MATH 146B: Ordinary and Partial Differential Equations

Midterm Review Problems

Problems in Homework 1

Review problems in the 1st homework for the topics roots of polynomials and solutions of ODE, method of undetermined coefficients, method of variation of parameters.

Taylor series and radius of convergence

Problem 1. Find the Taylor series centered at x_0 for the provided function. Additionally, determine the radius of convergence of the series.

(a) (1 point) $f(x) = \frac{5}{x^3}$ and $x_0 = -2$.

(b) (1 point) $g(x) = x + 3x^2 + e^{-x}$ and $x_0 = 0$.

Power series solutions near an ordinary point

Problem 2. Consider the equation $y'' + xy' + 2y = 0$ and the point $x_0 = 0$.

(a) (1 point) Seek a solution in the form $y = \sum_{n \geq 0} c_n(x - x_0)^n$ and determine the first four nonzero terms (expressions in the two parameters c_0 and c_1).

(b) (1 point) Let denote the solution corresponding to the values $c_0 = 5$ and $c_1 = -3$. Utilize your findings in part (a) to compute the values of (x_0), $'(x_0)$, and $''(x_0)$.

(c) (1 point) Let [donkey] denote the solution corresponding to the values $c_0 = -2$ and $c_1 = 4$. Compute the values of [donkey](x_0), [donkey]$'(x_0)$ and [donkey]$''(x_0)$.

(d) (1 point) Check if the functions $\left\{ [shrek], [donkey] \right\}$ form a fundamental set of solutions.

Hint: compute $W\left([shrek], [donkey] \right)(x_0)$.

Euler equation

Problem 3. Find the general solution for each of the following equations on the interval $(0, a)$, where $a > 0$. (see Lecture 8 notes).

(a) (1 point) $x^2 y'' + 7xy' + 9y = 0$.

(b) (1 point) $x^2 y'' - xy' + 3y = 0$.

(c) (1 point) $x^2 y'' - 5xy' + 8y = 0$.

Singularities

Problem 4. For each equation, provide a complete list of singular points and determine their types.

(*a*) (1 point) $(5x^2 + 3)(x - 4)(x + 3)^2 y'' + (x^2 - 16)y' + (x + 3)y = 0.$

(*b*) (1 point) $(x^2 + x^4)y'' + 5\sin(x)y' + 2024y = 0.$