Implementation of Common Non-Maxwellians Background Distributions in `GENE`

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1 Background and Motivation

In its base relase, the gyrokinetic code `GENE` has support for only Maxwellian backgrounds

$$f_{s0}[\mathbf{x};\mathbf{v}] = f_{s0}^{\mathrm{M}}[\psi; v_\parallel, \mu] \tag{1}$$

$$= n_s \sqrt{\frac{m_s}{2\pi k_B T_s}}^3 \exp\left[-\frac{1}{k_B T_s}\left(\frac{m_s}{2}v_\parallel^2 + B\mu\right)\right] \tag{2}$$

in species s, where:

- $v_\parallel$ = the parallel velocity
- $\mu = \left(\frac{m_s v_\perp^2}{2B}\right)$ = the magnetic moment

ψ is constant along flux tubes (using the notation as used in [Abe+13]) and the following are functions of ψ only, such that they are constant on flux tubes.

- $n_s(\psi)$ = the particle density
- $T_s(\psi)$ = the temperature

In reality, the space of background distributions for tokamak plasmas is much more rich than just Maxwellians, and different background distributions can have great effects on the behaviour of the model.

1.1 Shifted Maxwellians and Bi-Maxwellians

Examples include:

- Shifted Maxwellians

$$f_{s0}[\mathbf{x};\mathbf{v}] = f_{s0}^{\mathrm{SM}}[\psi; v_\parallel, \mu] \tag{3}$$

$$= n_s \sqrt{\frac{m_s}{2\pi k_B T_s}}^3 \exp\left[-\frac{1}{k_B T_s}\left(\frac{m_s}{2}(v_\parallel - u_{\parallel s})^2 + B\mu\right)\right] \tag{4}$$

for some parallel velocity $u_{\parallel s}$, allowing for travelling backgrounds. This is particularly relevant for beam injection simulations.

- Bi-Maxwellians

$$f_{s0}[\mathbf{x};\mathbf{v}] = f_{s0}^{\mathrm{BM}}[\psi; v_\parallel, \mu] \tag{5}$$

$$= n_s \sqrt{\frac{m_s}{2\pi k_B T_s}}^3 \exp\left[-\frac{1}{k_B}\left(\frac{m_s}{2T_{\parallel s}} v_\parallel^2 + \frac{B}{T_{\perp s}}\mu\right)\right] \tag{6}$$

for some parallel and perpendicular temperature fields $T_{\parallel s}$ and $T_{\perp s}$, allowing for backgrounds with differing variations in velocity in the parallel vs. perpendicular direction.

These two modified backgrounds were implementing in an old version of `GENE` as part of Alessandro Di Siena's 2020 thesis, [Di 20], for $u_{\parallel s}$, $T_{\parallel s}$, $T_{\perp s}$ functions of ψ. Unfortunately, the code in its form from the thesis is not available to the public, and is only partially maintained as part of the `GENE` GPU development branch- many portions of the code presume a Maxwellian background, and do not account for non-Maxwellian backgrounds.

1.2 Damped Distributions

Another common effect to consider is Alfvénic damping, wherein high-energy particles with velocities on the scale of the Alfvén velocity $v_A = \frac{B}{\sqrt{\mu_0 \rho}}$ lose energy due to resonance with the Alfvén waves, causing velocity distributions with sharper cut-offs than the $\mathcal{O}\left[\exp\left(-\text{const.}v^2\right)\right]$ tails in Maxwellians, shifted Maxwellians, bi-Maxwellians etc. A general way to consider such "damped distributions" is presented here.

Suppose, before damping, the background has some classic distribution, e.g. a shifted Maxwellian, with distribution function $f_{s0}^{\mathrm{pre}}\left[v_\parallel^{\mathrm{pre}}, \mu^{\mathrm{pre}}\right]$ at a given position. Consider this velocity being modified by some "damping function", $(v_\parallel^{\mathrm{pre}}, \mu^{\mathrm{pre}}) \mapsto (v_\parallel, \mu)$ that reduces the velocity of the high-velocity particles, but leaves the low-velocity particles unchanged. Denote this as:

$$v_\parallel(v_\parallel^{\mathrm{pre}}, \mu^{\mathrm{pre}}) \qquad \mu_\parallel(v_\parallel^{\mathrm{pre}}, \mu^{\mathrm{pre}}) \tag{7}$$

