Homework #3

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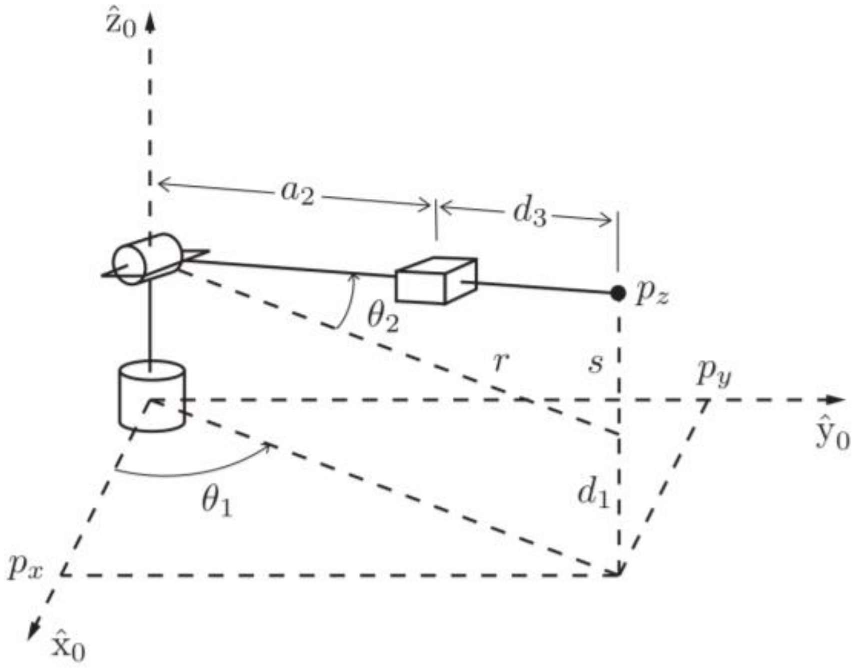
Github link: github.com/BorisAnimal/jacobian-practice

Forward kinematics



$$R_z(q_1) * T_z(d_1) * R_y(q_2) * T_x(a_2 + d_3)$$

$$r = (a_2 + d_3)cos(q_2)$$

$$s = (a_2 + d_3)sin(q_2)$$

$$z = d_1 + s = d_1 + (a_2 + d_3)S_2$$

$$x = r * cos(q_1) = (a_2 + d_3)C_1C_2$$

$$y = r * sin(q_1) = (a_2 + d_3)S_1C_2$$

Inverse kinematics

2 cases covered

$$x, y, z \to q_1, q_2, d_3$$

$$r = \sqrt{x^2 + y^2}$$

$$s = z - d_1$$

Case 1

$$q_{1,1} = atan2(y, x)$$

$$q_{2,1} = atan2(s, r)$$

$$q_{3,1} = \sqrt{r^2 + s^2} - a_2$$

Case 2

$$q_{1,2} = atan2(y, x) + \pi$$

$$q_{2,2} = \pi - atan2(s, r)$$

$$q_{3,2} = \sqrt{r^2 + s^2} - a_2$$

#4 Singularity analysis

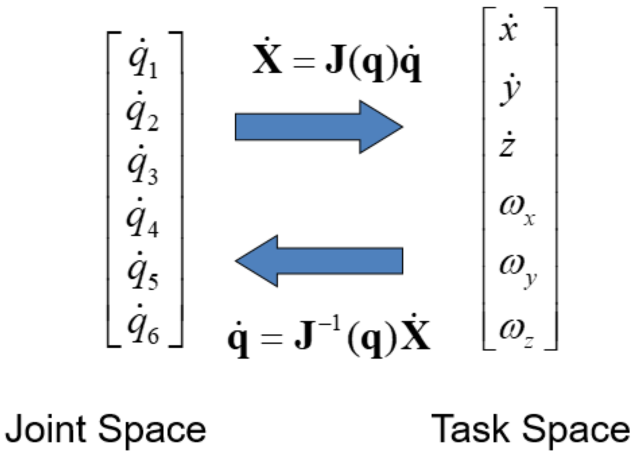
1) a2 + d3 = 0
    a. Means d3 translates EE directly to q2 position
2) q2 = pi/2
    a. Means arm oriented straight to top
    b. Due to this robot is 3 DOF, it’s XYZ mapping of Jacobian becomes singular

