angle formulas

Half-angl formulas

Addition and subtraction formulas

Lecture 25 More trigonometric formulas

MATH 0200

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Outline

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Doubleangle formula

Half-angle ormulas

Addition and subtraction Double-angle formulas

2 Half-angle formulas

3 Addition and subtraction formulas

• $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$;

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Addition and subtraction



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Addition and subtraction

- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$;
- $\cos(2\alpha) = \cos^2(\alpha) \sin^2(\alpha) = 2\cos^2(\alpha) 1 = 1 2\sin^2(\alpha);$

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Addition and subtraction formulas

- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$;
- $\cos(2\alpha) = \cos^2(\alpha) \sin^2(\alpha) = 2\cos^2(\alpha) 1 = 1 2\sin^2(\alpha);$
- $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$.

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Addition and subtraction

- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$;
- $\cos(2\alpha) = \cos^2(\alpha) \sin^2(\alpha) = 2\cos^2(\alpha) 1 = 1 2\sin^2(\alpha);$
- $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$.

Example

• Let's pretend that we don't know the value of $\sin(60^\circ)$, but know that $\sin(30^\circ) = \frac{1}{2}$ and $\cos(30^\circ) = \frac{\sqrt{3}}{2}$.

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Addition and subtraction formulas

- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$;
- $\cos(2\alpha) = \cos^2(\alpha) \sin^2(\alpha) = 2\cos^2(\alpha) 1 = 1 2\sin^2(\alpha);$
- $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$.

Example

• Let's pretend that we don't know the value of $\sin(60^\circ)$, but know that $\sin(30^\circ) = \frac{1}{2}$ and $\cos(30^\circ) = \frac{\sqrt{3}}{2}$. We get $\sin(60^\circ) = 2\sin(30^\circ)\cos(30^\circ) = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$.

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•
$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha);$$

- $\cos(2\alpha) = \cos^2(\alpha) \sin^2(\alpha) = 2\cos^2(\alpha) 1 =$ $1-2\sin^2(\alpha)$:
- $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$.

Example

- Let's pretend that we don't know the value of $\sin(60^{\circ})$, but know that $\sin(30^\circ) = \frac{1}{2}$ and $\cos(30^\circ) = \frac{\sqrt{3}}{2}$. We get $\sin(60^\circ) = 2\sin(30^\circ)\cos(30^\circ) = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$.
- but know that $\tan(30^\circ) = \frac{1}{\sqrt{3}}$.

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Addition and subtraction formulas

- $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$;
- $\cos(2\alpha) = \cos^2(\alpha) \sin^2(\alpha) = 2\cos^2(\alpha) 1 = 1 2\sin^2(\alpha);$
- $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$.

Example

- Let's pretend that we don't know the value of $\sin(60^\circ)$, but know that $\sin(30^\circ) = \frac{1}{2}$ and $\cos(30^\circ) = \frac{\sqrt{3}}{2}$. We get $\sin(60^\circ) = 2\sin(30^\circ)\cos(30^\circ) = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$.
- 2 Let's pretend that we don't know the value of $\tan(60^\circ)$, but know that $\tan(30^\circ) = \frac{1}{\sqrt{3}}$.

We get
$$\tan(60^\circ) = \frac{2\tan(30^\circ)}{1-\tan^2(30^\circ)} = \frac{\frac{2}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}(1-\frac{1}{3})} = \frac{2}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

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Doubleangle formulas

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formulas Addition

and subtraction formulas

•
$$\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$$

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Addition and subtraction

•
$$\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$$

•
$$\cos(\alpha/2) = \pm \sqrt{\frac{1+\cos(\alpha)}{2}};$$

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Addition and subtraction

•
$$\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$$

•
$$\cos(\alpha/2) = \pm \sqrt{\frac{1+\cos(\alpha)}{2}};$$

•
$$\tan(\alpha/2) = \frac{1-\cos(\alpha)}{\sin(\alpha)}$$
.

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Doubleangle formula:

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Addition and subtraction formulas • $\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$

•
$$\cos(\alpha/2) = \pm \sqrt{\frac{1+\cos(\alpha)}{2}};$$

•
$$\tan(\alpha/2) = \frac{1-\cos(\alpha)}{\sin(\alpha)}$$
.

