UNIVERSIDAD DE CHILE
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS
DEPARTAMENTO DE ASTRONOMÍA

STUDY OF THE ROLE OF GAS IN THE FORMATION OF SMBH SEED, THROUGH SIMULATIONS OF NSC WITH GAS EMBEDDED

TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS, MENCIÓN ASTRONOMÍA

BORIS CUEVAS GOMEZ

PROFESOR GUÍA:
Andrés Escala

MIEMBROS DE LA COMISIÓN:
PROFESOR 2
PROFESOR 3

Este trabajo ha sido parcialmente financiado por:
NOMBRE INSTITUCIÓN

SANTIAGO DE CHILE
2025

RESUMEN DE LA MEMORIA PARA OPTAR
AL TÍTULO DE MAGÍSTER EN CIENCIAS
DE LA INGENIERÍA
POR: BORIS CUEVAS GOMEZ
FECHA: 2025
PROF. GUÍA: NOMBRE PROFESOR

STUDY OF THE ROLE OF GAS IN THE FORMATION OF SMBH SEED, THROUGH SIMULATIONS OF NSC WITH GAS EMBEDDED

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Una frase de dedicatoria,
pueden ser dos líneas.

Saludos

Acknowledgments

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introducción

1.1. SMBH background

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.1.1. Una breve introducción

Este es un párrafo, puede contener múltiples 'Expresiones' así como fórmulas o referencias[1] como (1.1). A continuación se muestra un ejemplo de inserción de imágenes (como la Figura 1.1) con el comando \insertimage:

Figure 1.1: Where are you? de 'Internet'.

A continuación[2] se muestra un ejemplo de inserción de ecuaciones simples con el comando \insertequation:

$$a^k = b^k + c^k \quad \forall k > 2 \tag{1.1}$$

[1] Las referencias se hacen utilizando la expresión `\label`{etiqueta}.

[2] Como se puede observar las funciones `\insert`... añaden un párrafo automáticamente.

Este template ha sido diseñado para que sea completamente compatible con editores LaTeX para escritorio y de manera online(Overleaf, 2024). La compilación es realizada siempre usando las últimas versiones de las librerías, además se incluyen los parches oficiales para corregir eventuales *warnings*.

Este es un nuevo párrafo. Para crear un nuevo párrafo basta con usar \\ en el anterior, lo que fuerza una nueva línea. También se insertar un nuevo párrafo con el comando `\newp` si el compilador de latex arroja una alerta del tipo *Underfull \hbox (badness 10000) in paragraph at lines ...*

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

1.2. Aims and objectives

También puedes usar tablas, ¡Crearlas es muy fácil!. Puedes usar el plugin Excel2Latex (Excel2Latex, 2017) de Excel para convertir las tablas a LaTeX o bien utilizar el 'creador de tablas online' (Generator, 2024).

Table 1.1: Ejemplo de tablas.

Columna 1	**Columna 2**	**Columna 3**
ω	ν	δ
Φ	Θ	$\mathit{\Sigma}$
$\mathbb{R}$	$\mathbb{E}$	ψ

Table 1.2: Ejemplo de tablas con colores de filas.

Valor A	**Valor B**	**Valor C**	**Valor Esperado**
1	a	$3x$	Cumple
2	b	$6x$	No cumple
3	c	$3x + y$	Quizás
4	d	$5 \sin x$	No
5	e	0	Sí

1.3. Previous work

El template por defecto está configurado para trabajar con citas de la librería natbib, y se configuró al estilo *ieeetr*. Puedes usar otros estilos cambiando la configuración `\natbibrefstyle` si es que usas natbib. También se da soporte a las librerías **bibtex** y **apacite**, para ello puedes cambiar la configuración `\stylecitereferences`. Una completa guía de estilos la puedes consultar en https://latex.ppizarror.com/doc/bibstylescompared.pdf.

