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$\vec{X} = (x_1, \dots, x_n) \sim$ Мультиномиальное распределение с параметром $\vec{p}$.

$P(p) \sim \frac{1}{B(\alpha)} \cdot \prod x_i^{\alpha_i - 1}$ $\qquad P(p|D) = ?$

$P(X|p) = \frac{n!}{n_1! \cdot \ldots \cdot n_m!} \cdot p_1^{n_1} \cdots p_m^{n_m}$, $\qquad P(x|D) = ?$

$$P(p|D) \propto P(D|p) \cdot P(p) = \frac{n!}{\prod_{i=1}^{m} n_i!} \cdot \prod_{i=1}^{m} p_i^{n_i} \cdot \frac{1}{B(\alpha)} \cdot$$

$$\cdot \prod_{i=1}^{m} p_i^{\alpha_i - 1} \propto \prod p_i^{n_i + \alpha_i - 1} \sim Dir(n_1 + \alpha_1, n_2 + \alpha_2, \dots, n_m + \alpha_m)$$

$$P(p|D) \sim \frac{1}{B(\alpha + n)} \cdot \prod p_i^{(\alpha_i + n_i - 1)}$$

$$P(x|D) \sim \int_p P((x, p)|D) \cdot dp = \int_p P(x|p, D) \cdot P(p|D) \cdot dp =$$

$$= \int_p P(x|p) \cdot P(p|D) \cdot dp = \int_p \frac{n!}{\prod_{i=1}^{m} x_i!} \cdot \prod_{i=1}^{m} p_i^{x_i} \cdot \frac{1}{B(\alpha + n)} \cdot \prod_{i=1}^{m} p_i^{\alpha_i + n_i - 1} \, dp =$$

$$= \frac{n!}{\prod x_i!} \cdot \frac{1}{B(\alpha + n)} \cdot \int_p \prod p_i^{x_i + \alpha_i + n_i - 1} \, dp \; (=)$$

$$\int_{\substack{\sum p_i = 1 \\ p_i \geq 0}} \prod_{i=1}^{m} p_i^{\beta_i - 1} = B(\beta)$$

$$(=) \; \frac{n!}{\prod x_i!} \cdot \frac{1}{B(\alpha + n)} \cdot B(x + \alpha + n) \sim DirMult(n, \alpha + n)$$

Ответ: $P(p|D) \sim Dir(n_1 + \alpha_1, \dots, n_m + \alpha_m)$

$P(X|D) \sim DirMult(n, \alpha + n)$