

In []: **import** Analysis

```
#[ ..., [r,m,Num_bosons,sigma,Num_stars],...]
# Args = [
#     [0.5,1.0,0,1,10000],
#     [0.5,1.0,10000,1,10000],
#     [1,0.5,20000,1,10000],
#     [5,0.1,100000,1,10000],
#     [10,0.05,200000,1,10000],
#     [50,0.01,1000000,1,10000],
#     [0.5,1.0,10000,1,0]
# ]
```

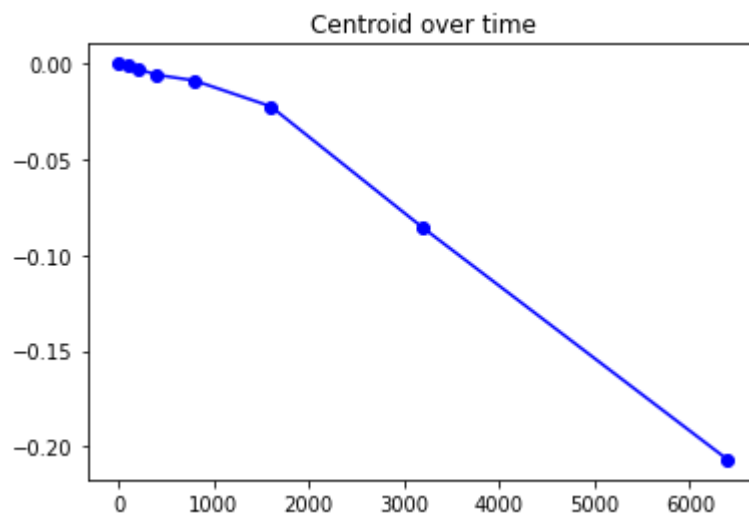
```
Args = [
    [1,1,0,0.001,1000],
    [1,1,0,0.0002,5000],
    [1,1,0,0.0001,10000],
    [1,1,0,0.00002,50000],
    [1,1,0,0.00001,100000]
]
```

```
for args in Args:
    print("-----New Analysis-----")
    print(
        f"r = {args[0]}",
        f"mu = {args[1]}",
        f"Num_bosons = {args[2]}",
        f"sigma = {args[3]}",
        f"Num_stars = {args[4]}"
    )
    Analysis.analysis(*args)
```

/home/boris/Documents/Research/FDM_n_Bodies/1D_Codes/Non-Dim/Analysis

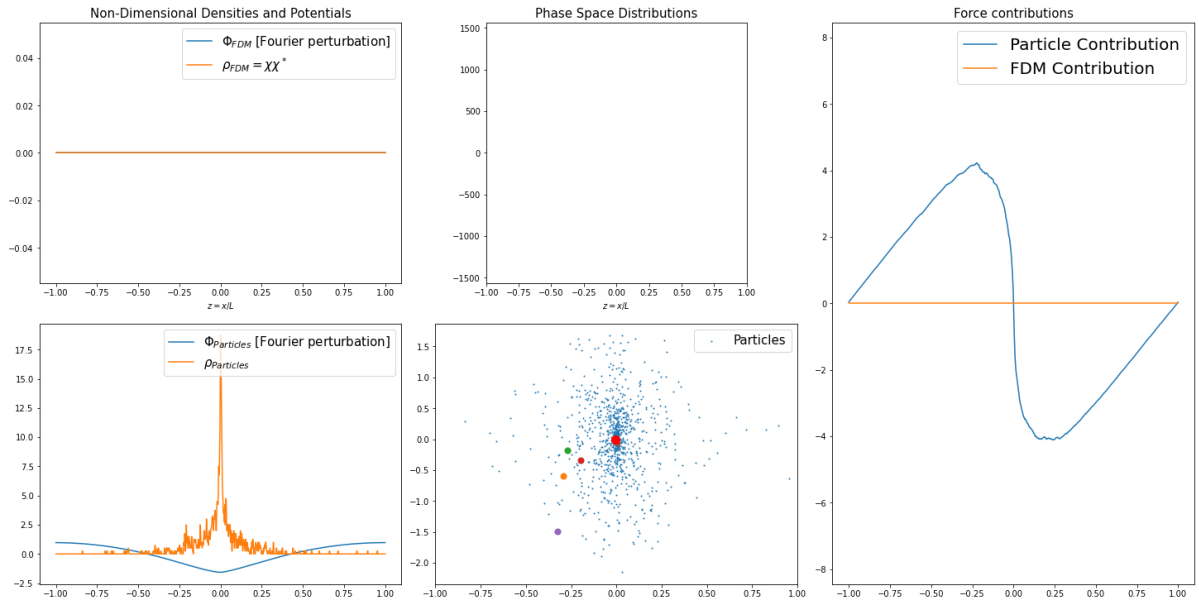
-----New Analysis-----

r = 1 mu = 1 Num_bosons = 0 sigma = 0.001 Num_stars = 1000



/home/boris/Documents/Research/FDM_n_Bodies/OneD/WaveNonDim.py:129: Runtime Warning: invalid value encountered in true_divide

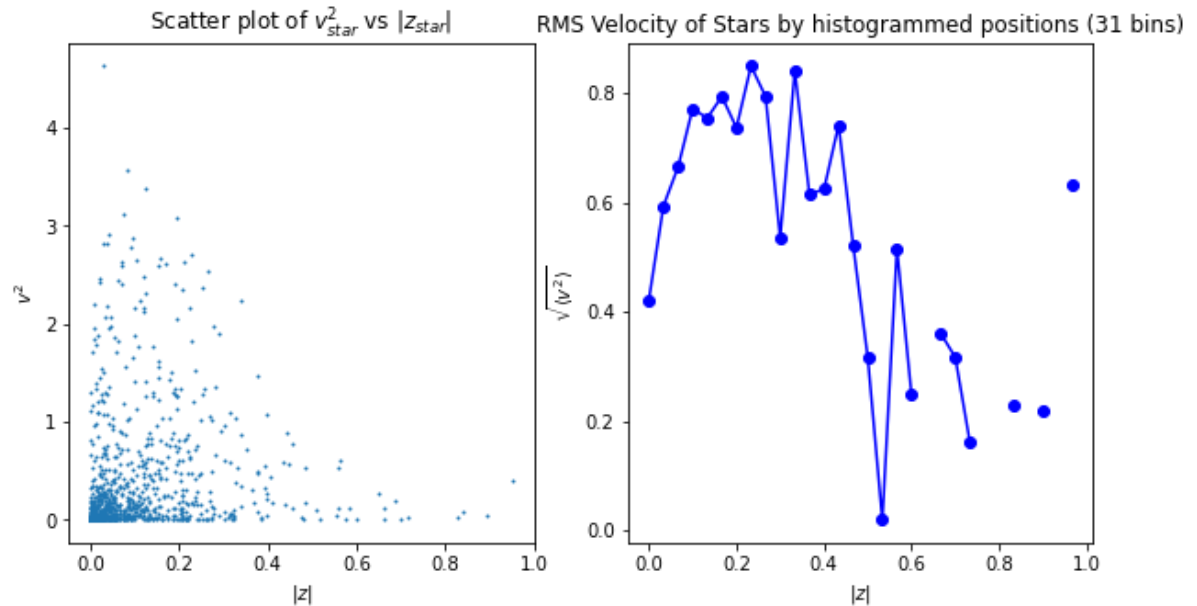
F_s = F_s/Norm_const

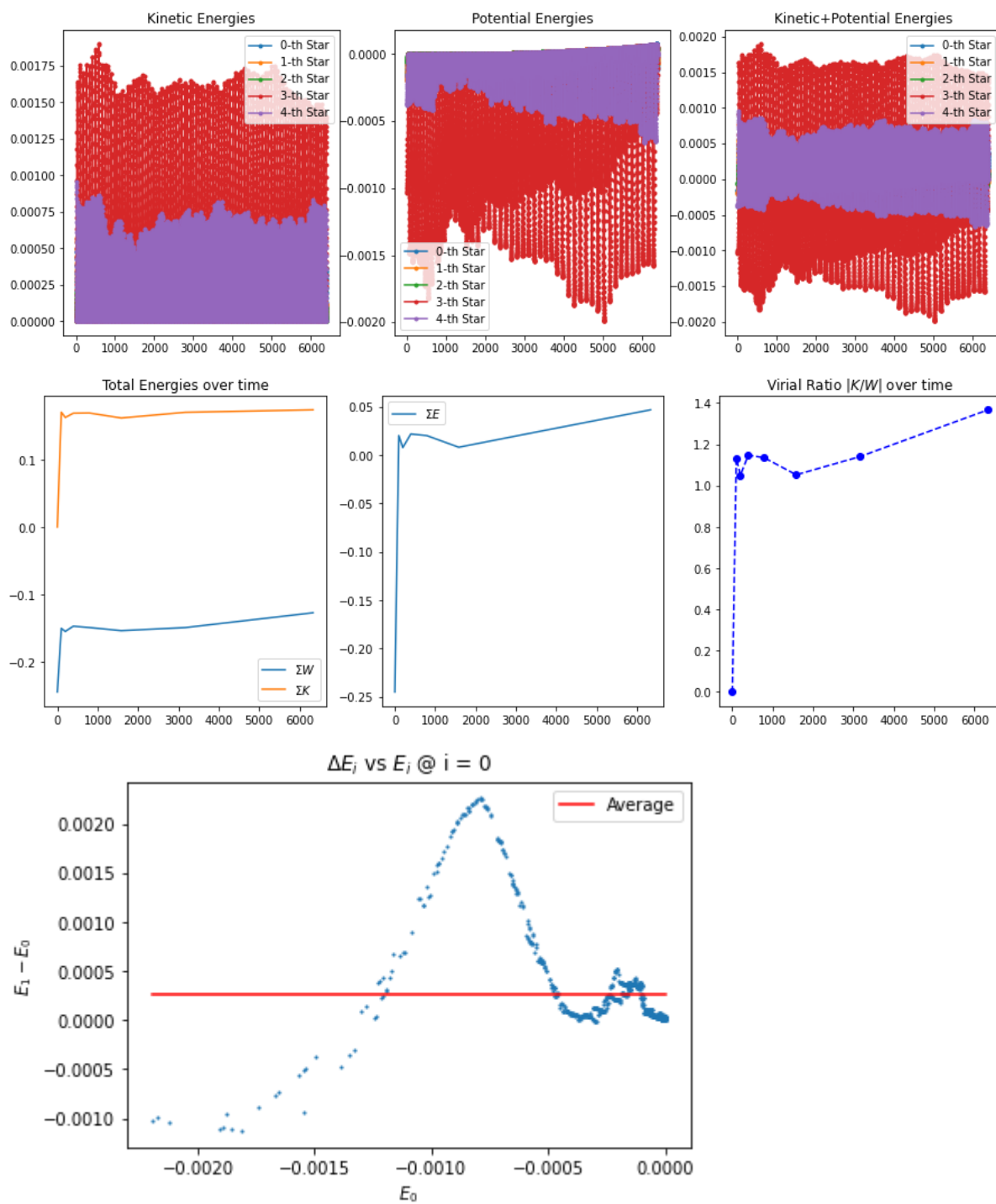


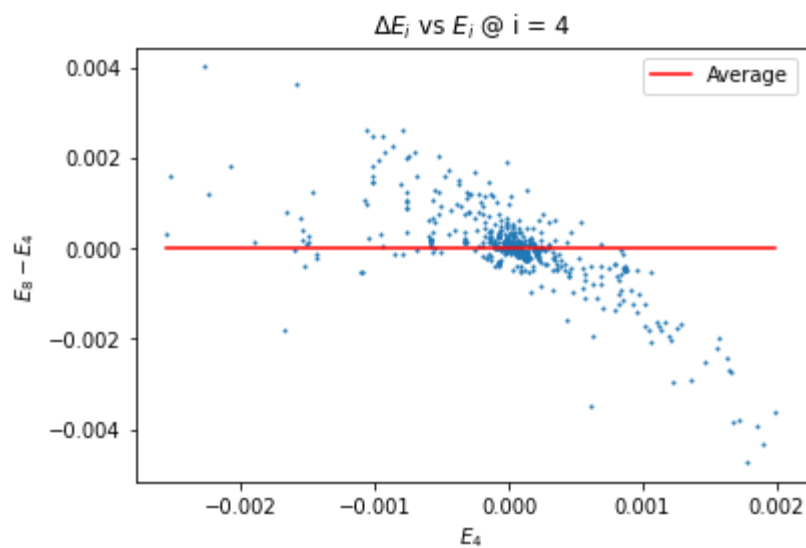
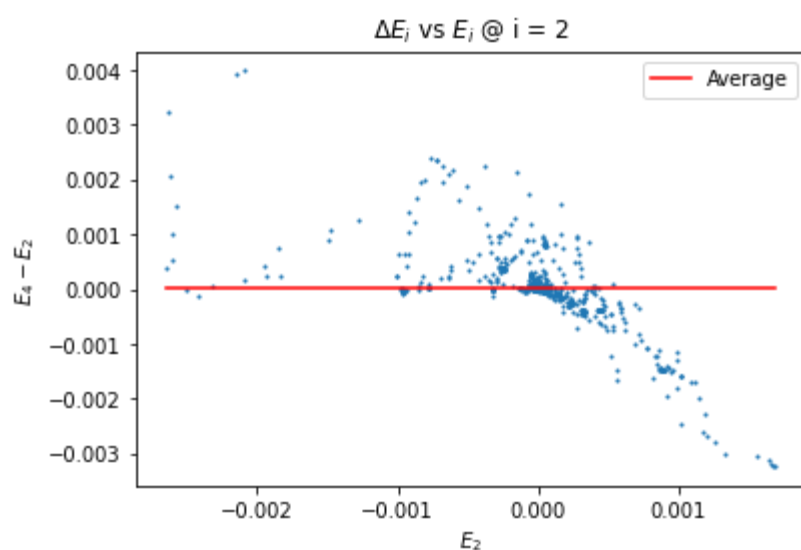
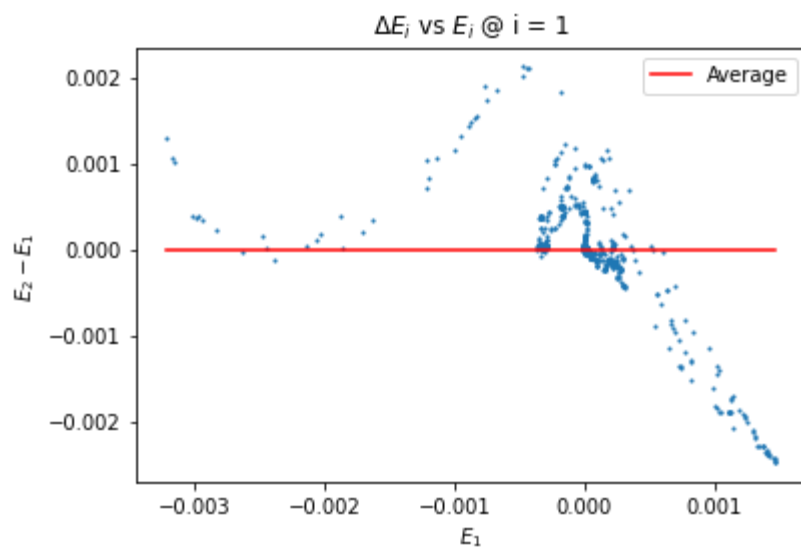
```
v_rms = 0.5923277051957598
z_rms = 0.15871675883738892
K_avg = 0.5*m*v_rms^2 = 0.1754260551712375 (m=1)
=> 2*K_avg = 0.350852110342475
W_avg = 158.7167588373889
```

