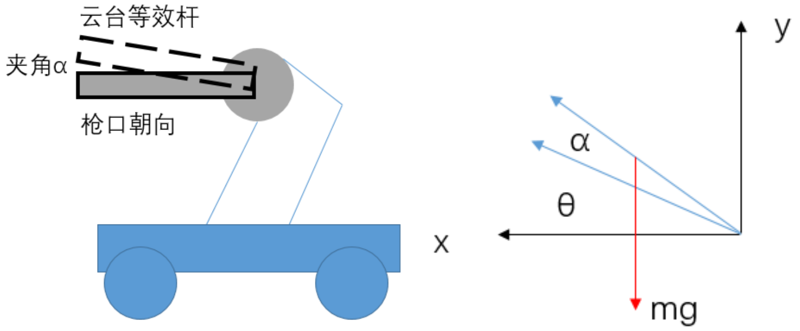
RM 控制算法公式推导

一、云台控制



云台分为 yaw 轴和 pitch 轴，其中 pitch 轴更加复杂一点，因为它还需要做重力补偿，因此以 pitch 轴为例进行分析。Pitch 轴建模如上图所示，因为云台不只是枪管，所以如果将云台抽象为一根杆的话，杆与枪管可能会存在夹角，设该夹角为α，设云台的转动惯量为J，云台等效杆质量为m，等效杆长为l，枪管与水平的夹角为θ，角速度为ω，电机输出扭矩为u，则有下列等式：

$$\begin{cases} \dot{\theta} = \omega \\ J\dot{\omega} = u - mg\dfrac{l}{2}\cos(\alpha + \theta) \end{cases}$$

其中$J\omega = u - mg\frac{l}{2}\sin(\alpha+\theta) = u - \frac{1}{2}mglcos\alpha cos\theta - \frac{1}{2}mglsin\alpha sin\theta = u - G_c cos\theta - G_s sin\theta$，令$J = \alpha_1$，$G_c = \alpha_1$，$G_s = \alpha_2$，则整理得：

$$\begin{cases} \dot{\theta} = \omega \\ \alpha_1\dot{\omega} = u - \alpha_2 cos\theta - \alpha_3 sin\theta \end{cases}$$

我们的任务是控制电机输入u，让枪管角度θ转动至目标角度θ_d，即让误差$e_\theta = \theta_d - \theta$趋近于 0。整个系统的工作过程是电机扭矩先改变枪管的角速度，然后角速度再改变角度，所以控制器的设计也分为两步：首先找到角速度的目标值ω_d，使得$\theta \to \theta_d$，第二步设计电机输入u，使得$\omega \to \omega_d$。

第一步：

对于$e_\theta = \theta_d - \theta$，构造 Lyapunov 函数$V_1 = \frac{1}{2}{e_\theta}^2$，$\dot{V_1} = e_\theta\dot{e_\theta} = e_\theta\left(\dot{\theta_d} - \omega\right) = -k_1{e_\theta}^2$，于是$\omega_d = \dot{\theta_d} + k_1e_\theta$。

第二步：

对于$e_\omega = \omega_d - \omega$，构造 Lyapunov 函数$V_2 = \frac{1}{2}{e_\theta}^2 + \frac{1}{2}{e_\omega}^2$，于是有：

$$\dot{V_2} = e_\theta\dot{e_\theta} + e_\omega\dot{e_\omega} = e_\theta\left(\dot{\theta_d} - \omega\right) + e_\omega\dot{e_\omega} = e_\theta\left(\omega_d - \dot{\theta_d} - e_\omega\right) + e_\omega\dot{e_\omega} = -k_1{e_\theta}^2 + e_\omega(\dot{e_\omega} + e_\theta)$$

于是有$\dot{e_\omega} + e_\theta = -k_2e_\omega$，而$\dot{e_\omega} = \dot{\omega_d} - \dot{\omega} = \dot{\omega_d} - \frac{1}{\alpha_1}(u - \alpha_2 cos\theta - \alpha_3 sin\theta)$，整理可得如下结果：

$$u = \alpha_1\dot{\omega_d} + \alpha_2 cos\theta + \alpha_3 sin\theta + \alpha_1 e_\theta + \alpha_1 k_2 e_\omega$$

然而云台的转动惯量、质量都是未知的，即$\alpha_{1\sim3}$是未知的，我们还需要进行系统辨识。与e_θ和e_ω类似，对于这三个系数，我们用$\hat{\alpha}_{1\sim3}$表示估计值，$\tilde{\alpha}_{1\sim3} = \alpha_{1\sim3} - \hat{\alpha}_{1\sim3}$表示估计误差，显然我们需要让$\tilde{\alpha}_{1\sim3} \to 0$，基于这个思路，构造 Lyapunov 函数$V_3 = \frac{1}{2}{e_\theta}^2 + \frac{1}{2}{e_\omega}^2 + \frac{1}{2\alpha_1\gamma_1}{\tilde{\alpha}_1}^2 + \frac{1}{2\alpha_1\gamma_2}{\tilde{\alpha}_2}^2 + \frac{1}{2\alpha_1\gamma_3}{\tilde{\alpha}_2}^2$，于是有：

$$\dot{V_3} = e_\theta \dot{e_\theta} + e_\omega \dot{e_\omega} + \frac{1}{\alpha_1\gamma_1}\tilde{\alpha}_1\dot{\tilde{\alpha}}_1 + \frac{1}{\alpha_1\gamma_2}\tilde{\alpha}_2\dot{\tilde{\alpha}}_2 + \frac{1}{\alpha_1\gamma_3}\tilde{\alpha}_2\dot{\tilde{\alpha}}_2$$