A continuación se detallan algunos links de ayuda para el uso de las referencias:

- Galería de estilos numéricos por corchetes
- Galería de estilos por autor/fecha
- Guía básica referencias Mendeley
- Guía completa de estilos

Chapter 2

Mathematical model

2.1. density criterion

2.2. Stars and gas in equilibrium

Chapter 3

A method for coupling gas and stars

In this work, we investigate the effect of gas in the process of seeds for SMBH formation. In contrast with other works, we do not take into account gas accretion by stars, since we are interested in the gravitational effect of gas over stars and vice versa.

We need, in principle, three codes: one for the stars, one for the gas and a coupling code that make gravitational interactions between gas and stars.

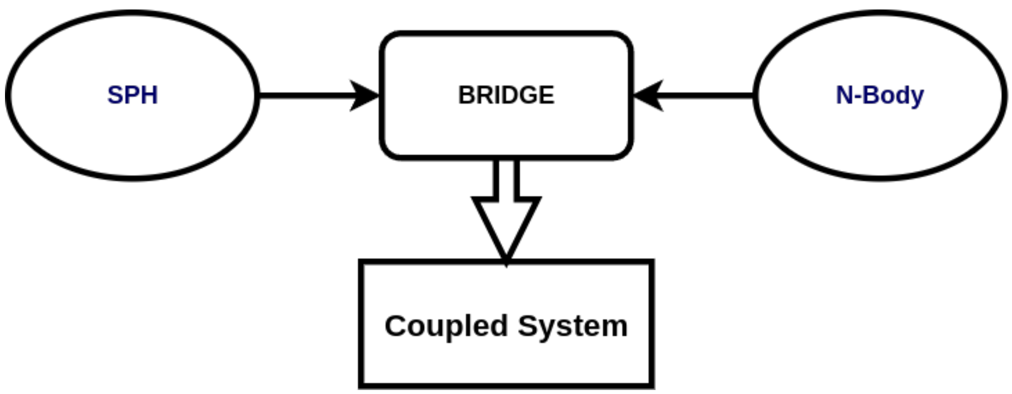

Figure 3.1: Codes scheme

The N-Body and SPH codes are used through the AMUSE interface, and are coupling with an external code.

3.1. N-body code: Ph4

The proposed scenarios presented in (referencia a capitulo) require the modeling of dense stellar clusters. And as we need to consider close encounters and collisions, is necessary a precise code.

We use the PH4 code from the AMUSE interface. In the present section, we include a description of the code, as some external routines.

3.1.1. Hermite integrator

The Ph4 code uses an fourth-order Hermite integrator (Makino & Aarseth, 1992) for calculating the position and velocities of stars due to gravity. This is a predictor-corrector method.

The fourth-order Hermite scheme is based in a precise calculation of the individual time step. Considering the particle i with own time t_i, time step Δt_i, position x_i, velocity v_i, acceleration a_i and jerk $\dot{a}_i$, calculated at time t_i. The algorithm of integration process proceeds as follows:

1. Select the particle with minimum $t_i + \Delta t_i$. Set the global time as $t = t_i + \Delta t_i$.
2. Calculate the predicted positions (x_p) and velocities (v_p) for all particles, using the actual values for x, v, a, $\dot{a}$.
3. Calculate the acceleration (a_i) and jerk ($\dot{a}_i$) for particle i at time $t_i + \Delta t_i$ using the predicted positions and velocities.
4. Calculate the second and third time derivative of acceleration ($a_i^{(2)}$ and $a_i^{(3)}$), using an Hermite interpolation.
5. Add the corrections to position and velocities of particle i.
6. Calculate and update the time step Δt_i.
7. Repeat the algorithm.

