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Abstract:
Bulgaria started the transition in the early 90's with a sovereign default and debt restructuring. Later on, under a strict fiscal discipline, the country succeeded to reduce significantly its debt burden and is currently among the top EU performers in that respect. The current debt outstanding is composed mainly of local currency treasuries issued on the domestic market as well as Eurobonds and Global bonds on the international one. These instruments give rise to two risky spreads - credit and currency. The Currency Board Arrangement and the fixed exchange rate regime the country follows prevent from a discretionary monetary policy and this gives relative stability to the bonds' yields and the risky spreads. Their financial role starts dominating over any macroeconomic one making them a natural object for investigation with financial engineering tools. The main focus of the paper is an analysis of the informational content of the risky spreads in a multifaceted way from no-arbitrage, financial, and macroeconomic points of view. For the purpose we build from scratch both a reduced form model in a Heath-Jarrow-Morton setting and a structural one which extends the classical Merton model to the case of a risky sovereign. We estimate the former model and extract the priced factors. Then we relate them to a set of indicators the latter model provides as well as to suitable macrofinancial and macroeconomic variables. The setting allows to have a better understanding of the spreads' risk drivers, which could be useful not only to the participants in the bond market, but also for drawing qualitative conclusions what kind of optimal debt issuance policy the country could follow.

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1. Introduction

The primary and the secondary markets of local currency government bonds (treasuries) are a vital part of the financial system of every market economy. In the case of Bulgaria, this market was one of the first that developed rapidly in the first years of transition in the early 90's. That was mainly due to the large need for an internal source of financing driven by the skyrocketing budget deficits and the exclusion of the country from the international capital markets after the moratorium of 1990 on the accumulated foreign debt from the communist times. After the Brady bonds deal of 1994 and the achieved foreign debt restructuring the country was back on the international markets. However, it was only after the financial crisis of 1997 and the introduction of the Currency Board Arrangement (CBA) when was Bulgaria on a stabilization track. In that period of budget balancedness and macroeconomic stability, the role of the treasuries changed fundamentally. Providing credit to the government was not any more the main objective. Further, the Central Bank (CB) is banned by law from monetizing the budget deficit under the CBA. All that made the treasuries become a liquid low risk instrument in which local and foreign players could invest without having been disturbed by any local monetary shocks from open market operations. That brought stability to the expectations and thus to the Bulgarian leva (BGN) yield curve. Actually, only after the CBA implementation can we talk about a proper yield curve evolvement. That is especially valid after 2002 when the true long end segment of the curve developed with the Ministry of Finance (MF) started issuing bonds with maturities of 10 and 15 years.

In respect to all above, an interesting dilemma appeared. If the state budget needed no debt financing until the outbreak of the 2008 crisis, then what policy the MF should follow. In these pre-crisis times, the development of the financial system, the high economic growth, and the macroeconomic stability increased the demand for high quality instruments from banks and large institutional investors, both local and foreign, for diversifying their portfolios. The optimal strategy for the Fiscal Authority (FA) seemed to be one of maintaining a stable low level of indebtedness by rolling over old debt in a way that does not create shocks to the system. Actually, exactly such kind of policy was followed and there was consistency in that respect among the governments that took office. So although it lost its macro function, from 1997 to 2008, the role of the treasuries, and the government debt in general, was broader compared to the primary one of financing the budget deficit. It was one of maintaining the financial system and the capital market, easing the allocation of risk and return in the economy, creating a low risk highly liquid benchmark asset setting the time value of the local currency. That is in accordance with the overall philosophy of the CBA. Namely, the setting relies on an automatic self-adjustment and no active accommodative macroeconomic policy. Such could be followed only indirectly either as a by-product of the financial policy or as a positive externality to the overall institutional and society development.

In respect to the former, the primary role of the CB and the FA is to keep the stability of the financial system and achieve low cost of financing of the public sector. That was exactly targeted during the boom years by regulation of the financial sector in controlling its risk exposure. Further, as already mentioned, if there was no need to finance the budget, from a financial point of view it was optimal the spare resources coming from the economic growth and the large foreign direct and portfolio investments to be channeled towards reducing the foreign debt. Last but not least, an interesting macrofinancial tool in respect to the fiscal reserve[1] appeared. Despite the pressure it to be discretionally used in the real economy or for social needs, its primary role in that period was to accumulate a buffer fund against economic

[1] It is formed by the surplusses the government has accumulated through time, stays deposited in the CB, and is treated like a liability on its balance sheet. Any change in this item causes an equivalent change in the monetary base and can be considered as a quasi-monetary policy.

downturns thus reducing the sovereign risk of the country. It acted counter-cyclically and in opposite to the accumulated current account deficit.

In respect to the latter, interestingly, factors like infrastructure, legal system, regulatory regimes, business practices, etc. have a macroeconomic impact through the investments inflow and the balance of payments. Under a CBA, this could be a policy tool with huge implications not only for the international position of the country, but also for its economic development.

From the outbreak of the 2008 crisis to now, the role of the treasuries remained again within their financial function and hardly can we say that any macro one re-emerged. The policy makers stuck to fiscal austerity by reducing the public spending in accordance to the shrunk GDP and foreign investments inflow, no changing of the tax rates, and use of the accumulated so far large fiscal reserve. Small budget deficits appeared but they were completely manageable and unlike many EU members didn't put the country under risk. The increase of the domestic debt was marginal. The economic policy followed was based in general on the philosophy that for a small open economy the harmful effect of any rise of the debt level would spur a credit risk surge which would completely crowd out any positive effects on the aggregate demand. Namely, there would be a transfer of risk from the sovereign to the corporate and consumer sectors both though the credit channel in terms of higher lending rates and reduced net foreign investments. Although there was a huge initial spike in the credit spreads in 2009 driven mainly by a global rise in risk aversion, especially towards the emerging markets segment, as well as uncertainty how the local authorities are going to fight against the crisis, Bulgaria achieved to have one of the lowest premiums of its treasuries across the CEE countries and a stable fiscal position. Further, the country presently covers formally the Maastricht convergence criteria in terms of the debt to GDP ratio, total amount of debt, and exchange rate. It is also very close to covering the interest rate and the inflation criteria giving it even a better position to some core Eurozone members.

If the local treasuries curve references the total sovereign risk, any securities the country has in foreign hard currency would allow separating it into credit and currency ones. Similar to many emerging market countries, Bulgaria went through a restructuring of its foreign debt. In 2002, it substituted its Brady bonds with a Eurobond that matured in 2007 and two Global bonds denominated in EUR and US dollar maturing in 2013 and 2015 respectively. These instruments lack the complicated cash flow structures of the old Brady bonds and have higher liquidity. Referenced to the German Bunds they instantly started to serve as a suitable benchmark for the credit risk of the country. Interestingly, they had a very positive influence on the domestic treasuries market by making possible to price the longer term segment of the curve with a mark up over them. The latter captures the currency risk. However, the bonds were just giving several dots of a hypothetical risky EUR curve and getting a benchmark for the credit risk for the whole maturity spectrum was not possible. Only after instruments like CDS became relatively liquid from 2003 does this become feasible.

Both the credit and the currency spreads serve as important indicators for the sovereign risk. They are characterized by complex dynamics and it is important their informational content to be well understood. This poses not only an empirical challenge, but also a theoretical one. Namely, since the interest rate is simultaneously a financial and a macroeconomic variable, very diverse forces act on the yield curves and the risky spreads. On one hand, imposing no-arbitrage puts certain restrictions on their shape and the evolution of the driving factors[2]. On the other hand, being items on the liability side of an aggregate sovereign balance sheet, the foreign bonds, the treasuries, and the monetary base must show an internal consistency if viewed as financial contracts with payoffs contingent on the

[2] like smoothness of the former and mean reversion of the latter

country’s assets. Last but not least, the rates and the spreads are part of important macro transmission channels that must be considered together with other macro variables. Such a multifaceted analysis is highly desirable and requires setting up formal models that can serve later as a base for empirical applications.

The paper will follow exactly such an approach. First, we build a coherent theoretical setting for analyzing the credit and currency spreads of a risky sovereign that follows a fixed exchange rate regime. This part is highly motivated by the explained above in length situation of Bulgaria, but it has its own separate reading as well. For the purpose we construct from scratch a no-arbitrage reduced form model which will allow us not only to understand better how to price the treasuries and the foreign bonds, but also to extract the driving factors. This is done in the most general Heath-Jarrow-Morton setting. We derive the no-arbitrage constitutions, analyze their content, and then move to a specific affine realization. Under it, we estimate the model and derive the priced factors. Then we move to a structural financial model in the spirit of the classical Merton model for a firm. We extend it suitably to the case of a risky sovereign with debt issues in different currencies. We consider formally the subordination and the possibility the country to monetize in avoiding nominal default on the domestic debt. This setting allows us to derive the fundamental values of the two spreads from a financial point of view. Further, it provides us with relevant financial indicators whose impact on the extracted factors from before must be considered. We calibrate the structural model, derive the indicators’ values, and analyze their influence together with a set of macro indicators. That is precisely the second aim of the paper where the focus is explicitly on Bulgaria.

Both from theoretical and empirical points of view the academic literature seems to be vague in investigating the issues that have been raised above. In terms of the former, a formal setting for analysis of the credit and currency spreads of an emerging market country by no arbitrage considerations seems to be lacking. If two countries' interest rate models are well-known in the academic literature, when credit risk kicks in, there are only a few. However, they either consider the problem from a reduced form point of view or from a structural one under a single currency debt issue which makes the approach incomplete. In terms of the latter, if there is research done on emerging market countries for single spreads, there is none to the author’s knowledge for both of them. That is valid for Bulgaria as well.

For a recent treatment of two countries interest rate models see Bjork (1998) and Slinko (2006). However, they focus only on the case of no default risk in a simple diffusion setting. Bjork, Kabanov, and Runggaldier (1995) and Bjork, Masi, et. al. (1997) consider a HJM framework when the dynamics of the assets is driven by jump diffusions, but again with no default risk. Schönbucher (1998) adjusts the HJM framework to the case of default risk but only in a single currency setting. Jankowitsch and Pichler (2005) show empirically in a cross-currency setting that there is dependence between the corporate credit spreads and the exchange rate dynamics but do not have formal analysis. Ehlers and Schönbucher (2006) give a cross-currency model for CDS of two obligors which accounts for such dependence and allows also for jumps in the exchange rate at the default time. However, their setting is different from ours due to two reasons. First, the exchange rate is between hard currencies which is not our case. Second, we have a single risky obligor issuing bonds with a possible monetization effect and macroeconomic forces that act too. So there is a need for a flexible reduced form model for the setting we concentrate on and we provide such. From a structural point of view, Frenkel et. al. (2004) modify the classical Merton model to the case of a country and demonstrate that the analysis does not change too much. Gray and Malone (2008) extend their model to a sovereign having both local currency and foreign currency debt issues. However, they do not provide explicit formulas for the two risky spreads and do not have any factor and sensitivity analysis for them. There are a few papers, see Hoffmaister, et al. (2010) for a review, devoted to the yield curves of CEE countries. However, they concentrate mainly on the local currency curves from a macroeconomic point of view. Arbitrage is not tackled formally and is not of main concern.

In the case of Bulgaria, there are several papers dealing with the yield curves and the risky spreads but only very indirectly and from specific points of view. Manchev and Budina (2000) investigate the determinants of the Bulgarian credit spread extracted from the old Brady bonds in a simple regression framework without any no-arbitrage and term structure considerations. Nenovsky and Chobanov (2004) concentrate on the short end of the treasuries yield curve aiming to extract the systematic risk of the CBA arrangement. Minassian (2005) provides a comprehensive analysis of the Brady bonds restructuring deal and to what extent it was beneficial for the country.

2. Reduced form setting

In this section, we first lay the foundations in brief for pricing of risky debt in a general reduced form setting. Then we move to our concrete model for pricing foreign and domestic debt of Bulgaria. We conclude by analysis of the no-arbitrage conditions and pave the way for moving to the structural view.

2.1. Risky bonds pricing

We consider a probability space $\left(\Omega,(G_t)_{t\geq 0},P\right)$ which supports an $n-$dimensional Brownian motion $W^P=(W_1,W_2,...,W_n)$ under the objective probability measure P and a marked point process:

$$\mu:(\Omega,B(R^+),\varepsilon)\to R^+$$

with markers $\left(\tau_i,X_i\right)$ in a measurable space $\left(E,\varepsilon\right)$, where $E=\left[0,1\right]$, and with ε we denote the Borel subsets of E. We assume that $\mu\left(\omega,dt,dx\right)$ has a separable compensator of the form:

$$\upsilon:(\Omega,B(R^+),\varepsilon)\to R^+$$
$$\upsilon(\omega,dt,dx)=h(\omega;t)F_t(\omega;dx)dt,$$

where

$$h(\omega;t)=\int_{R^+}\upsilon(\omega;t,dx)$$

is a G_t measurable intensity and the marks have a conditional distribution of the jumps of $F_t(\omega;dx)$. So we have $\int_E F_t(\omega;dx)=1$ to hold.

Effectively, the marked point process $\left(\tau_i,X_i\right)$ is characterized by the probability measure $\mu\left(\omega;dt,dx\right)$ which gives the number of jumps with size dx in a small time interval of dt. The compensator $\upsilon\left(\omega,dt,dx\right)$ provides a full probability characterization of the process. It incorporates in itself two effects. On one hand, we have the intensity $h(\omega;t)dt$ which gives the conditional probability of a jump of the process in a small time interval of dt based on the whole market information up to t. On the other hand, we have the conditional distribution $F_t(\omega;dx)$ of the markers X in the case of a jump realization.

We can look at the jumps of the marked point process as sequential defaults of an obligor at random times τ_i that lead to losses X_i at each of them. They can be considered also as a set of restructuring events leading to losses for the creditors. Under this general setting, the prices of the riskless and risky bonds are given by:

- **Riskless bond:**
$$P(t,T) = E^Q(\exp(-\int_t^T r(s)ds) \mid G_t) = \exp(-\int_t^T f(t,s)ds) \tag{1}$$
- **Risky bond:**
$$P^*(t,T) = E^Q(\exp(-\int_t^T r(s)ds) R(\omega;T) \mid G_t) = R(\omega;t)\exp(-\int_t^T f^*(t,s)ds), \tag{2}$$

where $r(t)$, $f(t,T)$, and $f^*(t,T)$ are the riskless spot, riskless forward, and risky forward rates respectively.

The pricing formula for the risky debt could be significantly simplified if assume a specific form of the recovery. Under a multiple defaults specification, we have that at every default there is a percentage mark down, q, from the previous recovery. This gives the form $R(\omega;\tau_i) = \left(1 - q(\omega;\tau_i, X_i\right) R(\omega;\tau_i-)$ and allows us to write:

$$\mu\left(\omega, dt, dx\right) = \sum_{s>0} 1_{\{s,\Delta N(\omega,s)>0\}}(dt, dx)$$
$$dR(\omega;t) = -R(\omega;t)\int_E q(\omega;t,x)\mu(\omega;dt,dx);\ R(\omega;0) = 1$$

If we assume no jumps of the intensity and the risk free rate at the default times, we would get the pricing formula:

$$\begin{aligned} P^*(t,T) &= E^Q(\exp(-\int_t^T r(s)ds) R(\omega;T) \mid G_t) \\ &= R(\omega;t)\exp(-\int_t^T (r(s) + h(s) \underbrace{\int_E q(\omega;s,x) F_s(dx) ds}_{q_e(s)}) \mid G_t) = R(\omega;t)\exp(-\int_t^T f^*(t,s)ds) \end{aligned}$$

It should be noted that there is no "last default" in this setting. So the default intensity does not to go to zero after any default. This combined with the continuity of the default process makes the general market filtration G_t behave like a background one. Thus we could avoid the complications connected with the general well-known Duffie, Schroeder, and Skiadas (1994) formula. Further, we have non-separability between the intensity and the recovery making the pricing formula to depend only on the generalized intensity $q_e(t) = \int_E q(\omega;t,x) F_t(dx)$.