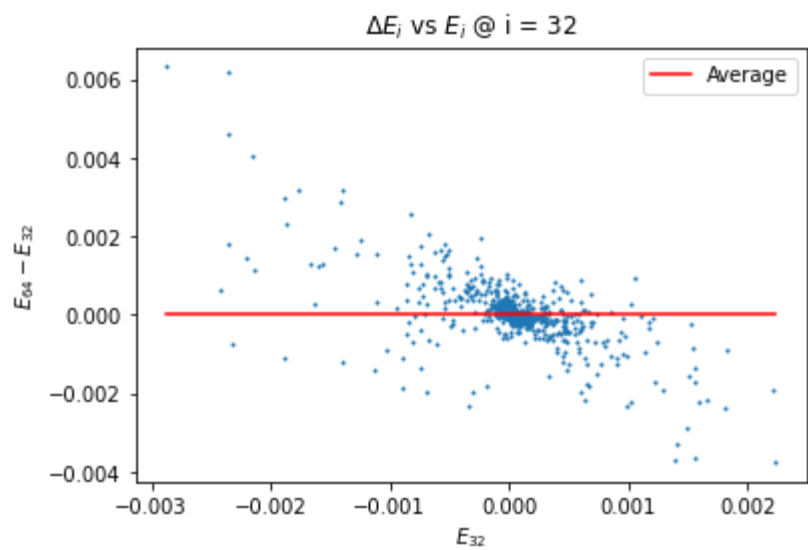
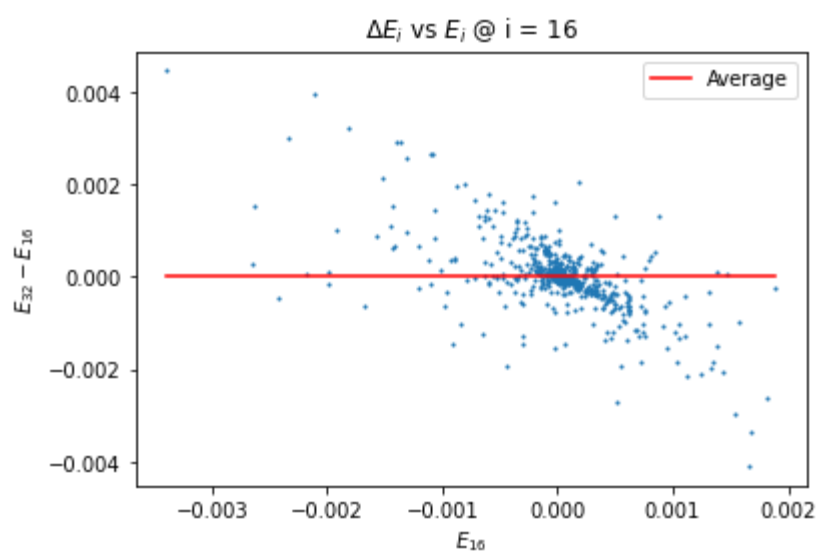
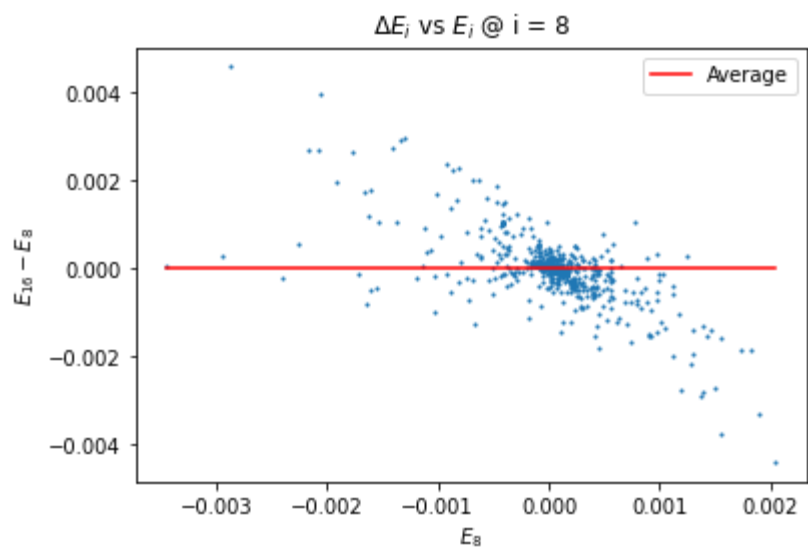
```
-----
K_tot = 0.17542605517123733
K_avg = 0.00017542605517123733
W_tot = -0.14596981745731766
W_avg = -0.00014596981745731765
```

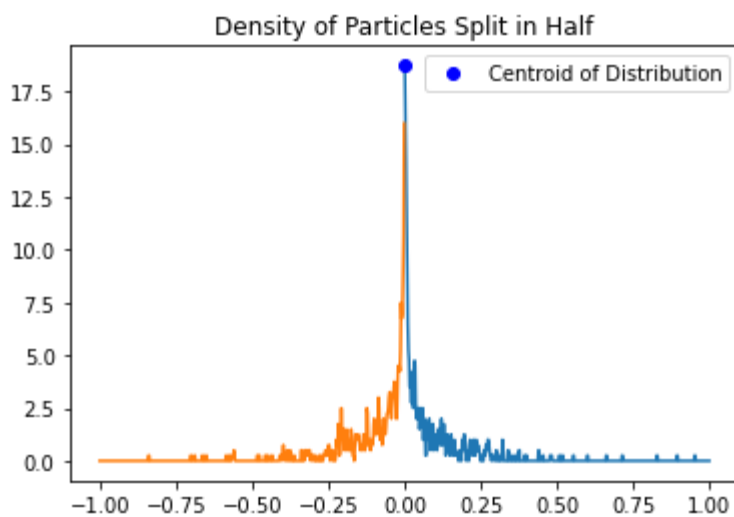
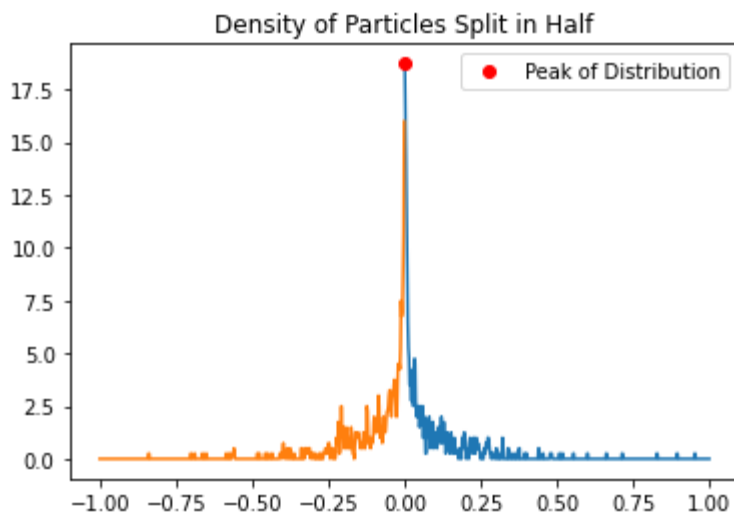
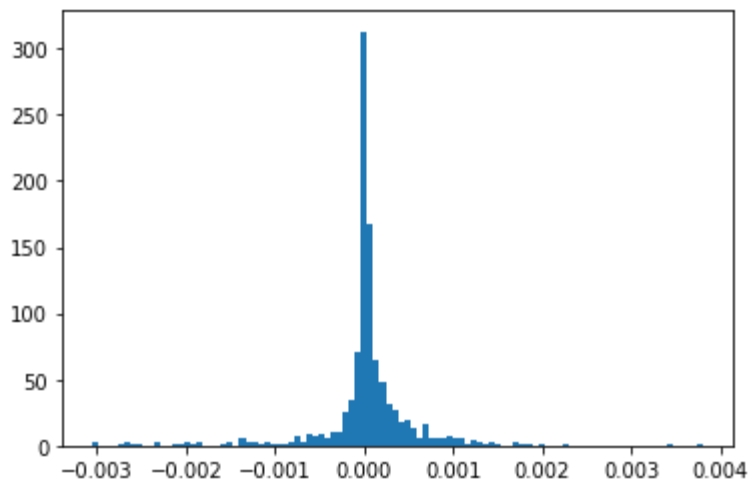
```
/home/boris/Documents/Research/FDM_n_Bodies/1D_Codes/Non-Dim/Analysis/Analysis.py:266: RuntimeWarning: invalid value encountered in true_divide
  v_rms_array = bins/bins_counts
```



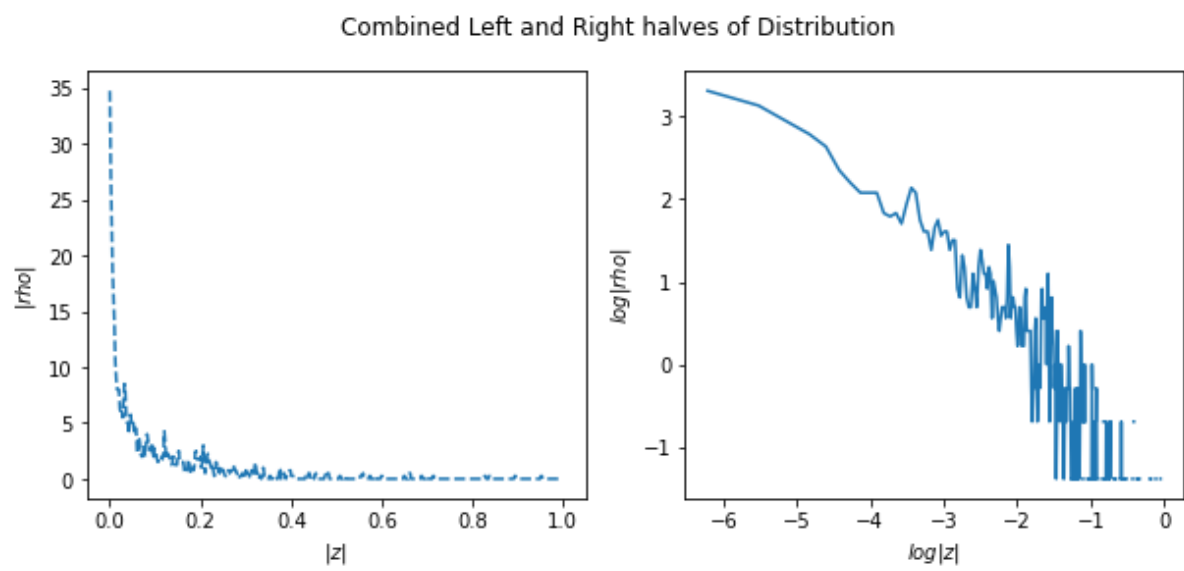




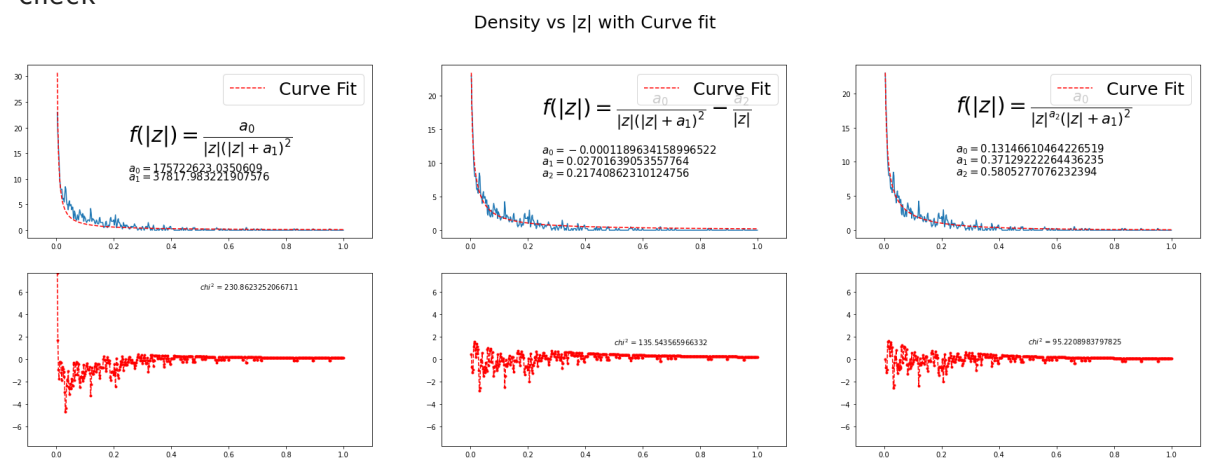




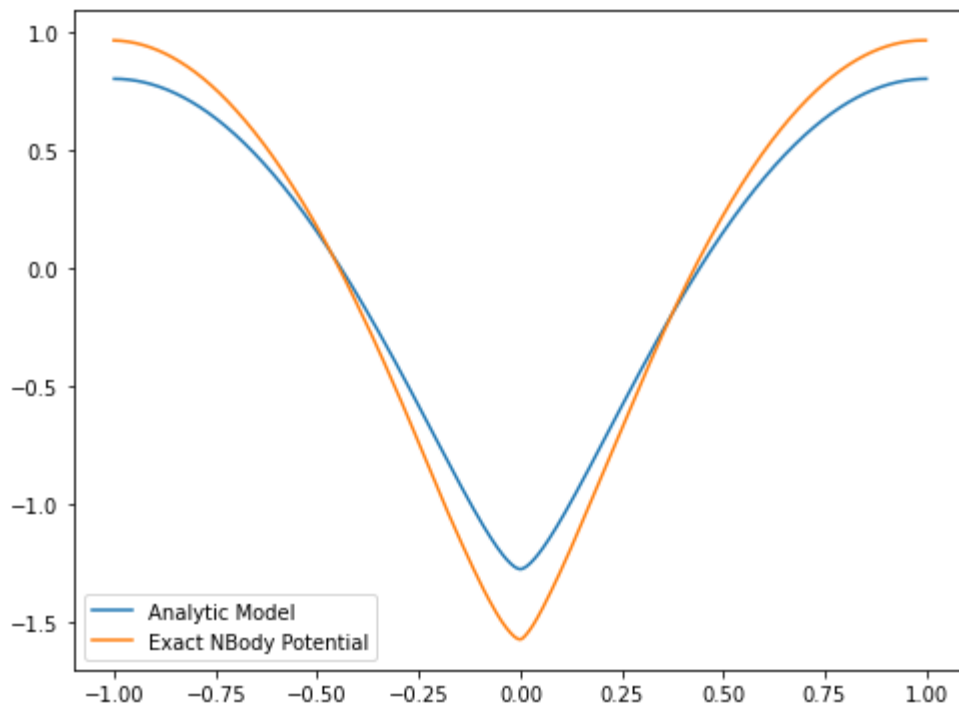
```
/home/boris/Documents/Research/FDM_n_Bodies/1D_Codes/Non-Dim/Analysis/Analysis.py:442: RuntimeWarning: divide by zero encountered in log
  ax[1].plot(np.log(z_right), np.log(rho_whole))
```



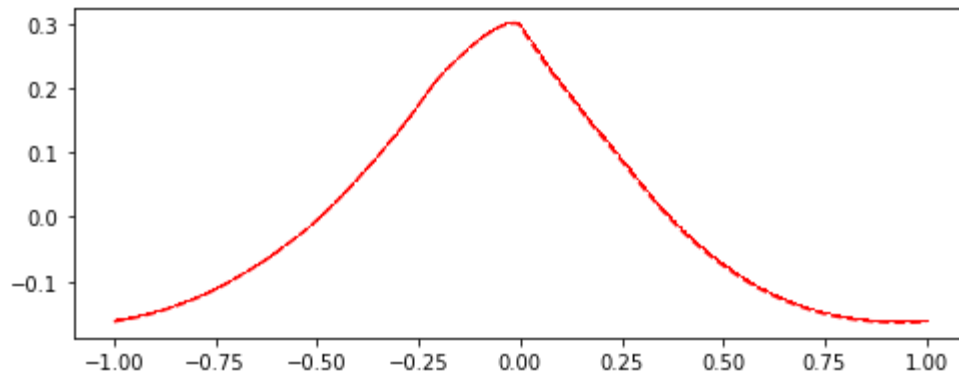
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Gravitational Potential in the Box



