UNIVERSITY OF CAMBRIDGE

Project Plan

Imaging Quantum Gravity with Supercomputers

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1 Introduction

The unification of Quantum Field Theory and General Relativity remains one of the largest unsolved problems in physics. One leading approach to developing theories of Quantum gravity uses computational models of lattice field theory on non-flat spacetimes to probe for critical points where real physics may arise. A promising lattice field theory (LFT) is Causal Dynamical Triangulation (CDT), which allows spacetime to evolve as a dynamical variable rather than be kept as a static background.

A challenge with CDT is the high dimensional scaling of the model, making large simulations prohibitively expensive to run. Therefore, a carefully selected algorithm must be used to run the simulations in order to collect physical results and one such algorithm well suited for this problem is Nested Sampling[1].

One aim of the project is to employ Nested Sampling to evolve a CDT model and extract observables with high precision. The initial problem will focus be on simple LFT models, which will present many challenges with using Nested Sampling. Once this has been acheived, the complexity of the model will gradually be increased, adding more elements of physics and pushing the computational limits.

Furthermore, the aim is then to produce visualisations of the Quantum gravity model, such as snapshots of the spacetime microstates and images of the different observables exploring new physics. This task will involve significant programming challenges in order to visualie higher dimensional manifolds and fractal structures.

2 Causal Dynamical Triangulation

Causal Dynamical Triangulation turns spacetime into a dynamic part of the model which can evolve according to a specified action. CDT discretises d-dimensional spacetime into a set of d-simplexes (2-simplex being a triangle) which are connected together along their edges. The triangles can switch connecting edges to create curvature in the spacetime with each configuration contributing to the Einstein-Hilbert action.

$$S = \frac{1}{G}\int d^4x\sqrt{-\det g}(R - 2\Lambda)$$

Where G is the gravitational constant, R is the Ricci curvature scalar computed from the triangle configuration, and Λ is the cosmological constant. This action can then be computationally minimised to find the distribution of spacetime microstates from which physical observables can be extracted.

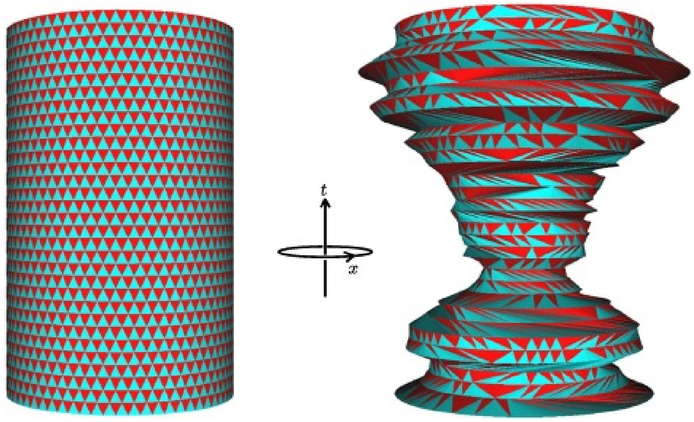

Figure 1: Causal Dynamical Triangulation Microstates for 1+1 spacetime segmented into green and red simplices. Flat spacetime (left) and critical spacetime (right).[3]

This approach has shown to be promising in 1+1 and 2+1 dimensions with numerical results agreeing with theoretical models. CDT solves the issue of pertubative non-renormalizability with a discretized spacetime triangulation, where the lattice can be made smaller and smaller to recover the continuous physical limit without blowing up to infinities.

3 ϕ^4 Theory

The most simple field theory which is solved analytically is ϕ^4 theory. This is a good starting point for research as the results obtained can be checked against known solutions.

The starting point is the action for ϕ^4 theory which is shown below

$$S = \int \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 d^4x$$

This action can be used to construct a path integral with the following general form.

$$\mathcal{Z} = \int [\mathcal{D}x]e^{-\frac{i}{\hbar}S[\phi]}$$

The path integral can be interpreted as a quantum mechanical function which fully characterises the system. It sums the contributions of every field configuration weighted by the action of that given configuration.

An interesting observation about this expression is the similarity of the action to the Boltzmann factor in thermodynamics. This draws analogs to using the Metropolis-Hastings (MH) algorithm, traditionally applied to statistical mechanics, for this application of lattice field theory.

3.1 Code Implementation

Solving ϕ^4 theory has already been implemented in C++ using MH with the code and results on `https://github.com/BorisDeletic/QuantumGravity`.

Below is a snippet of code for calculating the change in action for a change field configuration at a lattice point

```
double LatticeFieldTheory::scalarActionDifference
(Point &p, double newField, double kappa) {

    // compute change in the action for new field value at a point
    double oldField = fieldValue(p);
    double potentialChange = potential(oldField) - potential(newField);

    double actionChange = 2 * kappa * neighbourSum(p) *
                          (oldField - newField) - potentialChange;

    return actionChange;
}
```

Below is a snippet of the implementation of the Metropolis-Hastings algorithm as described.

```
int LatticeFieldTheory::stepMH(double kappa) {
    vector<int> rands;
    rands.push_back(pointRNG(gen));
    rands.push_back(pointRNG(gen));
    rands.push_back(pointRNG(gen));
    rands.push_back(pointRNG(gen));

    Point p = Point(rands);

    uniform_real_distribution<double> distr(-delta, delta);
    double rngField = distr(gen);

    double newField = fieldValue(p) + rngField;
    double actionChange = scalarActionDifference(p, newField, kappa);

    double r = uniformRNG(gen); // random in (0,1)

    if ((actionChange < 0) || r < exp(-actionChange)) {
        setField(p, newField);
        return 1;
    }
    return 0;
}
```

4 Nested Sampling

Nested Sampling is a bayesian inference algorithm which aims to find a set of input parameters that maximise a general likehood function. It's implementation has been developed largely by the Astrophysics group at Cambridge for the use in Cosmological models. It has many potential advantages over existing algorithms used for LFT simulations such as Metropolis-Hastings[2] and Hybrid Montecarlo.