2.2. Model formulation

2.2.1. General notes

Bulgaria, as already mentioned, has bonds denominated both in BGN and EUR. They give rise to two risky yield curves and thus to two risky spreads - credit and currency. In very general, the spreads arise due to the possibility the respective credit events to occur and their severity. To investigate them, a formal assumption is needed both about their characteristics and interdependence.

We will consider that the two types of debt have different priorities. The country is first engaged in meeting the foreign debt obligation from its limited international reserves. The impossibility this to be done leads to default or restructuring. In both cases, there is a credit event according to the ISDA classification. The foreign debt has a senior status. The spread that arises reflects the credit risk of the country. It is a function of 1) the probability of default; 2) the expected loss given default; 3) the risk aversion of the market participants to that event.

The domestic debt stays differently. It reflects the priority of the payments in hard currency and it incurs instantly the losses in case of insolvency. So this debt is the first to default and is subordinated. Technically, the credit event can be avoided because the country can always monetize it and pay the amounts due in local currency taking advantage of the fact that there is no resource constraint on it. However, the price for this is abandonment of the exchange rate regime, inflation pick-up, and exchange rate devaluation. This leads to real devaluation of the domestic debt as well. It is exactly the seigniorage and the dilution effect that cause the value to be lost. This resembles the case of a firm issuing more equity to avoid default. The spread of the domestic debt over the foreign one constitutes the currency spread. Its nature is very broad and it is not only due to the currency mismatch between the two types of debt. Namely it is a function of: 1) the probability of default and monetization; 2) the exchange rate devaluation after a monetization; 3) exchange rate devaluation solely due to an abandonment of the CBA; 4) the risk aversion of market participants to default, exchange rate devaluation, and the size of the devaluation. All these effects are captured by our model.

It must be noted that the effect of subordination should not be very strong for Bulgaria. It is much more plausible the domestic and foreign debt to receive an equal treatment. This is mainly due to the fact that the country initially pursued a goal to be a member of the EU which later transformed to become a member of the Eurozone. Letting a high currency spread will not only contradict to the interest rate criterion for joining the ERM II, but also would produce a strong negative signal for a CBA instability. However, changing the monetary regime always stay as an option exercising of which should be considered as a tail event.

2.2.2. Technical formulation

We use the setting of Subsection 2.1 modified to have two types of debt and exchange rate dynamics. First, we consider the case of no monetization and incurring of nominal losses. Then we introduce it and see how the analysis changes. Second, to avoid using of an additional marked point process, and thus of a second intensity, the default on the foreign debt is modeled indirectly. Namely, we will consider that default on the domestic debt leads to a default on the foreign debt as well, but due to the different priority of the two, we have just different losses incurred, respectively, recoveries. This means that by controlling recoveries we control default. If the insolvency to the domestic debt is so strong that leads to such for the foreign one, we incur zero recovery on the domestic debt and some positive one on the foreign debt. If the insolvency is mild, then we have only loss to the domestic debt, so we incur some positive recovery on the domestic debt and full one on the foreign debt. Third, we take as a benchmark Germany and EUR as the base hard currency.

We continue with the model setup. First, we give the suitable notation and assumptions. Then we move to pricing and derivation of the no-arbitrage conditions.

- **Notation**

$f_{EUR}(t,T)$ - nominal forward rate in EUR, Germany (Ger.)

$f^*_{EUR}(t,T)$ - nominal forward rate in EUR, Bulgaria (Bul.)

$f^*_{BGN}(t,T)$ - nominal forward rate in BGN, Bulgaria

$h^*_{EUR}(t,T) = f^*_{EUR}(t,T) - f_{EUR}(t,T)$ - credit spread, Bulgaria

$h^*_{BGN,EUR}(t,T) = f^*_{BGN}(t,T) - f^*_{EUR}(t,T)$ - currency spread, Bulgaria

$h^*_{BGN}(t,T) = f^*_{BGN}(t,T) - f_{EUR}(t,T)$ - general currency spread, Bulgaria

$P_{EUR}(t,T) = \exp(-\int_t^T f_{EUR}(t,s)ds)$ - domestic bond, Germany

$P^*_{f,EUR}(t,T) = R_{f,EUR}(t)\exp(-\int_t^T f^*_{EUR}(t,s)ds)$ - foreign bond, EUR, Bulgaria

$P^*_{d,BGN}(t,T) = R_{d,BGN}(t)\exp(-\int_t^T f^*_{BGN}(t,s)ds)$ - domestic bond, BGN, Bulgaria

$B_{EUR}(t) = \exp(\int_0^t r_{EUR}(s)ds)$ - bank account, EUR, Germany

$B^*_{f,EUR}(t) = R_{f,EUR}(t)\exp(\int_0^t r^*_{EUR}(s)ds)$ - bank account, EUR, Bulgaria

$B^*_{d,BGN}(t) = R_{d,BGN}(t)\exp(\int_0^t r^*_{BGN}(s)ds)$ - bank account, BGN, Bulgaria

$X(t)$ - exchange rate, EUR for 1 BGN

$1/X(t)$ - exchange rate, BGN for 1 EUR

$R_{f,EUR}(t)$ - recovery, foreign bond, EUR, Bulgaria

$R_{d,BGN}(t)$ - recovery, domestic bond, BGN, Bulgaria

The notation speaks for itself. We use asterisk to denote risk, the first letter (d or f) to denote domestic or foreign debt, and finally the currency of denomination is shown as EUR or BGN.

- **Currency denominations**

$P^*_{d,EUR}(t,T) = X(t)P^*_{d,BGN}(t,T)$ - domestic bond, EUR, Bulgaria

$B^*_{d,EUR}(t) = X(t)B^*_{d,BGN}(t)$ - domestic bank account, EUR, Bulgaria

$P^*_{f,BGN}(t,T) = \frac{1}{X(t)}P^*_{f,EUR}(t,T)$ - foreign bond, BGN, Bulgaria

$B^*_{f,BGN}(t) = \frac{1}{X(t)}B^*_{f,EUR}(t)$ - foreign bank account, BGN, Bulgaria

- **Intensities**

- Foreign debt	
intensity	$h_{EUR}(t) = h(t)$
generalized intensity	$h_{EUR}(t)q_{e,EUR}(t) = h(t)\int_E q_{f,EUR}(\omega;t,x)F_t(dx)$
- Domestic debt	
intensity	$h_{BGN}(t) = h(t)$
generalized intensity	$h_{BGN}(t)q_{e,BGN}(t) = h(t)\int_E q_{d,BGN}(\omega;t,x)F_t(dx)$

The generalized intensity characterizes default. Controlling in a suitable way the recovery, we can control it too and thus the default event. We turn attention now to the dynamics of the instruments under consideration.

- **Forward rates**

$$df_{EUR}(t,T) = \alpha_{EUR}(t,T)dt + \sum_{i=1}^{n} \sigma_{EUR,i}(t,T)dW_i^P(t)$$
$$df_{EUR}^*(t,T) = \alpha_{EUR}^*(t,T)dt + \sum_{i=1}^{n} \sigma_{EUR,i}^*(t,T)dW_i^P(t) + \int_E \delta_{EUR}^*(t,T,x)\mu(dt,dx)$$
$$df_{BGN}^*(t,T) = \alpha_{BGN}^*(t,T)dt + \sum_{i=1}^{n} \sigma_{BGN,i}^*(t,T)dW_i^P(t) + \int_E \delta_{BGN}^*(t,T,x)\mu(dt,dx)$$

We assume that in case of default there is a market turmoil leading to a jump in both curves. The maturity sector T of the euro curve jumps by an expected size of $\int_E \delta_{EUR}^*(t,T,x)\mu(dt,dx)$, and that of the local currency one by $\int_E \delta_{BGN}^*(t,T,x)\mu(dt,dx)$. The terms $\delta_{EUR}^*(t,T,x)$ and $\delta_{BGN}^*(t,T,x)$ show the jump sizes of the respective curves for every maturity.

- **Bank accounts**

$$dB_{EUR}(t) = r_{EUR}(t)B_{EUR}(t)dt$$
$$dB_{f,EUR}^*(t) = r_{EUR}^*(t)B_{f,EUR}^*(t)dt - \int_E q_{f,EUR}(t,x)\mu(dt,dx)$$
$$dB_{d,BGN}^*(t) = r_{BGN}^*(t)B_{d,BGN}^*(t)dt - \int_E q_{d,BGN}(t,x)\mu(dt,dx)$$

- **Recoveries**

$$\frac{dR_{f,EUR}(t)}{R_{f,EUR}(t)} = -\int_E q_{f,EUR}(t,x)\mu(dt,dx)$$
$$\frac{dR_{d,BGN}(t)}{R_{d,BGN}(t)} = -\int_E q_{d,BGN}(t,x)\mu(dt,dx)$$

After each default we have a devaluation of the bond by an expected value of $\int_E q(t,x)\mu(dt,dx)$. The stochasticity of the loss is captured by the random jump size $q(.,.)$.

- **Exchange rate**

$$\frac{dX(t)}{X(t)} = -\int_E \delta_X(t,x)\mu(dt,dx)$$

We assume that in case of default the market turmoil causes an exchange rate devaluation by an expected value of $\int_E \delta_X(t,x)\mu(dt,dx)$.

- **Bonds' prices**

$$P_{EUR}(t,T) = \exp(-\int_t^T f_{EUR}(t,s)ds) = E^{Q^f}(\exp(-\int_t^T r_{EUR}(s)ds) \mid G_t)$$
$$P_{f,EUR}^*(t,T) = R_{f,EUR}(t)\exp(-\int_t^T f_{EUR}^*(t,s)ds) = E^{Q^f}(\exp(-\int_t^T r_{EUR}(s)ds)R_{f,EUR}(T) \mid G_t)$$

$$P^*_{d,EUR}(t,T) = P^*_{d,BGN}(t,T)X(t) = R_{d,BGN}(t)X(t)\exp(-\int_t^T f^*_{BGN}(t,s)ds)$$
$$= E^{Q^f}(\exp(-\int_t^T r_{EUR}(s)ds)R_{d,BGN}(T)X(T) \mid G_t)$$

- **Arbitrage**

Under standard regularity conditions (see Bjork, Masi, Kabanov, and Runggaldier (1995)), the system to be free of arbitrage, all traded assets denominated in EUR must have a rate of return r_{EUR} under Q^f. This means that the processes:

$$\frac{P_{EUR}(t,T)}{B_{EUR}(t)}, \frac{B^*_{f,EUR}(t)}{B_{EUR}(t)}, \frac{P^*_{f,EUR}(t,T)}{B_{EUR}(t)}, \frac{B^*_{d,BGN}(t,T)X(t)}{B_{EUR}(t)}, \frac{P^*_{d,BGN}(t,T)X(t)}{B_{EUR}(t)} \qquad \text{(NoArb)}$$

must be local martingales under Q^f. For our purposes being martingales would be enough.

We move to deriving the no-arbitrage conditions. Taking the stochastic differential of the upper expressions, omitting the technicalities to Appendix 1, we can get:

$$r^*_{EUR}(t) - r_{EUR}(t) = h(t)\varphi_{q_{f,EUR}}(t) \qquad (3)$$
$$r^*_{BGN}(t) - r^*_{EUR}(t) = h(t)\left(\varphi_{\delta_X}(t) - \varphi_{q_{d,BGN},\delta_X}(t) + \varphi_{q_{d,BGN}}(t) - \varphi_{q_{f,EUR}}(t)\right) \qquad (4)$$
$$\alpha_{EUR}(t,T) = \sigma_{EUR}(t,T)\int_t^T \sigma_{EUR}(t,s)ds - \sigma_{EUR}(t,T)\phi(t) \qquad (5)$$
$$\alpha^*_{EUR}(t,T) = \sigma^*_{EUR}(t,T)\int_t^T \sigma^*_{EUR}(t,s)ds - \sigma^*_{EUR}(t,T)\phi(t) + h(t)\varphi^{q_{f,EUR},\delta_X}_{\theta^*_{EUR}}(t) \qquad (6)$$
$$\alpha^*_{BGN}(t,T) = \sigma^*_{BGN}(t,T)\int_t^T \sigma^*_{BGN}(t,s)ds - \sigma^*_{BGN}(t,T)\phi(t) + h(t)\varphi^{q_{d,BGN},\delta_X}_{\theta^*_{BGN}}(t), \qquad (7)$$

where we used the notation:

$$\theta^*_{EUR} = \exp(-\int_t^T \delta^*_{EUR}(t,s,x)ds), \ \theta^*_{BGN} = \exp(-\int_t^T \delta^*_{BGN}(t,s,x)ds) \qquad (8)$$
$$\varphi^{x,y,...}_{a,b,...}(t) = \int_E (ab...)((1-x)(1-y)...)\Phi(t,x)F_t(dx) \qquad (9)$$

and employed either vector notation or scalar products where necessary for simplicity.

By $\Phi(t,x)$ and $\phi(t)$ we denote the Girsanov's kernels of the counting process and the Brownian motion vector respectively when changing the probability measure from P to Q^f. The term $\varphi(t)$ gives scaled the expected jump sizes of the counting process. We can give interpretation of $\phi(t)$ as the market price of diffusion risk and of $\varphi(t)$ as the market price of jump risk. Parametrizing the volatilities and the market prices of risk, as well as imposing suitable dynamics on $h(t)$, we give a full characterization to our system. Further, the intensity could be a function of the underlying processes of the rates, so we could get correlation between the intensity, the interest rates, and the exchange rate.

2.2.3. Spreads diagnostics – reduced form view

It is important to give a deeper interpretation of the no-arbitrage conditions and see what factors drive the credit and the currency spreads. Equations (5) – (7) give modified the standard HJM drift restriction. The slight change from the classical riskless case is due to the jumps that take place at default. Equation (3) shows that the credit risk is proportional to the intensity of default and the scaled expected LGD by the coefficient controlling the risk aversion. The higher they are, the higher the spread is. Equation (4) gives the currency spread. The intensity of default and the difference between the two LGDs in BGN and EUR, scaled by the coefficient controlling the risk aversion, act like in the previous case. It is both important and interesting to note that inflation does not appear directly and it influences the spreads, as the next section shows, only through a secondary channel.

The monetization option in case of default needs a special analysis. In the above considerations, it was generally posed that there is a loss of $1-R_{d,LC}(T)$ on default of the domestic debt. However, if a full monetization is applied, then we would have $R_{d,LC}(T)=1$ and thus $\varphi_{\delta_X}(t)=\varphi_{q_{d,BGN},\delta_X}(t)=0$. If such a monetary injection is neutral to nominal values, it is not to real ones. Devaluation arises due to the abandonment of the CBA and the higher amount of money in circulation. Its effect can be measured differently based on what we take as a base - the price index or the exchange rate. Most naturally, we can expect both of them to depreciate due to the structural macrolinks that exist between these variables. For quantifying the amount we would need a macromodel which is beyond the scope of the reduced form model presented. The latter only shows what characteristics the market prices in general without imposing concrete macrolinks among them. Depending on what the base is we would have a direct estimation of certain type of indicators and an indirect one of the rest up to the amount they structurally influence the former. If the inflation is taken as a base, then we would have the comparison of inflation indexed bonds to the non-indexed ones. The spread between them would give an estimate for the expected inflation. Unfortunately, such an analysis is unrealistic in reality due to the fact that such bonds are issued very rarely in emerging market countries. In the case of Bulgaria, there is none. If the exchange rate is taken as a base, then we would have the comparison of domestic debt bonds to foreign debt bonds. The spread between them would give an estimate for the currency risk and the devaluation effect. The estimate for the inflation would be indirect and based on hypothetical structural links.

Whether the country would monetize or would declare a formal default is based on strategic considerations. It is a matter of structural analysis which option it would take. By all means, its decision is priced. In case of default, the pricing formula is equation (4). In case of monetization, we would have a jump in the exchange rate and let's denote its size[3] by $\hat{\delta}_X$. So we will get:

$$r^*_{BGN}(t)-r^*_{EUR}(t)=h(t)(\varphi_{\hat{\delta}_X}(t)-\varphi_{q_{f,EUR}}(t)) \tag{10}$$

There is not any arbitrage argument that $\varphi_{\hat{\delta}_X}(t)=\varphi_{\delta_X}(t)-\varphi_{q_{d,BGN},\delta_X}(t)+\varphi_{q_{d,BGN}}(t)$ must hold so that the two scenarios become equivalent. The only information we get from the market is an estimate for the generalized intensity being it $h(t)\varphi_{\hat{\delta}_X}(t)$ or $h(t)(\varphi_{\delta_X}(t)-\varphi_{q_{d,BGN},\delta_X}(t)+\varphi_{q_{d,BGN}}(t))$ not knowing which scenario will realize.

