# A note on the prime in SQISignHD

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**Abstract.** In this note, we propose new primes for the SQISignHD signature algorithm. SQISignHD uses primes of the form  $p=2^a3^bf-1$ . We argue that using primes p such that  $p^2-1$  has a smooth factor  $2^{a+1}T$ , with  $2^a>2\sqrt{p}\log p$ ,  $T\geq\sqrt{p}$  and T being very smooth allows a faster response verification. Contrarily to SQISign/BSIDH primes, these primes are easy to generate. Moreover, since one has access to smooth torsion of order larger than p, then the degrees of the secret isogenies can be chosen to be larger than p, as opposed to the current SQISignHD version where they are smaller than p.

**Keywords:** Supersingular Isogenies · SQISign · SQISignHD.

### 1 Introduction

SQISign [7,8] is a digital signature scheme whose design is inspired by the GPS [10] signature. Its security relies on the problem of computing a non trivial endomorphism of a random supersingular elliptic curve. In the identification scheme used in SQISign, a starting curve  $E_0$  with known endomorphism ring  $\mathcal{O}_0$  is fixed, the secret is an isogeny  $\tau: E_0 \to E_A$ . The commitment is a curve  $E_1$  obtained by computing a random isogeny  $\psi: E_0 \to E_1$ . The challenge is a random isogeny  $\varphi: E_1 \to E_2$ . The response consists of a random isogeny  $\sigma: E_A \to E_2$ . Figure 1 illustrates this identification protocol.

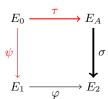


Fig. 1. The SQISign identification protocol.

The response computation heavily relies on the so called Deuring correspondence [9] that allows the owner of the secret key to recover the endomorphism rings  $\mathcal{O}_A$  and  $\mathcal{O}_2$  of  $E_A$  and  $E_2$  respectively, to compute the connecting ideal  $I = I(\mathcal{O}_A, \mathcal{O}_2)$ , to solve for an equivalent ideal  $J \sim I$  of smooth norm using an

improved version [7] of the the KLPT algorithm [12], and to translate the ideal J into an isogeny  $\sigma$ . The degree of the isogeny  $\sigma$  is very large (roughly  $p^{15/4}$ ). For these computations to be efficient, there are several requirements that need to be satisfied, among which, the prime p used needs to be twin smooth, also referred to as BSIDH primes in Isogeny-Based Cryptography. Finding such primes is not easy [4,7,8,5,1]. This has a negative impact on the efficiency of the scheme and makes it difficult to instantiate SQISign for security levels higher that 128 without efficiency loss.

In the recently proposed SQISignHD [6], the SQISign signature algorithm was redesigned using the SIDH attacks [2,13,16]. In fact, with these attacks, it has been shown by Robert [14] that an isogeny of generic degree can be represented using solely its action on some large enough smooth order torsion points. This means that in SQISign, instead of returning a random isogeny  $\sigma: E_A \to E_2$ of large smooth degree, one could return a random short isogeny  $\sigma_I: E_A \to E_2$ of generic degree using the new representation. Note that evaluating an isogeny of generic degree has exponential cost in general, but the signer can use the knowledge of the endomorphism rings of  $E_A$  and  $E_2$  to solve this task in polynomial time. This change in SQISign, together with other smart tricks, totally remove the many constraints on the base prime and allows to use SIDH primes  $p = 2^a 3^b f - 1$ , which make it easy to instantiate SQISignHD for any security level. Moreover, the signature scheme is more compact, and the security reduction is much more compelling. For efficiency reasons, the commitment isogeny is computed from  $E_A$  and the response isogeny is  $\sigma_I: E_1 \to E_2$ . This is illustrated in Figure 2.

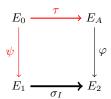


Fig. 2. The (high level) SQISignHD identification protocol.

The natural way to represent the isogeny response isogeny  $\sigma_I$  (using only its action on a power of 2 torsion) requires that  $\deg \sigma_I < 2^a$ . But, in SQISignHD [6, §5.2], it is shown that  $q := \deg \sigma_I \leq 2\sqrt{2p}/\pi =: q_{max}$  in general, hence  $2^a < q$  most of the time. For an efficient representation of the response isogeny in dimension 4, one further requires that  $2^{2r} - q$  is a prime congruent to 1 modulo 4, for some well chosen  $r \leq a$ . This implies that one should expect  $q_{max} < q$  with high probability. The SQISignHD signature algorithm represents the response isogeny  $\sigma_I$  using its evaluation on the  $2^r$ -torsion as far as  $q < 2^{2r}$  [6, §5.6].

