UNIVERSIDAD DE CHILE
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS
DEPARTAMENTO DE ASTRONOMÍA

STUDY OF THE ROLE OF GAS IN THE FORMATION OF SMBH SEED, THROUGH SIMULATIONS OF NSC WITH GAS EMBEDDED

TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS, MENCIÓN ASTRONOMÍA

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DE LA INGENIERÍA
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Una frase de dedicatoria,
pueden ser dos líneas.

Saludos

Acknowledgments

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Chapter 1

Introducción

1.1. SMBH background

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1.1.1. Una breve introducción

Este es un párrafo, puede contener múltiples 'Expresiones' así como fórmulas o referencias[1] como (1.1). A continuación se muestra un ejemplo de inserción de imágenes (como la Figura 1.1) con el comando \insertimage:

Figure 1.1: Where are you? de 'Internet'.

A continuación[2] se muestra un ejemplo de inserción de ecuaciones simples con el comando \insertequation:

$$a^k = b^k + c^k \quad \forall k > 2 \tag{1.1}$$

[1] Las referencias se hacen utilizando la expresión `\label`{etiqueta}.
[2] Como se puede observar las funciones `\insert`... añaden un párrafo automáticamente.

Este template ha sido diseñado para que sea completamente compatible con editores LaTeX para escritorio y de manera online(Overleaf, 2024). La compilación es realizada siempre usando las últimas versiones de las librerías, además se incluyen los parches oficiales para corregir eventuales *warnings*.

Este es un nuevo párrafo. Para crear un nuevo párrafo basta con usar \\ en el anterior, lo que fuerza una nueva línea. También se insertar un nuevo párrafo con el comando `\newp` si el compilador de latex arroja una alerta del tipo *Underfull \hbox (badness 10000) in paragraph at lines ...*

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

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1.2. Aims and objectives

También puedes usar tablas, ¡Crearlas es muy fácil!. Puedes usar el plugin Excel2Latex (Excel2Latex, 2017) de Excel para convertir las tablas a LaTeX o bien utilizar el 'creador de tablas online' (Generator, 2024).

Table 1.1: Ejemplo de tablas.

Columna 1	**Columna 2**	**Columna 3**
ω	ν	δ
Φ	Θ	Σ
$\mathbb{R}$	$\mathbb{E}$	ψ

Table 1.2: Ejemplo de tablas con colores de filas.

Valor A	**Valor B**	**Valor C**	**Valor Esperado**
1	a	$3x$	Cumple
2	b	$6x$	No cumple
3	c	$3x + y$	Quizás
4	d	$5 \sin x$	No
5	e	0	Sí

1.3. Previous work

El template por defecto está configurado para trabajar con citas de la librería natbib, y se configuró al estilo *ieeetr*. Puedes usar otros estilos cambiando la configuración `\natbibreferstyle` si es que usas natbib. También se da soporte a las librerías **bibtex** y **apacite**, para ello puedes cambiar la configuración `\stylecitereferences`. Una completa guía de estilos la puedes consultar en https://latex.ppizarror.com/doc/bibstylescompared.pdf.

A continuación se detallan algunos links de ayuda para el uso de las referencias:

- Galería de estilos numéricos por corchetes
- Galería de estilos por autor/fecha
- Guía básica referencias Mendeley
- Guía completa de estilos

Chapter 2

Mathematical model

2.1. density criterion

2.2. Stars and gas in equilibrium

Chapter 3

A method for coupling gas and stars

In this work, we investigate the effect of gas on the formation seeds of SMBHs. In contrast with other works, we do not take into account gas accretion by stars, since we are interested exclusively in the gravitational interaction between gas and stars.

We require, in principle, three codes: one for the stars, one for the gas and a coupling code that handles gravitational interactions between gas and stars.