From appendix A, the predicted positions and velocities are:

$$\mathbf{x}_{p,j} = \mathbf{x}_j + \mathbf{v}_j(t - t_j) + \frac{\mathbf{a}_{0,j}}{2!}(t - t_j)^2 + \frac{\dot{\mathbf{a}}_{0,j}}{3!}(t - t_j)^3 \tag{3.1}$$

$$\mathbf{v}_{p,j} = \mathbf{v}_j + \mathbf{a}_{0,j}(t - t_j) + \frac{\dot{\mathbf{a}}_{0,j}}{2!}(t - t_j)^2 \tag{3.2}$$

Where $\mathbf{x}_j$, $\mathbf{v}_j$ are the positions and velocities at time t_i. Note that the index i runs for all particles. The acceleration $\mathbf{a}_j$ and jerk $\dot{\mathbf{a}}_j$ are computed by equations:

$$\mathbf{a}_{0,j} = \sum_{k \neq j} G m_k \frac{\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} \tag{3.3}$$

$$\dot{\mathbf{a}}_{0,j} = \sum_{k \neq j} G m_k \left[\frac{\mathbf{v}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} - \frac{3(\mathbf{v}_{0,jk} \cdot \mathbf{r}_{0,jk})\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{5/2}} \right] \tag{3.4}$$

$$\mathbf{r}_{0,jk} = \mathbf{r}_k - \mathbf{r}_j \tag{3.5}$$

$$\mathbf{v}_{0,jk} = \mathbf{v}_k - \mathbf{v}_j \tag{3.6}$$

From the Hermite interpolation, we can have an estimation of the second and third time derivative of acceleration. This estimation is used then for correct the positions and velocities.

$$\mathbf{a}_{0,i}^{(2)} = \frac{-6(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) - \Delta t_i(4\dot{\mathbf{a}}_{0,i} + 2\dot{\mathbf{a}}_{1,i})}{{\Delta t_i}^2} \tag{3.7}$$

$$\mathbf{a}_{0,i}^{(3)} = \frac{12(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) + 6(\dot{\mathbf{a}}_{0,i} + \dot{\mathbf{a}}_{1,i})\Delta t_i}{{\Delta t_i}^3} \tag{3.8}$$

After that, the correction over position and velocity is as follows:

$$\mathbf{x}_i(t_i + \Delta t_i) = \mathbf{x}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)} \Delta t_i^{\ 4}}{24} + \frac{\mathbf{a}_{0,i}^{(3)} \Delta t_i^{\ 5}}{120} \tag{3.9}$$

$$\mathbf{v}_i(t_i + \Delta t_i) = \mathbf{v}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)} \Delta t_i^{\ 3}}{6} + \frac{\mathbf{a}_{0,i}^{(3)} \Delta t_i^{\ 4}}{24} \tag{3.10}$$

Where corrections of four order have been made. The advantage of this method is that is only needed to calculate the acceleration and jerk

3.1.2. Block time steps

3.1.3. Collisions and mergers

3.1.4. Escaping stars

3.2. Smoothed particles hydrodynamics: Fi

3.3. Coupling strategy: Bridge

Chapter 4

Results and analysis

Chapter 5

El desarrollo de la tesis

5.1. Aquí una nueva sección

5.1.1. Haciendo una tesis como un profesional

Figure 5.1: Apolo flotando a la izquierda.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Proin sit amet augue. Praesent lacus. Donec a leo. Ut turpis ante, condimentum sed, sagittis a, blandit sit amet, enim. Integer sed elit. In ultricies blandit libero. Proin molestie erat dignissim nulla convallis ultrices. Aliquam in magna. Etiam sollicitudin, eros a sagittis pellentesque, lacus odio volutpat elit, vel tincidunt felis dui vitae lorem. Etiam leo. Nulla et justo.

$$\int_a^b f(x)\,\mathrm{d}x = \frac{\partial^\eta f(x)}{\partial x^\eta} \cdot \textstyle\sum_{x=a}^b f(x) \cancelto{1+\frac{\epsilon}{k}}{\left(1+\Delta x\right)} \tag{5.1}$$

Ecuación sin sentido.

