d}E/\mathrm{d}x + \mathrm{TR}$) energy loss distribution produced by electrons with a momentum of 30 GeV/c in a gas mixture $0.35\mathrm{Xe} + 0.55\mathrm{He} + 0.15\mathrm{CH_4}$ with a thickness of 5 cm at pressure 1 atm. Closed circles are the experimental data [10], the histogram simulation according to the PAI model and the XTR model. XTR radiator: $350 \cdot (\mathrm{CH_2/CO_2} = 0.019/0.6)$ mm.

momentum of 20 GeV/c in a gas mixture $0.35\mathrm{Xe}+0.55\mathrm{He}+0.15\mathrm{CH}_4$ with a thickness of 5 cm at pressure 1 atm. Closed circles are the experimental data [10], the histogram simulation according to the PAI model and the XTR model. XTR radiator: $350\cdot(\mathrm{CH}_2/\mathrm{CO}_2=0.019/0.6)$ mm. The distributions are normalised on the same integral statistics in the comparison region. The experimental and simulated mean energy losses are 42.72 and 46.17 keV, respectively. One can see that the simulation overestimates the total energy loss and the difference between the mean energy loss is about 7%.

The current XTR energy loss models implemented GEANT4 simulation toolkit are described by the following classes:

(i) G4GammaXTRadiator for the description of XTR radiator with gamma distributed foil and gas gap thicknesses.
(ii) G4RegularXTRadiator for the description of XTR radiator with fixed foil and gas gap thicknesses taking into account the influence of XTR absorption.
(iii) G4TransparentRegXTRadiator for the description of simplified transparent XTR radiator with fixed foil and gas gap thicknesses.
(iv) G4StrawTubeXTRadiator for the description of XTR radiator in straw tube XTR detector.

Some of GEANT4 models show satisfactory agreement between experimental data and simulation and other require more comparisons with measurements.

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