Numerical Bifurcation Analysis for Heat Transfer by the Edge Plasma of a Tozmahok

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Abstract

BA: Abstract.

Contents

BA: I'm aware having a contents page is ridiculous, it's just nice to have for now for navigation purposes.

Introduction

BA: Motivation.

BA: Previous work and literature review.

BA: Results we might expect to find.

BA: Relevance and impact.

BA: Other topics to note mentioned on the transfer thesis requirements.

1 Physical Model and System of Equations

BA: Room for lots of pictures here.

BA: What physics characterise a (magnetised) neutral plasma? Quasi-neutral mix of *separated* electrically-charged phases. (Check out the plasma Wikipedia page.)

BA: Creates a coupled system with the EM field.

BA: What causes a fluid to turn into a plasma? Ionisation. (High heat/strong EM field.)

BA: What makes a tozmahok plasma special (give stats):

- *Massive* heat.
- *Massive* magnetic field- highly magnetised. (Talk about particle gyro-orbits and drifts.)
- *Minimal* density. (Why?)

.

BA: Will make the assumptions:

- Only 2 phases- this is generally the case in edge plasmas (i.e. outside the divertor) provided impurity effects are negligible. (A bold assumption?)
- Only (thermalising) Coulomb collisions are considered- these are generally dominant over the others in a tozmahok. (N.B. No fusion.)

BA: Complicated BCs in a tozmahok.

1.1 Kinetic Models

BA: What is a "kinetic" model.

1.1.1 Typical Fluids

BA: The resultant kinetic PDE. (Boltzmann equation.)

BA: How we traditionally convert that to a "fluid" model.

BA: Will use this simpler case as a reference study to develop the ideas for the more complicated tozmahok plasma case.

1.1.2 Tozmahok Plasmas

BA: Why the fluid/MHD model reductions aren't necessarily valid in tozmahok plasmas. (Incorrectly assumed dominant collisional term- get some estimates on the scale of these terms in the edge plasma. Good content under "Mathematical Descriptions" here.)

BA: Many effects not captured my MHD/2 fluid models (check out this diagram off Wikipedia, or again the content under "Mathematical Descriptions" here.):

- Most plasma waves
- Most plasma/kinetic instabilities
- Landau damping/bump-on-tail instability
- Leakage
- Structures (Beams/double layers)
- Anisotropic pressure

BA: The resultant kinetic PDE. (Boltzmann/Vlasov equations.)

BA: Talk about gyrokinetic model:

- The model's physical basis/mathematics. (Equations provide good insight into the origin of some behavioural effects, e.g. gyro-orbits/drifts.)
- Why we don't use it:
  - High mathematical (more terms in lower dimensions doesn't necessarily mean faster computation)/computational (really don't want to do a 5D simulation) complexity.
  - Errors from neglection of terms. (Non-physical behaviour over long times/resonances and adiabatic invariants can be lost.)

1.2 Coupled Maxwellian/Perturbation Decomposition

BA: How we can re-adapt the techniques that traditionally give a fluid model when the collision operator is non-dominant to get an accurate fluid model, to apply modern techniques in fluid simulation?

BA: Expand as a sum of a Maxwellian and some perturbation!

1.2.1 Maxwellian Background: A *Fluid* Model

BA: Ideas already well-developed!

BA: Perturbation contribuation not too problematic (hopefully).

1.2.2 Anisotropic Perturbation: A *Kinetic* Model

BA: Not just kicking the problem down the road- plasma is thermalised/Maxwellian in "most places" for "most physically relevant simulations", so the perturbations is (compartively) small in "most places".

BA: How do we model this:

- Lattice Boltzmann?
- Some series expansion?
- Particle-in-cell (PIC)?

2 Numerical Simulation and Preconditioning

2.1 Maxwellian Background: A Fluid Simulation

2.2 Anisotropic Perturbation: A Kinetic Simulation

BA: What approach?

2.2.1 Lattice Boltzmann?

2.2.2 Series Expansion?

2.2.3 Particle-in-Cell (PIC)?

3 Bifurcation Analysis

4 Summary and Further Work