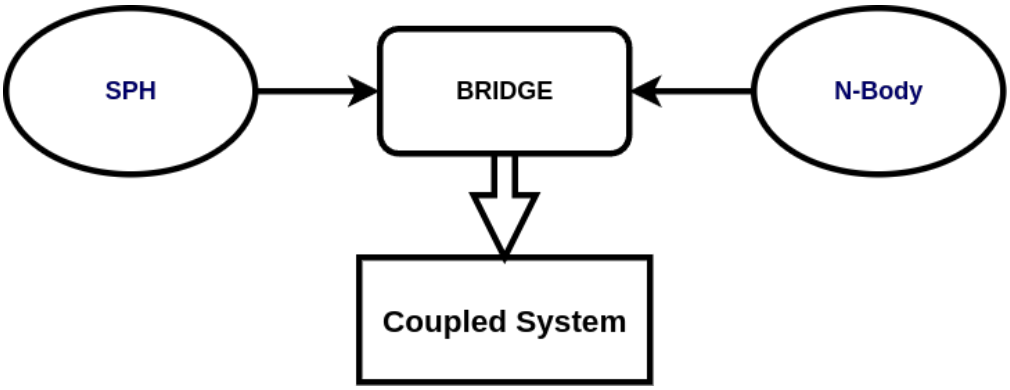

Figure 3.1: Schematic representation of the code coupling framework

The N-Body and SPH codes are accessed through the AMUSE interface, and coupled via an external code.

3.1. N-body code: Ph4

The proposed scenarios presented in (referencia a capitulo) require modeling dense stellar clusters. As we need to consider close encounters and collisions, a high-precision code is necessary.

The PH4 code from the AMUSE interface is used. In this section, we include a description of the code, and some external routines.

3.1.1. Hermite integrator

The Ph4 code uses a fourth-order Hermite integrator (Makino & Aarseth, 1992) for calculating the positions and velocities of stars due to gravity. This is a predictor-corrector method.

The fourth-order Hermite scheme is based on a precise calculation of the individual time

step. Considering the particle i with its own time t_i, time step Δt_i, position $\mathbf{x}_i$, velocity $\mathbf{v}_i$, acceleration $\mathbf{a}_i$ and jerk $\dot{\mathbf{a}}_i$, all calculated at time t_i. The integration algorithm proceeds as follows:

1. Select the particle with the minimum $t_i + \Delta t_i$. Set the global time to $t = t_i + \Delta t_i$.
2. Calculate the predicted positions ($\mathbf{x}_p$) and velocities ($\mathbf{v}_p$) for all particles, using the current values for $\mathbf{x}$, $\mathbf{v}$, $\mathbf{a}$, $\dot{\mathbf{a}}$.
3. Calculate the acceleration ($\mathbf{a}_i$) and jerk ($\dot{\mathbf{a}}_i$) for particle i at time $t_i + \Delta t_i$ using the predicted positions and velocities.
4. Calculate the second and third time derivatives of acceleration ($\mathbf{a}_i^{(2)}$ and $\mathbf{a}_i^{(3)}$) using Hermite interpolation.
5. Apply corrections to the position and velocity of particle i.
6. Calculate and update the time step Δt_i.
7. Repeat the algorithm.

A detailed derivation of the algorithm and the full set of equations are provided in Appendix A. From Eqs. (A.2) and (A.3), the predicted positions and velocities are:

$$\mathbf{x}_{p,j} = \mathbf{x}_j + \mathbf{v}_j(t - t_j) + \frac{\mathbf{a}_{0,j}}{2}(t - t_j)^2 + \frac{\dot{\mathbf{a}}_{0,j}}{3!}(t - t_j)^3 \tag{3.1}$$

$$\mathbf{v}_{p,j} = \mathbf{v}_j + \mathbf{a}_{0,j}(t - t_j) + \frac{\dot{\mathbf{a}}_{0,j}}{2!}(t - t_j)^2 \tag{3.2}$$