This representation is less efficient. We hence propose to change of the base p prime in such a way that the  $2^a$ -torsion available satisfies deg  $\sigma_I < 2^a$ . In

fact, we know that there always exists an ideal I of norm  $q < q_{max}$ . If we require that  $2^r - q$  is a prime, then we can heuristically find such an ideal with the norm  $q < q_{max} \log p$ . If we further require that the prime  $2^r - q$  is congruent to 1 modulo 4, then we can can hope to find such an ideal with with the norm  $q < 2q_{max} \log p$ . Hence, having  $2^a$  larger than  $2q_{max} \log p$  heuristically assures that the optimal isogeny representation will be used. Having  $2^a$  larger than  $2q_{max}\log p$  and  $p=2^a3^b-1$  means that  $3^b$  is considerably smaller than  $\sqrt{p} \approx 2^{\lambda}$ . This implies that the secret isogeny, the commitment isogeny and the challenge isogeny do not offer enough security. To address this, we compensate the loss created by increasing a with the smooth part of p-1. That is, we choose p such that  $p^2 - 1 = 2^{a+1}Tf$  where  $T > \sqrt{p}$  is odd and as smooth as possible, ideally T is divisible by a large power of 3. In the building blocks of SQISignHD, one replaces  $3^b$  by T. As opposed to SQISign/BSIDH primes, these primes are relatively easy to generate since there is no other requirement. We suggest a set of such primes for the various security levels (see Table 1). Moreover, since one has access to smooth torsion of order larger than p, then the degrees of the secret and commitment isogenies can be chosen to be larger than p, as opposed to the current SQISignHD version where they are smaller than p. This enables the public curve  $E_A$  and the commitment curve  $E_1$  to have a distribution which is slightly closer to the uniform distribution.

Even though the change potentially allows a more efficient signature algorithm (to be confirmed by future implementation), one drawback is that signature size is larger. In fact, since the representation of the isogeny  $\sigma_I$  now uses points of order  $\approx 2^{\log q}$  instead of  $\approx 2^{\frac{1}{2}\log q}$ , then the size of the part of the response (in the signature) representing these points will double. This will lead to signatures that are about 1.24 times larger, but still very compact; say  $\approx 140$  bytes.

### 2 Overview of SQISignHD

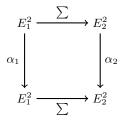
#### 2.1 New isogeny representation from Kani's theorem

Informally, a efficient representation of an isogeny  $\varphi: E \to E'$  is any string of polynomial size that allows to evaluate the isogeny  $\varphi$  on any point of E which is defined over a relatively small extension of  $\mathbb{F}_{p^2}$ . An isogeny  $\varphi: E \to E'$  of small prime degree  $\ell$  can be efficiently computed and evaluated from its kernel, or one of its kernel generator. This kernel generator is a point of order  $\ell$ , which is defined over an extension of  $\mathbb{F}_{p^2}$  of degree at most  $O(\ell)$ . This means that any kernel generator P of  $\varphi$  is an efficient representation of  $\varphi$ . If the isogeny  $\varphi$  has large prime power degree, say  $\ell^n$ , then  $\varphi$  can be decomposed as  $\varphi = \varphi_n \circ \varphi_{n-1} \circ \cdots \circ \varphi_2 \circ \varphi_1$ , where each  $\varphi_i$  has degree  $\ell$ . An efficient representation of  $\varphi$  can be obtained from that of the isogenies  $\varphi_i$ . Moreover, if n = kd and the the  $\ell^d$  torsion is defined over a small extension of  $\mathbb{F}_{p^2}$ , then one can represent  $\varphi$  more compactly by using a more compact decomposition  $\varphi = \psi_k \circ \psi_{k-1} \circ \cdots \circ \psi_2 \circ \psi_1$  where each  $\psi$  is represented by one of its kernel generators which is a point of order  $\ell^d$ . This is exactly how the response isogeny is returned in SQISIgn. In

fact, in SQISign, the response isogeny  $\sigma$  has degree  $2^n \approx p^{15/4}$ . Since one only has access to some  $2^d$  torsion where  $d < \log p$ ,  $\sigma$  is efficiently represented as a composition is isogenies of degree at most  $2^d$ . As you may notice, this kernel representation only works for smooth degree isogenies.