[3] It will be different from the no-monetarization one, δ_X, due to the different regimes that are followed.

2.3. Affine specification

The abstract formulation of the model in a HJM setting has made possible to give interpretations of the risky spreads from arbitrage point of view. For a concrete empirical estimation we need to specify the volatilities and the generalized intensities so that we could get convenient pricing formulas and make the model work. We turn to an affine setting where semi-closed form solutions for the prices of the bonds could be found. A further argument for using the latter specification is the well known fact that affine models are rich enough in the sense that they are the only family that allows finite dimensional realizations of the HJM dynamics and thus clear factor analysis. A theoretical discussion can be found in Filipovic and Teichmann (2002) for the diffusion case and in Tappe (2009) for the case allowing for jumps.

We impose the following simple dynamics under the risk neutral foreign measure:

- **Riskless country benchmark in EUR:**

$$r_{EUR}(t) = s_1(t) + s_2(t) \tag{11}$$

with latent factors having the dynamics:

$$ds_i(t) = k_i(\hat{\theta}_i - s_i(t))dt + \sigma_i dW_i^{Q^f}(t),\ i = 1, 2 \tag{12}$$

- **Credit spread**

$$h^*_{EUR}(t) = b_1 s_1(t) + b_2 s_2(t) + s_3(t) \tag{13}$$

where we implicitly assume a constant market price of jump risk and $s_3(t)$ is a new latent factor with dynamics:

$$ds_3(t) = k_3(\hat{\theta}_3 - s_3(t))dt + \sigma_3 dW_3^{Q^f}(t) \tag{14}$$

- **Currency spread**

$$h^*_{BGN}(t) = \bar{b}_1 s_1(t) + \bar{b}_2 s_2(t) + \bar{b}_3 s_3(t) + s_4(t) \tag{15}$$

where we again implicitly assume a constant market price of jump risk and $s_4(t)$ is a new latent factor with dynamics:

$$ds_4(t) = k_4(\hat{\theta}_4 - s_4(t))dt + \sigma_4 dW_4^{Q^f}(t) \tag{16}$$

Further, we have $\hat{\theta}_i = \theta_i - \frac{\sigma_i \phi_i}{k_i}$, $i = 1, 2, 3, 4$, where ϕ_i can be considered as market prices of diffusion risk.

Several general things must be observed about the above structure. First, we do not consider jumps in the factors and thus in the intensities. That would lead just to an overparametrization and is not needed. So the formulas (1) and (2) would be valid. Second, we do not stick to the classical canonical form of the processes as in Dai and Singleton (2000) because it would be difficult to give them a direct economic interpretation. So we use more general dynamics but reduce the number of parameters appropriately by controlling the weights of the factors. Thus, when we do not give a weight to a factor, this is due exactly to that reason. Such approach was followed in Pearson and Sun (1994) and Duffee (1999). From an econometric point of view the specification is equivalent to the canonical model in terms of richness. Third, the above factors' dynamics corresponds to the Gaussian (Vasicek) affine family. We preferred it to a CIR or to the mixed case due to its tractability. As many empirical studies have shown, it is much more flexible in fitting the data. Further, as we will see, there is a possibility for negative spreads and the Gaussian dynamics allows for this. Last but not least, we do not impose correlation on the Brownian motions of the factors, because we do not pursue to having a humped instantaneous forward rates maturity structure. The latter is an issue for interest rate option pricing, but not when we deal with bonds where we have an empirically decreasing volatility structure. See for a further discussion Brigo and Mercurio (2002). Closed for formulas for the bond prices could be also found there.

The factors' structure above is not specified ad hoc. It has a direct relation to the forward and exchange rate dynamics as well as to the no-arbitrage conditions (3) and (4). Equation (13) is just equation (3) but modified appropriately. We have:

$$h^*_{EUR}(t) = h(t)\varphi_{q_{f,EUR}}(t) = r^*_{EUR}(t) - r_{EUR}(t) = b_1 s_1(t) + b_2 s_2(t) + s_3(t) \tag{17}$$

where the coefficients b_1 and b_2 give the factors' weights and allow us to control the correlation between $r_{EUR}(t)$ and $h^*_{EUR}(t)$. The term $\varphi_{q_{f,EUR}}(t)$, encompassing the jump risk and the recovery, is assumed to be constant and stays implicitly incorporated in the weights and in the coefficients of the diffusions. In the same way, equation (15) is just equation (4). We have:

$$h^*_{BGN,EUR}(t) = h(t)(\varphi_{\hat{\delta}_X}(t) - \varphi_{q_{f,EUR}}(t)) = r^*_{BGN}(t) - r^*_{EUR}(t) = \bar{b}_1 s_1(t) + \bar{b}_2 s_2(t) + b_3 s_3(t) + s_4(t) \tag{18}$$

where the coefficients $\bar{b}_1$, $\bar{b}_2$, and b_3 give the factors' weights. This provides econometric flexibility and allows us to control the correlations among $r_{EUR}(t)$, $h^*_{EUR}(t)$, and $h^*_{BGN,EUR}(t)$. The term $\varphi_{q_{d,BGN}}(t)$ encompassing the jump risk and the recoveries, as well as the exchange rate depreciation size $\varphi_{\hat{\delta}_X}(t)$, is assumed to be constant and stays implicitly incorporated in the factor weights and the coefficients of the diffusions.

Special attention should be given to the process $s_4(t)$. If we consider the terms $h(t)\varphi_{q_{f,EUR}}(t)$ and $h(t)(\varphi_{\hat{\delta}_X}(t) - \varphi_{q_{f,EUR}}(t))$, we see that the currency spread should not be influenced by an additional factor over the credit spread unless we work with different stochastic recovery rates for the two bonds or/and stochastic size of the devaluation of the exchange rate. That latter is completely valid. Further, adding such an additional factor would be helpful from an econometric point of view too and will provide us with more flexibility in the estimation procedure.

2.4. Empirical analysis

In this subsection, we estimate empirically the reduced form model presented. First, we start with data description and general statistical analysis. Then we move to the concrete model estimation by Kalman filter. The empirical section can be viewed as an application of the theoretical setting we developed.

2.4.1. General notes

We take as a risk free rate benchmark the German bunds curve. If until recently that was an innocuous assumption, with the Eurozone turmoil provoked by the Greek debt problems started in April 2010, an interesting dilemma appears. Germany is the major donor in the rescue packages voted by the European Commission so some risk of the periphery countries got transferred to it. This is reflected in the German CDS quotes which from completely illiquid and in the rage of 0-10 bps from mid 2008 started an upward movement to reach levels of 60-90 bps during the peaks of the Lehman and the Eurozone crises. At the same time the yields of the German bunds exactly during these shock periods were falling. That was not only due to the expansionary monetary policy of the European Central Bank (ECB), but also to the fact that the German bunds served as a safe heaven. There was a massive flight to quality to them. So Germany as a sovereign started bearing a credit risk, but at the same time its treasuries become even less riskless. However, the participants in the Bulgarian secondary market for treasuries did not change their attitude to take the German bunds as a benchmark. So their use in our analysis seems to be the best choice.

We concentrate on the period 01/04/2004 – 31/12/2011 capturing both pre-crisis and post-crisis times. We build the EUR curve by using quotes for CDS. The Eurobond and the Global bonds outstanding characteristics are given in Table 1.

Table 1. Bulgarian foreign bonds

Instrument	Characteristics			
	Currency	Issue date	Maturity date	Coupon
Eurobond	EUR	01/11/2001	01/03/2007	7.25%
Global bond	EUR	22/03/2002	15/01/2013	7.50%
Global bond	USD	22/03/2002	15/01/2015	8.25%

It is clear that they are completely insufficient to have a full term structure. If before the crisis the liquidity of the CDS was not enough, from 2008 crisis, and especially after 2010 start of the Eurozone crisis their liquidity increased tremendously. As already mentioned in the introduction, initially the market participants were pricing the BGN treasuries referencing them to the Eurobond and the Global bonds. However, later they switched to the CDS. This gives support for our approach. To be completely precise, we take the CDS spreads, extract the probabilities of default, and then construct risky yields to maturity. That enables us to refer them to the German benchmark curve and derive the credit spread. Further, it must be emphasized that the phenomenon of basis appears between the CDS and the Eurobonds. However, as Figure 9 shows, hardly can we see any systematic pattern in its evolution. So sticking to CDS would not lead to a material loss of informational content. We take closing bid quotes from Bloomberg at 17.00 London time.

We build the Bulgarian BGN treasuries curve using coupon bonds and McCulloch (1977) smoothing procedure. It happens that it gives more stable results than the alternative Nelson-Siegel and Svensson procedures. We take closing bid quotes from Reuters and Bloomberg both of the outstanding and already matured bonds.

2.4.2. Market and statistical analysis

We begin with a visual plot of the spreads. Figure 1, Figure 2, and Figure 3 give the credit, currency, and general currency spreads evolution respectively. Figure 4 gives the two spreads by maturity spectrum. Figure 5 gives the yields by maturity spectrum of the German curve.