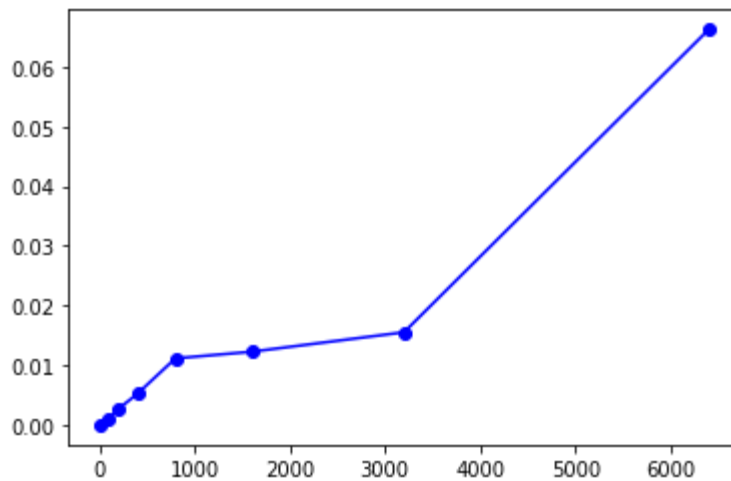
Residuals

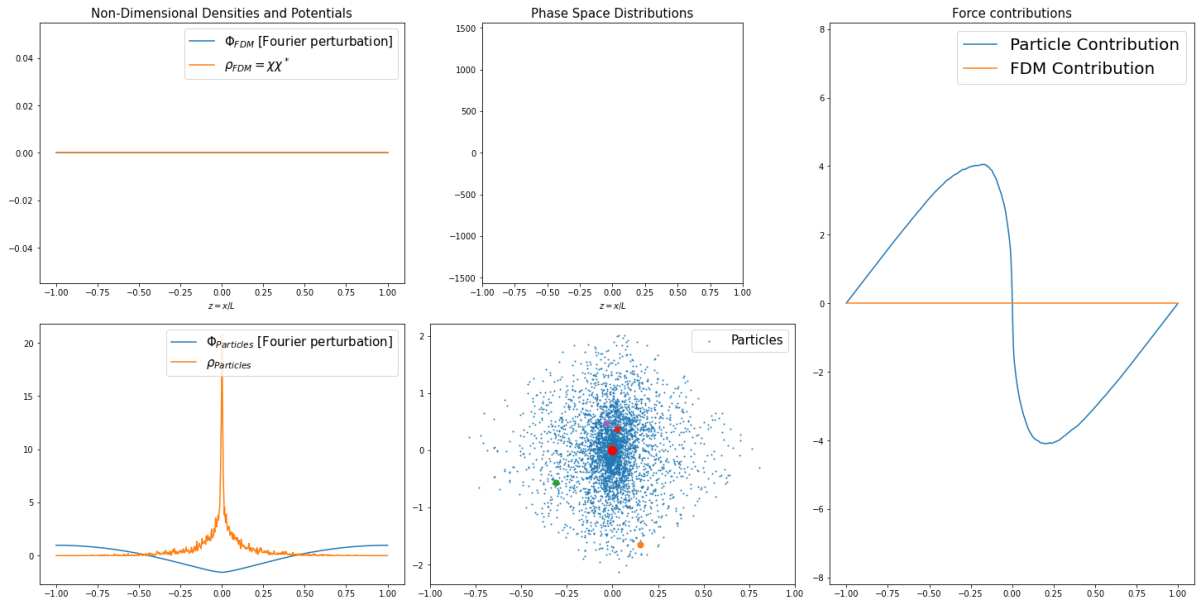


-----New Analysis-----

r = 1 mu = 1 Num_bosons = 0 sigma = 0.0002 Num_stars = 5000

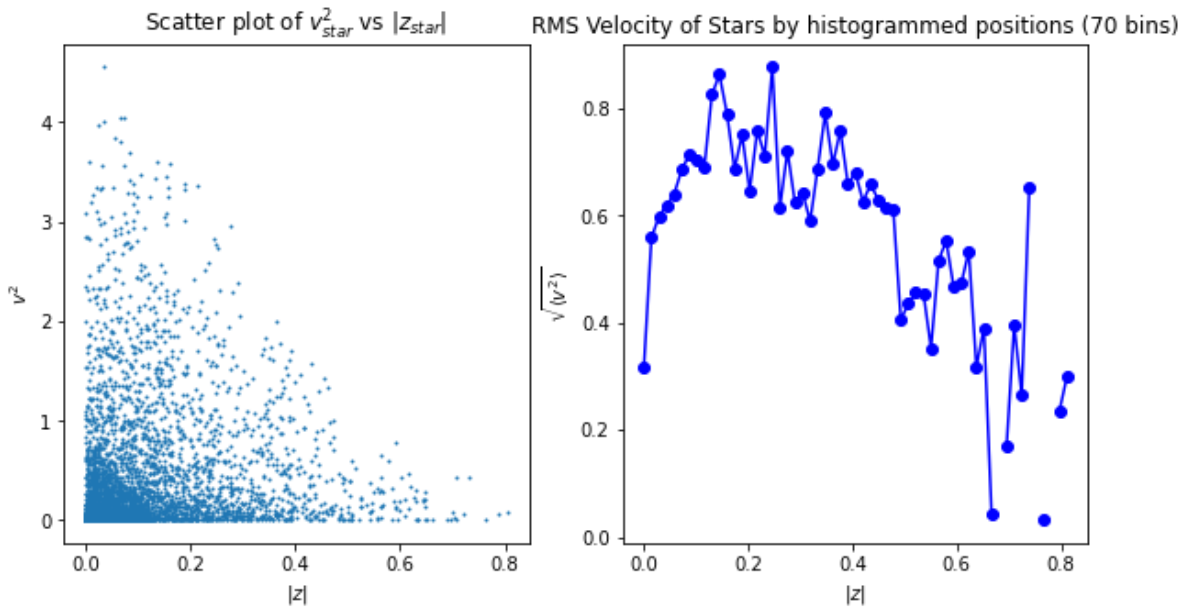
Centroid over time

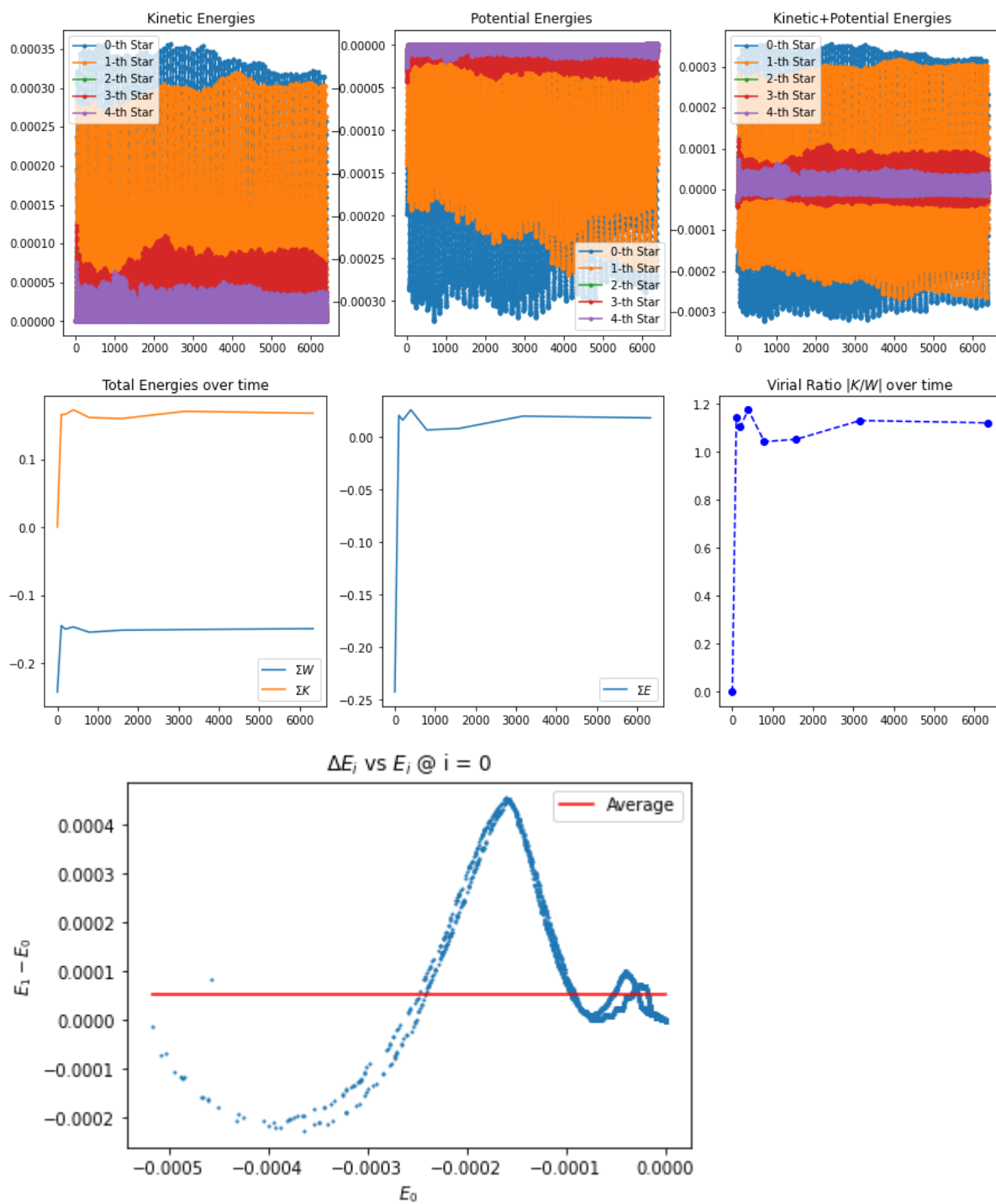


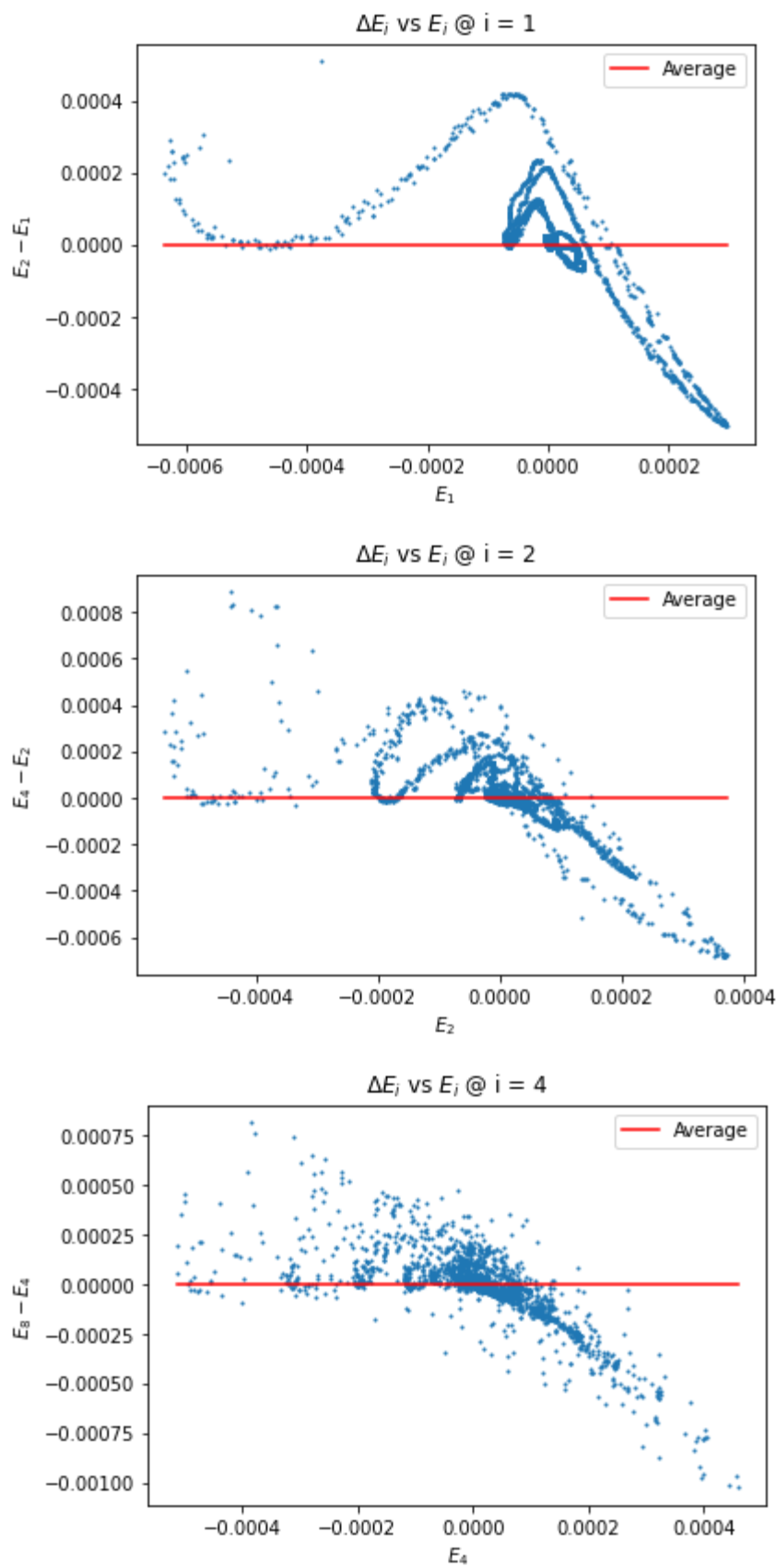


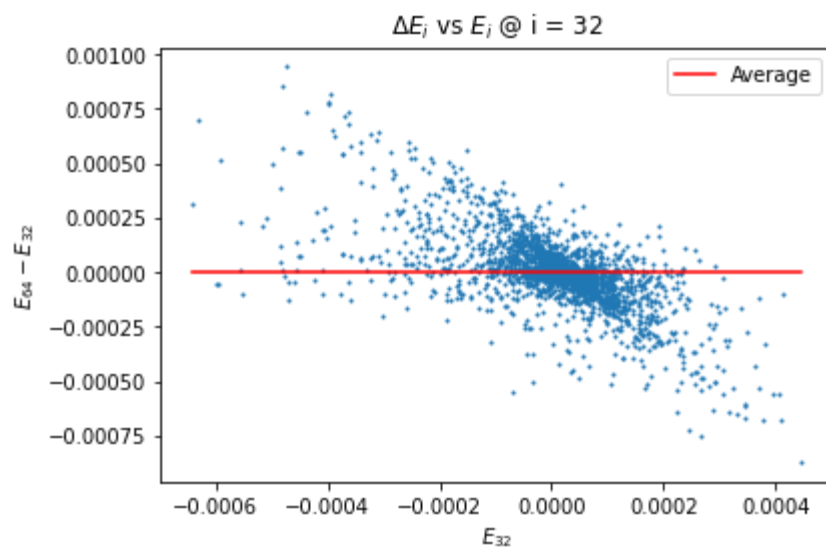
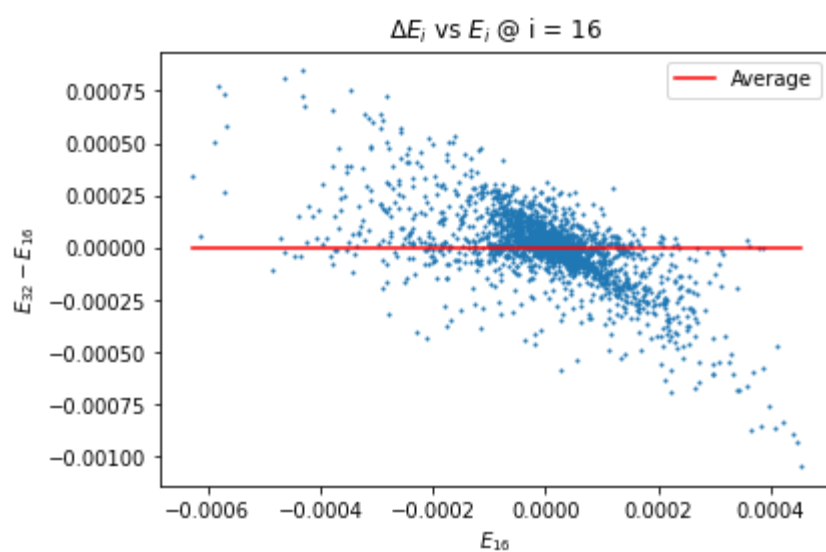
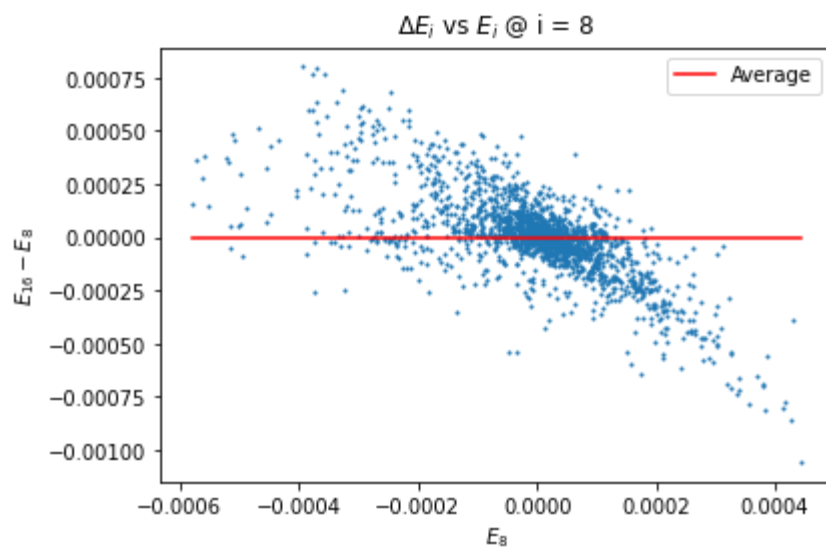
$v_{rms} = 0.5851737606787921$
 $z_{rms} = 0.1541549084183391$
 $K_{avg} = 0.5 * m * v_{rms}^2 = 0.17121416509348014 \quad (m=1)$
 $\Rightarrow 2 * K_{avg} = 0.3424283301869603$
 $W_{avg} = 770.7745420916955$

