Fuzzy Dark Matter Dynamics and the Quasi-Particle Hypothesis

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Introduction

Fuzzy Dark Matter (FDM) is an alternative to Λ-Cold-Dark-Matter that behaves as a density wave; ultra-light bosons (with mass $10^{-22}\mathrm{eV} \leq m_{\mathrm{FDM}} \leq 10^{-20}\mathrm{eV}$) can have a wavelength of astrophysical scale, and collections of such bosons form a coherent wave.

An interesting property of FDM is the appearance of density perturbations due to wave interference. In an FDM halo, these perturbations can dynamically heat stars in a manner similar to classical two-body relaxation. The *quasi-particle hypothesis* thus asserts that these perturbations are analogous to classical heavy particles in a mixed mass particle system. Formally, the perturbations are *quasi-particles* with effective mass proportional to the local phase-space density.

The QP-hypothesis has been used in the literature as a basis to exclude FDM as a dark matter candidate. In particular, analysis of the dynamical heating phenomenon yields $m_{\mathrm{FDM}} \approx 10^{-19}\mathrm{eV}$, outside the FDM mass range. However, the simulations used to arrive at these conclusions were not fully self-consistent, due to computational costs. We aim to directly test the quasi-particle hypothesis in one dimension using simulations of full numerical accuracy.

The System

The coupled FDM+Particle system is ruled by the *Schrödinger-Poisson-Particle* (SPP) System:

$$\begin{cases} i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi \\ \frac{d^2\mathbf{x}_i}{dt^2} = -\nabla\Phi(\mathbf{x}_i) \\ \nabla^2\Phi = 4\pi G(\rho_{\mathrm{FDM}} + \rho_{\mathrm{part}}) \end{cases}$$

where the mass densities are defined by:

$$\rho_{\mathrm{FDM}} = m|\psi|^2 \qquad \rho_{\mathrm{part}} = \sum_{i=1}^{N_{\mathrm{part}}} m_i\delta(\mathbf{x} - \mathbf{x}_i)$$

Here G is Newton's constant, $\hbar$ Planck's reduced constant, $m \equiv m_{\mathrm{FDM}}$ is the FDM boson mass, N_{part} the number of particles, m_i and $\mathbf{x}_i$ the mass and position of the i^{th} particle, and δ the Dirac delta. The SPP is made one-dimensional by making replacements $\mathbf{x} \mapsto x$ and $\nabla \mapsto \frac{d}{dx}$.

The first equation in the SPP is the Schrödinger equation that governs the dynamics of the FDM wavefunction $\psi = \psi(x,t)$. The second equation is just Newton's 2nd Law and governs the particles. The third equation is Poisson's equation, which provides the gravitational potential that couples the two.

We further define the *fuzziness* parameter:

$$r = \frac{\hbar}{2m}$$

as a control on the de Broglie wavelength of the FDM.

Numerical Techniques

We must simulate FDM and particle dynamics in a fully self-consistent manner. Part of this consistency is already contained in the Schrödinger-Poisson-Particle system, while the rest relies on how numerical solutions are found. We use a *Kick-Drift-Kick* algorithm for both the FDM and particle components:

$$\begin{array}{lll} \text{KICK 1:} & \psi \mapsto e^{-i\frac{\Delta t}{4r}\Phi(t+\Delta t,x)}\psi & v_i(t+\Delta t/2) = v_i(t) + a_i(t)\frac{\Delta t}{2} \\ \text{DRIFT:} & \psi \mapsto \left[\mathrm{FFT}^{-1} \circ e^{ir\Delta tk^2} \circ \mathrm{FFT}\right]\psi & x_i(t+\Delta t) = x_i(t) + \Delta t \cdot v_i(t+\Delta t/2) \\ \text{KICK 2:} & \psi \mapsto e^{-i\frac{\Delta t}{4r}\Phi(t,x)}\psi & v_i(t+\Delta t) = v_i(t+\Delta t/2) + \frac{\Delta t}{2}a_i(t+\Delta t/2) \end{array}$$

Between DRIFT and KICK 2, it is necessary to update the potential Φ by solving Poisson's equation, which we do using a Green's function convolution over the doubled numerical grid:

$$\Phi(x) = 4\pi\int_{-L/2}^{3L/2} G(x-x')\rho(x')dx'; \qquad G(x) = \frac{1}{2}|x|$$

where L is the length of the grid. To satisfy isolated boundary conditions, we set the density ρ to zero on the doubled portion of the grid. For initial conditions, we choose the lowered Spitzer distribution, visible in the figure below.

All of this is coded in Python3.