Figure 1. Credit spread evolution

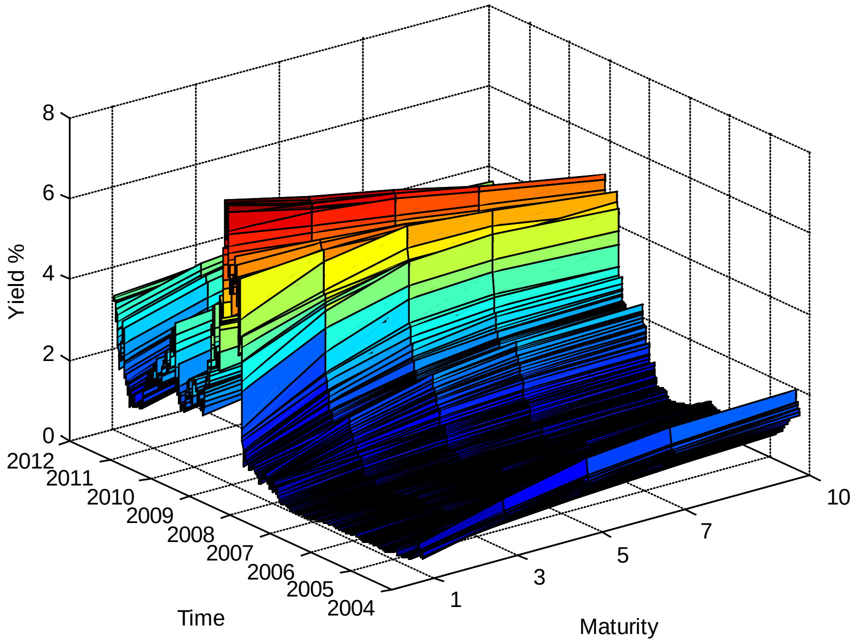


Figure 2. Currency spread evolution

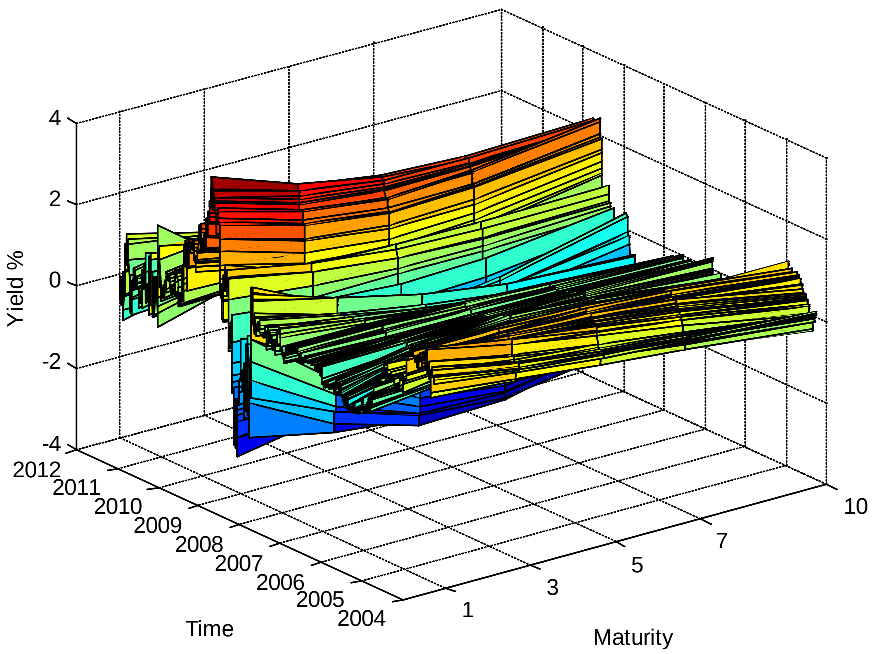


Figure 3. General currency spread evolution

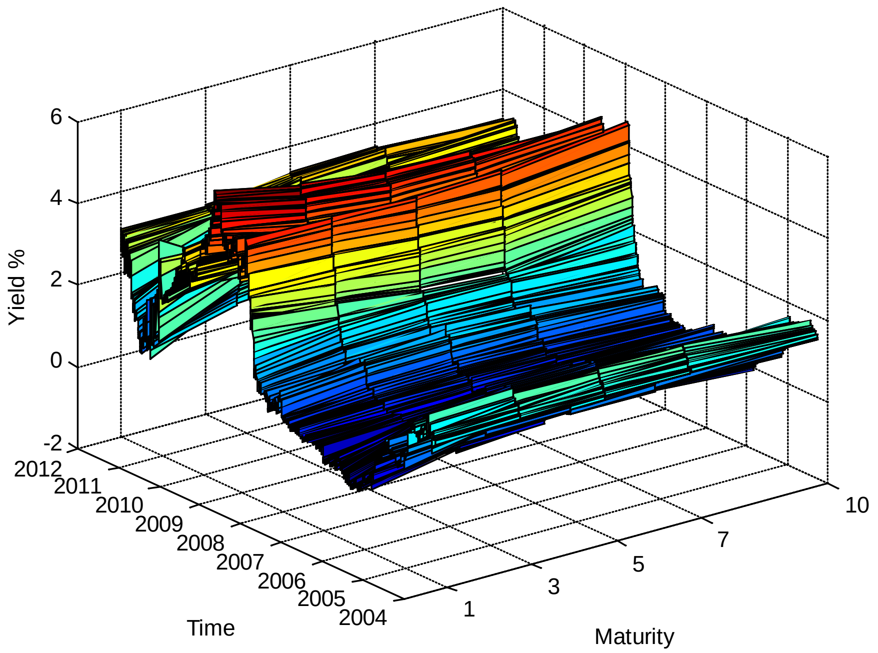


Figure 4. Risky spreads by maturity spectrum

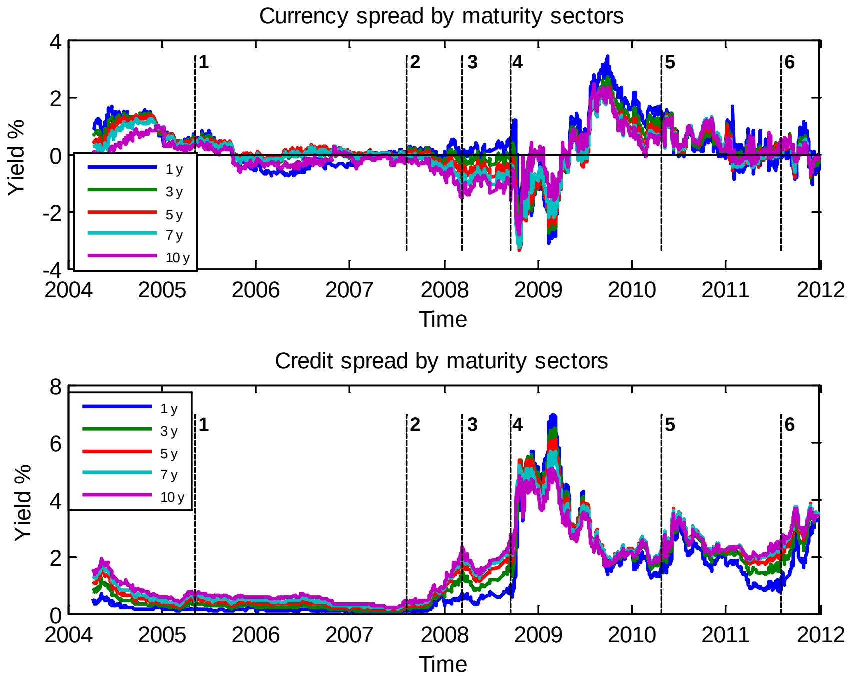
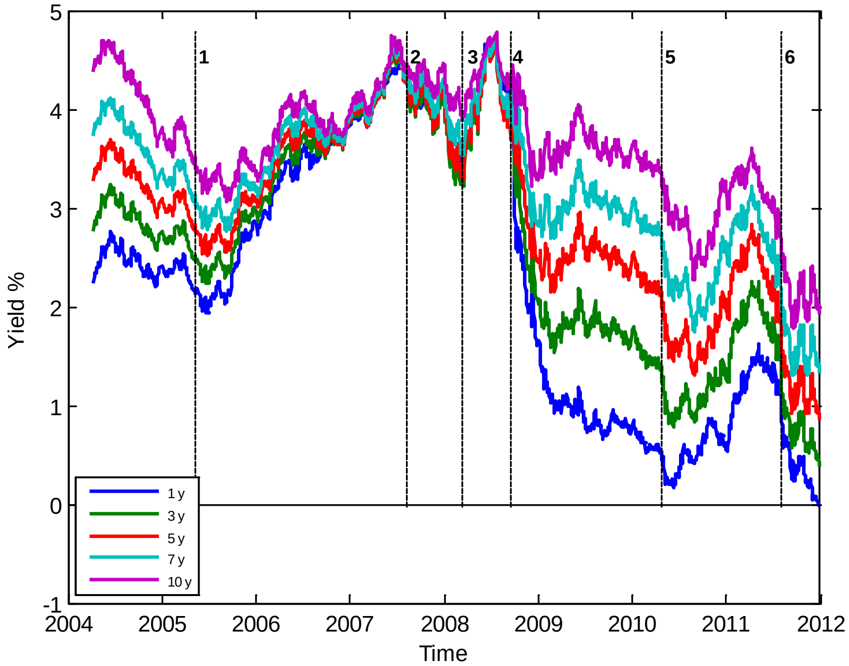


Figure 5. German yields by maturity spectrum



In all the figures, the events: 1 - GM and Ford ratings downgrade of May 09, 2005, 2 - Liquidity crisis of August 09, 2007, 3 - Bear Sterns default of March 14, 2008, 4 - Lehman default of September 15, 2008, 5 - Greek turmoil start of April 23, 2010, 6 – the US rating downgrade of August 05, 2011 are marked by the vertical dashed lines.

A Principal Components Analysis (PCA) of the driving factors of the curves is shown in Table 2. We can see that both for the risky spreads and the German yields three factors could be extracted with real significance in explaining the variance to be only two of them. The eigen vectors plot shows, as usual, that they can be interpreted as shift and slope. In the period under consideration, we have an average increase of all the yields and spreads making the shift slightly positive. The slope, defined as the difference between the long end and the short end of the curves, is positive on average for the German yields and negative for the risky spreads. In the former case, this is due to the downward move of the short end of the curve influenced by the significant decrease of the Eurozone refi rate as a major policy tool of the ECB for providing monetary stimulus during the crisis. Further, a flight to liquidity effect is also largely present. In the latter one, the turbulent times from the second half of the period led to a significant risk aversion towards the EM segment, fear of fiscal problems driven by the recession, and negative expectations for the Bulgarian creditworthiness. Since the default possibility and exchange rate devaluations were expected mainly in the short run, this caused the spreads rise sharply producing flat and inverted risky spreads curves. It is interesting to go beyond the average picture the eigenvectors plot provides and see the dynamics of the curves through time.

Table 2. PCA analysis

	Bulgaria		Germany
Factor % / Object	Credit spread	Currency spread	Yield
Shift	98.60	91.39	95.76
Slope	1.36	6.38	4.03
Rotation	0.03	2.20	0.21
4	0.00	0.03	0.00
5	0.00	0.00	0.00

Concretely, for the German curve we have a standard evolution over the business cycle. In the beginning of the period under consideration it was increasing. Then predicting the monetary policy tightening, the curve gradually flattened. During the very realizations of the refi rate hikes, it became almost flat. Hardly can we claim that the curve predicted the recession with a raise in its slope. During the Bear Sterns default and the first signals about the coming turmoil there was an erratic behavior. The ambivalence of the situation whether we would have a soft landing or a deep recession led to these fluctuations. Further, there was a fear of stagflation making the monetary easing not the best policy. After the Lehman default, it became clear that both the credit system and the fast slowing economy needed a huge liquidity injection so that a depression would be avoided. Interestingly, the long end of the curve dropped significantly too fueling expectations for a long recession and continued monetary easing.

For the credit spread three periods could be outlined - two normal ones and one extraordinary. The normal ones were during in the economic boom from 2004-2007 and after the end of the recession from 2009. During the former Bulgaria didn't follow a very different pattern from the other CEE countries. Namely, the prolonged low interest rate policies of ECB lead to high economic growth and boom in foreign investments in the whole region having at certain moments the characteristics of a bubble. With the exception of Hungary that led to strong budget positions. Servicing the foreign debt was significantly eased. All that led to a strong decrease in the credit spread. A further factor specific for Bulgaria was that the period also coincided with the country being on the final stage to a full EU membership. This contributed additionally to lowering the risk premium. As already discussed, Bulgaria also followed a macroeconomic policy that was neutral to any balance of payments or inflation considerations. The focus was only on achieving stability of the financial sector through prudential regulation. There were not any sterilization operations through issuing domestic debt. Thus the high investment inflow led to budget surpluses, accumulation of fiscal reserve that acted as a risk buffer to the current account deficits, and a significant reduction of the total debt to GDP ratio from 45.2% to 14.9% for the whole period. This helped to reduce the credit spread further. The spread curve itself had a positive slope. The usual term premium appeared pointing out that default was more likely in the long run. During the crisis times the CDS overreacted to news bringing an instant shift increase and slope decrease. The later even become negative and the credit curve was inverted for a short period. Then both factors gradually moved in a direction to restoring the pre-crisis time levels. The start of the economic recovery and the improvement of the fiscal position together with the general market sentiment helped for that too. However, the shift still remained higher due to the uncertainty in global economy and the start of the Eurozone crisis. The slope gained back its previous form.

The currency spread dynamics is more complicated. It does not incorporate any inflation expectations and is only indirectly influenced by any macro forces. In principle, it should just reflect the subordination of the local currency debt and the tail possibility of devaluation abandoning the CBA. These considerations seem to be valid but only during the crisis period. In general, it happened that other effects matter too and their influence needs a special analysis.

The period 2004-2005 was characterized initially for a short while with increasing spreads due to the political uncertainty what policy the new government elected in mid 2005 would take. Then there was a prolonged trend of a decreasing currency spread and an inverted shape of the spread curve. If the overall macro stability is the main driver for the former, the latter could be attributed to the pending EU membership. So any risks of CBA abandonment were viewed short termed. The membership itself was considered as an implicit guarantee for stability and access to European funding. Further factor for the quick decrease of the spreads was the significant accumulation of free cash by the large local institutional investors who had a significant home bias to investing in domestic treasuries. Some of them like the pension funds were also under a strict regulation to hold at least 50% of their portfolio in these instruments. The treasuries were also largely used as risk free collateral and thus had a special status granted by law. In this period, there were also made some changes to the legislation giving more liquidity of the domestic market. Such were the shortened settlement date of the local payment system and the abandonment of the bid ask spread for converting EUR to BGN.

At the end of 2005, a clear trend for raising the rates in the Eurozone emerged as a consequence of the attempt of the ECB to gradually cool the economy and curb the inflation. The BGN treasuries curve also followed that movement but with a small delay. Since the credit spread was already on a very low level, the phenomenon of negative currency spread appeared. It lasted till mid 2009. Interestingly, at every shock it became bigger. Four are the main drivers for that abnormal situation. First, the local treasuries market had started to have less liquidity since 2006. The reason for this was that the boom in the corporate and consumer credits and the bubble on the stock exchange were channeling the resources out of fixed income segment to these more profitable ones. The treasuries in most of the cases were subject to buy and hold strategy. At the same time, the CDS were increasing their liquidity and the price discovery about the sovereign risk of the country could be attributed to them. Although the BGN curve followed, in general, the dynamics of the benchmark, it was sticky to shocks. Thus every abrupt rise in the credit spread artificially reduced the currency one. Second, if the treasuries started suffering from liquidity draught, exactly the opposite was valid for the banking system and the financial system in general. Even offering lower yield, there was still demand for them for diversification needs and in their role as collateral. The issues by the MF were kept constant because of no need for positive net financing, but the cash in the banks and investors was increasing. Thus a segmented demand for the treasuries appeared and no other financial asset could substitute them. So this led the negative currency spread to prevail for a long time. This also explains the very erratic behavior of the spread till 2009. Third, as Figure 3 shows, the whole 2006 was marked by even a negative general currency spread for the maturity sector of 1 year. This did not lead to a negative credit spread, but just to a greater negative magnitude of the currency one. This could again be explained by the segmented demand for this sector. Namely, the very short maturity is preferred by some institutional investors aiming greater liquidity of their portfolios. This is especially valid for the Bulgarian Deposit Insurance Fund which should follow a very conservative strategy in investing its resources by by-laws. Fourth, there are transaction costs in implementing an arbitrage strategy to take advantage of the negative spread. If the domestic banks and funds are the main holders of treasuries, they have very limited access to instruments like CDS due to their general focus on the local market. Interestingly, even the negative currency spreads coming from the Eurobond and the Global bond were not taken advantage of. The arbitrage strategy is connected with taking a position in reverse repo in the treasuries and a simultaneous purchase of the Eurobond or the Global bond. However, there is a significant risk that once sold on the market, the treasuries cannot be bought back especially in a low liquidity market.

With Bulgaria entering the EU, no big change in the treasuries market was observed. There was also no huge inflow of foreign capital to that segment mainly due to the higher attractiveness of credit and equity. The curve did not change significantly its dynamics. From the turmoil of 2007 till the relative stabilization of mid 2009, the treasuries always underreacted to shocks. This led to an increase in the negative spread. Further, the curve inverted again discounting negative shocks in the short run. From mid 2009 till the Greek crisis, we can say that both spreads were close to equilibrium levels with the effect of undereaction been cleared. The currency spread was about 100 bp and was reflecting mainly the risk that the country could abandon the CBA in fighting with the crisis.

2.4.3. Estimation results

We estimate the model by Kalman filter. The joint estimation of equations of such kind, although giving more efficient parameter estimates, is numerically very costly in computer time. We keep to the simpler, but asymptotically equivalent, approach of equation by equation estimation as in Duffee (1999). For this purpose we first estimate equations (11) and (12) using the German yield curve data. Then we find estimates of $s_1(t)$ and $s_2(t)$ in terms of $\hat{s}_1(t)$ and $\hat{s}_2(t)$ by smoothing. Then we plug them into (13). Using the credit spread data, we estimate (13) and (14) to find the filtered factor $\hat{s}_3(t)$. Finally, we plug all the filtered latent factors $\hat{s}_1(t)$, $\hat{s}_2(t)$, and $\hat{s}_3(t)$ into (15), and use the currency spread data to estimate (15) and (16). The estimated coefficients are represented below.

Table 3. Estimation results

Country	Coefficients									
Ger.	$k_1(t)$	$\theta_1(t)$	$\sigma_1(t)$	$\phi_1(t)$	$k_2(t)$	$\theta_2(t)$	$\sigma_2(t)$	$\phi_2(t)$	h	
	0.26*	9.82*	0.77*	1.88*	-0.02*	-2.82	0.38*	-4.52*	0.002*	
	(0.00)	(1.24)	(0.02)	(0.40)	(0.00)	(7.43)	(0.00)	(0.37)	(0.00)	
Bul. credit spread	$k_2(t)$	$\theta_2(t)$	$\sigma_2(t)$	$\phi_2(t)$	$k_3(t)$	$\theta_3(t)$	$\sigma_3(t)$	$\phi_3(t)$	h	b_2
	-0.02*	-2.82	0.38*	-4.52*	0.11*	5.82*	0.40*	-0.08*	0.08*	-0.45*
	(0.00)	(7.43)	(0.00)	(0.37)	(0.002)	(0.54)	(0.002)	(0.18)	(0.001)	(0.02)
Bul. currency spread	$k_2(t)$	$\theta_2(t)$	$\sigma_2(t)$	$\phi_2(t)$	$k_4(t)$	$\theta_4(t)$	$\sigma_4(t)$	$\phi_4(t)$	h	$\bar{b}_2$
	-0.02*	-2.82	0.38*	-4.52*	0.13	11.8*	0.40*	2.53*	0.12*	0.35*
	(0.00)	(7.43)	(0.00)	(0.37)	(0.002)	(0.47)	(0.003)	(0.18)	(0.001)	(0.01)

The brackets represent the standard errors ; * signific. at 1%.

We observe significance of two factors for the German curve. The same is valid for the spread curves. For them one factor is coming from the German curve and the other is idiosyncratic. This is completely in line with the PCA analysis. However, it is interesting to see what exactly the relationship between the factors from the Kalman filter and the ones from the PCA analysis is. Theoretically, the former should be considered as more correctly representing the driving forces behind the curves, because they are based on a no-arbitrage model. Unfortunately, they lack any interpretation and represent just statistical quantities. We gradually move to analyzing their informational content.

It is good to have initially a geometric view on the filtered factors. Table 4 gives the correlations between them and the ones derived from the PCA analysis.

Table 4. Factors' correlations

Bulgaria			PCA						Kalman			
			Shift			Slope			Ger.		Spr. EUR	Spr. BGN
			Ger.	Spr. EUR	Spr. BGN	Ger.	Spr. EUR	Spr. BGN	fact. 1	fact. 2		
PCA	Shift	Ger.	1,00	0,24	0,34	0,00	-0,20	0,36	0,99	-0,38	-0,06	-0,20
		Spr. EUR	0,24	1,00	-0,25	-0,64	0,00	-0,02	0,25	-0,69	0,94	-0,41
		Spr. LC.	0,34	-0,25	1,00	-0,28	0,06	0,00	0,35	-0,39	-0,35	0,82
	Slope	Ger.	0,00	-0,64	-0,28	1,00	-0,17	0,29	0,00	0,92	-0,66	-0,30
		Spr. EUR	-0,20	0,00	0,06	-0,17	1,00	0,47	-0,20	-0,08	0,07	0,19
		Spr. LC.	0,36	-0,02	0,00	0,29	0,47	1,00	0,36	0,13	-0,14	-0,19
Kalman	Ger.	fact. 1	0,99	0,25	0,35	0,00	-0,20	0,36	1,00	-0,39	-0,06	-0,21
		fact. 2	-0,38	-0,69	-0,39	0,92	-0,08	0,13	-0,39	1,00	-0,59	-0,20
	Spr. EUR		-0,06	0,94	-0,35	-0,66	0,07	-0,14	-0,06	-0,59	1,00	-0,35
	Spr. LC.		-0,20	-0,41	0,82	-0,30	0,19	-0,19	-0,21	-0,20	-0,35	1,00

We see that the Kalman filter factors coming from the German curve are highly correlated with the shift and the slope coming from the PCA analysis. So we can give them exactly such an interpretation. The remaining factors of the credit and currency spreads are highly correlated with the corresponding shift factors, so they get such an interpretation. Further, it is the German slope factor that becomes relevant for the two spreads.

It is interesting to plot the evolution of the derived smoothed factors. We are going to do this in a special way. Namely, we construct indices starting from unity and then plot the relative performance. We do this to avoid possible miscalling that could arise due to numerical reasons coming from the specification of the affine model. Figure 6 presents the results for the credit spread factor and Figure 7 for the currency spread one.

