

Inter2. Prefixes of **abc** are $\{\lambda, a, ab, abc\}$.

Problem 2. Prove

$$\sum_{k=0}^n 2^k = 2^{n+1} - 1, n \geq 0$$

Basis $P(0)$ when $n = 0$

$$\sum_{k=0}^0 2^k = 2^0 = 1 = 2^1 - 1 = 2^{0+1} - 1$$

is true.

Assume $P(i)$ is true, $A_L = A_R$ when $n = i$

$$A_L = \sum_{k=0}^i 2^k = 2^{i+1} - 1 = A_R$$

Inductive step $P(i)$ to $P(i+1)$

$$\begin{aligned} \sum_{k=0}^{i+1} 2^k &= 2^{i+2} - 1 \\ &= A_L + 2^{i+1} \\ &= A_R + 2^{i+1} = 2^1 \cdot 2^{i+1} \\ &= 2^{i+2} - 1 \end{aligned}$$

$$\therefore \sum_{k=0}^n 2^k = 2^{n+1} - 1, n \geq 0$$