Spring 2021

MATH 1025: Introduction to Cryptography

Bonus 5

goldfish in the pond

Problem. Let X be a random variable on the probability space $\mathbb{Z}_{\geq 0} = \{0, 1, 2, \ldots,\}$, which has a Poisson distribution with parameter λ.

(a) [2 **pts**] $\mathbb{E}(X^2) = \sum_{k=0}^{\infty} k^2 P_X(k) = \sum_{k=0}^{\infty} k^2 \cdot \frac{\lambda^k}{k!} e^{-\lambda}$.

Find a differential operator $\mathcal{D}$, such that $\mathbb{E}(X^2) = \mathcal{D}\left(\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda}\right)$.[1]

(b) [2 **pts**] Using that $\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} = \sum_{k=0}^{\infty} P_X(k) = 1$, compute $\mathbb{E}(X^2) = \mathcal{D}(1)$ and find the variance $Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$ (recall that we computed $\mathbb{E}(X) = \lambda$ in class).

[1] **Hint:** take a look at the differential operator on the bottom of page 10, 'Week 11' notes and modify it accordingly.