Figure 6. Credit spread factor dynamics

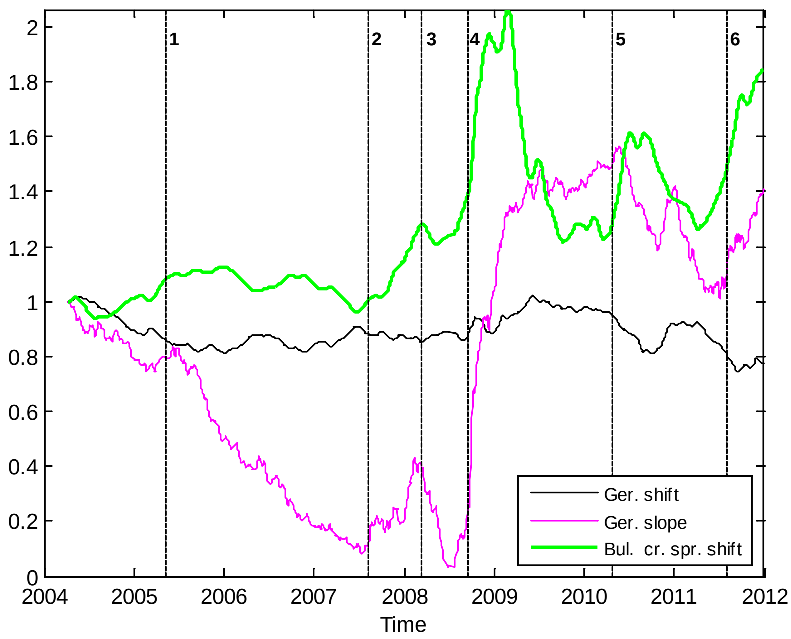
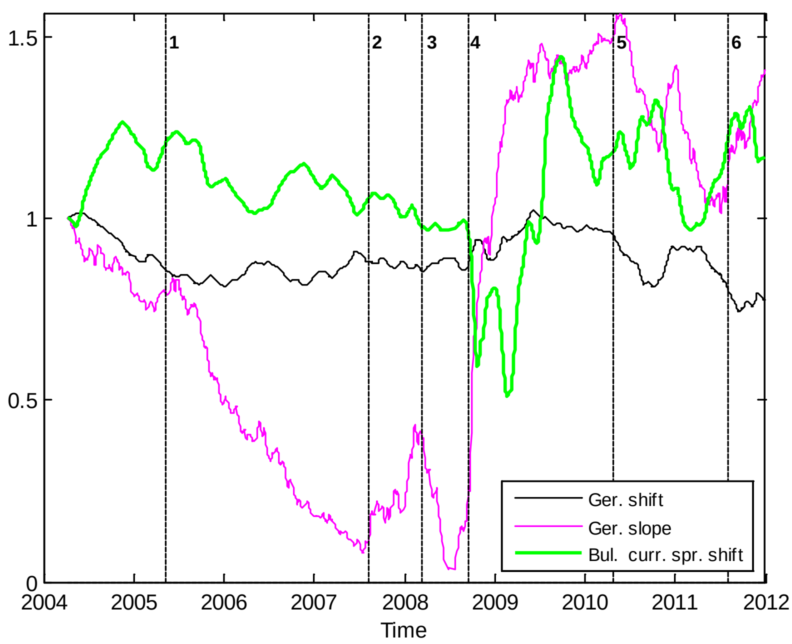


Figure 7. Currency spread factor dynamics



We see that the plot of the German factors evolution largely reflects the analysis we made above about the behavior of the shape of the yield curve through the period under consideration. Interestingly, only the slope was subject to a risk aversion by the market participants and this is reflected by the negative sign of the coefficient $\phi_2(t)$.

The credit spread shift factor dynamics shows an interesting pattern. Namely, there was a relative stability of it till the start of the credit crunch turmoil in mid 2007. So the observed general decrease in the credit spread was mainly due to the influence of the German slope. This can be interpreted in two ways. On one hand, the started rise in the Germen curve from late 2005 was not accompanied by a similar move of the Bulgarian CDS curve. So that led to a technical tightening of the credit spread. On the other hand, the latter was a reflection of the fact that both the economies were booming. So if the German needed cooling through interest rate hikes, the Bulgarian one was considered to becoming less risky. However, it remains an open question what economic forces stay behind the shift factor. It is expected to have a whole bunch of them. On one hand, they should reflect that the economic growth in Bulgaria was relatively higher making the country less risky compared to the German one. Here the general convergence argument could be applied. On the other hand, they should be a mix of local macro and financial indicators that need to be carefully selected. Last but not least, the impact of the global risk aversion factors needs to be included here. We do this a bit later.

From the market turmoil of 2007 on, the volatility of the shift factor increased tremendously. Further, it was the main driver for the increase of the credit spread. The German slope factor was still present, but its significance was lower. We see very strong spikes during the Lehman and the Greek crises periods.

The currency spread factor has a very erratic performance through the whole period under consideration. This reflected the same behavior of the currency spread itself. The most notable feature that can be observed is that, as mentioned before, the negative currency spread during the shocks in the crisis periods are due to an overreaction of the CDS curve and an undereaction of the treasuries one. At those times, the shift factors of the credit and currency spreads have opposite movements. It is interesting to note also that if there is some risk aversion towards the credit spread shift factor, there is no such towards the currency spread one. This is a further evidence for the artificially subdued values of it and is helpful in explaining the prolonged for long time negative spread.

It must be emphasized that the overreaction of the CDS to shocks was not only linked to this specific choice for representing the Bulgarian EUR yield curve. The same pattern is observed if we consider the credit and currency spreads derived from the Eurobond with maturity 2007 and the Global bond with maturity 2013. This is shown in Figure 8. Since the bonds are coupon ones, we use the standard concept of z-spread to derive the risky spreads. Further, since the bonds change their maturity at every observation through the considered period, we control that by taking the corresponding maturities of the interpolated German bunds benchmark curve and the BGN treasuries one.

Finally, as already mentioned, hardly can we claim that there is a very ostensible pattern in the basis that arises. It is shown in Figure 9 as a difference between the pure risky yield induced by the CDS and the z-spread of the two bonds.

Figure 8. Risky spreads of Euro/Global bond 2007/2013

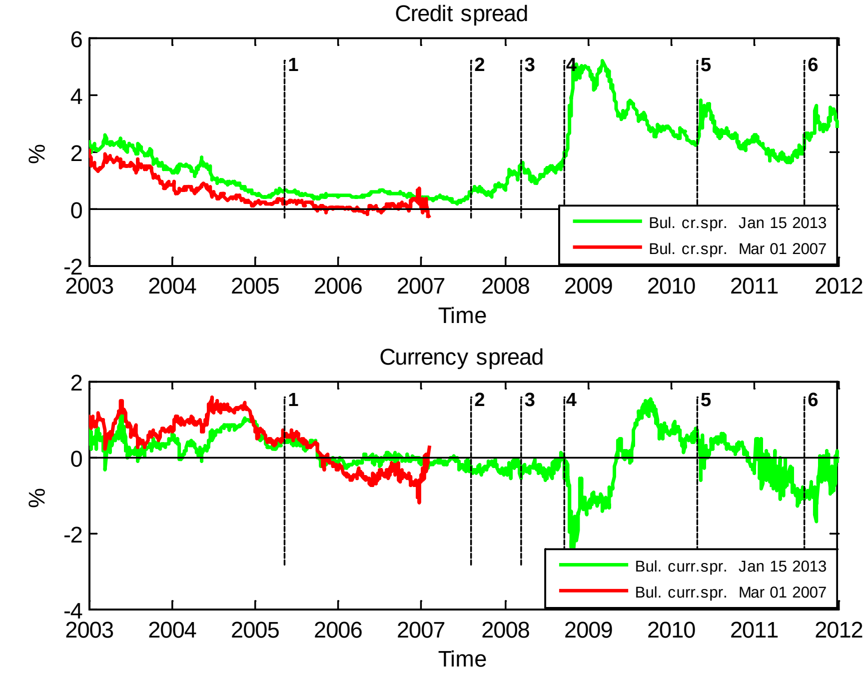
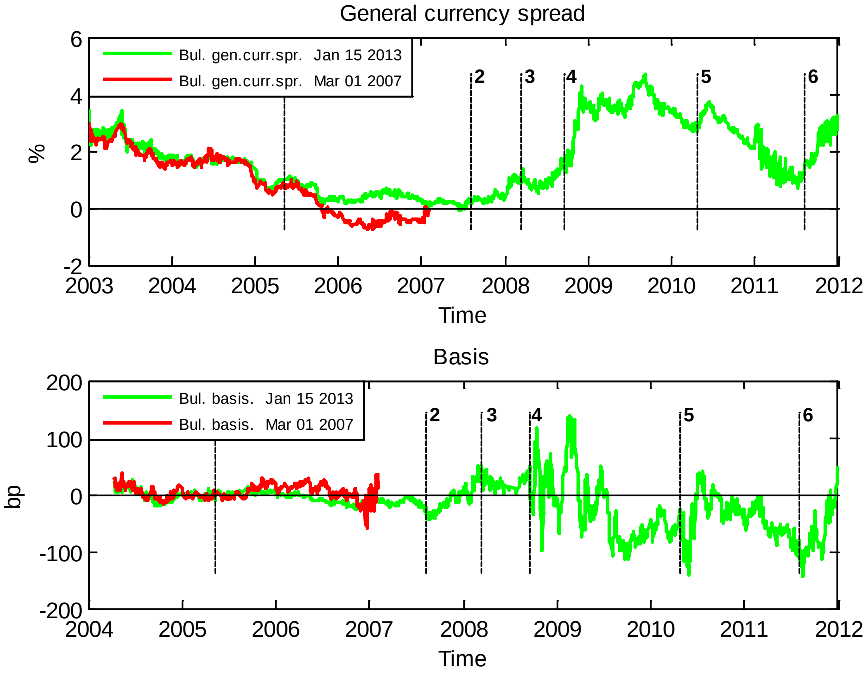


Figure 9. Basis of Euro/Global bond 2007/2013



3. Structural setting

In this section, we lay the foundations of the structural view on the domestic and foreign debt of a risky country under a fixed exchange rate regime. It could be considered as an extension of the classical Merton model. Then we analyze what explanatory power the setting could provide. We conclude the section with an application to Bulgaria.

3.1. Model formulation

We focus our attention on an aggregated balance sheet of Bulgaria encompassing the balances of the CB and the MF. Its liability side consists of three items in terms of seniority - foreign debt, domestic debt, and monetary base. In the reduced form model, we controlled the seniority by suitable distributions of the recovery rates. However, despite enough for the analysis there, this was artificial and does not provide good financial characteristics of the instruments. When we view the two types of debt as coexisting and being contingent claims on the country assets, a structural view is needed. Under such the subordination is made ostensible and is defined by the different default boundaries the three items have.

If in the reduced setting default could happen at any instant and was constructed as a totally inaccessible stopping time, here, in the structural one, it could take place only at the maturity of the debt[4] T in case the country assets fall below the face value of the debt. We assume that the country assets denominated in EUR are given by the sum:

$$A_{d,EUR}(t) = MM_{d,EUR}(t) + A_{d,BGN}(t)X(t), \tag{19}$$

where $MM_{d,EUR}(t)$ is any monetization used, $X(t)$, as before, is the exchange rate EUR for 1 unit of BGN considered fixed, and $A_{d,BGN}(t)$ are the assets in BGN. Further, we specify the face values of the foreign and the domestic debt to be $B_{f,EUR}$ and $B_{d,BGN}$ respectively. The subordination of the domestic debt to the foreign one is made explicit by assuming that that the default boundary of the foreign debt is $B_{f,EUR}$ and that of the domestic one is $B_{f,EUR} + X(T-)B_{d,BGN}$. The monetization could happen only at the maturity of the debt and increases the assets so that the nominal losses on the domestic debt would be fully covered[5]. This gives the following dynamics:

$$\begin{aligned} \frac{dMM_{d,EUR}(t)}{MM_{d,EUR}(t-)} &= \frac{X(t-)B_{d,LC}}{MM_{d,EUR}(t-)}\delta_{\{t=T\}\wedge\{A_{d,EUR}(T-)<B_{f,EUR}\}}(dt) \\ &+ \frac{B_{f,EUR} + X(t-)B_{d,LC} - A_{d,EUR}(t-)}{MM_{d,EUR}(t-)}\delta_{\{t=T\}\wedge\{B_{f,EUR}\le A_{d,EUR}(T-)<B_{f,EUR}+X(T-)B_{d,BGN}\}}(dt) \end{aligned} \tag{20}$$

The exchange rate is fixed. However, it is assumed that the macroeconomic equilibrium is of neoclassical type. Thus, any monetization leads instantly to exchange rate devaluation with its equilibrium amount to be determined in short. This leads to the following dynamics under Q^f:

$$\frac{dX(t)}{X(t-)} = -\rho^1\delta_{\{t=T\}\wedge\{B_{f,EUR}\le A_{d,EUR}(T-)<B_{f,EUR}+X(T-)B_{d,BGN}\}}(dt) - \rho^2\delta_{\{t=T\}\wedge\{A_{d,EUR}(T-)<B_{f,EUR}\}}(dt), \tag{21}$$

where we have a devaluation by ρ_1, when the default is only on the domestic debt, and by ρ_2 when it is also on the foreign one with $\rho_2 > \rho_1$. Considering the diffusion pattern of the exchange rate we can assume the following dynamics for the assets themselves:

[4] This does not pose a significant restriction for the purposes of the current analysis. Namely, to have an indication for the fundamental values of the domestic and foreign debt from a structural financial point of view and to derive indicators with a good explanatory power for the spread. Considering default before the debt maturity is possible, but unlike any dynamic capital structure theory for the firm, where it is needed (e.g. see Black and Cox (1976), Saa-Requejo and Santa Clara (1997), Leland and Toft (1996) among the others), here it will only burden the exposition.

[5] The losses on the foreign debt are impossible to be covered, because of the limited foreign reserves.

$$\frac{dA_{d,EUR}(t)}{A_{d,EUR}(t-)} = \mu_{A_{d,EUR}} dt + \sigma_{A_{d,EUR}} dW^{Q^f}(t) \\ -\rho_1 \delta_{\{t=T\}\wedge\{B_{f,EUR} \le A_{d,EUR}(T-) < B_{f,EUR} + X(T-)B_{d,BGN}\}}(dt) - \rho_2 \delta_{\{t=T\}\wedge\{A_{d,EUR}(T-) < B_{f,EUR}\}}(dt) \tag{22}$$

The drift contains already the monetization effects. The latter two terms can be incorporated into it too. So assuming that $A_{d,EUR}(t)$ is traded, we can get:

$$\frac{dA_{d,EUR}(t)}{A_{d,EUR}(t)} = r_{EUR} dt + \sigma_{A_{d,EUR}} dW^{Q^f}(t) \tag{23}$$

As implied above, the values of the coefficients ρ_1 and ρ_2 are chosen such that the money emission offsetting the losses on the domestic debt lead to an exchange rate depreciation exactly in an amount to have no jumps[6] in $A_{d,EUR}(t)$. So we have:

$$\left(B_{f,EUR} + X(t-)B_{d,BGN} - A_{d,EUR}(t-)\right) 1_{\{t=T\}\wedge\{B_{f,EUR} \le A_{d,EUR}(T-) < B_{f,EUR} + X(T-)B_{d,BGN}\}} = \\ = \rho_1 A_{d,EUR}(t-) 1_{\{t=T\}\wedge\{B_{f,EUR} \le A_{d,EUR}(T-) < B_{f,EUR} + X(T-)B_{d,BGN}\}} \tag{24}$$

$$X(t-)B_{d,BGN} 1_{\{t=T\}\wedge\{A_{d,EUR}(T-) < B_{f,EUR}\}} = \rho_2 A_{d,EUR}(t-) 1_{\{t=T\}\wedge\{A_{d,EUR}(T-) < B_{f,EUR}\}} \tag{25}$$

Note that default is defined before any monetizations and jumps realizations, so we take where necessary the assets and the exchange rate at $T-$.