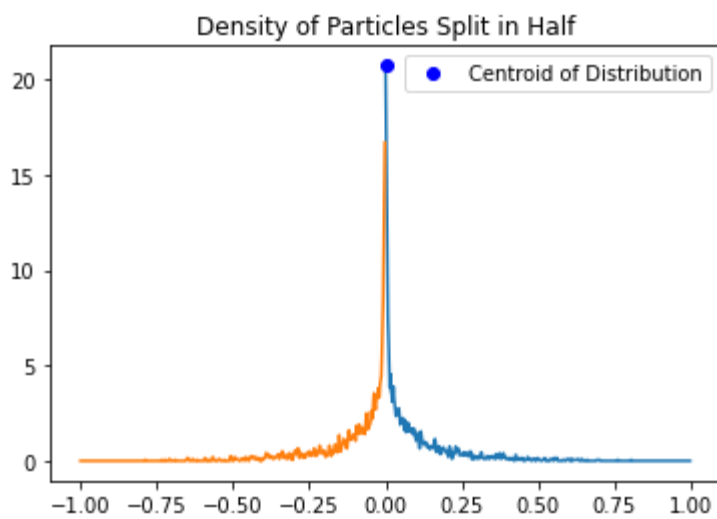
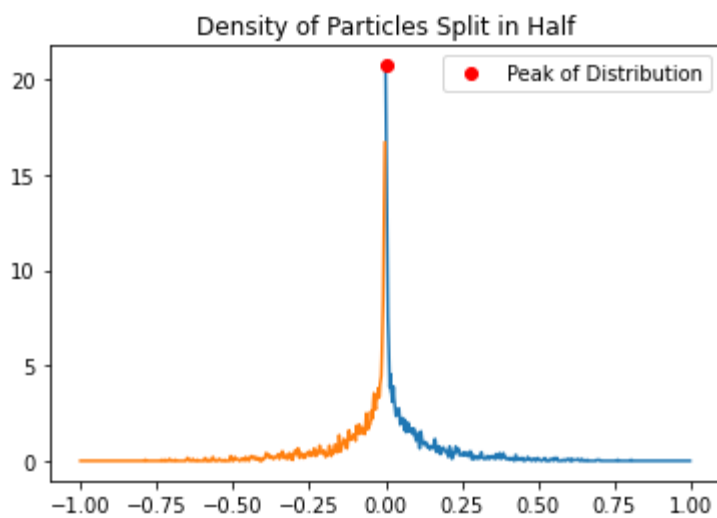
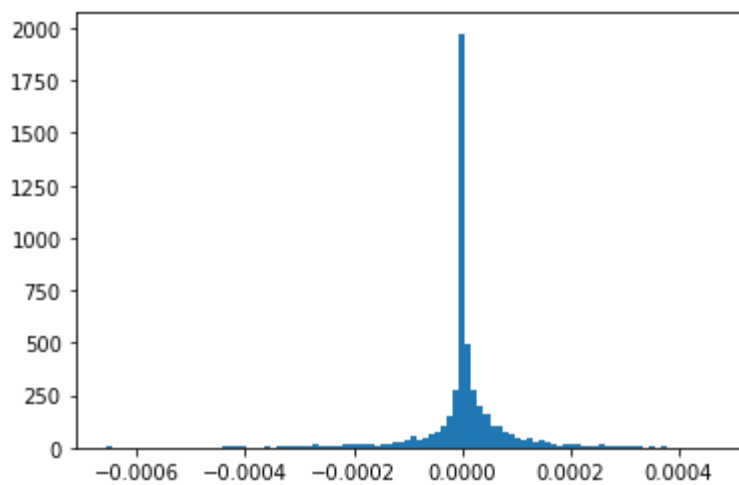
$K_{tot} = 0.1712141650934794$
 $K_{avg} = 3.424283301869588e-05$
 $W_{tot} = -0.14558193639607853$
 $W_{avg} = -2.9116387279215706e-05$

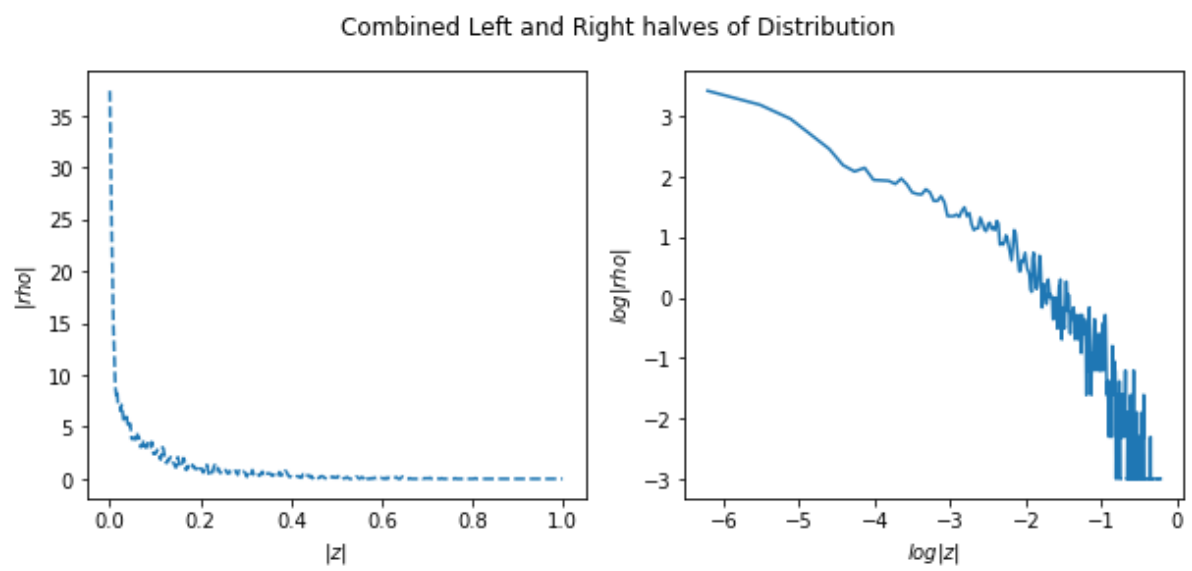




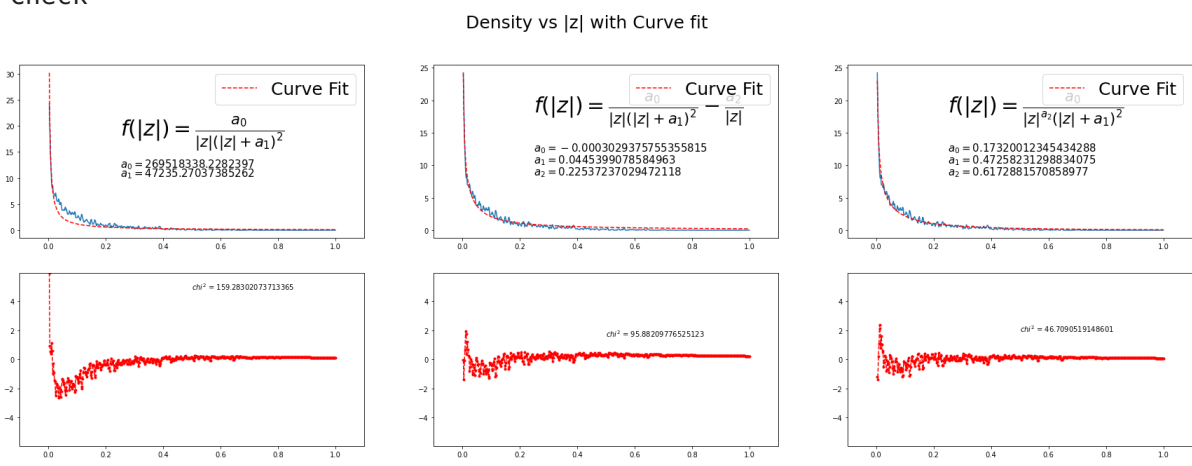




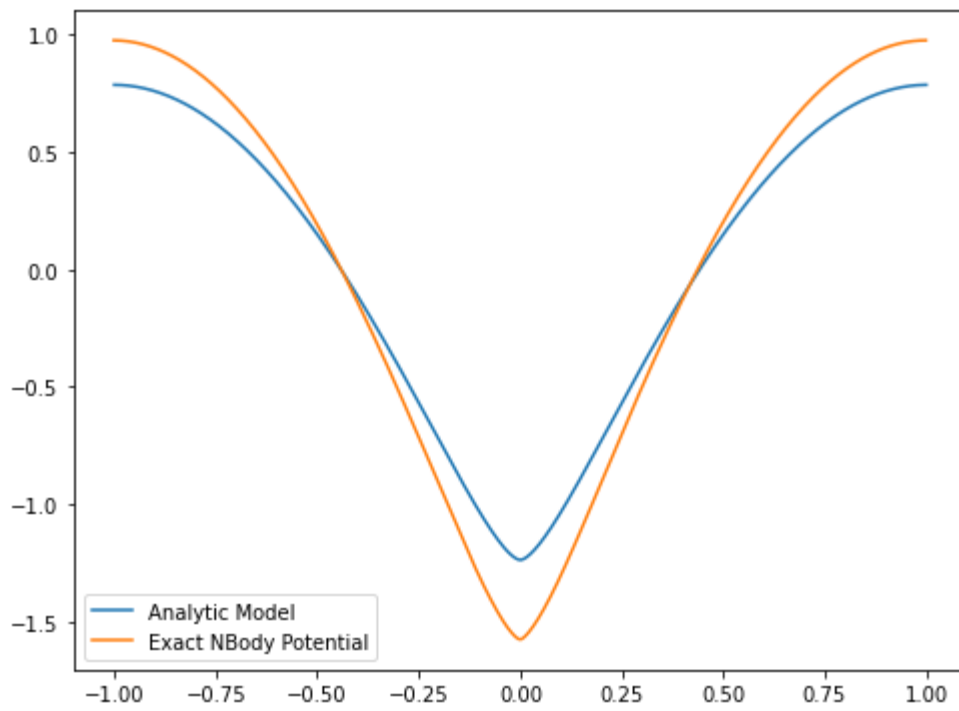




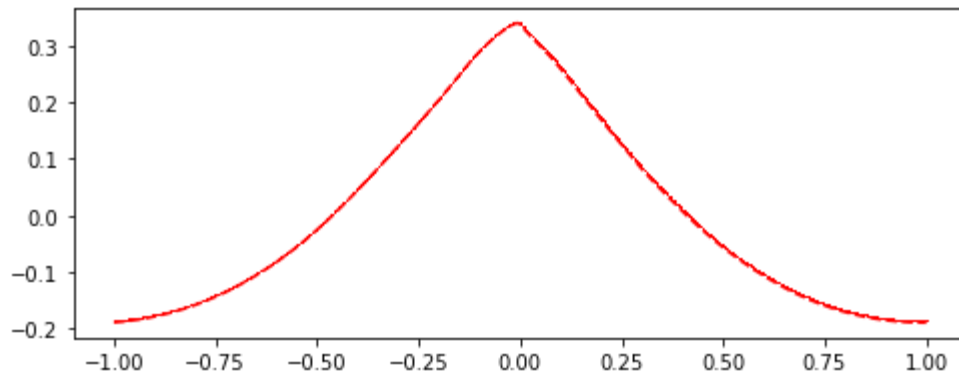
Check
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Gravitational Potential in the Box



