Homework #2. Report

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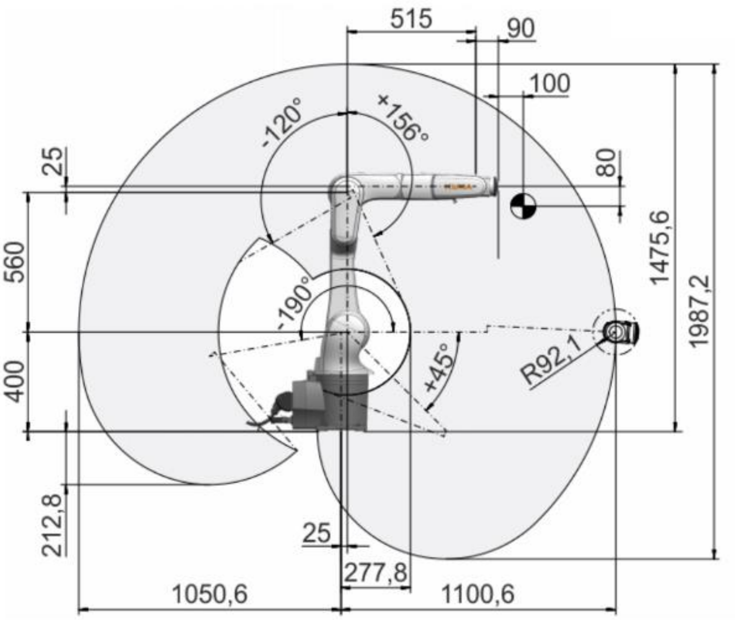
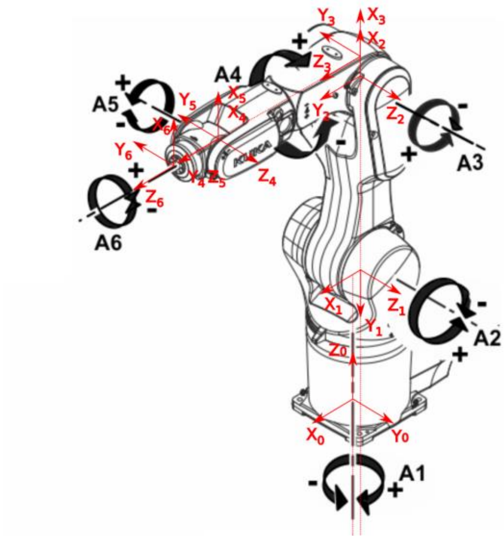
Github project: https://github.com/BorisAnimal/ik-fk-computing

Description of KUKA KR 10 R1100-2





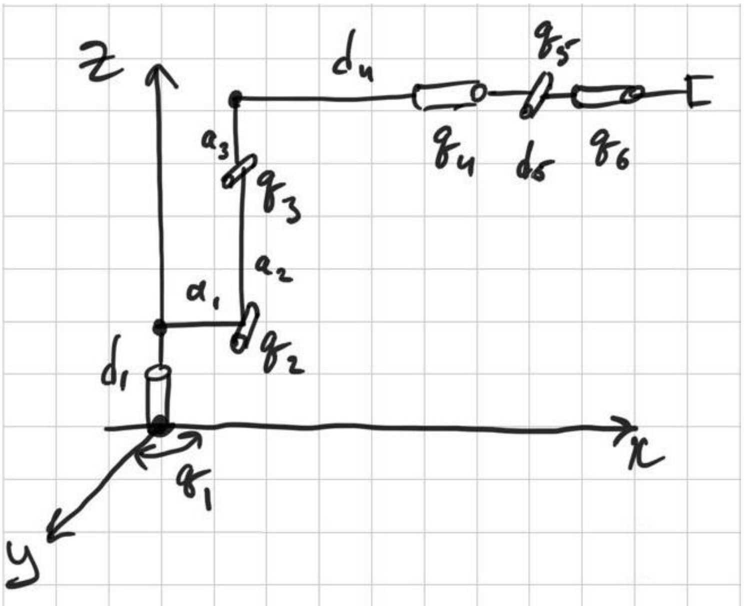
DH parameters

	Θ, degree	d, mm	a, mm	α, degree
1	0	400	25	-90
2	-90	0	560	0
3	0	0	25	-90
4	0	515	0	90
5	0	0	0	-90
6	0	90	0	0