Let's turn attention now to the payoffs of the three balance sheet items as derivatives on the country assets $A_{d,EUR}(t)$. Keeping to the notation from the reduced form model we have:

- **Foreign debt** $P^*_{f,EUR}(t,T)$:
  $B_{f,EUR} - \max(B_{f,EUR} - A_{d,EUR}(T))$
- **Domestic debt** $P^*_{d,BGN}(t,T)$:
  $\max(A_{d,EUR}(T) - B_{f,EUR}, 0) - \max(A_{d,EUR}(T) - B_{f,EUR} - B_{d,BGN} X(T-), 0)$
- **Monetary base** $M_{d,BGN}(t)$:
  $\max(A_{d,EUR}(T) - B_{f,EUR} - B_{d,BGN} X(T-), 0)$

The foreign bond represents a long position in a risk free bond and a short position in a put option with strike $B_{f,EUR}$. For its price we get:

$$\begin{aligned} P^*_{f,EUR}(t,T) &= B_{f,EUR} e^{-c^*_{EUR}(t,T)(T-t)} \\ &= E^{Q^f}(e^{-r_{EUR}(T-t)}(B_{f,EUR} - \max(B_{f,EUR} - A_{d,EUR}(T), 0) \mid G_t) \\ &= B_{f,EUR} e^{-r_{EUR}(T-t)} - P(t, T, B_{f,EUR}, A_{d,EUR}(t)) \\ &= A_{d,EUR}(t) N(-d_1^{B_{f,EUR}}) + B_{f,EUR} e^{-r_{EUR}(T-t)} N(d_2^{B_{f,EUR}}) \end{aligned} \tag{26}$$

[6] Note that by default we have no such in $A_{d,BGN}(t)$ too.

The foreign yield to maturity is:

$$c^*_{EUR}(t,T) = -\frac{1}{T-t}\log\left(\frac{P^*_{f,EUR}(t,T)}{B_{f,EUR}}\right)$$

and the credit spread is:

$$\begin{aligned} s^*_{EUR}(t,T) = c^*_{EUR}(t,T) - r_{EUR} &= -\frac{1}{T-t}\log\left(N(d_2^{B_{f,EUR}}) + \frac{1}{\rho_{f,EUR}(t,T)}N(-d_1^{B_{f,EUR}})\right) \\ = -\frac{1}{T-t}\log\left(\frac{f(\rho_{f,EUR}(t,T))}{\rho_{f,EUR}(t,T)}\right) &= -\frac{1}{T-t}\log\big(h(\rho_{f,EUR}(t,T))\big), \end{aligned} \tag{27}$$

Where $d_{1,2}^{B_{f,EUR}} = \dfrac{\log\left(\dfrac{1}{\rho_{f,EUR}(t,T)}\right) \pm \dfrac{1}{2}\sigma^2_{A_{d,EUR}}(T-t)}{\sigma_{A_{d,EUR}}\sqrt{T-t}}$, $\rho_{f,EUR}(t,T) = \dfrac{e^{-r_{EUR}(T-t)}B_{f,EUR}}{A_{d,EUR}(t)}$ is the foreign quasi-leverage ratio, and the functions $f(.)$ and $g(.)$ will be defined in the next section.

The domestic debt represents a long position in a call option with stochastic strike $B_{f,EUR} + B_{d,BGN}X(T-)$. For its price we get:

$$\begin{aligned} &P^*_{d,BGN}(t,T) = B_{d,BGN}e^{-c^*_{BGN}(t,T)(T-t)} \\ &= C_{BGN}(t,T,B_{f,EUR},A_{d,EUR}(t)) - C_{BGN}(t,T,B_{f,EUR} + X(T-)B_{d,BGN},A_{d,EUR}(t)) \end{aligned} \tag{28}$$

$$\begin{aligned} &= \frac{A_{d,EUR}(t)}{X(t)}(N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}})) - \frac{B_{f,EUR}}{X(t)}e^{-r_{EUR}(T-t)}(N(d_2^{B_{f,EUR}}) - N(d_2^{B_{f,EUR},B_{d,BGN}}) \\ &-B_{d,BGN}e^{-r_{EUR}(T-t)} \end{aligned}$$

where $d_{1,2}^{B_{f,EUR},B_{f,BGN}} = \dfrac{\log\left(\dfrac{1}{\rho_{f,EUR}(t,T)+\rho_{d,EUR}(t,T)}\right) \pm \dfrac{1}{2}\sigma^2_{A_{d,EUR}}(T-t)}{\sigma_{A_{d,EUR}}\sqrt{T-t}}$ and

$\rho_{d,EUR}(t,T) = \dfrac{e^{-r_{EUR}(T-t)}X(t)B_{d,BGN}}{A_{d,EUR}(t)}$ is the domestic quasi-leverage ratio.

The domestic yield to maturity is:

$$c^*_{BGN}(t,T) = -\frac{1}{T-t}\log\left(\frac{P^*_{d,BGN}(t,T)}{B_{d,BGN}}\right)$$

and the currency spread could be written as:

$$
\begin{aligned}
s^*_{BGN,EUR}(t,T) &= \underbrace{c^*_{BGN}(t,T) - c^*_{EUR}(t,T)}_{\text{currency spread}} = \underbrace{(c^*_{BGN}(t,T) - r_{EUR})}_{\text{general currency spread}} - \underbrace{(c^*_{EUR}(t,T) - r_{EUR})}_{\text{credit spread}} \\
&= \underbrace{(c^*_{BGN}(t,T) - r^*_{BGN})}_{\text{term general currency spread}} - \underbrace{(c^*_{EUR}(t,T) - r^*_{EUR})}_{\text{term credit sprerad=credit spread}} + \underbrace{r^*_{BGN} - r^*_{EUR}}_{\text{short rates currency spread}}
\end{aligned}
\tag{29}
$$

The first representation is standard. The second representation gives the currency spread as a difference between the general currency spread and the credit spread. Note that the latter two spreads have as a base the riskless EUR curve which is flat in the standard structural models. So we have an equality of the riskless EUR yield to maturity to the short EUR riskless rate for any maturity. The third representation adds as an additional element the term structure of the general currency spread and the credit spread.

Everywhere the short rates are defined in a standard way:

$$
\begin{aligned}
&c_{EUR}(t,T) \xrightarrow{T\downarrow t} c_{EUR}(t,t) = r_{EUR},\ c^*_{EUR}(t,T) \xrightarrow{T\downarrow t} c^*_{EUR}(t,t) = r^*_{EUR}, \\
&c^*_{BGN}(t,T) \xrightarrow{T\downarrow t} c^*_{BGN}(t,t) = r^*_{BGN}
\end{aligned}
\tag{30}
$$

Since in structural models as the current one the default time is defined as a predictable stopping time, we have the standard result that there is no infinitesimal credit spread. So $r^*_{EUR} = r_{EUR}$ is valid. Analogously, there should be no infinitesimal currency spread. So $r^*_{BGN} = r_{BGN}$ is valid. However, we haven't defined r_{BGN} yet, and exactly the latter could be considered as a definition. It represents the riskless rate in BGN. Both r_{EUR} and r_{BGN} are exogenous parameters to the model. We will come back to that analysis in the next section and conceptually close it there.

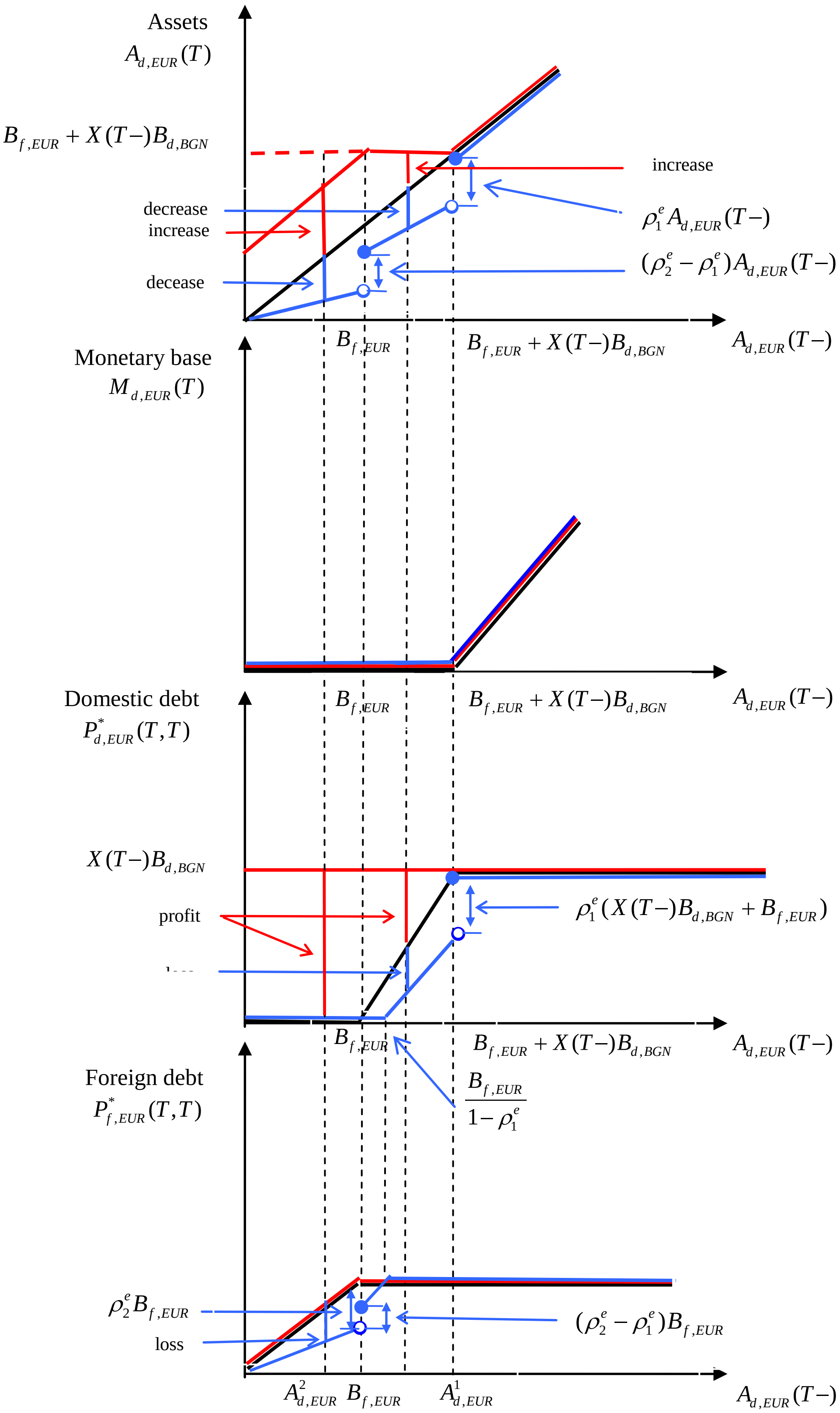
Further, from (30) and the definition of $f(.)$ that will follow, we have:

$$
s^*_{BGN,EUR}(t,T) = -\frac{1}{T-t}\log\left(\frac{\rho_{f,EUR}}{\rho_{d,EUR}}\frac{f(\rho_{f,EUR}+\rho_{d,EUR}) - f(\rho_{f,EUR})}{f(\rho_{f,EUR})}\right)
\tag{31}
$$

The payoffs, the balance sheet positions, and scenarios for their evolution are represented in Figure 11. The country could choose two possibilities at default – to incur the loss given default (LGD) or to monetize. The assets in EUR before any exchange rate devaluation are represented on the x-axis. We denote them by $A_{d,EUR}(T-)$. Then we distinguish between three cases for the fair values of the instruments: 1) no monetization and a full incur of the LGD (in black), 2) full monetization to avoid default and no exchange rate devaluation (in red), 3) no monetization with a full exchange rate devaluation (in blue). The first scenario is the realistic benchmark one and it stays in between the next too which could be considered as the border cases. We illustrate them just for completeness.

The figure represents what happens when there is a full monetization and what the equilibrium values of the devaluation are so that effectively scenario 1) is in effect. So for any asset value $A^i_{d,EUR}$, $i=1,2$, there are equilibrium values of the devaluations ρ_i, $i=1,2$, such that at that asset values the gains and losses under scenarios 2) and 3) are equalized.

Figure 10. Balance sheet positions[7] under different scenarios



[7] Note that $MM_{d,EUR}(t)+M_{d,EUR}(t)+P^*_{f,EUR}(t,T)+P^*_{d,EUR}(t,T)=A_{d,EUR}(t)$ must hold at every t incl. $t=T$. The discontinuity in the blue line for the two types of debt has mainly a technical character.

3.2. Technical notes

In this subsection, we will present several technical results which will provide a better understanding of the model setting.

First, as already seen, an important role in our analysis would have the function $f(x) = N(d_2(x))x + N(-d_1(x))$, where x denotes the leverage, so $d_{1,2}(x) = \dfrac{\log\left(\frac{1}{x}\right) + \frac{1}{2}\sigma^2_{A_{d,EUR}}(T-t)}{\sigma_{A_{d,EUR}}\sqrt{T-t}}$. It is straightforward to prove that $f(x)$ is increasing and concave for $x > 0$. We have also $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 1$. The concavity of $f(x)$ gives $\lim_{x\to 0} f^{'}(x) = 1$ and $\lim_{x\to\infty} f^{'}(x) = 0$. Further, if we consider $h(x) = \dfrac{f(x)}{x}$, the concavity of $f(x)$ gives that $h(x)$ is decreasing for $x > 0$. We have also $\lim_{x\to 0} h(x) = 1$ and $\lim_{x\to\infty} h(x) = 0$. As far as the concavity of $h(x)$ is concerned, the situation is more delicate. Despite conceptually straightforward, it is technically demanding to prove by direct differentiation that $h(x)$ is concave for $x \in (0,\zeta]$ and convex for $x \in (\zeta,+\infty)$ for some point ζ. Analogously, we have that $\log(h(x))$ is concave for $x \in (0,\lambda]$ and convex for $x \in (\lambda,+\infty)$ for some point λ. A further result, technically involved but conceptually straightforward, is that $\dfrac{xf^{'}(x)}{f(x)}$ is decreasing for $x > 0$.

Second, we can get also easily that the following equations are valid:

$$N(d_1^{B_{f,EUR}})A_{d,EUR}(t) = E^{Q^f}(e^{-r_{EUR}(T-t)}A_{d,EUR}(T)1_{\{A_{d,EUR}(T)\geq B_{f,EUR}\}} \mid G_t) \tag{32}$$

$$N(d_2^{B_{f,EUR}}) = Q^f(A_{d,EUR}(T) \geq B_{f,EUR} \mid G_t) \tag{33}$$

Let’s turn back now to equations (24) and (25). Since ρ_1 and ρ_2 are assumed to be fixed and determined at the initial time t, we can take expectations in these equations and together with (32) and (33) we obtain:

$$\begin{aligned}&\left(N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}})\right)B_{f,EUR} + \left(N(d_2^{B_{f,EUR}}) - N(d_2^{B_{f,EUR},B_{d,BGN}})\right)X(t)B_{d,BGN}\\ &= (\rho_1+1)A_{d,EUR}(t)\left(N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}})\right)\end{aligned} \tag{34}$$

$$X(t)B_{d,BGN}N(-d_2^{B_{f,EUR}}) = \rho_2 A_{d,EUR}(t)N(-d_1^{B_{f,EUR}})) \tag{35}$$

Solving the latter we get the equilibrium values for the exchange rate devaluations ρ_1^e and ρ_2^e.

