NODAL EXPANSION IN A REAL MATTER PLASMA★

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The Mayer-Salpeter nodal expansion is extended to two-components high-temperature plasmas.

A well-known long-standing paradigm in the classical perturbation theory of equilibrium thermal properties of systems with short as well as long-ranged two-body forces is its limitation to purely repulsive interactions [1], in order to prevent a possible collapse due to a lack of stability arising from the strong attractive interactions evidenced by the appearance of bound states. As a consequence, a widespread popular belief tends to relegate systems with attractive forces in a quantum mechanical framework exclusively. With explicit thermodynamical results explained only in the $T \to 0$ limit. Our purpose is to show on an important example that such a restrictive point of view is not always necessary. The above noticed limitations, to a large extent, may be removed, provided one puts the emphasis on a high enough mean kinetic energy regime, in order to allow for a complete compensation of the bound states by the scattering states through Levinson theorem [2]. The system we have in mind is the real two-components nonrelativistic electron proton plasma, i.e. real matter in bulk taken in the high temperature regime $k_B T > 1$ Ry. The corresponding two-body sum-over-states for unlike charges may be estimated phenomenologically on sound heuristic basis with the aid of the screened Debye interaction $-e^2 \exp(-r/\lambda_D)/r$ [3], and it is shown that the corresponding bound states sum is negligible. So, the bound states formation do not plague anymore through the appearance of uncontrolled infinite quantities, the use of the perturbative nodal expansion [4] with respect to the plasma parameter $\Lambda = e^2/k_B T \lambda_D$. Incidentally, the same line of reasoning applies a fortiori to two-components plasmas with a smaller dimensionality.

Starting from the standard one-component plasma (hereafter referred to as ocp) nodal expansion [4], we are allowed to extend it in a straightforward way to the present situation. In so doing, we are led to parallel to some extent a formalism already used for charged hard spheres systems [5], used in the theory of symmetrical electrolytes. However, in contradiction to this particular situation, we are not restricted to a second-order in Λ nodal expansion of the thermal properties, because the short-range behavior of the proton-electron interaction is well taken into account in the high temperature regime through the soft pseudopotential [6]

$$W_{e-i}(r) = -\frac{e^2}{r}(1 - e^{-Cr}), \quad C = \frac{1}{\lambda_{ei}}, \quad \lambda_{ei} = \frac{\hbar}{\sqrt{2 m_{ei} k_B T}}. \tag{1}$$

The ocp nodal rules [1, 4] have to be modified as follows ($Z_1 = 1, Z_2 = -1$)

(a) λ_D^2 becomes $k_B T/(4\pi n e^2 (C_1 Z_1^2 + C_2 Z_2^2))$, where $C_i = N_i/N$ so that each field point gets an extra factor $(C_1 Z_1^2 + C_2 Z_2^2)^{-1}$.

(b) The interaction between particles i and j is proportional to $z_i z_j$, so that each field point is again factorized with $C_1 Z_1^{M_k} + C_2 Z_2^{M_k}$, where M_k denotes the number of lines merging into the field point k. The root points have to be given a factor $Z_1^{M_1} Z_2^{M_2}$.

(c) The Debye lines $(k^2+1)^{-1}$, in momentum space, corresponding to the bare Coulomb interaction r^{-1} have to be replaced by a normalized sum of the three modified Debye lines attached to the direct interaction ion-ion, ion-electron, electron-electron respectively. This is easily performed once we notice that the Debye interaction resuming the long range behavior of the effective interaction (1) may be written as [7] ($C\lambda_D > 2$)

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$$\frac{e^2}{r\sqrt{1-4/C^2\lambda_D^2}}\left(e^{-\alpha_1 r}-e^{-\alpha_2 r}\right),\quad \frac{1}{k^2+\alpha_1^2}-\frac{1}{k^2+\alpha_2^2}, \tag{2}$$

in r-space and k-space respectively, with

$$\alpha_{1,2}=\frac{C}{\sqrt{2}}\left(1\mp\sqrt{1-\frac{4}{C^2\lambda_D^2}}\right)^{1/2}.$$

(d) Putting altogether the above rules for the field points, we see that each nodal graph with k field points has to be given a factor $(Z_1^{Mk-1}-Z_2^{Mk-1})/(Z_1-Z_2)$ when the overall neutrality condition $C_1Z_1+C_2Z_2=0$ is taken into account. An immediate by-product of this condition is the vanishing to all order of the longest chains building up the long range behavior of the ocp pair correlation function in the HNC approximation. Also, the number of graphs at a given order is drastically reduced[‡1]. The remaining ones are linear combinations of the corresponding ocp topologies, explained with the aid of eq. (2) in a subsequent work.

‡1 In second-order, only the 2-bubble is surviving the $g_2(r)$ nodal expansion.

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