Here, $\mathbf{x}_j$ and $\mathbf{v}_j$ are the position and velocity of particle j at time t_j. The current acceleration $\mathbf{a}_{0,j}$ and jerk $\dot{\mathbf{a}}_{0,j}$ are computed as:

$$\mathbf{a}_{0,j} = \sum_{k \neq j} G m_k \frac{\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} \tag{3.3}$$

$$\dot{\mathbf{a}}_{0,j} = \sum_{k \neq j} G m_k \left[\frac{\mathbf{v}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} - \frac{3(\mathbf{v}_{0,jk} \cdot \mathbf{r}_{0,jk})\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{5/2}} \right] \tag{3.4}$$

$$\mathbf{r}_{0,jk} = \mathbf{r}_k - \mathbf{r}_j \tag{3.5}$$

$$\mathbf{v}_{0,jk} = \mathbf{v}_k - \mathbf{v}_j \tag{3.6}$$

Using Hermite interpolation, the second and third time derivatives of the acceleration can be estimated. This estimation is subsequently used to correct the positions and velocities. The relevant expressions, derived in the appendix, are:

$$\mathbf{a}_{0,i}^{(2)} = \frac{-6(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) - \Delta t_i(4\dot{\mathbf{a}}_{0,i} + 2\dot{\mathbf{a}}_{1,i})}{{\Delta t_i}^2} \tag{3.7}$$

$$\mathbf{a}_{0,i}^{(3)} = \frac{12(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) + 6(\dot{\mathbf{a}}_{0,i} + \dot{\mathbf{a}}_{1,i})\Delta t_i}{{\Delta t_i}^3} \tag{3.8}$$

The final corrected position and velocity of particle i are then given by:

$$\mathbf{x}_i(t_i + \Delta t_i) = \mathbf{x}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)} {\Delta t_i}^4}{24} + \frac{\mathbf{a}_{0,i}^{(3)} {\Delta t_i}^5}{120} \tag{3.9}$$

$$\mathbf{v}_i(t_i + \Delta t_i) = \mathbf{v}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)} {\Delta t_i}^3}{6} + \frac{\mathbf{a}_{0,i}^{(3)} {\Delta t_i}^4}{24} \tag{3.10}$$

These corrections incorporate terms up to fourth order. The main advantage of this method is that it only requires the explicit calculation of the acceleration and jerk at each step, with higher-order derivatives being obtained through the interpolation scheme.

3.1.2. Block Time Steps

An important part of the Hermite integrator is the time step calculation. The time step controls precision, energy conservation and integration time. While time steps are individual (each particle has its own), they are grouped into blocks for efficiency.

Time steps are set in two steps. First, a dynamic time step is calculated. Then, the time step is assigned using a block scheme. For the dynamical time step, the standard formula (A.35) is used:

$$\Delta t_i = \sqrt{\eta \frac{|\mathbf{a}_{1,i}||\mathbf{a}_{1,i}^{(2)}| + |\dot{\mathbf{a}}_{1,i}|^2}{|\dot{\mathbf{a}}_{1,i}||\mathbf{a}_{1,i}^{(3)}| + |\mathbf{a}_{1,i}^{(2)}|^2}} \tag{3.11}$$

As can be seen, $\mathbf{a}_{1,i}$ and $\dot{\mathbf{a}}_{1,i}$ are calculated in each step, while $\mathbf{a}_{1,i}^{(3)}$ is constant. Only $\mathbf{a}_{1,i}^{(2)}$ needs to be estimated:

$$\mathbf{a}_{1,i}^{(2)} = \mathbf{a}_{0,i}^{(2)} + \Delta t_i \mathbf{a}_{1,i}^{(3)} \tag{3.12}$$

When the simulation begins, the initial time steps are calculated using (A.37):

$$\Delta t = \eta_s \frac{|\mathbf{a}|}{|\dot{\mathbf{a}}|} \tag{3.13}$$

The main algorithm of the Hermite 4th-order scheme evolves the particle with the minimum $t_j + \Delta t_j$. As it is expensive to evolve particles one by one, the goal is to assign particles to discrete time steps. In Ph4, time steps are set to powers of two of the N-body time units.