In SQISignHD, the picture changes completely, one uses a new representation [15] inspired by the SIDH attacks [2,13,16]. In fact, an isogeny can be represented by its action on some smooth rational torsion points regardless of whether its degree is smooth or not. This representation is a consequence of Kani's theorem [11], and it involves a higher dimension (2, 4 or 8) isogeny. In this note, we are interested in representations that use isogenies in dimension 4. We refer to [6, §3], [16] and [15] for further details.

Dimension 4 representations. Let  $\sigma: E_1 \to E_2$  be an isogeny of degree q and let N > q be a smooth integer such that  $E_1[N]$  defined over a small extension of  $\mathbb{F}_{p^2}$  and  $a = N - q = a_1^2 + a_2^2$  is the sum of two squares. Set  $\sum = diag(\sigma, \sigma) \in Hom(E_1^2, E_2^2)$  and  $\alpha = \begin{bmatrix} a_1 & a_2 \\ -a_2 & a_1 \end{bmatrix}$ . Then we have the following diagram, also known as isogeny diamond, where  $\alpha_1$  and  $\alpha_2$  are the endomorphisms whose matrix is  $\alpha$ .



Then the higher dimension endomorphism

$$F = \begin{pmatrix} \alpha_1 \widehat{\sum} \\ -\sum \widehat{\alpha_2} \end{pmatrix} \in \operatorname{End}(E_1^2 \times E_2^2)$$

has degree N=a+q, and we denote this endomorphism by  $F(\sigma,a_1,a_2)$ . Its kernel is given by

$$\ker(F) = \left\{ \left(\widehat{\alpha}_1(P), \sum(P)\right) \mid P \in E_1^2[N] \right\}.$$

In practice, one chooses N to be as smooth as possible, say a power of 2. For this representation to be possible, N-q needs to be the sum of two squares. One way to achieve this is to require that N-q is a prime number congruent to 1 modulo 4, and to use Cornacchia's algorithm [3] to find  $a_1$  and  $a_2$ . When one cannot write N-q as the sum of two squares, one is obliged to do a dimension 8 representation instead. A much less efficient version of SQISignHD uses this

diemension 8 representation. But here we discuss only the fast version that uses the dimension 4 representation since it is the practically efficient one.

This means that the response isogeny in SQISign no more needs to be a smooth degree isogeny. In SQISignHD, the very long smooth degree response isogeny is replaced by a short isogeny  $\sigma_I: E_1 \to E_2$  of generic degree. In fact, since the signer knows the endomorphism rings  $\mathcal{O}_1$  and  $\mathcal{O}_2$  of  $E_1$  and  $E_2$  respectively, he can sample a short ideal I of norm q connecting  $\mathcal{O}_1$  and  $\mathcal{O}_2$ , evaluate the corresponding isogeny  $\sigma_I$  on a canonical basis (P,Q) of  $E_1[N]$ , and return  $(\sigma_I(P), \sigma_I(Q), q)$  as a representation of  $\sigma_I$ . The verification consists in checking that  $(\sigma_I(P), \sigma_I(Q), q)$  leads to an endomorphism  $F(\sigma_I, a_1, a_2) \in \operatorname{End}(E_1^2 \times E_2^2)$  of the correct form. We refer to [6] for more details.

#### 2.2 The SQISignHD identification protocol

In what follows, we provide a simplified description of the SQISignHD identification protocol. Several technical details are omitted since they are not relevant for our exposition. We refer to the SQISignHD paper [6] for more details.

Setup. Let  $p=2^a3^bf-1$  be a prime. Consider the supersingular curve  $E_0:y^2=x^3+x$  defined over  $\mathbb{F}_p$  and let  $\mathcal{O}_0\simeq \mathrm{End}(E_0)$  be its endomorphism ring.

Key generation. Compute a random double-path made of two isogenies  $\tau: E_0 \to E_A$  and  $\tau': E_0 \to E_A$  of degree  $3^{2b''} \sim p$  and  $2^{2a''} \sim p$  respectively. The public key is  $\mathsf{pk} = E_A$  and the secret key is  $\mathsf{sk} = (\tau, \tau')$ .

Commitment. Sample a random isogeny  $\psi: E_0 \to E_1$  of degree  $3^{2b'} \sim p$ . Return  $\mathsf{com} = E_1$ .