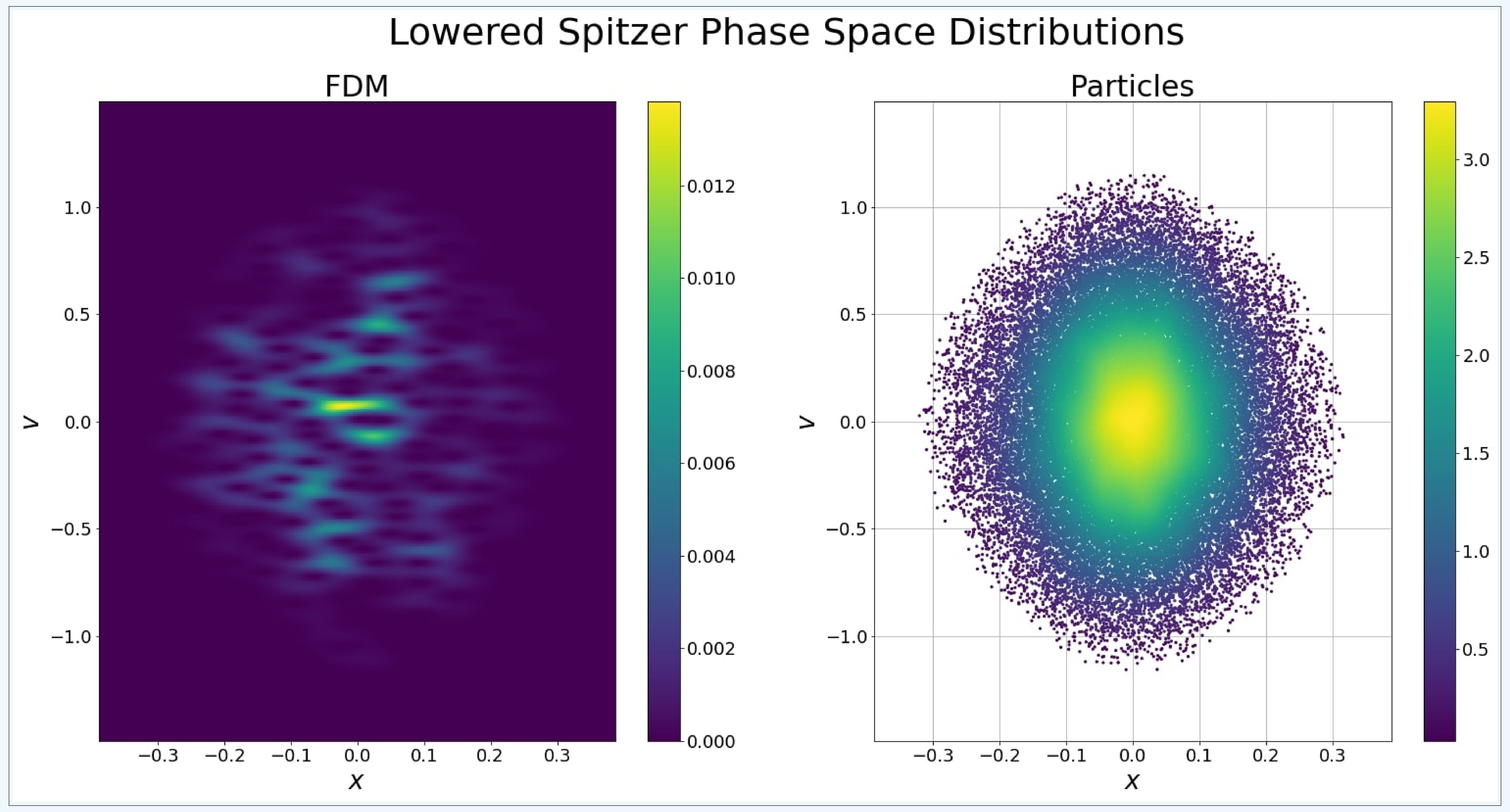


Figure: $f(x,v_x) = f_0(e^{-(E_x-E_0)/\sigma_x^2} - 1)I(0 \leq E_x \leq E_0)$, where $E_x = \frac{1}{2}v_x^2 + \Phi(x)^2$. We set $f_0 = 0.1$, $E_0 = 0.7$ and $\sigma_x = 0.5$.

Methods

We approach the quasi-particle hypothesis with two independent tests:

Test 1

Quantify the oscillations in kinetic energy from simulations of two different regimes:

1. Pure Particles
2. Pure FDM

Test 2

Fit the energy-transfer curves in two regimes:

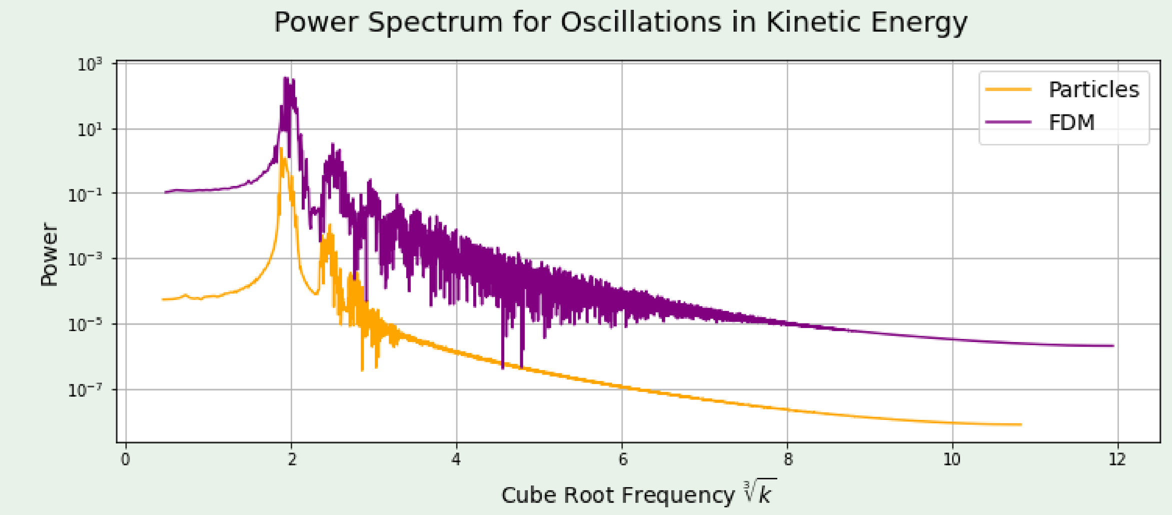
1. Mixed Particles (Heavy + Light masses)
2. FDM and Particles

Total mass is split equally between components.

In both tests, we search for an empirical law that provides a correspondence between the FDM fuzziness r and the effective mass of quasi-particles m_{eff}.

Test 1 Results

Naturally occurring oscillations in both pure FDM and pure particle systems appear to have corresponding power spectra.

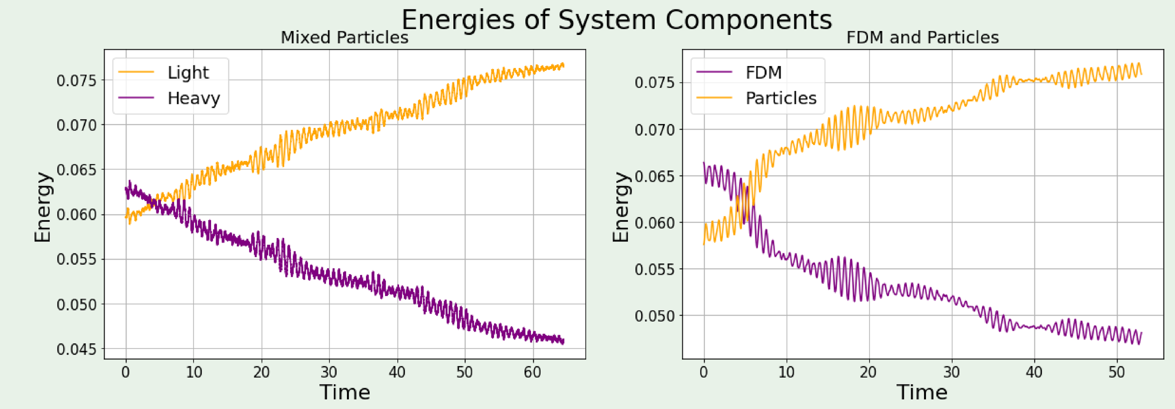


By curve-fitting the root-mean-square (RMS) amplitude for many simulations as a function of N_{part} and r in either regime, we find a power law:

$$N_{\mathrm{eff}}(r) = \alpha\left(\frac{1}{r}\right)^{\beta}$$

where $\alpha \approx 1 \times 10^{-6}$ and $\beta \approx 3.3$.

Test 2 Results



Dynamical heating of (light) particles, such as in the figure above, occurs across all simulations of the two regimes. That is, we see similar results for varying values of the heavy:light particle mass ratio μ and fuzziness r.

It is possible to fit the energy-transfer curves with:

$$\frac{\Delta E}{E_{\mathrm{initial}}}(t;p) = a(p)(\ln(t+1))^{b(p)}.$$

where the secondary parameter is either $p := \mu$ or $p := r$. Further fitting the parameters a and b with respect to μ and r is suggestive of a power law:

$$\mu_{\mathrm{eff}} = \xi r^{\gamma}$$

where μ_{eff} is the effective mass ratio quasi-particle:particle. However, current data does not yield definitive values for ξ and γ.

Discussion

Test 1 gives a convincing empirical effective quasi-particle mass: $m_{\mathrm{eff}} = \frac{M}{\alpha}r^{\beta}$, where M is the system's total mass. We would like our second test to provide an identical result. This will be possible with more data or a better fitting formula. The goal of our future research is to complete these refinements.

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