During initialization, the time step is calculated using Equation (3.13), and then set. Given the initial time step $\Delta t_{0,j}$, the N-body time τ, and an integer n:

$$0.5 < \frac{\tau 2^n}{\Delta t_{0,j}} < 1 \tag{3.14}$$

The time step is then set to $\tau 2^n$

As the model evolves, the new time step is computed using (3.11). The time step is halved if the calculated time step is smaller than the current time step, and doubled if it is larger

than twice the current time step.

The advantage of this time step criterion is that the time step in the code is equal to or lower than that required by the dynamics. Moreover, since particles share the same time steps, they can be integrated simultaneously. Additionally, all particles can be synchronized, as their times are multiples of powers of two of the same time unit.

3.1.3. Collisions and mergers

3.1.4. Escaping stars

3.2. Smoothed particles hydrodynamics: Fi

3.3. Coupling strategy: Bridge

Chapter 4

Results and analysis

Chapter 5

El desarrollo de la tesis

5.1. Aquí una nueva sección

5.1.1. Haciendo una tesis como un profesional

Figure 5.1: Apolo flotando a la izquierda.

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$$\int_a^b f(x)\,\mathrm{d}x = \frac{\partial^\eta f(x)}{\partial x^\eta} \cdot \sum_{x=a}^{b} f(x) \cancelto{1+\frac{\epsilon}{k}}{\left(1+\Delta x\right)} \tag{5.1}$$

Ecuación sin sentido.

Definition 5.1 (ver (Einstein, 1905)) *Definición definitiva*

$$\frac{d}{dx}\int_a^x f(y)dy = f(x)$$

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justo.

5.1.2. Otros párrafos más normales

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$$\begin{aligned} \Lambda_f &= \frac{L \cdot f}{W} \cdot \frac{{Q_e}^2}{8\pi^2 W^4 g} + \sum_{i=1}^{l} \frac{f \cdot \Big(M - d\Big)}{l \cdot W} \cdot \underbrace{\frac{\Big(Q_e - i \cdot Q\Big)^2}{8\pi^2 W^4 g}}_{\sim \mathcal{A}} \\ Q_e &= 2.5Q \cdot \int_0^e V(x)\,\mathrm{d}x + \sin^{-1}\left(1 + \frac{1}{1-e}\right) \end{aligned} \tag{5.2}$$

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

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$$\boxed{f(x) = \frac{\partial^2 u}{\partial t^2}} \tag{5.3}$$

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eu enim. Vestibulum pellentesque felis eu massa.

5.1.3. Ejemplos de inserción de código fuente

El template permite la inserción de los siguientes lenguajes de programación de forma nativa: ABAP, Ada, Assembler x64, Assembler x86[masm], Awk, Bash, Basic, C, Caml, CMake, Cobol, C++, C#, CSS, CSV, CUDA, Dart, Docker, Elisp, Elixir, Erlang, Fortran, F#, GLSL, Gnuplot, Go, Haskell, HTML, INI, Java, Javascript, JSON, Julia, Kotlin, LaTeX, Lisp, LLVM, Lua, Make, Maple, Mathematica, Matlab, Mercury, Modula-2, Objective-C, Octave, OpenCL, OpenSees, Pascal, Perl, PHP, Texto plano, PostScript, Powershell, Prolog, Promela, Pseudocódigo, Python, Q#, R, Racket, Reil, Ruby, Rust, Scala, Scheme, Scilab, Simula, SPARQL, SQL, Swift, TCL, VBScript, Verilog, VHDL y XML.

Para insertar un código fuente se debe usar el entorno `sourcecode`, o el entorno `sourcecodep` si es que se quiere utilizar parámetros adicionales. A continuación se presenta un ejemplo de inserción de código fuente en Python (Código 5.1), Java y Matlab:

Code 5.1: Ejemplo en Python.

```python
import numpy as np
def incmatrix(genl1, genl2):
  m = len(genl1)
  n = len(genl2)
  M = None # Comentario 1
  VT = np.zeros((n*m, 1), int) # Comentario 2
```