Definition 5.1 (ver (Einstein, 1905)) *Definición definitiva*

$$\frac{d}{dx}\int_a^x f(y)dy = f(x)$$

Proin sit amet augue. Praesent lacus. Donec a leo. Ut turpis ante, condimentum sed, sagittis a, blandit sit amet, enim. Integer sed elit. In ultricies blandit libero. Proin molestie erat dignissim nulla convallis ultrices. Aliquam in magna. Etiam sollicitudin, eros a sagittis pellentesque, lacus odio volutpat elit, vel tincidunt felis dui vitae lorem. Etiam leo. Nulla et justo.

5.1.2. Otros párrafos más normales

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

$$\begin{gathered}\Lambda_f = \frac{L \cdot f}{W} \cdot \frac{{Q_e}^2}{8\pi^2 W^4 g} + \sum_{i=1}^{l} \frac{f \cdot \left(M - d\right)}{l \cdot W} \cdot \underbrace{\frac{\left(Q_e - i \cdot Q\right)^2}{8\pi^2 W^4 g}}_{\sim\mathcal{A}} \\ Q_e = 2.5Q \cdot \int_0^e V(x)\,\mathrm{d}x + \sin^{-1}\left(1 + \frac{1}{1-e}\right)\end{gathered} \tag{5.2}$$

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

$$\boxed{f(x) = \frac{\partial^2 u}{\partial t^2}} \tag{5.3}$$

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

5.1.3. Ejemplos de inserción de código fuente

El template permite la inserción de los siguientes lenguajes de programación de forma nativa: ABAP, Ada, Assembler x64, Assembler x86[masm], Awk, Bash, Basic, C, Caml, CMake, Cobol, C++, C#, CSS, CSV, CUDA, Dart, Docker, Elisp, Elixir, Erlang, Fortran, F#, GLSL, Gnuplot, Go, Haskell, HTML, INI, Java, Javascript, JSON, Julia, Kotlin, LaTeX, Lisp, LLVM, Lua, Make, Maple, Mathematica, Matlab, Mercury, Modula-2, Objective-C, Octave, OpenCL, OpenSees, Pascal, Perl, PHP, Texto plano, PostScript, Powershell, Prolog, Promela, Pseudocódigo, Python, Q#, R, Racket, Reil, Ruby, Rust, Scala, Scheme, Scilab, Simula, SPARQL, SQL, Swift, TCL, VBScript, Verilog, VHDL y XML.

Para insertar un código fuente se debe usar el entorno **sourcecode**, o el entorno **sourcecodep** si es que se quiere utilizar parámetros adicionales. A continuación se presenta un ejemplo de inserción de código fuente en Python (Código 5.1), Java y Matlab:

Code 5.1: Ejemplo en Python.

```python
import numpy as np
def incmatrix(genl1, genl2):
    m = len(genl1)
    n = len(genl2)
    M = None # Comentario 1
    VT = np.zeros((n*m, 1), int) # Comentario 2
```

Code 5.2: Ejemplo en Java.

```java
import java.io.IOException;
import javax.servlet.*;

public class Hola extends GenericServlet { // Hola mundo
    public void service(ServletRequest request, ServletResponse response)
    throws ServletException, IOException{
        response.setContentType("text/html");
        PrintWriter pw = response.getWriter();
        pw.println("Hola, mundo!");
    }
}
```

Code 5.3: Ejemplo en Matlab.

```matlab
% Se crea gráfico
f = figure(1);
title('Espectro de pulso de desplazamiento');

for j = 1:length(BETA)
    fad = ones(1, NDATOS); % Arreglo para el FAD
    for i = 1:NDATOS
        [t, u_t, ~, ~] = main(BETA(j), r(i), M, K, F0, 0);
        fad(i) = max(abs(u_t)) / uf0;
```

```
    end
end
```

5.1.4. Agregando múltiples imágenes

El template ofrece el entorno images que permite insertar múltiples imágenes de una manera muy sencilla. Para crear imágenes múltiples se deben usar las siguientes instrucciones:

```
\begin{images}[\label{imagenmultiple}]{Ejemplo de imagen múltiple.}
  \addimage[\label{ciudadfoto}]{ejemplos/test-image}{width=6.5cm}{Ciudad}
  \addimageanum{ejemplos/test-image-wrap}{height=4cm}
  \imagesnewline
  \addimage{ejemplos/test-image}{width=11cm}{Ciudad más grande}
\end{images}
```

Obteniendo así:

(a) Ciudad

(b) Ciudad más grande

Figure 5.2: Ejemplo de imagen múltiple.