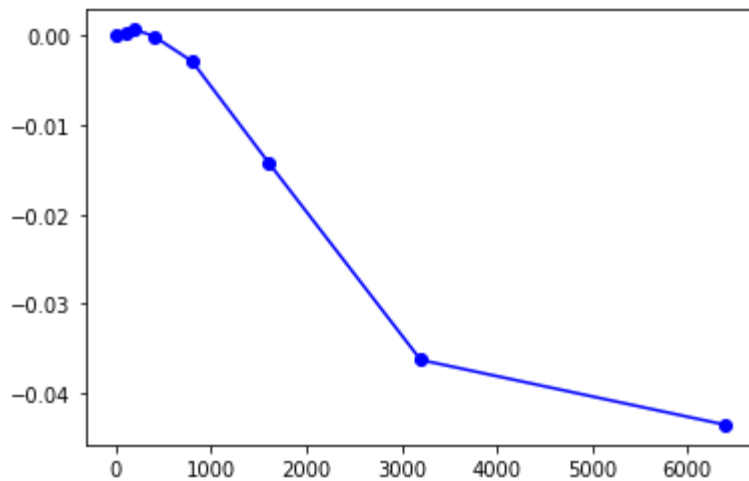
Residuals

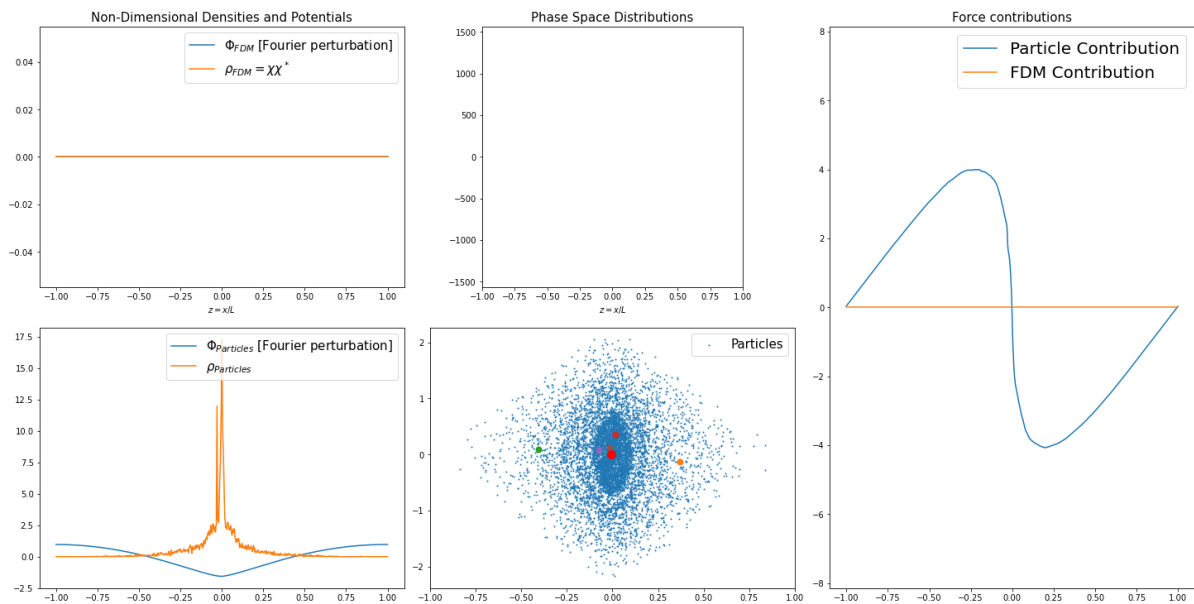


-----New Analysis-----

$r = 1$ $\mu = 1$ Num_bosons = 0 $\sigma = 0.0001$ Num_stars = 10000

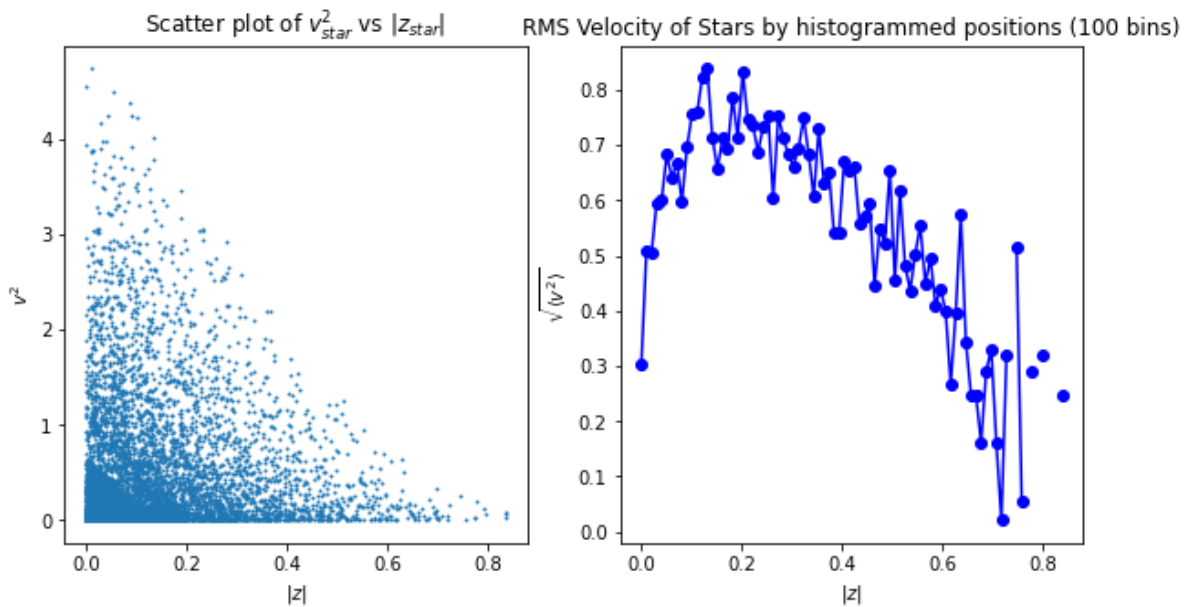
Centroid over time

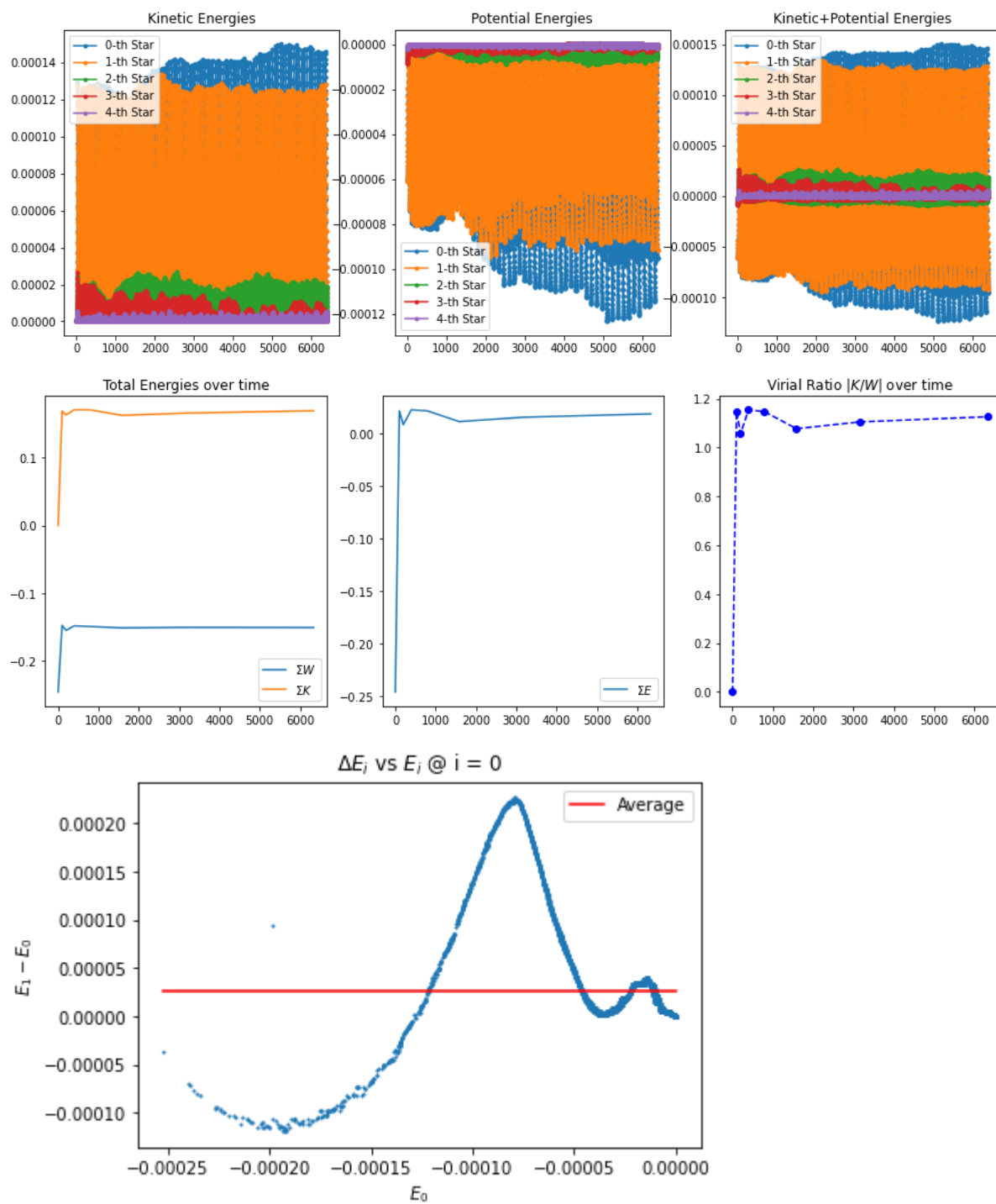


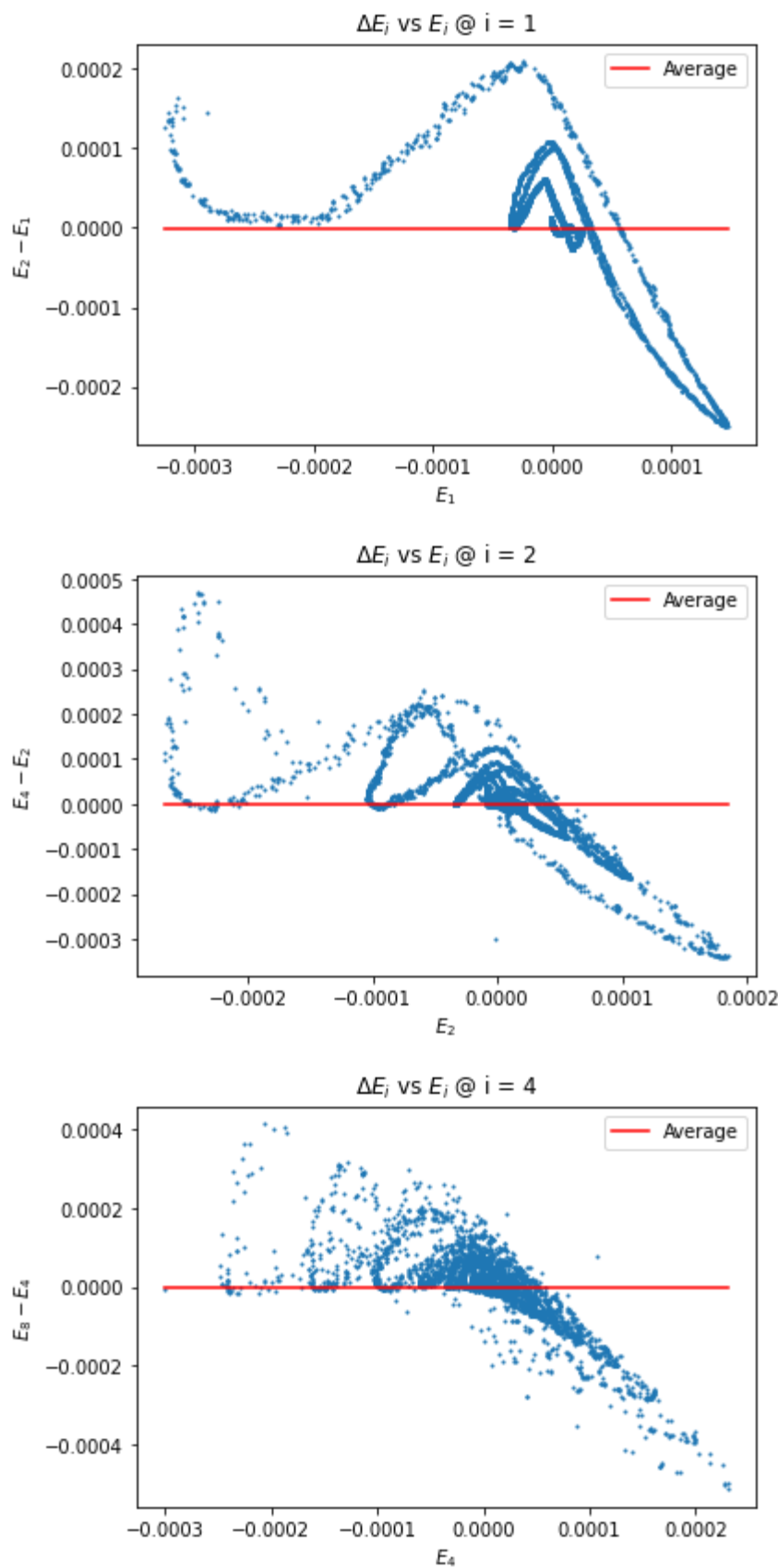


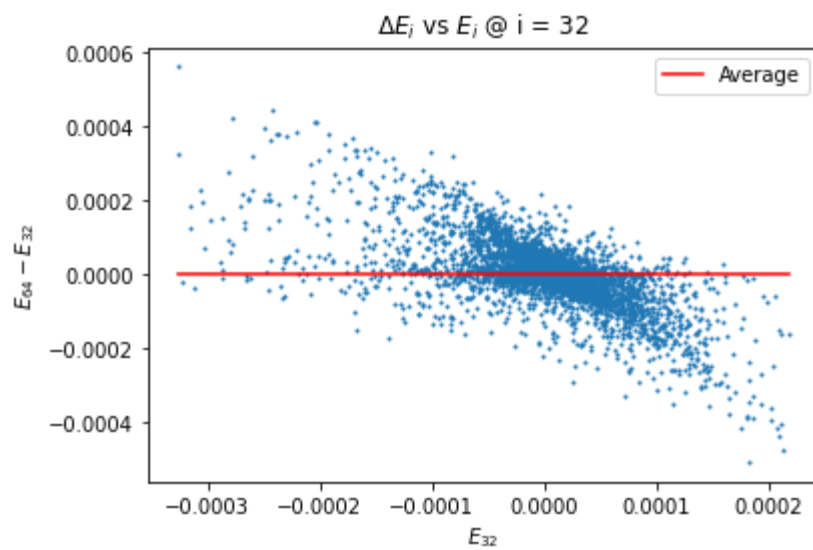
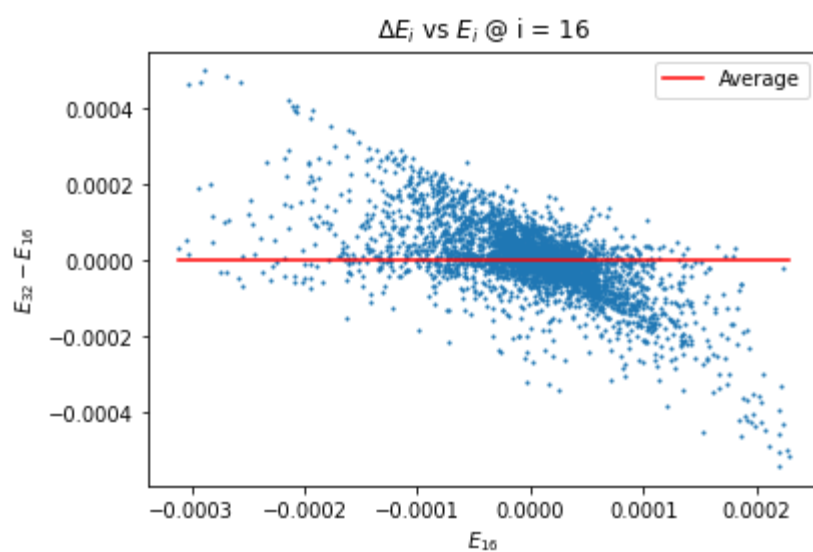
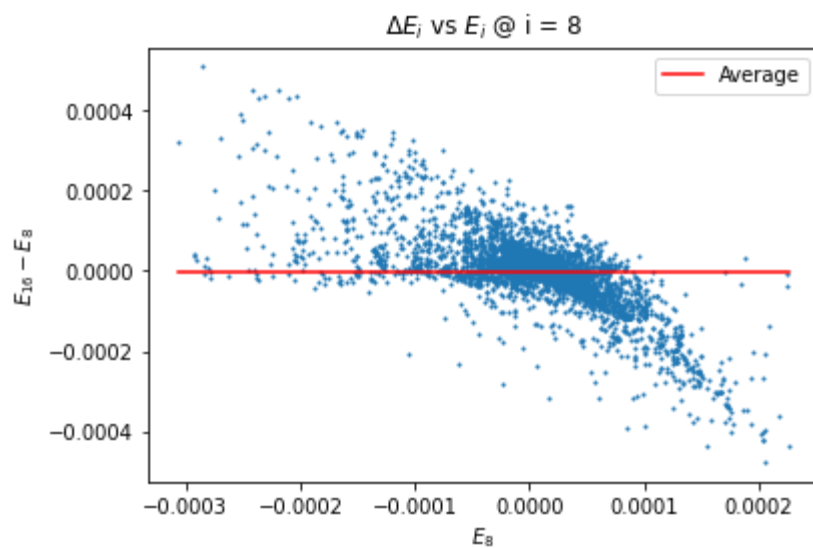
$v_{rms} = 0.581649317774426$
 $z_{rms} = 0.15731951911022882$
 $K_{avg} = 0.5 * m * v_{rms}^2 = 0.16915796443372758 \quad (m=1)$
 $\Rightarrow 2 * K_{avg} = 0.33831592886745515$
 $W_{avg} = 1573.1951911022882$

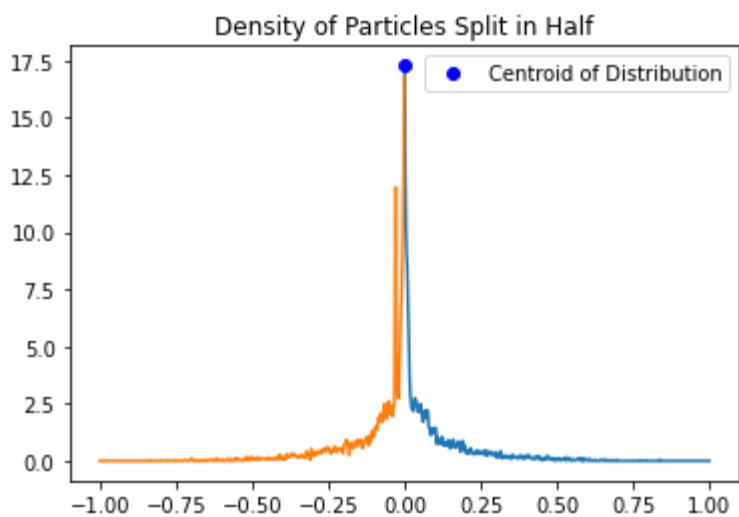
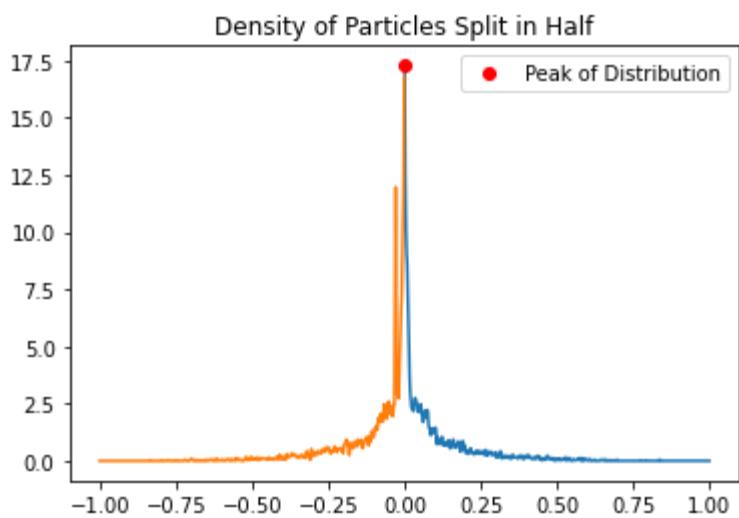
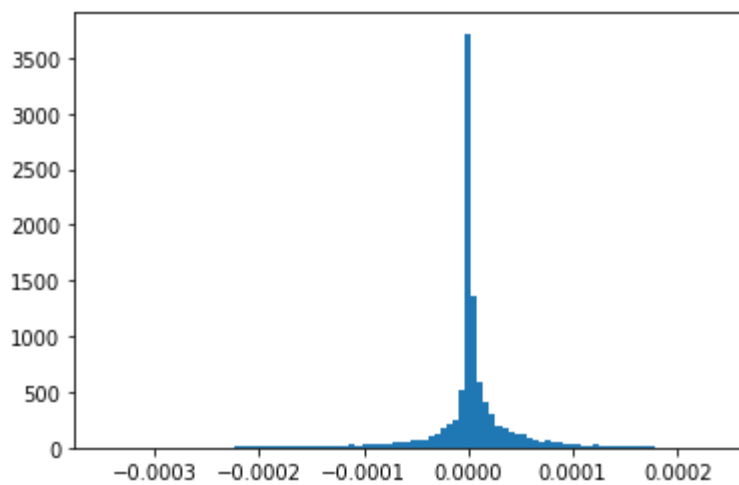
$K_{tot} = 0.1691579644337266$
 $K_{avg} = 1.691579644337266e-05$
 $W_{tot} = -0.15082252821733694$
 $W_{avg} = -1.5082252821733694e-05$

