Numerical Bifurcation Analysis for Heat Transfer by the Edge Plasma of a Tozmahok

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March 2022

Abstract

BA: Abstract.

Contents

Introduction

BA: Introduction.

BA: Motivation.

BA: Previous work and literature review.

BA: Results we might expect to find.

BA: Relevance and impact.

BA: Other topics to note mentioned on the transfer thesis requirements.

BA: Summary.

Part I

Research

Chapter 1

Physical Model and System of Equations

BA: Introduction.

BA: Room for lots of pictures here.

BA: What physics characterise a (magnetised) neutral plasma? Quasi-neutral mix of *separated* electrically-charged phases. (Check out the plasma Wikipedia page.)

BA: Creates a coupled system with the EM field.

BA: What causes a fluid to turn into a plasma? Ionisation. (High heat/strong EM field.)

BA: What makes a tozmahok plasma special (give stats):

- *Massive* heat.
- *Massive* magnetic field- highly magnetised. (Talk about particle gyro-orbits and drifts.)
- *Minimal* density. (Why?)

.

BA: Will make the assumptions:

- Only 2 phases- this is generally the case in edge plasmas (i.e. outside the divertor) provided impurity effects are negligible. (A bold assumption?)
- Only (thermalising) Coulomb collisions are considered- these are generally dominant over the others in a tozmahok. (N.B. No fusion.)
- Relativistic effects are negligible. (Is observing that the velocity scale is much smaller than c sufficient justification? Imperial guy at the NEPTUNE workshop appeared to think not.)

BA: Complicated BCs in a tozmahok.

1.1 Kinetic Models

BA: What is a “kinetic” model.

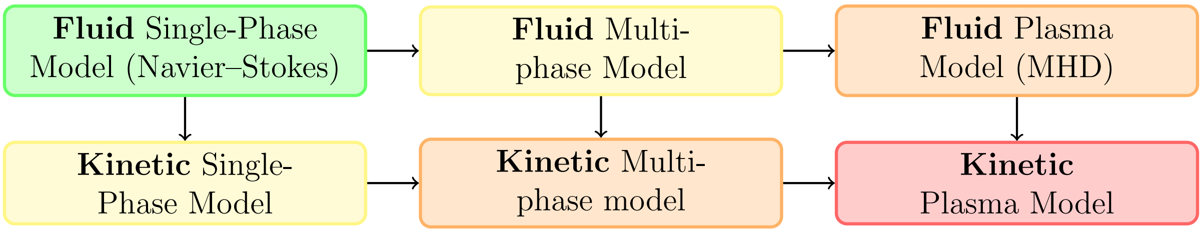

Figure 1.1: BA: Diagram of workflow for creating a kinetic plasma model to account for pressure anisotropy.

1.1.1 Single-Phase Fluids

BA: The resultant kinetic PDE. (Boltzmann equation.)

BA: How we traditionally convert that to a "fluid" model.

BA: Will use this simpler case as a reference study to develop the ideas for the more complicated tozmahok plasma case.

1.1.2 Multiphase Fluids

BA: Similar analysis for a multiphase fluid, in preparation for handling the tozmahok plasmas.

1.1.3 Tozmahok Plasmas

BA: Why the fluid/MHD model reductions aren't necessarily valid in tozmahok plasmas. (Incorrectly assumed dominant collisional term- get some estimates on the scale of these terms in the edge plasma. Good content under "Mathematical Descriptions" here.)

BA: Many effects not captured my MHD/2 fluid models (check out this diagram off Wikipedia, or again the content under "Mathematical Descriptions" here.):

- Most plasma waves
- Most plasma/kinetic instabilities
- Landau damping/bump-on-tail instability
- Leakage
- Structures (Beams/Double layers)
- Anisotropic pressure

BA: The resultant kinetic PDE. (Boltzmann/Vlasov equations.)

BA: Talk about gyrokinetic model:

- The model's physical basis/mathematics. (Equations provide good insight into the origin of some behavioural effects, e.g. gyro-orbits/drifts.)
- Why we don't use it on the general kinetic equation:

- High mathematical (more terms in lower dimensions doesn't necessarily mean faster computation)/computational (really don't want to do a 5D simulation) complexity.
  - Errors from neglection of terms. (Non-physical behaviour over long times/resonances and adiabatic invariants can be lost.)

- We *will* however use it for the PIC correction.

1.2 Coupled Maxwellian/Correction Decomposition

BA: How we can re-adapt the techniques that traditionally give a fluid model when the collision operator is non-dominant to get an accurate fluid model, to apply modern techniques in fluid simulation?

BA: Expand as a sum of a Maxwellian and some correction!

1.2.1 Maxwellian Background: A *Fluid* Model

BA: Ideas already well-developed!

BA: Correction contribution not too problematic (hopefully).

1.2.2 Anisotropic Correction: A *Kinetic* Model

BA: Not just kicking the problem down the road- plasma is thermalised/Maxwellian in "most places" for "most physically relevant simulations", so the correction is (compartively) small in "most places".

BA: How do we model this:

- Lattice Boltzmann?
- Some series expansion?
- Particle-in-cell (PIC)?