Más ejemplos

Listas y Enumeraciones

Hacer listas enumeradas con LaTeX es muy fácil con el template[3], también puedes revisar el manual (Pizarro, 2024), para ello debes usar el comando `\begin{enumerate}`, cada elemento comienza por `\item`, resultando así:

1. Grecia
2. Abracadabra
3. Manzanas

También se puede cambiar el tipo de enumeración, se pueden usar letras, números romanos, entre otros. Esto se logra cambiando el **label** del objeto `enumerate`. A continuación se muestra un ejemplo usando letras con el estilo `\alph`[4], números romanos con `\roman`[5] o números griegos con `\greek`[6]:

a) Peras

b) Manzanas

c) Naranjas

i) Rojo

ii) Café

iii) Morado

α) Matemáticas

β) Lenguaje

γ) Filosofía

Para hacer listas sin numerar con LaTeX hay que usar el comando `\begin{itemize}`, cada elemento empieza por `\item`, resultando:

– Peras

– Manzanas

– Naranjas

* Rojo

* Café

* Morado

• Árboles

• Pasto

• Flores

[3] También puedes revisar el manual de las enumeraciones en https://latex.ppizarror.com/doc/enumitem.pdf.
[4] Con `\Alph` las letras aparecen en mayúscula.
[5] Con `\Roman` los números romanos salen en mayúscula.
[6] Una característica propia del template, con `\Greek` las letras griegas están escritas en mayúscula.

Chapter 6

Conclusions

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

References

Aarseth, S. J. (1985). Direct methods for n-body simulations. In J. U. Brackbill & B. I. Cohen (Eds.), *Multiple Time Scales* (pp. 377–418). Academic Press.

Einstein, A. (1905). Zur elektrodynamik bewegter körper. *Annalen der Physik*, 322(10), 891–921.

Excel2Latex (2017). Plugin para hacer tablas con excel.

Generator, T. (2024). Creador de tablas online para latex.

Makino, J. & Aarseth, S. J. (1992). On a hermite integrator with ahmad-cohen scheme for gravitational many-body problems. *Publications of the Astronomical Society of Japan*, 44(2), 141–151.

Overleaf (2024). Editor online para latex.

Pizarro, P. (2024). Template informe en latex.

Appendices

Appendix A. Fourth-Order Hermite Integration Scheme

The N-body code used in this work, `ph4`, is based on a fourth-order Hermite integrator. The following subsections describe in detail the individual time step scheme (ITS) from (Makino & Aarseth, 1992), including the time step criterion in the N-body code.

A.1. Step 1: Particle selection

The first step is to select the next particle to be advanced. At any given time, each particle j has its own mass m_j, time t_j, time step Δt_j, coordinates $\mathbf{x}_j$ and $\mathbf{v}_j$, acceleration $\mathbf{a}_j$ and jerk $\dot{\mathbf{a}}_j$ (with $\dot{\mathbf{a}}_j = \partial \mathbf{a}_j / \partial t$). The selected particle i is the one satisfying the condition:

$$i = \arg\min_j (t_j + \Delta t_j) \tag{A.1}$$

Note that always $t_j \leq t_i + \Delta t_i$, for all j. The current simulation time is then set to $t = t_i + \Delta t_i$.