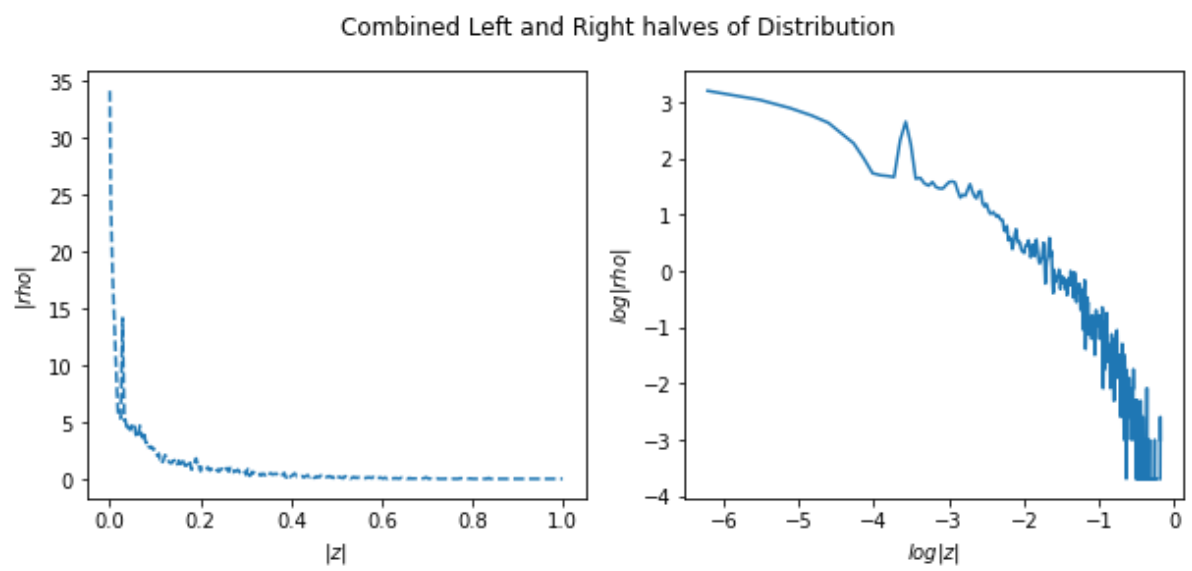




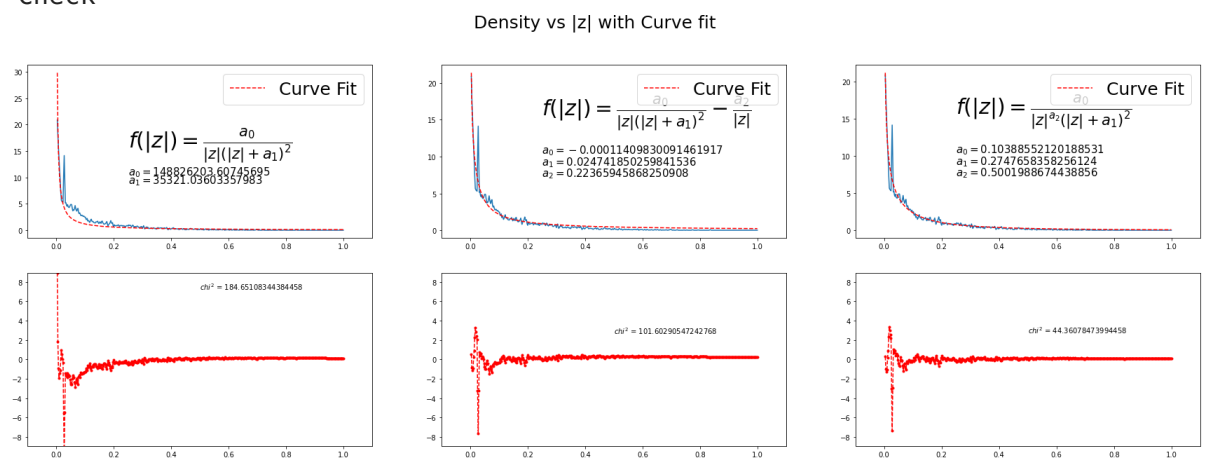




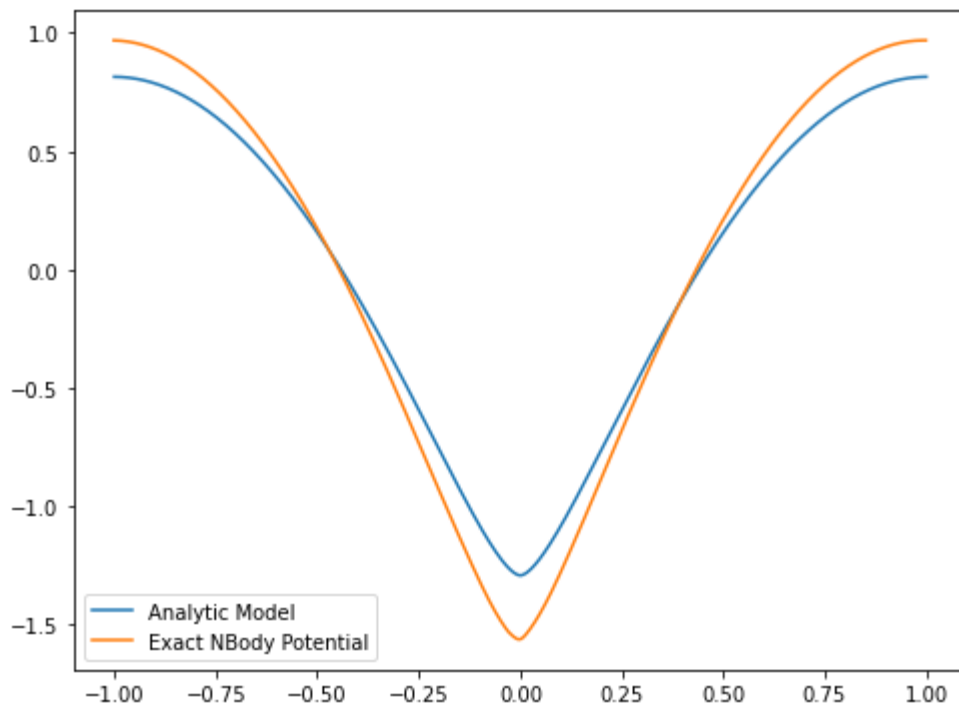




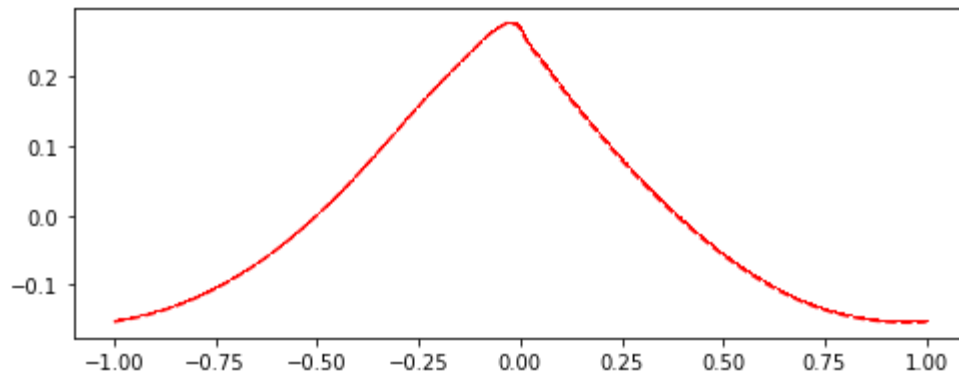
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Gravitational Potential in the Box