1.3 Edge Plasma Model

BA: Little about the edge plasma model we're going to consider:

- Looking for symmetric structures on a low aspect ratio?
- Idk there's a lot of ways to consider this- I could very well end up doing different kinds of simulations with all different kinds of BCs in the final product, and seeing how my results match up.

Summary

BA: Summary.

Chapter 2

Numerical Simulation and Preconditioning

BA: Introduction.

2.1 Maxwellian Background: A Fluid Simulation

BA: Important thing of note here is the fact that this is necessarily a *compressible*, and therefore partially *hyperbolic* system, which can cause a lot of difficulties for creating good discretisations and simulations. (C.F. Numerical dissipation.)

2.1.1 Augmented Lagrangian (AL) Preconditioning

BA: Very high Reynolds, so these fluid equations are *primed* for augmented Lagrangian preconditioning.

BA: Problem is, these equations are necessarily compressible- AL preconditioners have never been done for *compressible* fluid simulations before. How to transfer the ideas across is not immediate.

BA: Crucially: Need exact satisfaction of the mass conservation equations- can tackle this by using the vector of momentum, $\rho\mathbf{u}$, instead of velocity, $\mathbf{u}$.

Stationary State Simulations

Transient (State) Simulations

2.1.2 Fast Diagonalisation Method (FDM)

BA: NEPTUNE interested in high-order methods- why?:

- Better approximation properties (provided sufficient regularity- N.B. *Not* necessarily the case with funky BCs in a tozmahok, but *should* be fine in my case if I pick a nice model with nice BCs on a nice domain).

- Better numerical approximation properties. (Recall that diagram from the NEPTUNE workshop with the travelling bump).
- Better suited to modern computer architectures. (Ask Pablo for more clarification here.)

Why not?:

- Massively worse computational complexity- very dense matrices, unless we find some way to mitigate this...

2.2 Anisotropic Correction: A Kinetic Simulation

BA: What approach?

2.2.1 Lattice Boltzmann?

2.2.2 Series Expansion?

2.2.3 Particle-in-Cell (PIC)?

BA: Would be a great opportunity to work in the ideas of gyrokinetic theory, especially the more mathematical aspects such as the Lie transformations overviewed in Lapillone's thesis (I'm sure there's a more original source for these ideas of course).

BA: Will Saunders at NEPTUNE said there's evidence thatthis decomposition gives a "low-noise" PIC simulation- quite what this means I don't know — potentially the reduction of high-wavenumber perturbations in the fluid portion? that's the impression I got off James Cook in his NEPTUNE presentation — *however*, the key takeaway is there is *solid evidence* that this is *the way to go*. Just google "low noise particle in cell" and you find there's a *solid* amount of literature on the idea.

Summary

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Chapter 3

Bifurcation Analysis

BA: Introduction.

BA: Lots to be said about the search for symmetry and ill-posedness here.

BA: Also worthy of note is the fact that the way I handle the PIC side could end up being non-deterministic, which would maybe mess up the deflation algorithm. (Maybe use some kind of deterministic seed- I could run the bifurcation analysis a few times with different seeds, and compare the results?)

Summary

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Chapter 4

Numerical Implementation

BA: Introduction.

Support for:	Firedrake	Nektar++	NGSolve	deal.ii	Bespoke
Open source	✔		✔		✔
Portable	✔	✔	✔	✔	✘
FEEC elements	✔	✘	✱[5]		✱[1]
Block/AL preconditioning	✔				✱[1]
Multigrid preconditioning	✔				✱[1]
> 3 dimensions	✘			✔	✱[1]
Fluid/PIC interaction	✱[2]	✱[3]			✱[1]
Deflation	✱[4]	✘			✱[1]

Figure 4.1: BA: Applicability of different numerical implementation frameworks. BA: Details about what the ✔'s, ✘'s and ✱'s mean.

BA: What each of the ✱'s means:

✱[1] Obviously all of this is supported in a bespoke implementation, in as far as I would have to implement it, however I would have much more control over how it was done in this case.

✱[2] There's various possibilities here. Patrick mentioned `DMSwarm` in `PETSc`,[1] Pablo mentioned a few other ideas and people to get in contact with about this- I'm confident

[1]Will Saunders has concerns about the way `DMWarm` passed particles between MPI ranks- the particles are distributed from one rank to *all possible receiving ranks* — computationally intense already! — then has no checks for existence and uniqueness for a receiving rank. Apparently solving this would be a parallelisation nightmare too.

that some of the ideas I'd like to try should already be supported in some form, *without* me having to delve into the nitty-gritty of parallelising the PIC.

- ✱[3] Some of the fluid/PIC integration is implemented in *certain* cases within the ExCALIBUR NEPTUNE project in `NESO` (link), `NESO-Particles` (link) and `NESO-Spack` (link) within `Nektar++`- works on GPUs and everything!

- ✱[4] Firedrake has DefCon! Obviously, that's going to require some tweaks — as detailed in the Bifurcation Analysis chapter — for the applications I'd like to apply it to, but that's going to be the case for any premade deflation package.

- ✱[5] I'm not sure if `NGSolve` has support for FEEC elements on *non-simplicial* domains- it's unclear from the documentation.

Summary

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Part II

Project Overview

Chapter 5

Research Plan

BA: Introduction.

BA: What have I/others done so far, and what is there now to do.

BA: Flowchart!

Summary

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Summary

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