A.2. Step 2: Calculate the predicted position and velocities for all particles

The positions and velocities of all particles are predicted at time t, using a Taylor expansion up to third-order (jerk):

$$\mathbf{x}_{p,j} = \mathbf{x}_j + \mathbf{v}_j(t - t_j) + \frac{\mathbf{a}_{0,j}}{2!}(t - t_j)^2 + \frac{\dot{\mathbf{a}}_{0,j}}{3!}(t - t_j)^3 \tag{A.2}$$

$$\mathbf{v}_{p,j} = \mathbf{v}_j + \mathbf{a}_{0,j}(t - t_j) + \frac{\dot{\mathbf{a}}_{0,j}}{2!}(t - t_j)^2 \tag{A.3}$$

The acceleration is due only to gravitational force. The expression for each particle j is:

$$\mathbf{a}_{0,j} = \sum_{k \neq j} G m_k \frac{\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} \tag{A.4}$$

where ϵ is the softening parameter, and

$$\mathbf{r}_{0,jk} = \mathbf{r}_k - \mathbf{r}_j \tag{A.5}$$

Differentiating Eq. (A.4) with respect to time yields the jerk:

$$\dot{\mathbf{a}}_{0,j} = \frac{\partial \mathbf{a}_j}{\partial t} \tag{A.6}$$

$$= \sum_{k \neq j} G m_k \frac{\partial}{\partial t} \frac{\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} \tag{A.7}$$

$$\dot{\mathbf{a}}_{0,j} = \sum_{k \neq j} G m_k \left[\frac{\mathbf{v}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} - \frac{3(\mathbf{v}_{0,jk} \cdot \mathbf{r}_{0,jk})\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{5/2}} \right] \tag{A.8}$$

where

$$\mathbf{v}_{0,jk} = \mathbf{v}_k - \mathbf{v}_j \tag{A.9}$$

A.3. Step 3: Calculate acceleration and jerk for particle i

Similar to equations (A.4) and (A.8), the acceleration and jerk are calculated using the predicted positions and velocity:

$$\mathbf{a}_{1,i} = \sum_{j \neq i} G m_j \frac{\mathbf{r}_{ij}}{(\mathbf{r}_{ij}^2 + \epsilon^2)^{3/2}} \tag{A.10}$$

$$\dot{\mathbf{a}}_{1,i} = \sum_{j \neq i} G m_j \left[\frac{\mathbf{v}_{ij}}{(\mathbf{r}_{ij}^2 + \epsilon^2)^{3/2}} - \frac{3(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij})\mathbf{r}_{ij}}{(\mathbf{r}_{ij}^2 + \epsilon^2)^{5/2}} \right] \tag{A.11}$$

$$\mathbf{r}_{ij} = \mathbf{r}_{p,j} - \mathbf{r}_{p,i} \tag{A.12}$$

$$\mathbf{v}_{ij} = \mathbf{v}_{p,j} - \mathbf{v}_{p,i} \tag{A.13}$$

In this notation, the subscript 0 denotes quantities evaluated at the beginning of the time step (t_i), while the subscript 1 denotes quantities evaluated at the end of the time step (t), using the predicted position and velocity ($\mathbf{x}_{p,i}$ and $\mathbf{v}_{p_i}$).

A.4. Step 4: Second and third time derivative of acceleration, using Hermite interpolation

To obtain a high-order correction of $\mathbf{x}$ and $\mathbf{v}$, the acceleration of particle i is modeled as a third-order polynomial in time:

$$\mathbf{a}(\tau) = \alpha\tau^3 + \beta\tau^2 + \gamma\tau + \delta \tag{A.14}$$