Code 5.2: Ejemplo en Java.

```java
import java.io.IOException;
import javax.servlet.*;

public class Hola extends GenericServlet { // Hola mundo
  public void service(ServletRequest request, ServletResponse response)
  throws ServletException, IOException{
    response.setContentType("text/html");
    PrintWriter pw = response.getWriter();
    pw.println("Hola, mundo!");
  }
}
```

Code 5.3: Ejemplo en Matlab.

```matlab
% Se crea gráfico
f = figure(1);
title('Espectro de pulso de desplazamiento');

for j = 1:length(BETA)
  fad = ones(1, NDATOS); % Arreglo para el FAD
  for i = 1:NDATOS
    [t, u_t, ~, ~] = main(BETA(j), r(i), M, K, F0, 0);
    fad(i) = max(abs(u_t)) / uf0;
```

```
  end
end
```

5.1.4. Agregando múltiples imágenes

El template ofrece el entorno images que permite insertar múltiples imágenes de una manera muy sencilla. Para crear imágenes múltiples se deben usar las siguientes instrucciones:

```
\begin{images}[\label{imagenmultiple}]{Ejemplo de imagen múltiple.}
  \addimage[\label{ciudadfoto}]{ejemplos/test-image}{width=6.5cm}{Ciudad}
  \addimageanum{ejemplos/test-image-wrap}{height=4cm}
  \imagesnewline
  \addimage{ejemplos/test-image}{width=11cm}{Ciudad más grande}
\end{images}
```

Obteniendo así:

(a) Ciudad

(b) Ciudad más grande

Figure 5.2: Ejemplo de imagen múltiple.

Más ejemplos

Listas y Enumeraciones

Hacer listas enumeradas con LaTeX es muy fácil con el template[3], también puedes revisar el manual (Pizarro, 2024), para ello debes usar el comando `\begin{enumerate}`, cada elemento comienza por `\item`, resultando así:

1. Grecia
2. Abracadabra
3. Manzanas

También se puede cambiar el tipo de enumeración, se pueden usar letras, números romanos, entre otros. Esto se logra cambiando el **label** del objeto `enumerate`. A continuación se muestra un ejemplo usando letras con el estilo `\alph`[4], números romanos con `\roman`[5] o números griegos con `\greek`[6]:

a) Peras
b) Manzanas
c) Naranjas

i) Rojo
ii) Café
iii) Morado

α) Matemáticas
β) Lenguaje
γ) Filosofía

Para hacer listas sin numerar con LaTeX hay que usar el comando `\begin{itemize}`, cada elemento empieza por `\item`, resultando:

– Peras
– Manzanas
– Naranjas

* Rojo
* Café
* Morado

• Árboles
• Pasto
• Flores

[3] También puedes revisar el manual de las enumeraciones en https://latex.ppizarror.com/doc/enumitem.pdf.
[4] Con `\Alph` las letras aparecen en mayúscula.
[5] Con `\Roman` los números romanos salen en mayúscula.
[6] Una característica propia del template, con `\Greek` las letras griegas están escritas en mayúscula.

Chapter 6

Conclusions

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Appendices

Appendix A. Fourth-Order Hermite Integration Scheme

The N-body code used in this work, `ph4`, is based on a fourth-order Hermite integrator. The following subsections describe in detail the individual time step scheme (ITS) from (Makino & Aarseth, 1992), including the time step criterion in the N-body code.

A.1. Step 1: Particle selection

The first step is to select the next particle to be advanced. At any given time, each particle j has its own mass m_j, time t_j, time step Δt_j, coordinates $\mathbf{x}_j$ and $\mathbf{v}_j$, acceleration $\mathbf{a}_j$ and jerk $\dot{\mathbf{a}}_j$ (with $\dot{\mathbf{a}}_j = \partial \mathbf{a}_j / \partial t$). The selected particle i is the one satisfying the condition:

$$i = \arg\min_j (t_j + \Delta t_j) \tag{A.1}$$

Note that always $t_j \leq t_i + \Delta t_i$, for all j. The current simulation time is then set to $t = t_i + \Delta t_i$.