4.1 Metropolis-Hastings Algorithm

Metropolis-Hastings is a rejection based Monte Carlo Markov Chain (MCMC) method. The outline of the algorithm applied to ϕ^4 theory for example is as follows

1. Pick initial field configuration $\phi_0(x)$
2. Generate random new field by slighting pertubing the previous one according to $g(\phi'|\phi_t)$
3. Calculate the acceptance probability $A(\phi') = e^{-\frac{1}{\hbar}S[\phi']}$
4. Accept or Reject new state. Generate random number $u \in [0, 1]$.
   If $u < A(\phi')$ Reject the state.
   If $u > A(\phi')$ Accept the state, set $\phi_{t+1} = \phi'$.
5. Repeat from step 2.

4.2 Topological Freezing

One main potential advantage of Nested Sampling over Metropolis-Hastings is the resistance to topological freezing, which occurs near the transition critical point. This is an effect where the system gets stuck in local minima and rejection rates of the monte carlo methods become very high. This is a well known phenomenon and is an active area of research.[4] Nested Sampling overcomes topological freezing with its use of clustering and multimodal exploration of a distribution. The has been recent work in new sampling schemes to overcome this issue, in particular normalising flows[5]. These new methods have similar limitations with dimensionality as nested sampling, providing motivation to explore nested sampling as an alternative in this context.

4.3 NS Algorithm

The basic approach of the algorithm is to keep multiple instances of input parameters and calculate the likelihood given each set of inputs. The instance with the lowest likelihood is then sampled again subject to a hard likelihood constraint, guaranteeing the new instance has a greater likelihood. Iteratively repeating this step will cause the points to converge in parameter space around peaks in the likelihood function. This approach is much more effective at exploring bimodal distributions which are important for characterising phase transitions.

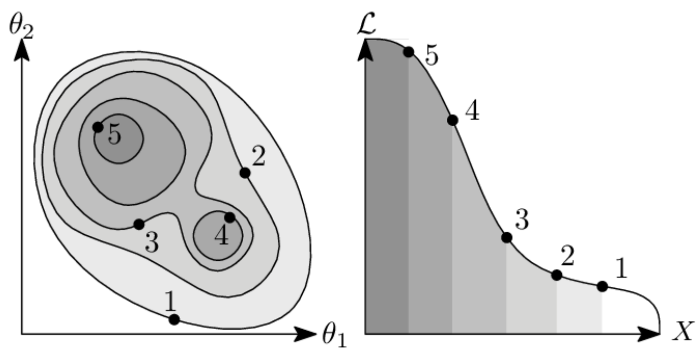

Figure 2: Nested Sampling Multimodal Clustering. Iso-Likelihood lines in parameter space (left), likelihood maximisation given input parameters (right).

The challenges involved with applying nested sampling to lattice field theory, and subsequently CDT will relate to the large dimensionality of the models. Every lattice point has multiple parameters associated in the model and must all be explored by nested sampling which itself does not scale well with dimension.

5 Timeline Plan

The project aims outlined above will be acheived by completing the following tasks with the estimated time for each goal.

1. Implement ϕ^4 theory using Nested Sampling in C++.
   2 weeks - finish during Christmas holidays

2. Explore dimensionality constraints of Nested Sampling and investigate alternate sampling schemes and their efficacy to LFT.
   2-4 weeks

3. Implement CDT in nested sampling. Explore the physics of the model and reconcile results with literature. This is a novel piece of work as CDT has not been solved with Nested Sampling before.
   4-6 weeks

4. Write program to visualise states and observables of CDT. (Additional outcome to supplement previous goals)
   2 weeks

5. Extend CDT to incorporate fermionic matter and higher dimensional physics. (Optional if time permits)
   2-4 weeks

This plan outlines the core objectives of the project and the estimated time to acheive them.

The main success of the project is to demonstrate a new algorithmic approach to LFT and Quantum Gravity with Nested Sampling. This will be a novel result which will have useful applications to the wider community. Having physically accurate visualisations will also provide a new useful tool for analysis of these models which may provide a point of extension for future work.

The extended aims provide a goal to explore new physics in addition to the novel algorithmic research.

6 Conclusion

The application of Nested Sampling to Causal Dynamical Triangulation will be a novel computational approach to Quantum Gravity. The techniques explored will have unique advantages over current methods and will be a valuable research project for the lattice QCD and Quantum Gravity community.

References

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[4] Hasenbusch, Martin. (2017). Fighting topological freezing in the two-dimensional CPN1 model. https://arxiv.org/abs/1709.09460

[5] Hackett. C. Daniel, Et al. (2021). Flow-based sampling for multimodal distributions in lattice field theory. https://arxiv.org/abs/2107.00734