Similarly, this has the inverse:

$$v_\parallel^{\mathrm{pre}}(v_\parallel, \mu) \qquad \mu_\parallel^{\mathrm{pre}}(v_\parallel, \mu) \tag{8}$$

A reasonable condition on this damping function is that it leaves the velocity of low-energy particles unchanged, namely:

$$\partial_{v_\parallel^{\mathrm{pre}}} v_\parallel(0,0) = 1 \qquad \partial_{v_\parallel^{\mathrm{pre}}} \mu(0,0) = 0 \tag{9}$$

$$\partial_{\mu^{\mathrm{pre}}} v_\parallel(0,0) = 0 \qquad \partial_{\mu^{\mathrm{pre}}} \mu(0,0) = 1 \tag{10}$$

Similarly, for the inverse:

$$\partial_{v_\parallel} v_\parallel^{\text{pre}}(0,0) = 1 \qquad \partial_{v_\parallel} \mu^{\text{pre}}(0,0) = 0 \tag{11}$$
$$\partial_{\mu} v_\parallel^{\text{pre}}(0,0) = 0 \qquad \partial_{\mu} \mu^{\text{pre}}(0,0) = 1 \tag{12}$$

The distribution function f_{s0} for the damped distribution will then evaluate as

$$f_{s0}[v_\parallel, \mu] = J(v_\parallel, \mu) f_{s0}^{\text{pre}} \left[v_\parallel^{\text{pre}}(v_\parallel, \mu), \mu^{\text{pre}}(v_\parallel, \mu) \right] \tag{13}$$

where J is the Jacobian

$$J(v_\parallel, \mu) := \partial_{v_\parallel} v_\parallel^{\text{pre}}(v_\parallel, \mu) \partial_\mu \mu^{\text{pre}}(v_\parallel, \mu) - \partial_\mu v_\parallel^{\text{pre}}(v_\parallel, \mu) \partial_{v_\parallel} \mu^{\text{pre}}(v_\parallel, \mu) \tag{14}$$

As such, to create a damped velocity distribution, we need only 3 “ingredients”:

- $f_{s0}^{\text{pre}} \left[v_\parallel^{\text{pre}}, \mu^{\text{pre}} \right]$, the velocity distribution *before damping.*
- $v_\parallel^{\text{pre}}(v_\parallel, \mu)$, the parallel velocity a damped particle had *before damping.*
- $\mu^{\text{pre}}(v_\parallel, \mu)$, the magnetic moment a damped particle had *before damping.*

Example. (Uniform damping) *Consider a damping function wherein a particle's pitch angle remains unaffected by the damping, but the speed $v = \sqrt{v_\parallel^2 + \frac{2B}{m_s}\mu}$ is damped uniformly among all pitch angles through some function $v^{\text{pre}}(v)$, such that:*

$$v_\parallel^{\text{pre}}(v_\parallel, \mu) = \frac{v^{\text{pre}}(v)}{v} v_\parallel \tag{15}$$
$$\mu^{\text{pre}}(v_\parallel, \mu) = \frac{v^{\text{pre}}(v)}{v} \mu \tag{16}$$

Such a $v^{\text{pre}}(v)$ would need to:

- *satisfy the low-velocity condition $\partial_v v^{\text{pre}}(0) = 1$.*
- *go to ∞ much faster than v for $v > v_A$ (or whatever the velocity scale on which the damping takes effects may be).*

Such a function may for example take the form

$$v^{\text{pre}}(v) := \left[1 + \left(\frac{v}{v_A} \right)^r \right] v \tag{17}$$

2 Implementation and Usage

References

[Abe+13] I. G. Abel et al. “Multiscale Gyrokinetics for Rotating Tokamak Plasmas: Fluctuations, Transport and Energy Flows”. In: *Reports on Progress in Physics* 76.11 (Oct. 2013), p. 116201. DOI: 10.1088/0034-4885/76/11/116201. URL: https://doi.org/10.1088/0034-4885/76/11/116201.

[Di 20] A. Di Siena. “Implementation and Investigation of the Impact of Bifferent Background Distributions in Gyrokinetic Plasma Turbulence Studies”. en. PhD thesis. Universität Ulm, Apr. 2020, p. 247. ISBN: 9781695601505. DOI: 10.18725/OPARU-29528.