A.2. Step 2: Calculate the predicted position and velocities for all particles

The positions and velocities of all particles are predicted at time t, using a Taylor expansion up to third-order (jerk):

$$\mathbf{x}_{p,j} = \mathbf{x}_j + \mathbf{v}_j(t - t_j) + \frac{\mathbf{a}_{0,j}}{2!}(t - t_j)^2 + \frac{\dot{\mathbf{a}}_{0,j}}{3!}(t - t_j)^3 \tag{A.2}$$

$$\mathbf{v}_{p,j} = \mathbf{v}_j + \mathbf{a}_{0,j}(t - t_j) + \frac{\dot{\mathbf{a}}_{0,j}}{2!}(t - t_j)^2 \tag{A.3}$$

The acceleration is due only to gravitational force. The expression for each particle j is:

$$\mathbf{a}_{0,j} = \sum_{k \neq j} G m_k \frac{\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} \tag{A.4}$$

where ϵ is the softening parameter, and

$$\mathbf{r}_{0,jk} = \mathbf{r}_k - \mathbf{r}_j \tag{A.5}$$

Differentiating Eq. (A.4) with respect to time yields the jerk:

$$\dot{\mathbf{a}}_{0,j} = \frac{\partial \mathbf{a}_j}{\partial t} \tag{A.6}$$

$$= \sum_{k \neq j} G m_k \frac{\partial}{\partial t} \frac{\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} \tag{A.7}$$

$$\dot{\mathbf{a}}_{0,j} = \sum_{k \neq j} G m_k \left[\frac{\mathbf{v}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{3/2}} - \frac{3(\mathbf{v}_{0,jk} \cdot \mathbf{r}_{0,jk})\mathbf{r}_{0,jk}}{(\mathbf{r}_{0,jk}^2 + \epsilon^2)^{5/2}} \right] \tag{A.8}$$

where

$$\mathbf{v}_{0,jk} = \mathbf{v}_k - \mathbf{v}_j \tag{A.9}$$

A.3. Step 3: Calculate acceleration and jerk for particle i

Similar to equations (A.4) and (A.8), the acceleration and jerk are calculated using the predicted positions and velocity:

$$\mathbf{a}_{1,i} = \sum_{j \neq i} G m_j \frac{\mathbf{r}_{ij}}{(\mathbf{r}_{ij}^2 + \epsilon^2)^{3/2}} \tag{A.10}$$

$$\dot{\mathbf{a}}_{1,i} = \sum_{j \neq i} G m_j \left[\frac{\mathbf{v}_{ij}}{(\mathbf{r}_{ij}^2 + \epsilon^2)^{3/2}} - \frac{3(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij})\mathbf{r}_{ij}}{(\mathbf{r}_{ij}^2 + \epsilon^2)^{5/2}} \right] \tag{A.11}$$

$$\mathbf{r}_{ij} = \mathbf{r}_{p,j} - \mathbf{r}_{p,i} \tag{A.12}$$

$$\mathbf{v}_{ij} = \mathbf{v}_{p,j} - \mathbf{v}_{p,i} \tag{A.13}$$

In this notation, the subscript 0 denotes quantities evaluated at the beginning of the time step (t_j), while the subscript 1 denotes quantities evaluated at the end of the time step (t), using the predicted position and velocity ($\mathbf{x}_{p,i}$ and $\mathbf{v}_{p_i}$).

A.4. Step 4: Second and third time derivative of acceleration, using Hermite interpolation

To obtain a high-order correction of $\mathbf{x}$ and $\mathbf{v}$, the acceleration of particle i is modeled as a third-order polynomial in time:

$$\mathbf{a}(\tau) = \alpha\tau^3 + \beta\tau^2 + \gamma\tau + \delta \tag{A.14}$$