Challenge. Sample a uniformly random isogeny  $\varphi: E_A \to E_2$  of degree  $3^b$ . Return chal  $= \varphi$ .

Response. Translate  $\psi$  into an ideal  $I_{\psi}$ . Use  $\tau'$  and  $I_{\tau'}$  to translate  $\varphi$  into an ideal  $I_{\varphi}$ . Compute the ideal  $J = \overline{I_{\psi}}I_{\tau}I_{\varphi}$ . Compute a random short ideal  $I \sim J$  such that  $2^{2r} - q$  is the sum of two squares for some  $r \leq a$ , where q is the norm of I. Use  $\mathcal{O}_0$ ,  $\tau$ ,  $\psi$  and  $\varphi$  to evaluate the isogeny  $\sigma_I : E_1 \to E_2$  corresponding to I on a canonical basis (P,Q) of  $E_1[2^r]$ . Return  $\mathsf{resp} = (\sigma_I(P), \sigma_I(Q), q)$ .

Verification. Verify that the response resp parses as (R, S, q) and that this is a valid representation of an isogeny from  $E_1$  to  $E_2$  of degree q. Return 1 if the verification succeeds and 0 if it fails.

### 2.3 The actual representation and verification in SQISignHD

As you may have notice, in SQISignHD, one represents an isogeny of degree q using its action on the  $2^r$  torsion points where  $2^r < q < 2^{2r}$ . The trick here is that

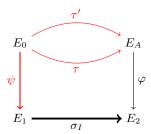


Fig. 3. The actual SQISignHD identification protocol.

a cyclic isogeny  $\phi: E \to E'$  of degree  $d = d_1^2 d_2$  can be recovered from its action on the  $d_1 d_2$ -torsion group. In fact, one writes  $\phi = \phi_2 \circ \phi_1$ , where  $\deg \phi_1 = d_1 d_2$  and  $\deg \phi_2 = d_1$ . Then  $\ker \phi_1 = E[d_1 d_2] \cap \ker \phi$  and  $\ker \widehat{\phi_2} = \phi(E[d_1])$ . The same applies to the higher dimension isogeny F. This means that with access to the  $2^a$  torsion points, the degree of F can be chosen to be  $2^{2r}$  with  $r \leq a$ , and one decomposes F as  $F = F_2 \circ F_1$  where  $F_1: E_1^2 \times E_2^2 \to C_1$  and  $\widehat{F_2}: E_1^2 \times E_2^2 \to C_2$  have degree  $2^r$ , with  $C_1$  and  $C_2$  be two isomorphic surfaces.

The verification consists in computing the isogenies  $F_1$  and  $F_2$  from the action of  $\sigma_I$  on the  $2^r$  torsion, checking that  $C_1$  and  $C_2$  are isomorphic, and that  $F = F_2 \circ F_1$  is of the correct form: by mapping a point of the form (Q, 0, 0, 0) through the isogeny F and checking that its image is of the form  $([a_1]Q, -[a_2]Q, \star, 0)$ . For an efficient isomorphism check between  $C_1$  and  $C_2$ , one needs to have  $r \leq a-2$  [6, Remark 4.2].

This representation is less optimal. In the next section we suggest to use different primes that will allow to have the optimal representation as presented in Section 2.1.

### 3 New primes for SQISignHD

### 3.1 Overview

We describe a variant of SQISignHD that uses primes p such that  $p^2-1$  has a very smooth part which is larger than p. The aim here is to choose  $2^a$  such that the degree of the response isogeny is always smaller than  $2^a$ . As explained earlier, the prime p is such that  $p^2-1=2^{a+1}Tf$  where  $2^a$  is larger than  $2q_{max}\log p$ , and  $T>\sqrt{p}$  is very smooth. We did a search of such primes and we found the ones described in Table 1 that could be used.

## 3.2 The new identification protocol

Setup. Let  $p \equiv 3 \mod 4$  be a prime such that  $p^2 - 1 = 2^{a+1}Tf$  as described above. Consider the supersingular curve  $E_0: y^2 = x^3 + x$  defined over  $\mathbb{F}_p$  and let  $\mathcal{O}_0 \simeq \operatorname{End}(E_0)$  be it endomorphism ring.