Example

• Find $\sin(22.5^{\circ})$.

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Doubleangle formulas

Half-angle formulas

Addition and subtraction formulas • $\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$

• $\cos(\alpha/2) = \pm \sqrt{\frac{1+\cos(\alpha)}{2}};$

• $\tan(\alpha/2) = \frac{1-\cos(\alpha)}{\sin(\alpha)}$.

Example

• Find $\sin(22.5^{\circ})$.

We get
$$\sin(22.5^\circ) = \sin((45/2)^\circ) = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$
.

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Doubleangle formulas

Half-angle formulas

Addition and subtraction • $\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$

•
$$\cos(\alpha/2) = \pm \sqrt{\frac{1 + \cos(\alpha)}{2}};$$

•
$$\tan(\alpha/2) = \frac{1-\cos(\alpha)}{\sin(\alpha)}$$
.

Example

• Find $\sin(22.5^{\circ})$.

We get
$$\sin(22.5^\circ) = \sin((45/2)^\circ) = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$
.

 \bigcirc Find $\cos(75^{\circ})$.

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Addition and subtraction formulas • $\sin(\alpha/2) = \pm \sqrt{\frac{1-\cos(\alpha)}{2}};$

• $\cos(\alpha/2) = \pm \sqrt{\frac{1+\cos(\alpha)}{2}};$

• $\tan(\alpha/2) = \frac{1-\cos(\alpha)}{\sin(\alpha)}$.

Example

• Find $\sin(22.5^{\circ})$.

We get
$$\sin(22.5^\circ) = \sin((45/2)^\circ) = \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$
.

 \circ Find $\cos(75^{\circ})$.

Notice that 75 = 150/2 and

$$\cos(150^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$
. We get

$$\cos(75^\circ) = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}.$$

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Doubleangle formula:

Half-ang

Addition and subtraction formulas

Addition

 $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

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formulas

Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$
- $\cos(a+b) = \cos(a)\cos(b) \sin(a)\sin(b).$

Subtraction

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Addition

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

Subtraction

 $\bullet \sin(a-b) = \sin(a)\cos(b) - \sin(b)\cos(a);$

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

Subtraction

- $\bullet \sin(a-b) = \sin(a)\cos(b) \sin(b)\cos(a);$

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

Subtraction

- $\bullet \sin(a-b) = \sin(a)\cos(b) \sin(b)\cos(a);$
- $\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b).$

Example

• Find $\sin(105^\circ)$.

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

Subtraction

- $\bullet \sin(a-b) = \sin(a)\cos(b) \sin(b)\cos(a);$

Example

• Find $\sin(105^\circ)$.

We notice that 105 = 60 + 45, therefore $\sin(105^\circ) = \sin(60^\circ)\cos(45^\circ) + \sin(45^\circ)\cos(60^\circ) = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

Subtraction

- $\bullet \sin(a-b) = \sin(a)\cos(b) \sin(b)\cos(a);$

Example

• Find $\sin(105^{\circ})$.

We notice that 105 = 60 + 45, therefore $\sin(105^\circ) = \sin(60^\circ)\cos(45^\circ) + \sin(45^\circ)\cos(60^\circ) =$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

② Express $\cos\left(a + \frac{\pi}{3}\right)$ in terms of $\sin(a)$ and $\cos(a)$.

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Doubleangle formula:

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Addition

- $\bullet \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a);$

Subtraction

- $\bullet \sin(a-b) = \sin(a)\cos(b) \sin(b)\cos(a);$
- $\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b).$

Example

• Find $\sin(105^\circ)$.

We notice that 105 = 60 + 45, therefore $\sin(105^\circ) = \sin(60^\circ)\cos(45^\circ) + \sin(45^\circ)\cos(60^\circ) =$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

- Express $\cos\left(a + \frac{\pi}{3}\right)$ in terms of $\sin(a)$ and $\cos(a)$. $\cos\left(a + \frac{\pi}{3}\right) = \cos(a)\cos\left(\frac{\pi}{3}\right) - \sin(a)\sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$
 - $\frac{1}{2}\cos(a) \frac{\sqrt{3}}{2}\sin(a).$