The successive time derivatives of $\mathbf{a}(\tau)$ are then:

$$\dot{\mathbf{a}}(\tau) = 3\alpha\tau^2 + 2\beta\tau + \gamma \tag{A.15}$$

$$\mathbf{a}^{(2)}(\tau) = 6\alpha\tau + 2\beta \tag{A.16}$$

$$\mathbf{a}^{(3)}(\tau) = 6\alpha \tag{A.17}$$

The coefficients α, β, γ, and δ are determined by using the known values of acceleration and jerk at times t_i and t. This provides a system of four equations:

$$\mathbf{a}(t_i) = \mathbf{a}_{0,i} = \alpha t_i^3 + \beta t_i^2 + \gamma t_i + \delta \tag{A.18}$$

$$\mathbf{a}(t) = \mathbf{a}_{1,i} = \alpha t^3 + \beta t^2 + \gamma t + \delta \tag{A.19}$$

$$\dot{\mathbf{a}}(t_i) = \dot{\mathbf{a}}_{0,i} = 3\alpha t_i^2 + 2\beta t_i + \gamma \tag{A.20}$$

$$\dot{\mathbf{a}}(t) = \dot{\mathbf{a}}_{1,i} = 3\alpha t^2 + 2\beta t + \gamma \tag{A.21}$$

where $t = t_i + \Delta t_i$ is the current time. As the system of equations is linearly independent, the four coefficients can be determined. However, determining α and β is enough to get the second and third time derivative of acceleration. From (A.21):

$$\gamma = \dot{\mathbf{a}}_{1,i} - 3\alpha t^2 - 2\beta t \tag{A.22}$$

Subtracting Eq. (A.20) from (A.21):

$$\beta = \frac{(\dot{\mathbf{a}}_{1,i} - \dot{\mathbf{a}}_{0,i}) - \alpha 3(t^2 - t_i^2)}{2(t - t_i)} \tag{A.23}$$

Subtracting Eq. (A.18) from (A.19), and using results from (A.22) and (A.23) gives an expression for α:

$$\alpha = \frac{(\dot{\mathbf{a}}_{1,i} + \dot{\mathbf{a}}_{0,i})(t - t_i) - 2(\mathbf{a}_{1,i} - \mathbf{a}_{0,i})}{(t - t_i)^3} \tag{A.24}$$

Replacing this result in Eq. (A.23):

$$\beta = \frac{-\dot{\mathbf{a}}_{1,i}(t + 2t_i) - \dot{\mathbf{a}}_{0,i}(2t + t_i)}{(t - t_i)^2} + \frac{3(\mathbf{a}_{1,i} - \mathbf{a}_{0,i})(t + t_i)}{(t - t_i)^3} \tag{A.25}$$

The second and third time derivatives of the acceleration at time t_i can now be evaluated. For the Eq. (A.17), and using $t - t_i = \Delta t_i$:

$$\mathbf{a}_{0,i}^{(3)} = \mathbf{a}^{(3)}(t_i) = 6\alpha \tag{A.26}$$

$$\mathbf{a}_{0,i}^{(3)} = 6\frac{(\dot{\mathbf{a}}_{1,i} + \dot{\mathbf{a}}_{0,i})\Delta t_i - 2(\mathbf{a}_{1,i} - \mathbf{a}_{0,i})}{{\Delta t_i}^3} \tag{A.27}$$

$$\mathbf{a}_{0,i}^{(3)} = \frac{12(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) + 6(\dot{\mathbf{a}}_{0,i} + \dot{\mathbf{a}}_{1,i})\Delta t_i}{{\Delta t_i}^3} \tag{A.28}$$

Evaluating the second time derivative in t_i, and replacing the obtained for α and β:

$$\mathbf{a}_{0,i}^{(2)} = \mathbf{a}^{(2)}(t_i) = 6\alpha t_i + \beta \tag{A.29}$$

$$\mathbf{a}_{0,i}^{(2)} = \frac{-6(\mathbf{a}_{0,i} - \mathbf{a}_{1,i}) - \Delta t_i(4\dot{\mathbf{a}}_{0,i} + 2\dot{\mathbf{a}}_{1,i})}{{\Delta t_i}^2} \tag{A.30}$$