Forward kinematics calculation formula

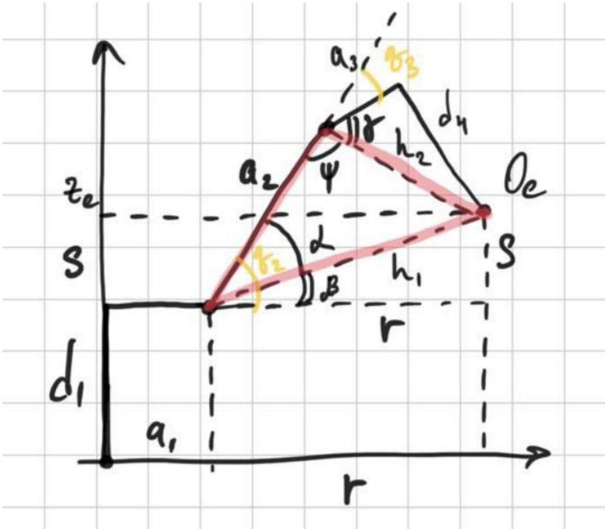
$$Tr = R_z(q_1) * T_z(d_1) * T_x(a_1) * R_y(q_2) * T_z(a_2) * R_y(q_3) * T_z(a_3) * T_x(d_4) * R_x(q_4) *$$
$$* R_y(q_5) * T_x(d_6) * R_x(q_6)$$

Kinematic scheme with description of the parameters.



Step by step explanation of inverse kinematics solution.

Arm orientation:



If O - expected end effector position and R - derived from RPY rotation matrix,

$$O_c = O - R * d_5$$

$$r = \sqrt{x_c^2 + y_c^2} - a_1$$

$$s = z_c - d_1$$

$$\gamma = atan2(d_4, a_3)$$

$$h_1 = \sqrt{r^2 + s^2}$$

$$h_2 = \sqrt{d_4^2 + a_3^2}$$

$$cos\psi = \frac{h_2^2 + a_2^2 - h_1^2}{2h_2a_2} = D$$

$$\psi = atan2(\pm\sqrt{1 - D^2}, D)$$

$$q_3 = 180 - \psi - \gamma, (2\ cases)$$

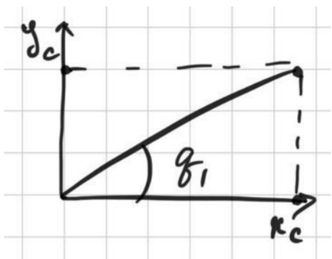
$$\beta = atan2(s, r)$$

$$sin\alpha = \frac{sin\psi * h_2}{h_1} = B,\ (see\ red\ triangle\ on\ pic)$$

$$\alpha = atan2(B, \pm\sqrt{1 - B^2}),\ (2\ cases\ above)$$

$$q_2 = \alpha + \beta$$

$$q_1 = atan2(y_c, x_c)$$



Wrist orientation:

$$R_6^0 = R_3^0 * R_6^3$$

$$R_6^3 = (R_3^0)^T * R$$

$$R_6^3 = \begin{pmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 \\ -s_5 c_6 & s_5 s_6 & c_5 \end{pmatrix}$$

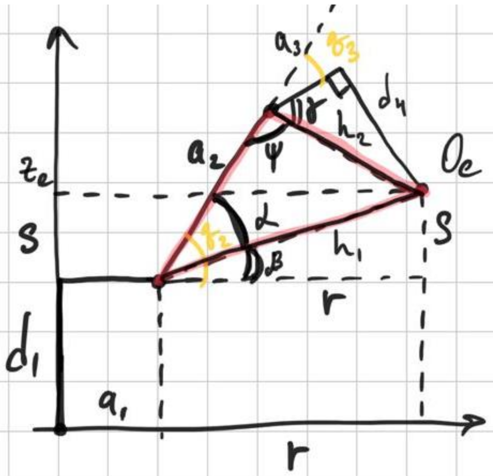
From here we can derive general case:

$$q_4 = atan2(R_{2,3}, R_{1,3})$$

$$q_5 = atan2(R_{2,3}, -R_{3,3} * S(q_3))$$

$$q_6 = atan2(-R_{3,2}, R_{3,1})$$

Direct calculations of Forward kinematics



$$h_2 = \sqrt{a_3^2 + d_4^2}$$

$$\angle\gamma = \arcsin\left(\frac{d_4}{a_3}\right)$$

$$\angle\psi = 180^\circ - \angle q_3 - \angle\gamma$$

$$h_1 = \sqrt{h_2^2 + a_2^2 - 2h_2 a_2 \cdot \cos\psi}$$

$$\angle\alpha = \arcsin\left(\frac{h_2 \cdot \sin\psi}{h_1}\right)$$

$$\angle\beta = \angle q_2 - \angle\alpha$$

$$s = h_1 \cdot \sin\beta$$

$$r = h_1 \cdot \cos\beta$$

$$z_c = d_1 + s$$

$$x_c = (a_1 + r) \cdot \cos q_1$$

$$y_c = (a_1 + r) \cdot \sin q_1$$

$$O = (z_c, x_c, y_c)^T + R_6^3 \cdot \vec{d_6}$$