#5 Plot discussion

Given:

$$q(t) = [sin(t), cos(2t), sin(3t)]^T$$

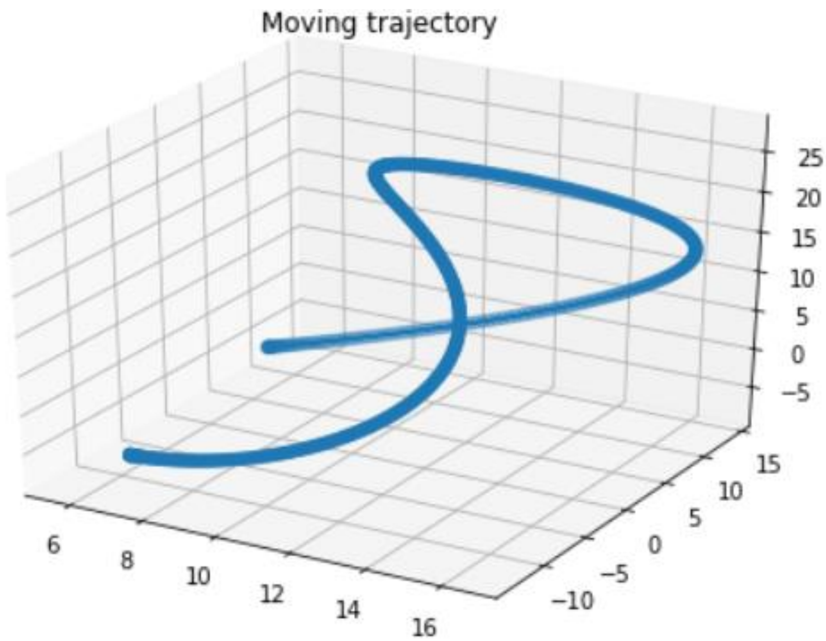
According to definition of Jacobian matrix