Third, it is interesting to take a deeper look at the dynamics of the instruments in the reduced form model. Exactly the same no-arbitrage conditions as the one from the reduced form model hold and we write them again for completeness. The processes:

$$\frac{P_{EUR}(t,T)}{B_{EUR}(t)}, \frac{B^*_{f,EUR}(t)}{B_{EUR}(t)}, \frac{P^*_{f,EUR}(t,T)}{B_{EUR}(t)}, \frac{B^*_{d,BGN}(t,T)X(t)}{B_{EUR}(t)}, \frac{P^*_{d,BGN}(t,T)X(t)}{B_{EUR}(t)} \tag{NoArb}$$

must be local martingales under Q^f. For the first two the condition is trivially met because we have $r^*_{EUR} = r_{EUR}$. The third and the fourth processes determine the drift of the bonds dynamics. Their volatility follows directly from the Ito's lemma as shown in Appendix 2. For the foreign bond we have:

$$\frac{dP^*_{f,EUR}(t,T)}{P^*_{f,EUR}(t,T)} = r_{EUR}dt + \frac{\sigma_{A_{d,EUR}}}{1+\rho_{f,EUR}(t,T)\dfrac{N(d_2^{B_{f,EUR}})}{N(-d_1^{B_{f,EUR}})}} dW^{Q^f}(t) \tag{36}$$

and for the domestic one:

$$\frac{dP^*_{d,EUR}(t,T)}{P^*_{d,EUR}(t,T)} = r_{EUR}dt \tag{37}$$

$$+\frac{\sigma_{A_{d,EUR}}(N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}}))}{N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}}) - \rho_{f,EUR}(N(d_2^{B_{f,EUR}}) - N(d_2^{B_{f,EUR},B_{d,BGN}})) + \rho_{d,EUR}N(d_2^{B_{f,EUR},B_{d,BGN}})} dW^{Q^f}(t)$$

The fifth process gives the drift of the risk neutral dynamics of the exchange rate. We have that in case of no default $r_{BGN} = r_{EUR}$ is valid. In case of default, the BGN rate jumps:

$$r_{BGN} = r_{EUR} + \rho_1 \delta_{\{t=T\}\wedge\{B_{f,EUR} \le A_{d,EUR}(T) < B_{f,EUR} + X(T-)B_{d,BGN}\}} + \rho_2 \delta_{\{t=T\}\wedge\{A_{d,EUR}(T) < B_{f,EUR}\}} \tag{38}$$

3.3. Spreads diagnostics

In this subsection, we go deeper into the analysis of the origin of the two spreads and what factors drive them.

The Merton model in its classical form is enough to characterize the credit spread. As discussed in Cossin and Pirotte (2000), the latter appears because the holders of foreign debt have in effect a short position in a put option which reflects in general the expected probability of default and recovery. The credit spread can be explained concisely by three indicators - 1) the foreign debt quasi-leverage ratio $\rho_{f,EUR}$, 2) the assets' volatility $\sigma_{A_{d,EUR}}$, and 3) the debt maturity. It is increasing in the former two and this is straightforward to be proved from the formulas above. Further, since they increase the risk, the economic intuition is in accordance. As far as the maturity $T-t$ is concerned, the situation is a little bit more complicated. This is due to the fact that the default is very probable in the short run with the assets close to the default boundary. We could only expect with the increase in maturity to postpone the payment of the face value and this to have a positive impact on the spread. If $\rho_{f,EUR}(t,T) < 1$, then the spread is initially increasing with respect to $T-t$ and after a certain point starts to be decreasing. This is expression of the fact that higher maturity leads to higher local volatility of the bond and this causes the spread rise. However, postponing the payment of the face value starts to dominate the first effect after a threshold. In essence, the bond becomes close to perpetuity, and the spread drops.

The situation with the currency spread is more complicated. In general, from a structural point of view, it arises due to three effects that add up to each other: 1) probability of default on the domestic debt, 2) subordination of the domestic debt to the foreign one, 3) likely monetization in case of pending default on domestic debt and exchange rate devaluation. A technical analysis of the sensitivities similar to the one from the previous paragraph is possible but not needed for the current setting and will go beyond the scope of the paper. What is important is to see from (31) that the currency spread is always positive. Further, straightforward differentiation and using the properties of the functions $f(.)$ and $h(.)$ gives that it is increasing in the total leverage $\rho_{EUR} = \rho_{f,EUR} + \rho_{d,EUR}$ and the domestic leverage $\rho_{d,EUR}$. For low starting values of the foreign leverage $\rho_{f,EUR}$ it is increasing in it and then starts to decrease. This is due to the fact that the credit spreads starts bearing most of the burden of the increased leverage.

3.4. Empirical analysis

3.4.1. General notes

In this subsection, we first estimate the structural model and evaluate its performance. The logic behind the procedure, as discussed in Gray and Malone (2008), is from market values for the domestic debt and the monetary base, which form the local currency liabilities $LCL_{d,EUR}$, to derive the implied values of the country assets and their volatility. It is good these most junior claims to be considered together, because they behave like an equity tranche with a fixed detachment point in foreign currency in the face of the default barrier of foreign debt. Thus, for $LCL_{d,EUR}$ both the intuition and machinery of the classical Merton model can be used. So their value is equal to that of a call option with strike the default boundary of the foreign debt $B_{f,EUR}$. We get:

$$
\begin{aligned}
LCL_{d,EUR}(t) &= (M_{d,BGN}(t) + B_{d,BGN} e^{-c^*_{BGN}(t,T)(T-t)}) X(t) = (M_{d,BGN}(t) + \underbrace{D^*_{d,BGN}(t)}_{P^*_{d,BGN}(t,T)}) X(t) \\
&= M_{d,EUR}(t) + P^*_{d,EUR}(t,T) = C(t,T,B_{f,EUR}, A_{d,EUR}(t)) = \\
&= A_{d,EUR}(t) N(d_1^{B_{f,EUR}}) - B_{f,EUR} e^{-r_{EUR}(T-t)} N(d_2^{B_{f,EUR}})
\end{aligned}
\tag{39}
$$

where $c^*_{BGN}(t,T)$ is the risky yield to maturity in local currency, $D^*_{d,BGN}(t)$ is the market value of domestic debt denominated in BGN, $D^*_{d,EUR}(t)$ is the market value of domestic debt denominated in EUR. Since that value is the one of a traded instrument, namely, the local currency bond, we can also denote it by $D^*_{d,EUR}(t) = P^*_{d,EUR}(t,T)$. Note that we make explicit the dependence on the face value. This is because in the structural setting the price of the bond is not only homogenous of degree one solely in $B_{d,BGN}$, but also simultaneously in $B_{d,BGN}$ and $A_{d,BGN}(t)$. This is a deviation from the usual model free pricing and the one of the reduced form models as well. So writing this dependence is important. However, for notational convenience, we will always use $P^*_{d,EUR}(t,T)$ keeping in mind what the actual dependence is.

The purpose of the model is from market values for the domestic debt, and thus $LCL_{d,EUR}$, to infer the value of the assets $A_{d,EUR}(t)$ and their volatility $\sigma_{A_{d,EUR}}$. This can be done using the system:

$$
\begin{aligned}
LCL_{d,EUR}(t) &= A_{d,EUR}(t) N(d_1^{B_{f,EUR}}) - B_{f,EUR} e^{-r_{EUR}(T-t)} N(d_2^{B_{f,EUR}}) \\
LCL_{d,EUR}(t)\sigma_{LCL_{d,EUR}} &= A_{d,EUR}\sigma_{A_{d,EUR}} N(d_1^{B_{f,EUR}})
\end{aligned}
\tag{40}
$$

where the second equation comes from comparing the dynamics of $LCL_{d,EUR}(t)$ to what the Ito's lemma implies for it. The system can be solved iteratively to infer $A_{d,EUR}(t)$ and $\sigma_{A_{d,EUR}}$.

The input for the variables participating in the processes are represented in Table 5.

Table 5. Key model variables

Variable	Legend
$M_{d,BGN}$	Monetary base BGN
r_{EUR}	Riskless short rate in EUR ($= r_{EUR}$)
r^*_{EUR}	Risky short rate in EUR
$c^*_{EUR}(t,T)$	Risky yield to maturity in EUR
r^*_{BGN}	Risky short rate in BGN
$c^*_{BGN}(t,T)$	Risky yield to maturity in BGN
$B_{f,EUR}$	Default boundary of the foreign debt
$B_{d,BGN}$	Default boundary of the domestic debt
X	Spot exchange rate
$\sigma_{P^*_{d,EUR}} / \sigma_{P^*_{d,EUR}}$	Volatility of domestic/foreign debt in EUR
$\rho_{M_{EUR},P^*_{d,EUR}}$	Correlation between the monetary base and the domestic debt
$\sigma_{M_{d,EUR}}$	Volatility of the monetary base denominated in EUR

Further, we have the identities:

1. $B_{d,BGN} =$ short term debt + $\alpha_{d,BGN}$long term debt + 1 year interest rates
2. $B_{d,EUR} =$ short term debt + $\alpha_{f,EUR}$long term debt + 1 year interest rates
3. $M_{d,EUR} = M_{d,BGN} X$
4. $LCL_{d,EUR} = M_{d,EUR} + P^*_{d,EUR}$
5. $\sigma^2_{LCL_{d,EUR}} = \left(\frac{M_{d,EUR}}{M_{d,EUR} + P^*_{d,EUR}} \right) \sigma^2_{M_{d,EUR}} + \left(\frac{P^*_{d,EUR}}{M_{d,EUR} + P^*_{d,EUR}} \right) \sigma^2_{P^*_{d,EUR}}$

$$+2\rho_{M_{d,EUR},P^*_{d,EUR}} \left(\frac{M_{d,EUR}}{M_{d,EUR} + P^*_{d,EUR}} \right) \left(\frac{P^*_{d,EUR}}{M_{d,EUR} + P^*_{d,EUR}} \right) \sigma_{M_{d,EUR}} \sigma_{P^*_{d,EUR}}$$

It must be noted that we have changed the notation. Now $B_{d,BGN}$ and $B_{f,EUR}$ represent the default boundaries of the two types of debt instead of their face values. The difference comes because it is realistic to assume that the short term debt and only a fraction of the long term one participate in forming the boundary. The parameter α is to be determined but an empirically motivated value is 0.5.

3.4.2. Model estimation

We use monthly data for the face value of domestic debt, face value of foreign debt, and the monetary base. The monetary base is taken to be seasonally adjusted so that additional noise is avoided. Further, we treat the fiscal reserve as a part of it which is plausible assumption from a structural point of view. We estimate the market value of the domestic debt by taking data for all bonds outstanding at the corresponding point in time and price them to the zero yield curve already constructed. Our data sources are statistical departments of the Bulgarian National Bank and the Ministry of Finance.

In Figure 11, we present the estimated risk neutral probabilities of default of the foreign debt and, for comparison reasons, in Figure 12, we show also those implied by the market values of the credit spread.

Figure 11. Default probabilities by maturity sector (implied by the market)

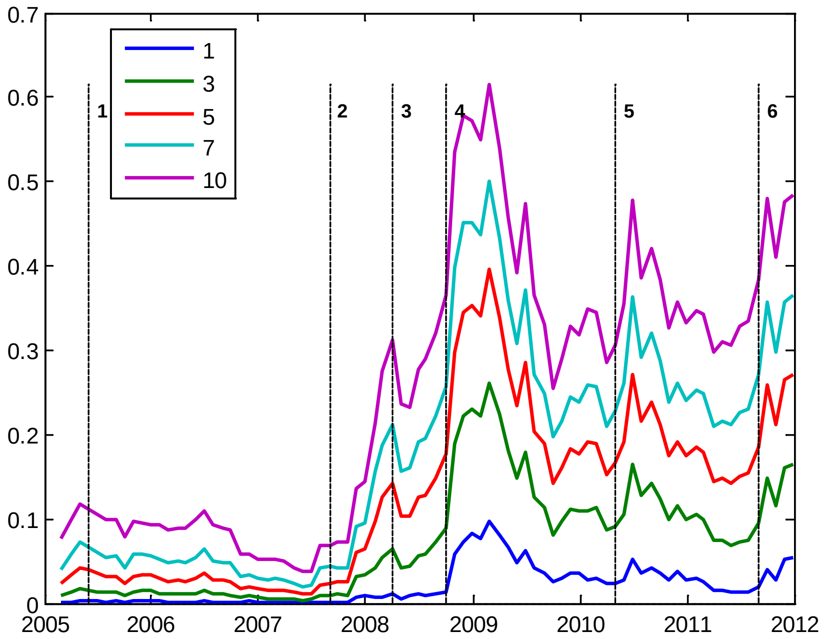
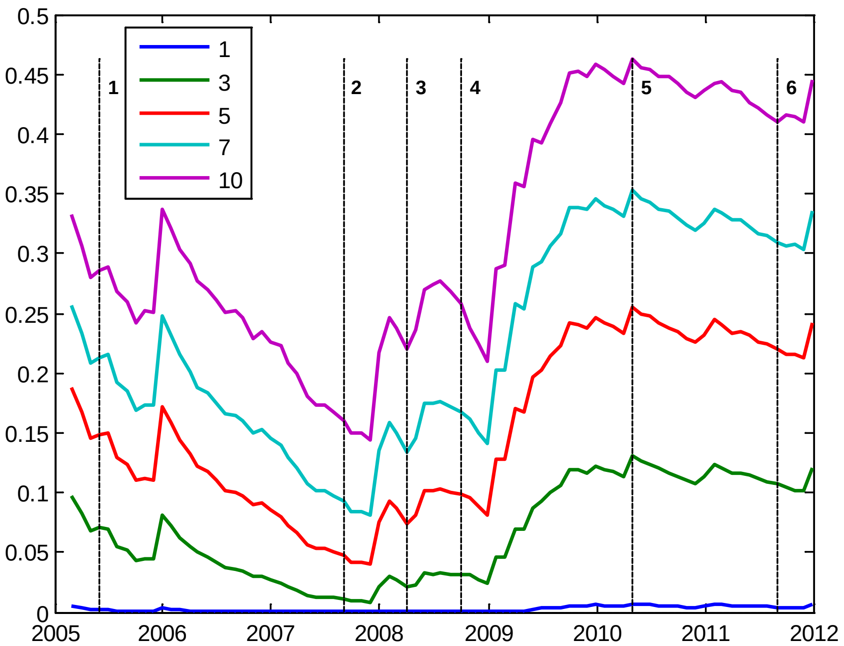


Figure 12. Default probabilities by maturity sector (structural model)



The figures present and evaluate in the best possible way how and to what extent the structural model works. It should be emphasized that to estimate the default probabilities of foreign debt, the Merton model relies only on information about the domestic bonds and the monetary base[8]. Then the implied assets of the country and their volatilities are estimated. Finally, we find the probability of default of the foreign debt together with the implied market leverage and other risky indicators of interest.

As expected, we see that the probabilities of default the structural model gives are much smoother than the ones implied by the market which reflects the gradual evolution of the risk fundamentals based on the macro-financial balance sheet information about the sovereign. Further, they are always a little bit lower in crisis periods producing a stable evolution where market sentiment playing a minor roles. Hardly can we expect sudden shifts in the view about the country's fundamentals. However, this is possible for the market quotes because additional factors like contagion, market sentiment, and global risk aversion play a significant role. Anyway, we see visually that the fundamentals are quite an important driver for the default probabilities showing consistency in pricing of the two types of debt. They capture to a satisfactory degree also the crisis periods indicated by the vertical lines.

After this preliminary analysis, we move to more formal one linking together macro, financial, and no-arbitrage information.

4. Integrated factor analysis

In this section, we focus on giving economic interpretation of the priced factors for the credit and currency risks that we got from the reduced form model. We use three types of variables having different informational content.

First, from market sentiment point of view, proxies for the global risk aversion are considered. Since the latter is a very broad concept, to capture its heterogeneity, we refer to several indices that got popularity. We take the VIX and the ITRAXX Europe IG to proxy the overall global uncertainty and the one of the corporate segment respectively. To proxy the risk of the banking system, we take the Euribor OIS spread[9]. Lastly, we refer to the JP Morgan’s EMBI+ Europe and EMBI Global indices to capture the risk of the emerging markets segment.

Second, from structural financial point of view, we take advantage of the risky indicators from the previous section: the domestic and foreign leverage as well as the asset volatility.

Third, we resort to macro variables like the GDP growth, the balance of payments position, and the budget deficit of Bulgaria. They complete the picture and give an overall macro characterization of the sovereign risk of the country. As additional macro variables, we take also the short end of the BGN and EUR curves. They are proxied by the BGN overnight rate (Leonia), EUR overnight rate (Eonia), and the interbank deposit rates from one week to three weeks.

