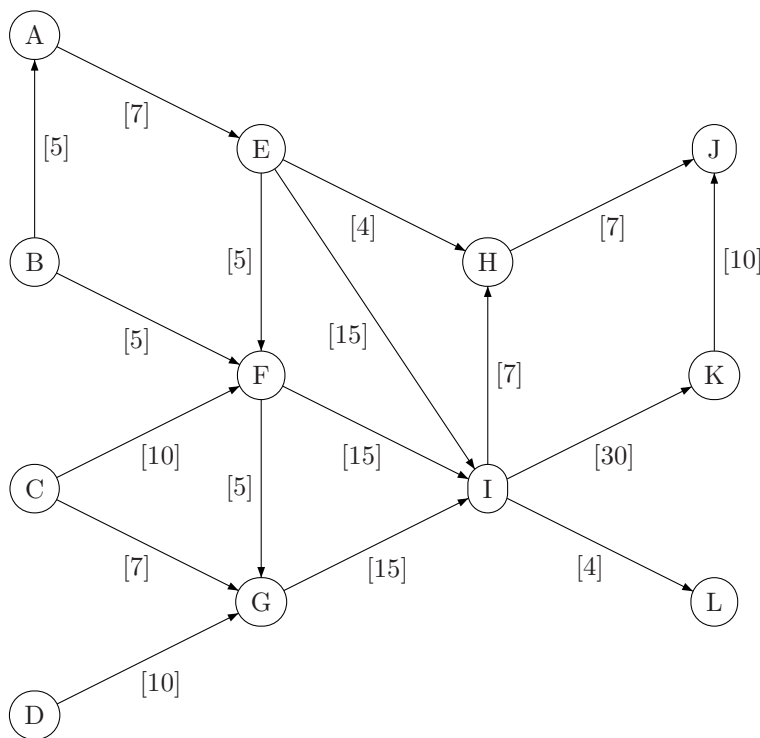


## Water pipes

### 1 Context

There are 4 water reserves A,B,C and D available for three cities J, K and L. The daily reserves availabilities are 15 thousand  $m^3$  for A, 10 B, 15 C and 15 for D. The redistribution system consists of both Roman aqueducts as well as recent pipelines. The whole system can be schematically represented by the graph below (the maximum flow rates are indicated on each arc in thousands of  $m^3$  per day) :



These three cities want to improve changing their redistribution system to meet the future needs. A study was made to determine the probable maximum daily demands, namely, for city J : 15 thousand  $m^3$ , for city K : 20 and 15 for city L.

### 2 Questions

1. Write a program that determines the value of the maximum flow in the current network and provides corresponding minimum cut.
2. The value of this flow is inadequate ; Municipalities of these three towns decide to redesign the pipes (A, E) and (I, L). Determine the optimal capacities for these two pipes and the value of the new optimal flow. Justify your answers.

3. Given the importance of the work, the municipalities decide not to change the two pipes at the same time. In which order should the municipalities make the replacement in order to increase the flow after each phase of the work? Justify your answer.
4. What is the value of the optimal flow after each phase of work?

Remark : your program should be general enough to be applied to any graph.