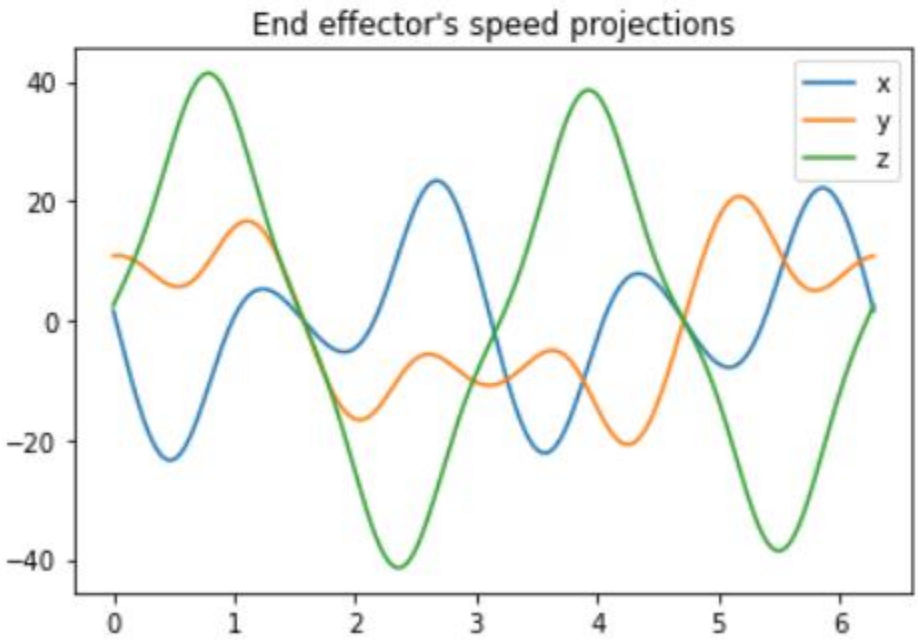


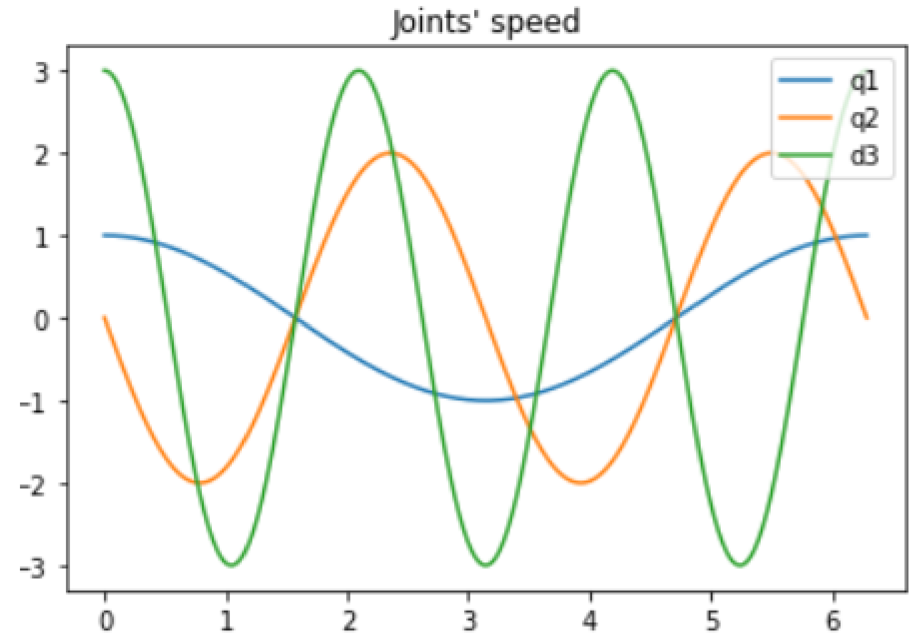
we can produce following plots, but let's starting from deriving

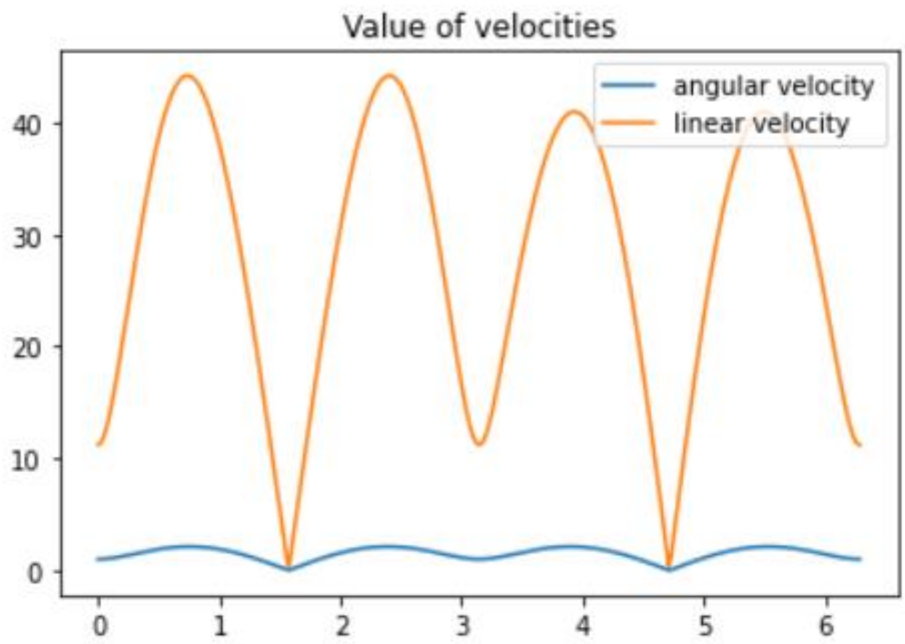
$$\dot{q}\,(t) = [cos(t), -2sin(2t), 3cos(3t)]^T$$

Plots




End effector's speed projections
x
y
z
40
20
0
−20
−40
0
1
2
3
4
5
6


Joints' speed
q1
q2
d3
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6


Value of velocities
angular velocity
linear velocity
40
30
20
10
0
0
1
2
3
4
5
6