For all the series that have a term structure we work with their shift and slope factors which could be viewed as a dimensionality reduction. We show regression results in daily and monthly frequencies with only the latter allowing taking advantage of the full set of variables from above. There we use monthly data for all the series so that there is a frequency consistency between the different sorts of data. We convert the quarterly data of the three main macro variables from above to monthly by linear interpolation. Further, for the daily series we take end of the month values.

We do not include further macroeconomic variables due to two main reasons. First, their frequency is usually on quarterly basis, so this would be in a dissonance with the majority of our main series that are given on daily basis. Thus the focus would be shifted to quarterly data which is not enough for robust statistical analysis with series available over several years only. Second, and more important, the macroeconomic information usually is

[8] Under a CBA the lack of volatility in the exchange rate is compensated by volatility in the monetary base.
[9] Computed for the short maturity segment as a difference between the Euribor and the OIS rate.

priced much in advance by the financial markets. So we would need to use an appropriate lag structure which is not clear what to be a priori. Technically, this would require resorting to a full fledged Var-Vec model which is possible to be employed as a toolkit but it would complicate too much the analysis and would enter into conceptual contradiction to the tools of the arbitrage analysis. Namely, the latter is based on the Efficient Market Hypothesis and usually does not consider lag and autocorrelation structures. We would leave such an extension together with further diagnostics of the informational content of the factors and their interactions with the macro variables for future research.

We regress the factors by the variables proposed. There is a clear indication of non-stationarity of the levels of the series especially during the crisis period. So we use first differences to avoid any spurious regression problem. Robust regression and control for any kinds of heteroscedasticity are employed.

We start with a daily frequency regression and use only the financial variables. The results for the credit and currency spreads' factors are shown in Table 6 and Table 7 respectively:

Table 6. Credit spread factors[10] (daily data)

explanatory/dependent	Δshift_credit_spread
c	0.0001* (0.0000)
Δshift_BGN_rate	0.0332 * (0.0058)
Δshift_ITRAXX	0.0050 * (0.0013)
Δshift_Euribor_OIS_spread	0.0006* (0.0001)
Δshift_EMBI_Global	-0.2196* (0.0450)
ΔVIX	0.0001* (0.0000)
ΔEMBI+_Europe	0.0014* (0.0005)
Δslope_ITRAXX	-0.0008* (0.0003)
Δslope_Euribor_OIS_spread	-0.0038* (0.0012)
Δslope_EMBI_Global	0.0021** (0.0001)
R^2	0.14
* signific. at 1% , * *signific. at 5%, * **signific. at 10%	

Table 7. Currency spread factors (daily data)

explanatory/dependent	Δshift_currency_spread
c	-0.0001* (0.0000)
Δshift_BGN_rate	-0.0001 *** (0.0062)
Δshift_Euribor_OIS_spread	-0.0024** (0.0012)
Δshift_EMBI_Global	0.2910* (0.04778)
ΔEMBI+_Europe	-0.0014*** (0.0005)
Δslope_Euribor_OIS_spread	0.0021*** (0.0013)
Δslope_EMBI_Global	-0.0053* (0.0010)
R^2	0.13
* signific. at 1% , * *signific. at 5%, * **signific. at 10%	

For both spreads we see high significance of large amount of the risk factors with relatively high R^2 for such type of regressions. All the coefficients have the right sign in the credit spread factor regression. Namely, the shifts of the risky factors push upwards the Bulgarian credit spread one so their coefficients are positive. Further, a higher slope of them is characterized by a boom and smooth functioning of the global economy which leads to a

[10] The standard errors are shown in the brackets

lower credit spread in Bulgaria. This explains the negative sign of the slopes' coefficients. The EMBI+ Europe is the only exception where the coefficients have opposite signs to what is expected. Most probably this is due to the fact that this index captures in a more focused way the forces that play at the emerging markets segment. So the flight to quality effect in the turbulent times is reflected. It outweighs any increase of the global riskiness.

Interestingly, in the currency spread regression, despite less, we have also a good amount of significant explanatory factors. At first glance, this seems paradoxical to the previous analysis that this spread is mainly driven by liquidity factors and more or less should have a random evolution. So the relatively high explanatory power seems to be strange. However, if we look at the signs of the coefficients, they have opposite signs to the ones of the previous regression. This gives an explanation what is going on. We have a purely technical phenomenon. Namely, the risky factors explain quite well both the EUR and BGN risky curves of Bulgaria but are only subject to different weights. Due to the already discussed higher relative elasticity of the EUR curve to shocks than the BGN one, the credit and currency spreads have in turbulent times opposite movement. So it is natural to expect different signs of the coefficients. Yet, if we add them, we still get correct signs for the general currency spread.

We continue now with the monthly data regression adding macrofinancial and macro variables in the analysis. The results are shown in Table 8 for the credit spread and in Table 9 for the currency one.

Table 8. Credit spread factors (monthly data)

explanatory/dependent	Δshift_credit_spread
c	0.0057 (0.0057)
Δshift_ITRAXX	0.1156* (0.0167)
Δshift_Euribor_OIS_spread	0.0272* (0.0010)
Δslope_ITRAXX	-0.0174* (0.0042)
Δslope_foreign_leverage	-0.0619* (0.0284)
R^2	0.64
* signific. at 1% , * *signific. at 5%, * **signific. at 10%	

Table 9. Currency spread factors (monthly data)

explanatory/dependent	Δshift_currency_spread
c	-0.0197 (0.0068)
Δshift_EMBI_Global	1.0550* (0.0120)
ΔGDP_rate	2.43** (1.1091)
R^2	0.51
* signific. at 1% , * *signific. at 5%, * **signific. at 10%	

We see a picture helping us to better understand the origin of the two spreads, but interestingly, a bit differently from its daily data counterpart above. In both cases, the explanatory power of the regression is quite high and the reason for this is that we are closer to the fundamentals and the noise from daily observations is eliminated.

For the credit spread we see significance of the global risk aversion factors similar to the daily data case. So both the risk of the corporate sector and the one of the banking system are transferred to the CDS premiums of the emerging markets countries segment with Bulgaria being part of the latter. Surprisingly, the structural indicators do not play a role. Neither the leverage ratios shifts, nor the asset volatility shift is significant. Actually, as we have seen above, the images of the default probabilities from structural and reduced form point of view are very similar. So, despite not shown, in a regression in levels, the leverage matters, but as already mentioned, there are econometric problems with a good identification. The same is valid for the macro variables too. As already mentioned, an analysis in levels would require Var-Vec methods and would go beyond the scope of the current paper and could be postponed for future research where a cross country comparison would be necessary too[11]. The significance of the slope of the foreign leverage ratio has a technical character. Since the structural model is estimated by the term structure of the BGN curve and the monetary base, it is natural to expect the term structure to be reflected in the output for the credit spread.

For the currency spread we see a striking result. The global risk aversion coming from the corporate and banking sectors is not significant. Only the one coming from the emerging markets plays a role. This is consistent with the daily data regression where we have similar picture. So all these shows that the currency spread is not only due to liquidity and does not only have a technical character. Namely, the EUR curve dynamics[12] is predominantly subject to risk factors coming from the corporate and banking sectors especially considering data on monthly basis which captures better the fundamentals picture. On the opposite, the BGN curve dynamics is driven by factors coming from the other emerging markets and especially common factors driving the bonds as an asset class. Idiosyncratic macro and macrofinancial variables seem to be more related to the levels than to the dynamics of the curves and the spreads. The significance of the GDP growth rate and its positive weight are reflecting the just hinted difference between the CDS and the bonds as asset classes. During the boom period both the credit and the currency spreads were falling. However, at the same time, the GDP growth was making the BGN treasuries lose attractiveness because equities and credit were giving a much higher return. When the crisis hit the Bulgarian economy the treasuries were the safe heaven and there was a flight to quality effect. The Bulgarian CDS and the EUR curve did not have such a status.

5. Conclusion

The paper studied in a comprehensive way the local currency and hard currency yield curves of Bulgaria. This was done from a no-arbitrage, structural financial and macroeconomic points of view. The formal no-arbitrage conditions that need to hold are the main result of the paper. They give a natural basis for analysis of the factors that drive the credit and currency risks of the country. We provided a set of indicators that should play a central role. The theoretical analysis has a leading role. Our empirical considerations have a complementary one and provide scope for further discussions and research. However, we found that in the Bulgarian case the specially build theoretical setting works quite well and the factors derived could explain to a large extent the regularities the country is subject to in a time period characterized by financial innovation, economic growth, market turmoil, financial crisis, and recession.

[11] For grasping to a large extent the economic forces behind the curves our setting is pretty enough.

[12] So the CDS too.

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Appendix 1

We prove here the formulas about the bonds' prices dynamics of the reduced form model. Applying the Ito's lemma and the Girsanov's theorem to the definition of the bonds' prices we can get:

$$\frac{dP^*_{f,EUR}(t,T)}{P^*_{f,EUR}(t-,T)} = \left(-\int_t^T \alpha^*_{EUR}(t,s)ds + r^*_{EUR}(t) + \frac{1}{2}\left\| \int_t^T \sigma^*_{EUR}(t,s)ds \right\|^2 \right) dt$$
$$-\left(\int_t^T \sigma^*_{EUR}(t,s)ds \right) dW^P(t)$$
$$+\int_E (1-q_{f,EUR}(x,t)) \left(\exp(-\int_t^T \delta^*_{EUR}(x,t,s)ds) - 1 \right) \mu(dx,dt) - \int_E q_{f,EUR}(x,t)\mu(dx,dt)$$

$$\frac{dP^*_{d,BGN}(t,T)}{P^*_{d,BGN}(t-,T)} = \left(-\int_t^T \alpha^*_{BGN}(t,s)ds + r^*_{BGN}(t) + \frac{1}{2}\left\| \int_t^T \sigma^*_{BGN}(t,s)ds \right\|^2 \right) dt$$
$$-\left(\int_t^T \sigma^*_{BGN}(t,s)ds \right) dW^P(t)$$
$$+\int_E (1-q_{d,BGN}(x,t)) \left(\exp(-\int_t^T \delta^*_{BGN}(x,t,s)ds) - 1 \right) \mu(dx,dt) - \int_E q_{d,BGN}(x,t)\mu(dx,dt)$$

Further, we have the dynamics of the exchange rate:

$$\frac{dX(t)}{X(t-)} = -\int_E \delta_X(x,t)\mu(dx,dt)$$

So using the no-arbitrage conditions and equating the expected local drifts to the risk free rate, we get the results shown in the main text.

Appendix 2

We prove here the formulas about the bonds' prices dynamics of the structural form model. For the foreign debt we have that the price is a function of the assets, i.e. it holds that $P^*_{f,EUR}(t,T) = F(t, A_{d,EUR}(t))$. Further, we have the dynamics for the assets to be $\frac{dA_{d,EUR}(t)}{A_{d,EUR}(t-)} = r_{EUR}dt + \sigma_{A_{d,EUR}} dW^{Q^f}(t)$. So by the Ito's lemma we get:

$$dF(t, A_{d,EUR}(t)) = \frac{F_t + r_{EUR}A_{d,EUR}(t)F_A + 0.5\sigma^2_{A_{d,EUR}} A^2_{d,EUR}(t)F_{AA}}{F} F(t, A_{d,EUR}(t))dt$$
$$+\frac{\sigma_{A_{d,EUR}} A_{d,EUR}(t)F_A}{F} F(t, A_{d,EUR}(t)) dW^{Q^f}(t)$$

But for the price itself we could also assume the dynamics:

$$\frac{dP^*_{f,EUR}(t,T)}{P^*_{f,EUR}(t-,T)} = r_{EUR}dt + \sigma_{P^*_{f,EUR}}(t,T)dW^{Q^f}$$

Making the corresponding volatility terms equal we get:

$$\sigma_{P^*_{f,EUR}}(t,T) = \frac{\sigma_{A_{d,EUR}} A_{d,EUR}(t) F_A}{F}$$

For the domestic debt we have that the price is a function of the stochastic processes of the assets, exchange rate, and monetization. It holds that $P^*_{d,EUR}(t,T) = F(t, A_{d,EUR}(t), X(t), MM_{d,EUR}(t))$. As before we have for the monetization the dynamics:

$$\frac{dMM_{d,EUR}(t)}{MM_{d,EUR}(t-)} = \frac{X(t-)B_{d,LC}}{MM_{d,EUR}(t-)} \delta_{\{t=T\}\wedge\{A_{d,EUR}(T-)<B_{f,EUR}\}}(dt)$$
$$+ \frac{B_{f,EUR} + X(t-)B_{d,LC} - A_{d,EUR}(t-)}{MM_{d,EUR}(t-)} \delta_{\{t=T\}\wedge\{B_{f,EUR} \le A_{d,EUR}(T-) < B_{f,EUR} + X(T-)B_{d,BGN}\}}(dt)$$

and for the exchange rate:

$$\frac{dX(t)}{X(t-)} = -\rho^1 \delta_{\{t=T\}\wedge\{B_{f,EUR} \le A_{d,EUR}(T-) < B_{f,EUR} + X(T-)B_{d,BGN}\}}(dt) - \rho^2 \delta_{\{t=T\}\wedge\{A_{d,EUR}(T-)<B_{f,EUR}\}}(dt)$$

By the Ito's lemma we get:

$$dF(A_{d,EUR}(t), X(t), MM_{d,EUR}(t)) =$$
$$= \frac{F_t + r_{EUR} A_{d,EUR}(t) F_A + 0.5\sigma^2_{A_{d,EUR}} A^2_{d,EUR}(t) F_{AA}}{F} F(A_{d,EUR}(t), X(t), MM_{d,EUR}(t))dt$$
$$+ \frac{\sigma_{A_{d,EUR}} A_{d,EUR}(t) F_A}{F} F(A_{d,EUR}(t), X(t), MM_{d,EUR}(t)) dW^{Q^f}(t) + \Delta F \delta_{\{t=T\}}(dt)$$

For the price of the domestic bond we could also assume the dynamics:

$$\frac{dP^*_{d,EUR}(t,T)}{P^*_{d,EUR}(t-,T)} = r_{EUR}dt + \sigma_{P^*_{d,EUR}}(t,T) dW^{Q^f}$$

Making the respective terms equal, gives us:

$$\sigma_{P^*_{d,EUR}}(t,T) = \frac{\sigma_{A_{d,EUR}} A_{d,EUR}(t) F_A}{F}$$

In general, the jump effects $\Delta F \delta_{\{t=T\}}(dt)$ are incorporated into the drift, but since we assumed that the exchange rate depreciation offsets the monetization, actually, we do not have jumps. Further, we can see that F_A is the Black-Scholes delta of the portfolio of the two options: $C_{BGN}(t,T,B_{f,EUR}, A_{d,EUR}(t)) - C_{BGN}(t,T,B_{f,EUR} + X(T-)B_{d,BGN}, A_{d,EUR}(t))$ which in fact gives the price of the bond. We found that it is:

$$P^*_{d,EUR}(t,T) = C_{EUR}(t,T,B_{f,EUR},A_{d,EUR}(t)) - C_{EUR}(t,T,B_{f,EUR} + X(T-)B_{d,BGN},A_{d,EUR}(t))$$
$$= A_{d,EUR}(t)(N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}})) - B_{f,EUR}e^{-r_{EUR}(T-t)}(N(d_2^{B_{f,EUR}}) - N(d_2^{B_{f,EUR},B_{d,BGN}}))$$
$$-X(t)B_{d,BGN}e^{-r_{EUR}(T-t)}$$

The homogeneity of degree one of $P^*_{d,EUR}(t,T)$ in respect to $A_{d,EUR}(t)$, $B_{f,EUR}$, and $X(t)$, together with the Euler's theorem gives directly the delta:

$$F_A = N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}})$$

So we get:

$$\sigma_{P^*_{d,EUR}}(t,T) = \frac{\sigma_{A_{d,EUR}} A_{d,EUR}(t) F_A}{F} =$$
$$= \frac{\sigma_{A_{d,EUR}}(N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}}))}{N(d_1^{B_{f,EUR}}) - N(d_1^{B_{f,EUR},B_{d,BGN}}) - \rho_{f,EUR}(N(d_2^{B_{f,EUR}}) - N(d_2^{B_{f,EUR},B_{d,BGN}})) + \rho_{d,EUR}N(d_2^{B_{f,EUR},B_{d,BGN}})}$$

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