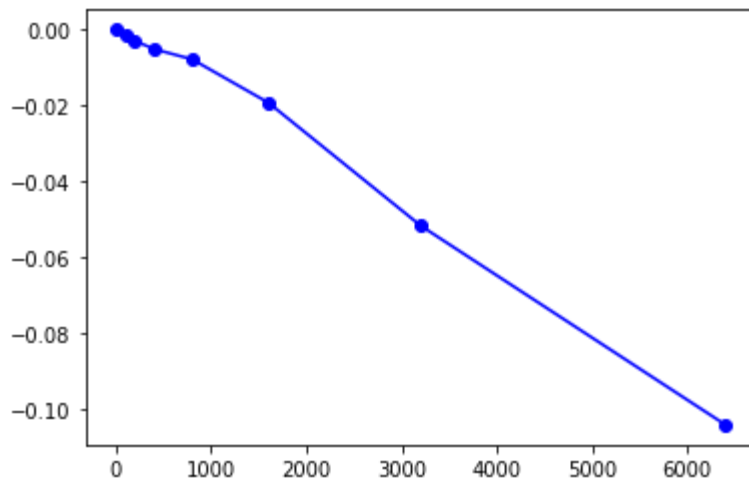
Residuals

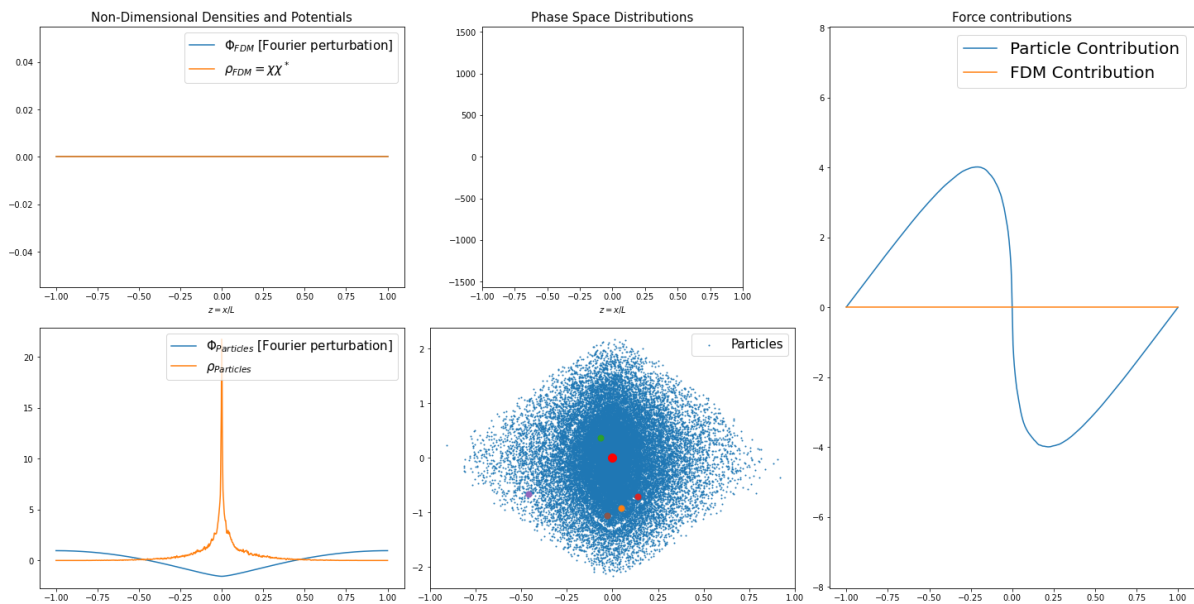


-----New Analysis-----

r = 1 mu = 1 Num_bosons = 0 sigma = 2e-05 Num_stars = 50000

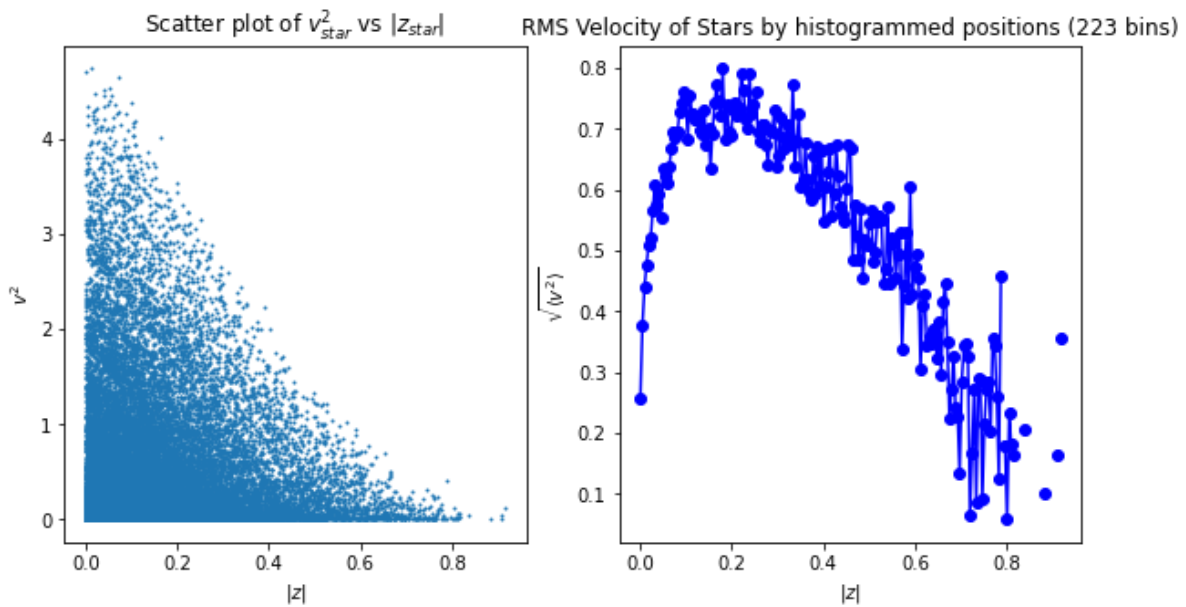
Centroid over time

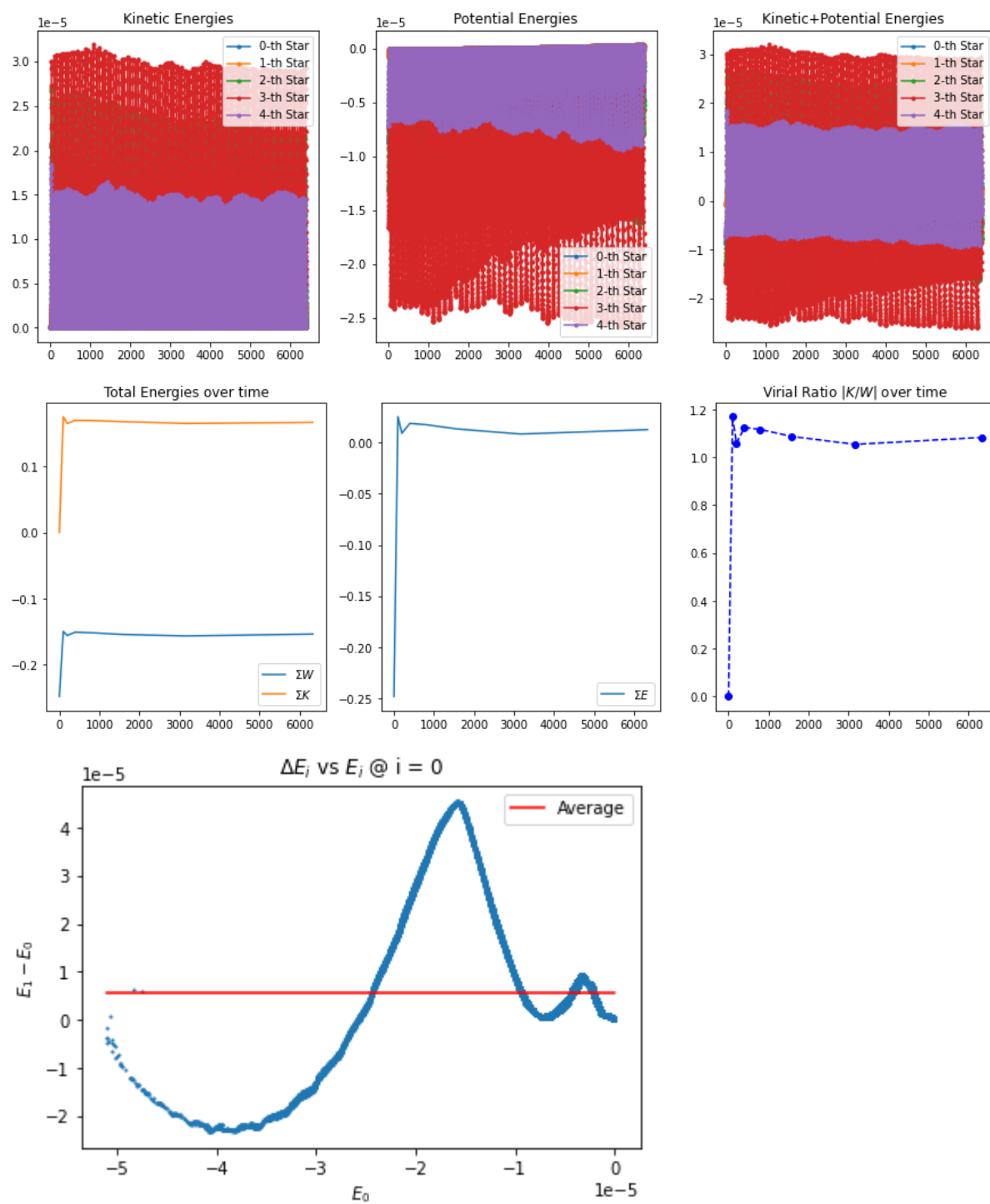


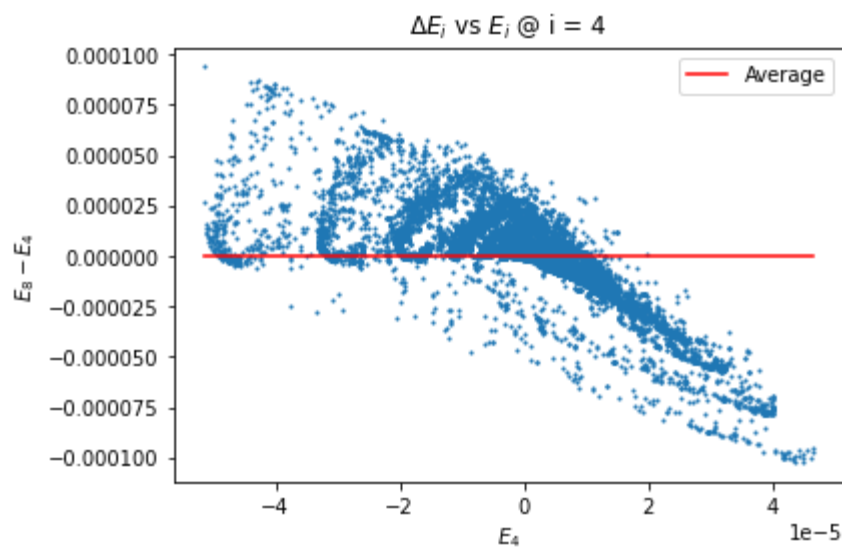
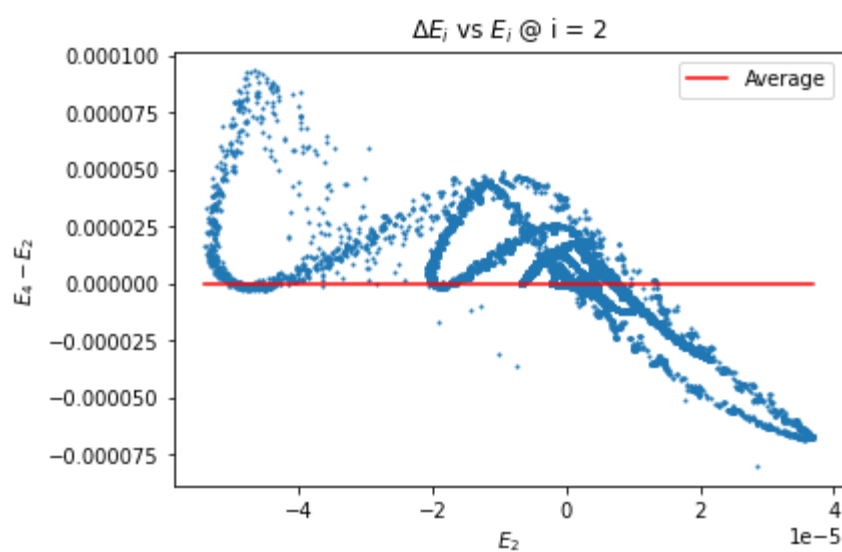
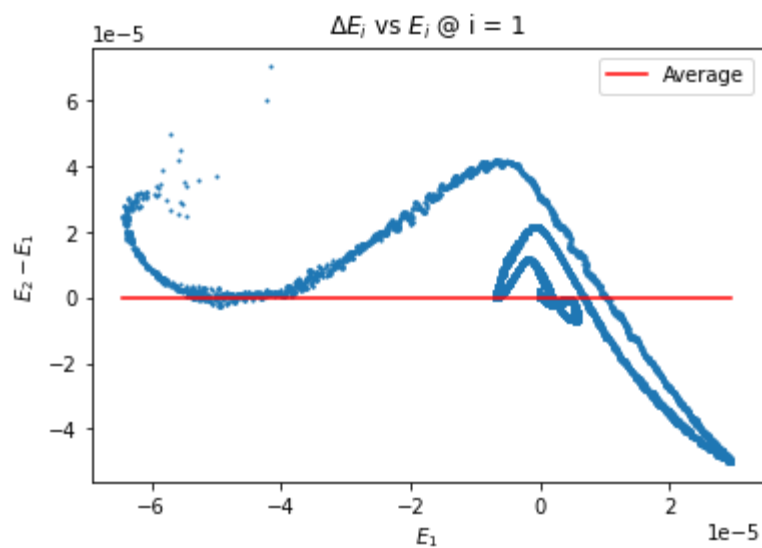


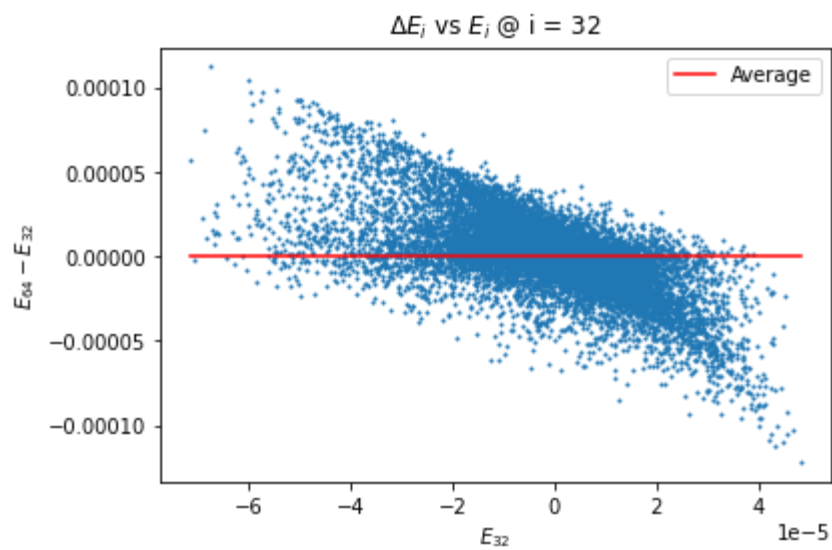
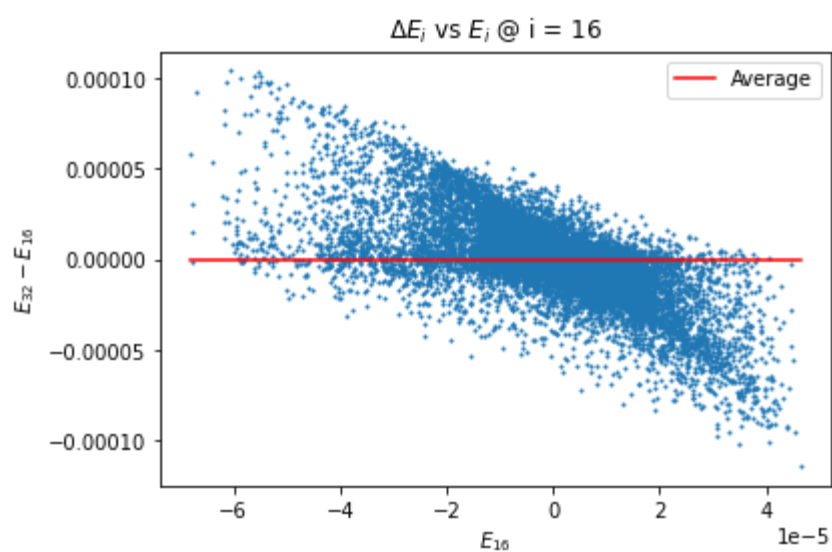
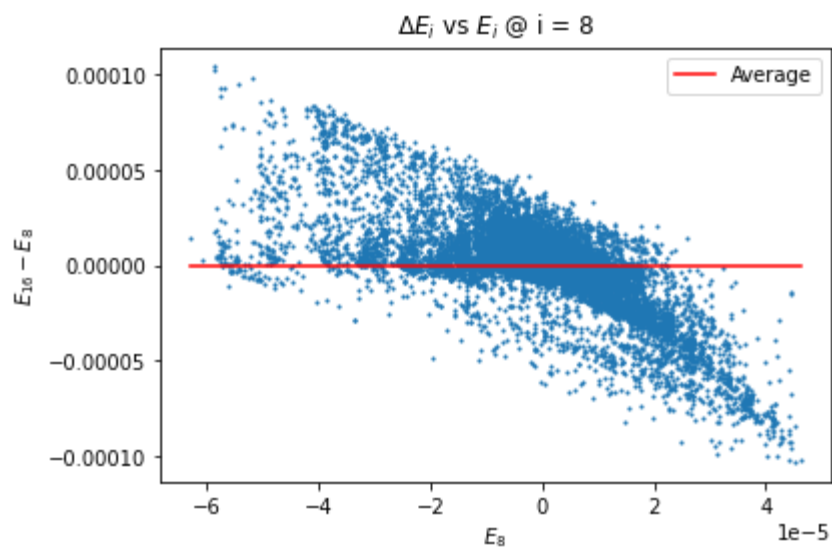
$v_{rms} = 0.5731593297761615$
 $z_{rms} = 0.15992388602013577$
 $K_{avg} = 0.5 * m * v_{rms}^2 = 0.1642558086547293 \quad (m=1)$
 $\Rightarrow 2 * K_{avg} = 0.3285116173094586$
 $W_{avg} = 7996.194301006788$

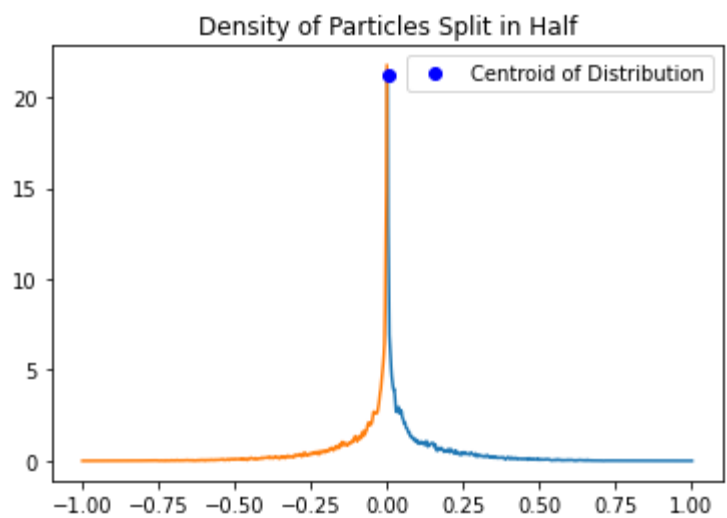
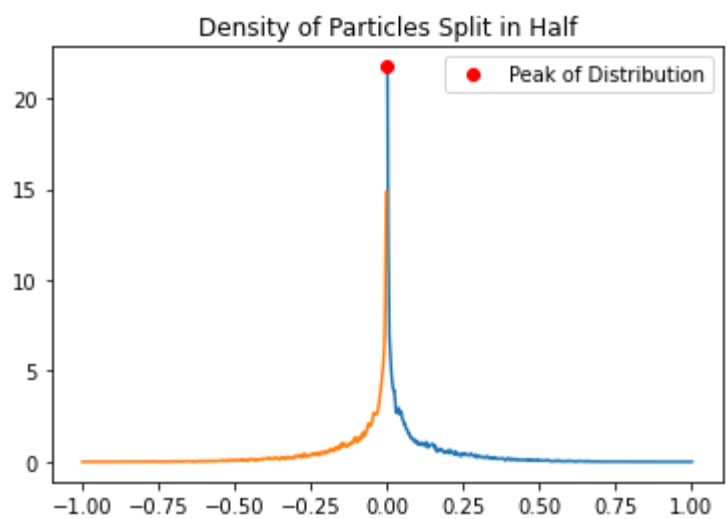
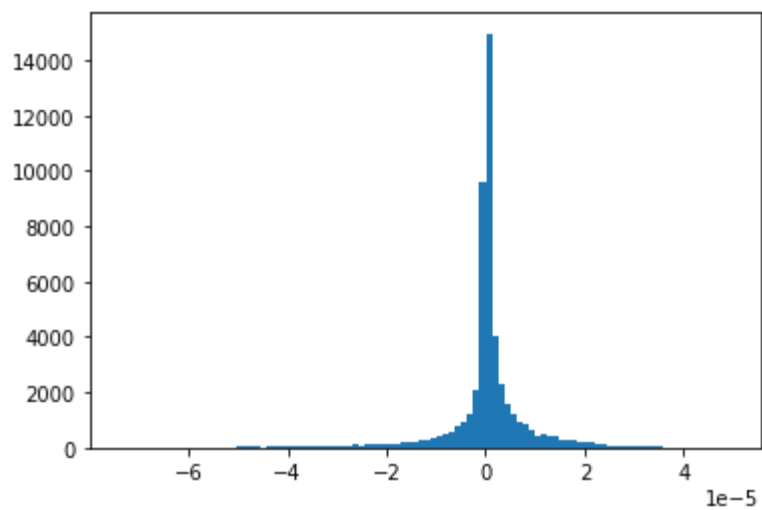
$K_{tot} = 0.16425580865473047$
 $K_{avg} = 3.285116173094609e-06$
 $W_{tot} = -0.1556381101749108$
 $W_{avg} = -3.112762203498216e-06$