λ	$\log p$	a	$T_1$	$T_2$
128	254	140	$5^6 * 7^3$	$3^{64} * 23 * 29$
192	382	203	$7^2 * 11^2$	$3^{111} * 5$
256	510	269	$5^3 * 19$	$3^{153} * 13$

**Table 1.** Suggested primes for SQISignHD. We have  $p+1=2^aT_1f_1$  and  $p-1=2T_2f_2$ , where  $T=T_1T_2$ ,  $f_1$  and  $f_2$  are co-factors. These primes  $p_{254}$ ,  $p_{382}$  and  $p_{510}$  are listed in Appendix A

Key generation. Compute a random double-path made of two isogenies  $\tau: E_0 \to E_A$  and  $\tau': E_0 \to E_A$  of degree  $T''^2 \sim T^2$  and  $2^{2a''} \sim 2^{2a}$  respectively. The public key is  $\mathsf{pk} = E_A$  and the secret key is  $\mathsf{sk} = (\tau, \tau')$ .

Commitment. Sample a random isogeny  $\psi: E_0 \to E_1$  of degree  $T'^2 \sim T^2$  where T'|T. Return  $\mathsf{com} = E_1$ .

Challenge. Sample a uniformly random isogeny  $\varphi: E_A \to E_2$  of degree T. Return  $\mathsf{chal} = \varphi$ .

Response. Translate  $\psi$  into an ideal  $I_{\psi}$ . Use  $\tau'$  and  $I_{\tau'}$  to translate  $\varphi$  into an ideal  $I_{\varphi}$ . Compute the ideal  $J = \overline{I_{\psi}}I_{\tau}I_{\varphi}$ . Compute a random short ideal  $I \sim J$  such that  $2^a - q$  is a sum of two squares where q is the norm of I. Use  $\mathcal{O}_0$ ,  $\tau$ ,  $\psi$  and  $\varphi$  to evaluate the isogeny  $\sigma_I : E_1 \to E_2$  corresponding to I on a canonical basis (P,Q) of  $E_1[2^a]$ . Return resp  $= (\sigma_I(P), \sigma_I(Q), q)$ .

Verification. Verify that the response resp parses as (R, S, q) and that this is a valid representation of an isogeny from  $E_1$  to  $E_2$  of degree q. Return 1 if the verification succeeds and 0 if it fails.

Algorithmic wise, only the response computation and the verification algorithm are modified as presented above. In the scheme in general, the degrees of some isogenies are changed, but they are still very smooth, hence we do not expect them to impact efficiency that much. On the other hand, the higher dimension isogeny F is directly computed and evaluated without splitting it into two. This will allow a faster response verification. Another advantage is that we have access to the  $2^aT > p+1$  torsion, hence our isogenies are slightly longer, hence offering better security guaranties.

One drawback with computing the higher dimension isogeny in one go is that it is represented with points of order  $\approx 2^{\log q}$  instead of  $\approx 2^{\frac{1}{2}\log q}$ . This implies an increase in the size of the signature. In fact, the returned signature is of the form  $||n_2||n_2||q||c_1||c_2||c_3|$  where  $n_1$  and  $n_2$  are the  $\mathbb{F}_p$ -coordinates of  $j(E_1)$ ,  $c_1$ ,  $c_2$ , and  $c_3$  are three of the four coordinates of  $\sigma_I(P)$  and  $\sigma_I(Q)$  in a canonical basis. The size of the signature is  $2\lceil \log(p)\rceil + \log(q) + 3u$  where the order of the points P and Q is  $2^u$ . In SQISignHD, the points P and Q have order  $2^{\frac{1}{2}\log q}$  roughly, while in our variant, their order is  $2^{\log q}$  roughly. Hence the size of of  $c_1$ ,

 $c_2$ , and  $c_3$  is doubled. This leads to an increase of about 24% in the signature signature size. We stress that the SQISignHD signature algorithm is divinely compact (for example, the signatures for NIST level I security are about 110 bytes), hence this increase in the signature size is affordable provided that the change enables a faster signature algorithm. A future implementation work will help verify these claims.

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# A Suggested primes

$p_{254}$	0x311a894ff82fdb307cedb44ecd84affffffffffffffffffffffffffffffffffff
$p_{382}$	0x2df5a3a93966d718e5975170efb0d1408fd4bfa4bc0737fffffffffffffffffffffffffffffffffff
	<b>FFFFFFFFFFFFF</b>
$p_{510}$	0x304d3218257f38bca253529e57639d710def7954b3ae134fdc7bc152f44ed
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Table 2. The new primes suggested for SQISignHD