The successive time derivatives of $\mathbf{a}(\tau)$ are then:

$$\dot{\mathbf{a}}(\tau) = 3\alpha\tau^2 + 2\beta\tau + \gamma \tag{A.15}$$

$$\mathbf{a}^{(2)}(\tau) = 6\alpha\tau + \beta \tag{A.16}$$

$$\mathbf{a}^{(3)}(\tau) = 6\alpha \tag{A.17}$$

The coefficients α, β, γ, and δ are determined by using the known values of acceleration and jerk at times t_i and t. This provides a system of four equations:

$$\mathbf{a}(t_i) = \mathbf{a}_{0,i} = \alpha t_i^3 + \beta t_i^2 + \gamma t_i + \delta \tag{A.18}$$
$$\mathbf{a}(t) = \mathbf{a}_{1,i} = \alpha t^3 + \beta t^2 + \gamma t + \delta \tag{A.19}$$
$$\dot{\mathbf{a}}(t_i) = \dot{\mathbf{a}}_{0,i} = 3\alpha t_i^2 + 2\beta t_i + \gamma \tag{A.20}$$
$$\dot{\mathbf{a}}(t) = \dot{\mathbf{a}}_{1,i} = 3\alpha t^2 + 2\beta t + \gamma \tag{A.21}$$

where $t = t_i + \Delta t_i$ is the current time. As the system of equations is linearly independent, the four coefficients can be determined. However, determining α and β is enough to get the second and third time derivative of acceleration. From (A.21):

$$\gamma = \dot{\mathbf{a}}_{1,i} - 3\alpha t^2 - 2\beta t \tag{A.22}$$

Subtracting Eq. (A.18) from (A.19) and using Eq. (A.22) gives:

$$\beta = \frac{(\dot{\mathbf{a}}_{1,i} - \dot{\mathbf{a}}_{0,i}) - \alpha 3(t^2 - t_i^2)}{2(t - t_i)} \tag{A.23}$$

Subtracting Eq. (A.18) from (A.19), and using (A.23) gives an expression for α:

$$\alpha = \frac{(\dot{\mathbf{a}}_{1,i} + \dot{\mathbf{a}}_{0,i})(t - t_i) - 2(\mathbf{a}_{1,i} - \mathbf{a}_{0,i})}{(t - t_i)^3} \tag{A.24}$$

Replacing this result in Eq. (A.23):

$$\beta = \frac{-\dot{\mathbf{a}}_{1,i}(t + 2t_i) - \dot{\mathbf{a}}_{0,i}(2t + t_i)}{(t - t_i)^2} + \frac{3(\mathbf{a}_{0,i} - \mathbf{a}_{1,i})(t + t_i)}{(t - t_i)^3} \tag{A.25}$$

The second and third time derivatives of the acceleration at time t_i can now be evaluated. For the Eq. (A.17), and using $t - t_i = \Delta t_i$:

$$\mathbf{a}_{0,i}^{(3)} = \mathbf{a}^{(3)}(t_i) = 6\alpha \tag{A.26}$$
$$\mathbf{a}_{0,i}^{(3)} = 6\frac{(\dot{\mathbf{a}}_{1,i} + \dot{\mathbf{a}}_{0,i})\Delta t_i - 2(\mathbf{a}_{1,i} - \mathbf{a}_{0,i})}{\Delta t_i{}^3} \tag{A.27}$$
$$\mathbf{a}_{0,i}^{(3)} = \frac{12(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) + 6(\dot{\mathbf{a}}_{0,i} + \dot{\mathbf{a}}_{1,i})\Delta t_i}{\Delta t_i{}^3} \tag{A.28}$$

Evaluating the second time derivative in t_i, and replacing the obtained for α and β:

$$\mathbf{a}_{0,i}^{(2)} = \mathbf{a}^{(2)}(t_i) = 6\alpha t_i + \beta \tag{A.29}$$
$$\mathbf{a}_{0,i}^{(2)} = \frac{-6(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) - \Delta t_i(4\dot{\mathbf{a}}_{0,i} + 2\dot{\mathbf{a}}_{1,i})}{\Delta t_i{}^2} \tag{A.30}$$