$$= -k_1{e_\theta}^2 + e_\omega(\dot{e_\omega} + e_\theta) - \frac{1}{\alpha_1}(\frac{1}{\gamma_1}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 + \frac{1}{\gamma_2}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 + \frac{1}{\gamma_3}\tilde{\alpha}_3\dot{\hat{\alpha}}_3)$$

$$\dot{e_\omega} = \dot{\omega_d} - \dot{\hat{\omega}} = \dot{\omega_d} - \frac{1}{\alpha_1}(\hat{u} - \alpha_2 cos\theta - \alpha_3 sin\theta)，\hat{u} = \hat{\alpha}_1\dot{\omega_d} + \hat{\alpha}_2 cos\theta + \hat{\alpha}_3 sin\theta + \hat{\alpha}_1 e_\theta + \hat{\alpha}_1 k_2 e_\omega$$

$$\dot{V_3} = -k_1{e_\theta}^2 - k_2{e_\omega}^2 + \frac{1}{\alpha_1}(e_\omega(\tilde{\alpha}_1(\dot{\omega_d} + e_\theta + k_2 e_\omega) + \tilde{\alpha}_2 cos\theta + \tilde{\alpha}_3 sin\theta) - \frac{1}{\gamma_1}\tilde{\alpha}_1\dot{\hat{\alpha}}_1 - \frac{1}{\gamma_2}\tilde{\alpha}_2\dot{\hat{\alpha}}_2 - \frac{1}{\gamma_3}\tilde{\alpha}_3\dot{\hat{\alpha}}_3)$$

令$\dot{\hat{\alpha}}_1 = \gamma_1 e_\omega(\dot{\omega_d} + e_\theta + k_2 e_\omega)$，$\dot{\hat{\alpha}}_2 = \gamma_2 e_\omega cos\theta$，$\dot{\hat{\alpha}}_3 = \gamma_3 e_\omega sin\theta$，则$\dot{V_3} = -k_1{e_\theta}^2 - k_2{e_\omega}^2$，为负定，因此三个系数可以通过下列积分得到：

$$\begin{cases} \hat{\alpha}_1 = \int \gamma_1 e_\omega(\dot{\omega_d} + e_\theta + k_2 e_\omega)dt \\ \hat{\alpha}_2 = \int \gamma_2 e_\omega cos\theta dt \\ \hat{\alpha}_3 = \int \gamma_3 e_\omega sin\theta dt \end{cases}$$

到此所有公式推导完毕，整理一下：

$$u = \hat{\alpha}_1\dot{\omega_d} + \hat{\alpha}_2 cos\theta + \hat{\alpha}_3 sin\theta + \hat{\alpha}_1 e_\theta + K_P e_\omega + K_I\int e_\omega dt$$

由于真实的物理机构更加复杂，该模型可能会有一些微小扰动没有考虑进去，所以在上式中加入了最后一项积分，以控制这些微小扰动。θ_d为目标角度的输入，$\theta\ \ \omega$可由编码器或陀螺仪解算得到，$\dot{\theta_d}\ \ \dot{\omega_d}$可由跟踪微分器或其他微分器得到。

二、底盘电机控制

步骤基本与云台控制相同，而且比云台的更加简单。设轮子的转动惯量为J，各种摩擦力矩之和为M_f，电机输出力矩为u，角速度为ω，则有以下等式成立：

$$J\dot{\omega} = u - M_f$$

给定轮子的目标角速度ω_d，对于误差$e_\omega = \omega_d - \omega$，要让它趋近于 0，构造 Lyapunov 函数$V = \frac{1}{2}{e_\omega}^2$，

$\dot{V} = e_\omega\dot{e_\omega} = -k{e_\omega}^2$，于是有$\dot{e_\omega} = \dot{\omega_d} - \frac{1}{J}\left(u - M_f\right) = -ke_\omega$，整理得到：

$$u = M_f + J\dot{\omega_d} + Jke_\omega$$

对于M_f, J，设其估计值为$\widehat{M_f}, \hat{J}$，对应的误差为$\widetilde{M_f}, \tilde{J}$，构造 Lyapunov 函数$V_1 = \frac{1}{2}{e_\omega}^2 + \frac{1}{2J\gamma_1}{\widetilde{M_f}}^2 + \frac{1}{2J\gamma_2}\tilde{J}^2$，

有：

$$\dot{V_1} = e_\omega\dot{e_\omega} + \frac{1}{J\gamma_1}\widetilde{M_f}\dot{\widetilde{M_f}} + \frac{1}{J\gamma_2}\tilde{J}\dot{\tilde{J}}，\dot{e_\omega} = \dot{\omega_d} - \frac{1}{J}\left(\hat{u} - M_f\right)，\hat{u} = \widehat{M_f} + \hat{J}\dot{\omega_d} + \hat{J}ke_\omega$$

整理可得：$\dot{V_1} = -k{e_\omega}^2 + \frac{e_\omega}{J}\left(\widetilde{M_f} + \tilde{J}\dot{\omega_d} + \tilde{J}ke_\omega\right) - \frac{1}{J}(\frac{1}{\gamma_1}\widetilde{M_f}\dot{\widehat{M_f}} + \frac{1}{\gamma_2}\tilde{J}\dot{\hat{J}})$，令$\dot{\widehat{M_f}} = \gamma_1 e_\omega$，$\dot{\hat{J}} = \gamma_2 e_\omega(\dot{\omega_d} + ke_\omega)$，则$\dot{V_1} = -k{e_\omega}^2$，负定。故：

$$\begin{cases} \hat{J} = \int \gamma_1 e_\omega(\dot{\omega_d} + ke_\omega)dt \\ \widehat{M_f} = \int \gamma_2 e_\omega dt \end{cases}$$

完毕。