A.5. Step 5: Add corrections

The next step in the scheme is to correct position and velocity of particle i at time t. Using a higher-order Taylor expansion around t_i:

$$\mathbf{x}_i(t_i + \Delta t_i) = \mathbf{x}_i(t_i) + \dot{\mathbf{x}}_i(t_i)\Delta t_i + \frac{\mathbf{x}_i^{(2)}(t_i)\Delta {t_i}^2}{2!} + ... + \frac{\mathbf{x}_i^{(5)}(t_i)\Delta {t_i}^5}{5!} \tag{A.31}$$

$$= \mathbf{x}_i + \mathbf{v}_i\Delta t_i + \frac{\mathbf{a}_{0,i}\Delta {t_i}^2}{2} + \frac{\dot{\mathbf{a}}_{0,i}\Delta {t_i}^3}{6} + \frac{\mathbf{a}_{0,i}^{(2)}\Delta {t_i}^4}{24} + \frac{\mathbf{a}_{0,i}^{(3)}\Delta {t_i}^5}{120} \tag{A.32}$$

The first four terms of this expansion correspond to the predicted position $\mathbf{x}_{p,i}$ from Eq. (A.2). Therefore, the corrected position can be written as:

$$\mathbf{x}_i(t_i + \Delta t_i) = \mathbf{x}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)}\Delta {t_i}^4}{24} + \frac{\mathbf{a}_{0,i}^{(3)}\Delta {t_i}^5}{120} \tag{A.33}$$

In a similar way, the corrected velocity is:

$$\mathbf{v}_i(t_i + \Delta t_i) = \mathbf{v}_{p,i} + \frac{\mathbf{a}_{0,i}^{(2)}\Delta {t_i}^3}{6} + \frac{\mathbf{a}_{0,i}^{(3)}\Delta {t_i}^4}{24} \tag{A.34}$$

The advantage of this scheme is that achieving high integration accuracy only requires the explicit calculation of the acceleration and jerk for all particles, and the predicted acceleration and jerk for the particle being advanced. This results in an integration scheme that is fourth-order accurate in time ($\mathcal{O}(a^{(3)})$).

A.6. Step 6: Time step update

After updating the position and velocity of particle i, a new time step Δt_i must be calculated. This step is crucial for controlling the integration error. A proven and stable criterion is given by the standard formula (Aarseth, 1985):

$$\Delta t_i = \sqrt{\eta \frac{|\mathbf{a}_{1,i}||\mathbf{a}_{1,i}^{(2)}| + |\dot{\mathbf{a}}_{1,i}|^2}{|\dot{\mathbf{a}}_{1,i}||\mathbf{a}_{1,i}^{(3)}| + |\mathbf{a}_{1,i}^{(2)}|^2}} \tag{A.35}$$

The values for $\mathbf{a}_{1,i}$ and $\dot{\mathbf{a}}_{1,i}$ are known from the direct calculation in Step 3. The value of $\mathbf{a}^{(3)}(t)$ is constant, since a third-order polynomial interpolation is used. Only $\mathbf{a}_{1,i}^{(2)}$ should be calculated:

$$\mathbf{a}_{1,i}^{(2)} = \mathbf{a}_{0,i}^{(2)} + \Delta t_i \mathbf{a}_{0,i}^{(3)} \tag{A.36}$$

During the initialization of the algorithm, the higher-order derivatives of the acceleration are not available, so an alternative formula for time step can be used in the startup:

$$\Delta t = \eta_s \frac{|\mathbf{a}|}{|\dot{\mathbf{a}}|} \tag{A.37}$$

The suggested value for the startup parameter is $\eta_s \sim 0.01$ (Makino & Aarseth, 1992).

Appendix B. Cálculos realizados

B.1. Metodología

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Figure B.1: Imagen en anexo.

B.2. Resultados

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Table B.1: Tabla de cálculo.

Elemento	ϵ_i	**Valor**	**Descripción**
A	10	$3{,}14\pi$	Valor muy interesante[a]
B	20	6	Segundo elemento
C	30	7	Tercer elemento[1]
D	150	10	Sin descripción
E	0	0	Cero

[a] Este elemento tiene una descripción debajo de la tabla
[1] Más comentarios