A.5. Step 5: Add corrections

The next step in the scheme is to correct position and velocity of particle i at time t. Using a higher-order Taylor expansion around t_i:

$$\mathbf{x}_i(t_i+\Delta t_i) = \mathbf{x}_i(t_i) + \dot{\mathbf{x}}_i(t_i)\Delta t_i + \frac{\mathbf{x}_i^{(2)}(t_i)\Delta t_i^{\,2}}{2!} + ... + \frac{\mathbf{x}_i^{(5)}(t_i)\Delta t_i^{\,5}}{5!} \tag{A.31}$$

$$= \mathbf{x}_i + \mathbf{v}_i\Delta t_i + \frac{\mathbf{a}_{0,i}\Delta t_i^{\,2}}{2} + \frac{\dot{\mathbf{a}}_{0,i}\Delta t_i^{\,3}}{6} + \frac{\mathbf{a}_{0,i}^{(2)}\Delta t_i^{\,4}}{24} + \frac{\mathbf{a}_{0,i}^{(3)}\Delta t_i^{\,5}}{120} \tag{A.32}$$

The first four terms of this expansion correspond to the predicted position $\mathbf{x}_{p,i}$ from Eq. (A.2). Therefore, the corrected position can be written as:

$$\mathbf{x}_i(t_i+\Delta t_i) = \mathbf{x}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)}\Delta t_i^{\,4}}{24} + \frac{\mathbf{a}_{0,i}^{(3)}\Delta t_i^{\,5}}{120} \tag{A.33}$$

In a similar way, the corrected velocity is:

$$\mathbf{v}_i(t_i+\Delta t_i) = \mathbf{v}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)}\Delta t_i^{\,3}}{6} + \frac{\mathbf{a}_{0,i}^{(3)}\Delta t_i^{\,4}}{24} \tag{A.34}$$

The advantage of this scheme is that achieving high integration accuracy only requires the explicit calculation of the acceleration and jerk for all particles, and the predicted acceleration and jerk for the particle being advanced. This results in an integration scheme that is fourth-order accurate in time ($\mathcal{O}(a^{(3)})$).

A.6. Step 6: Time step update

After updating the position and velocity of particle i, a new time step Δt_i must be calculated. This step is crucial for controlling the integration error. A proven and stable criterion is given by the standard formula (Aarseth, 1985):

$$\Delta t_i = \sqrt{\eta \frac{|\mathbf{a}_{1,i}||\mathbf{a}_{1,i}^{(2)}| + |\dot{\mathbf{a}}_{1,i}|^2}{|\dot{\mathbf{a}}_{1,i}||\mathbf{a}_{1,i}^{(3)}| + |\mathbf{a}_{1,i}^{(2)}|^2}} \tag{A.35}$$

The values for $\mathbf{a}_{1,i}$ and $\dot{\mathbf{a}}_{1,i}$ are known from the direct calculation in Step 3. The value of $\mathbf{a}^{(3)}(t)$ is constant, since a third-order polynomial interpolation is used. Only $\mathbf{a}_{1,i}^{(2)}$ should be calculated:

$$\mathbf{a}_{1,i}^{(2)} = \mathbf{a}_{0,i}^{(2)} + \Delta t_i \mathbf{a}_{0,i}^{(3)} \tag{A.36}$$

During the initialization of the algorithm, the higher-order derivatives of the acceleration are not available, so an alternative formula for time step can be used in the startup:

$$\Delta t = \eta_s \frac{|\mathbf{a}|}{|\dot{\mathbf{a}}|} \tag{A.37}$$

The suggested value for the startup parameter is $\eta_s \sim 0.01$ (Makino & Aarseth, 1992).

Appendix B. Cálculos realizados

B.1. Metodología

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Figure B.1: Imagen en anexo.

B.2. Resultados

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.

Table B.1: Tabla de cálculo.

Elemento	ϵ_i	**Valor**	**Descripción**
A	10	$3{,}14\pi$	Valor muy interesante[a]
B	20	6	Segundo elemento
C	30	7	Tercer elemento[1]
D	150	10	Sin descripción
E	0	0	Cero

[a] Este elemento tiene una descripción debajo de la tabla
[1] Más comentarios