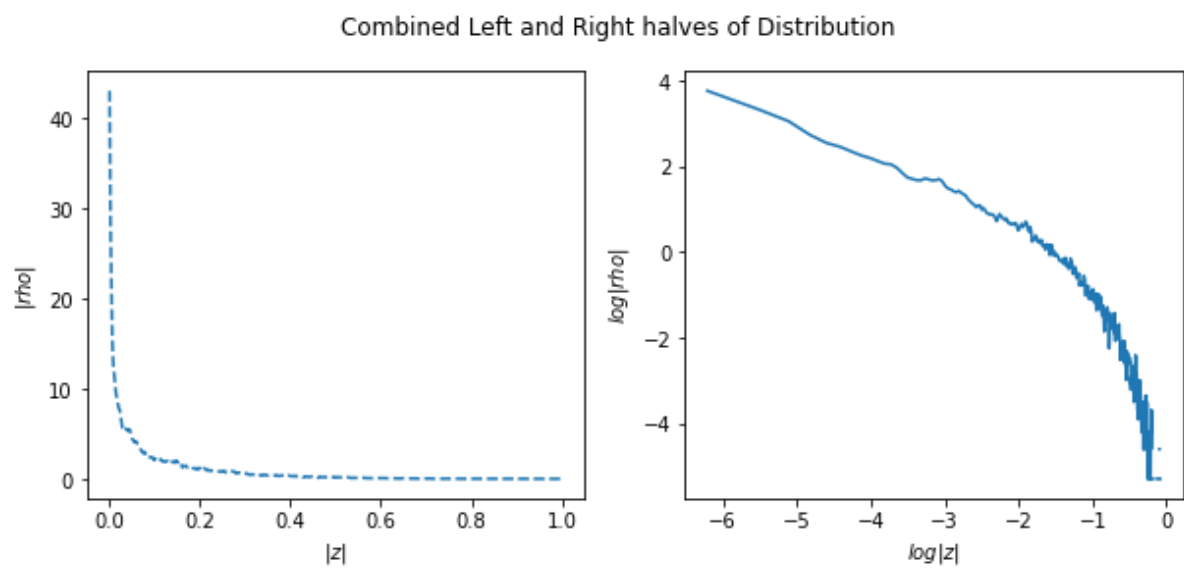




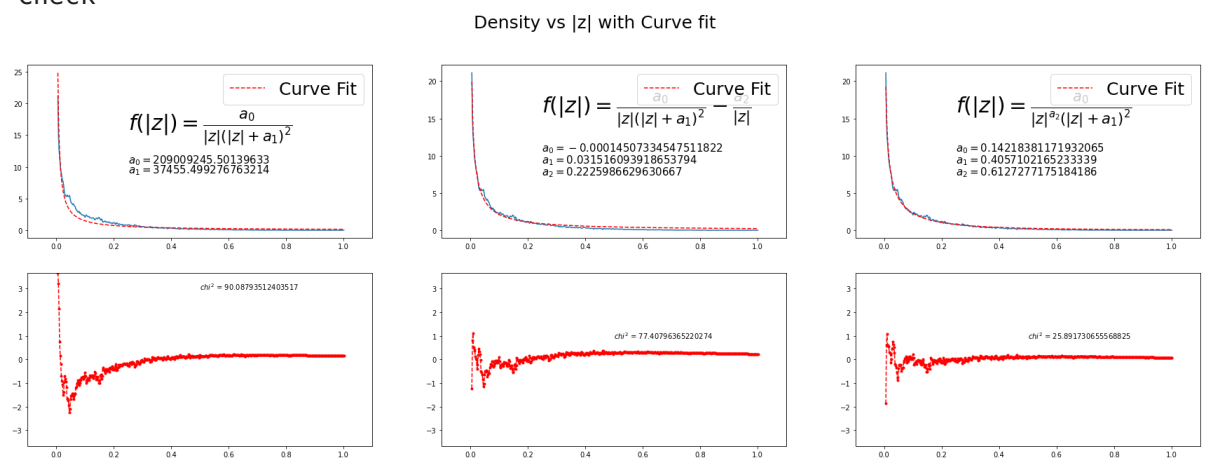




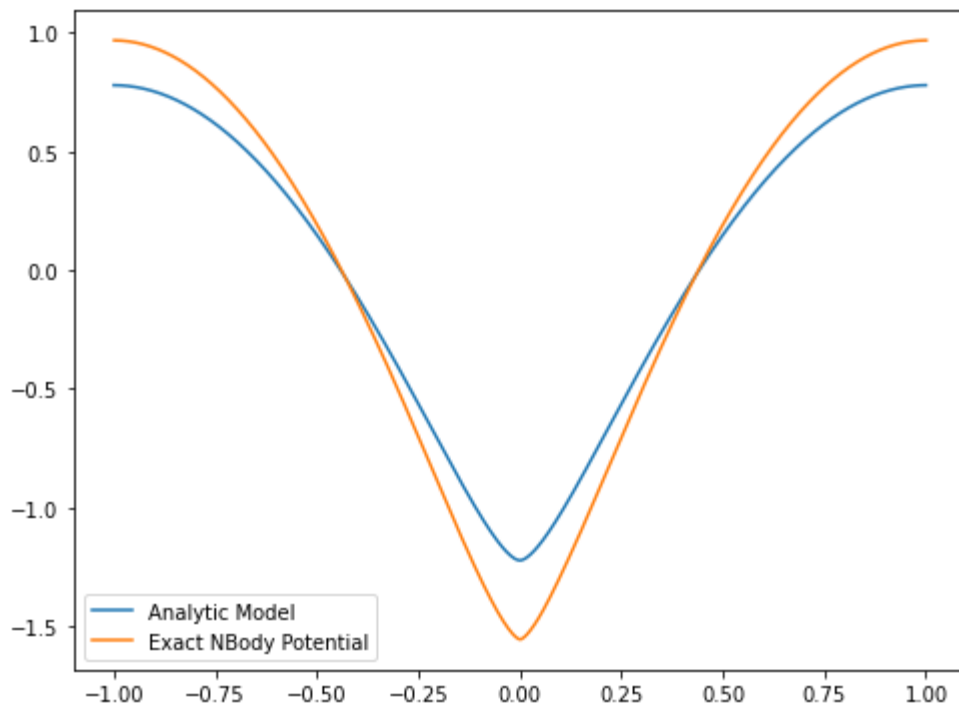




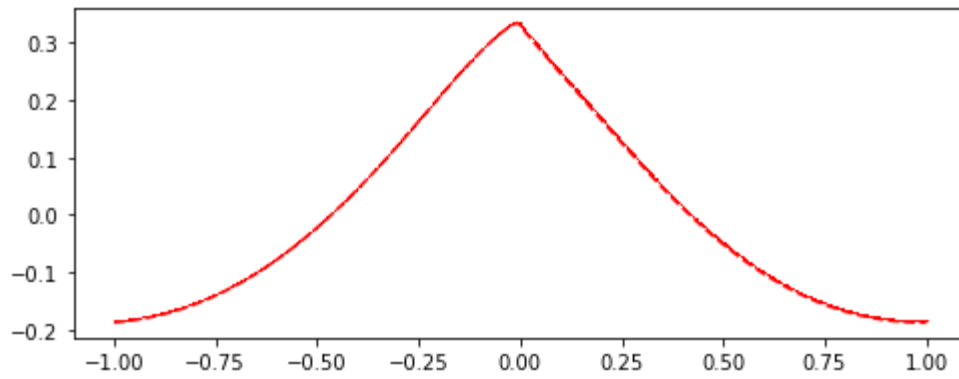
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Gravitational Potential in the Box



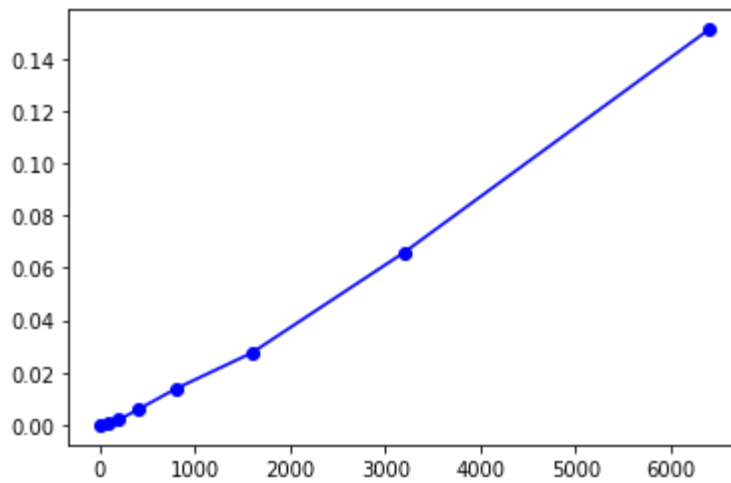
Residuals

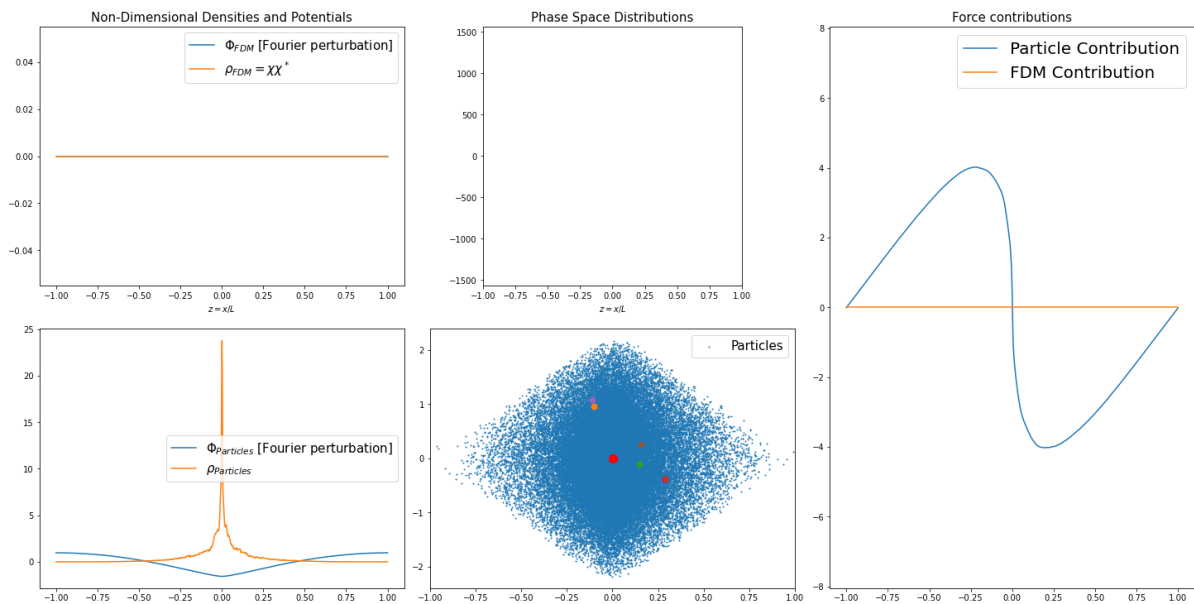


-----New Analysis-----

r = 1 mu = 1 Num_bosons = 0 sigma = 1e-05 Num_stars = 100000

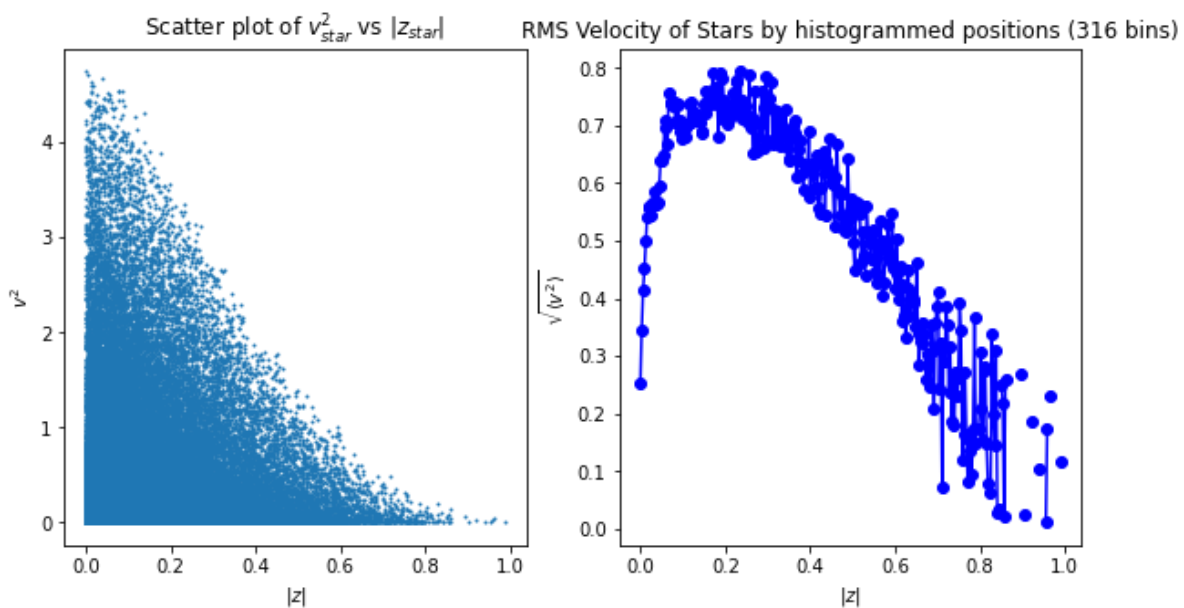
Centroid over time

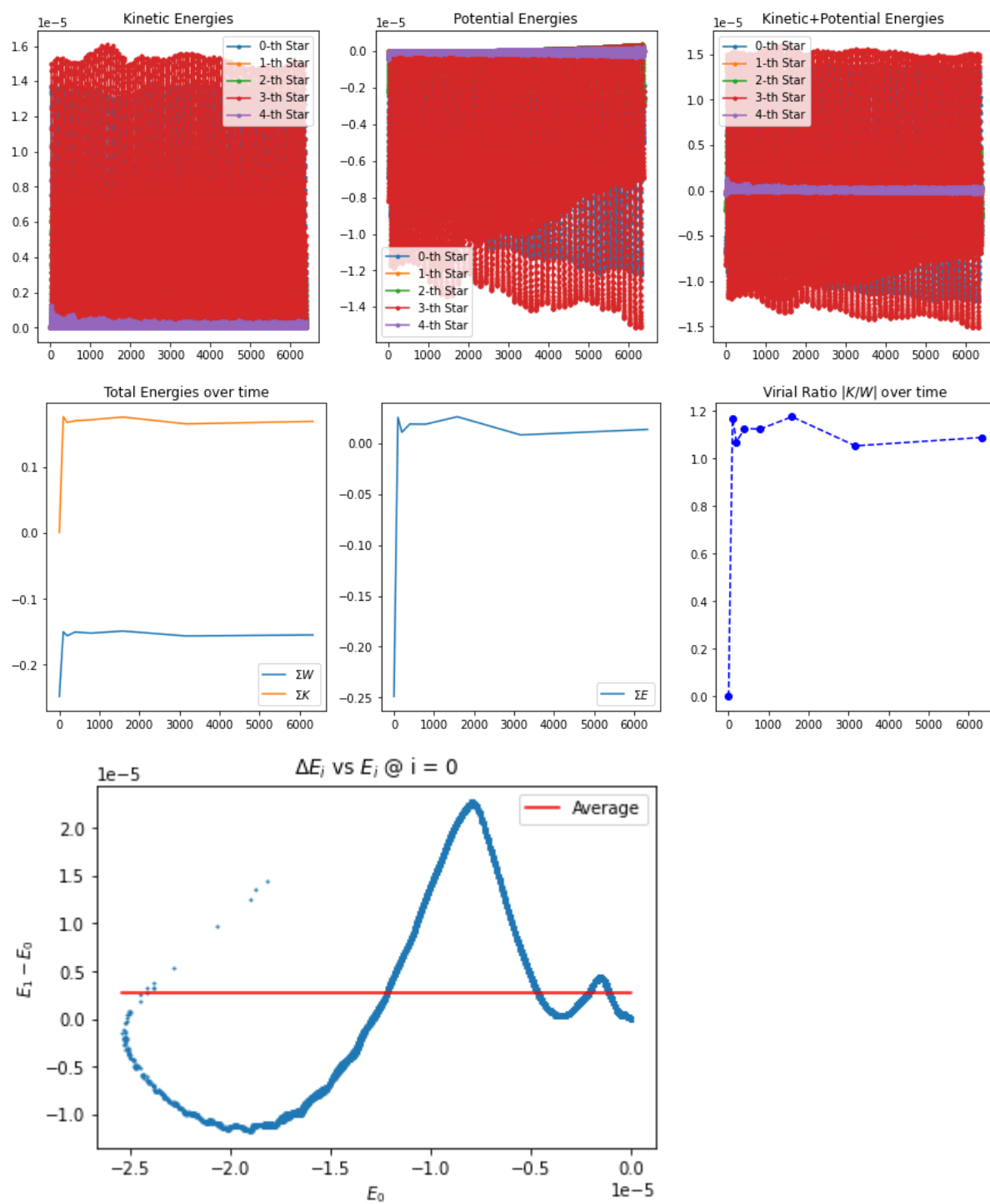


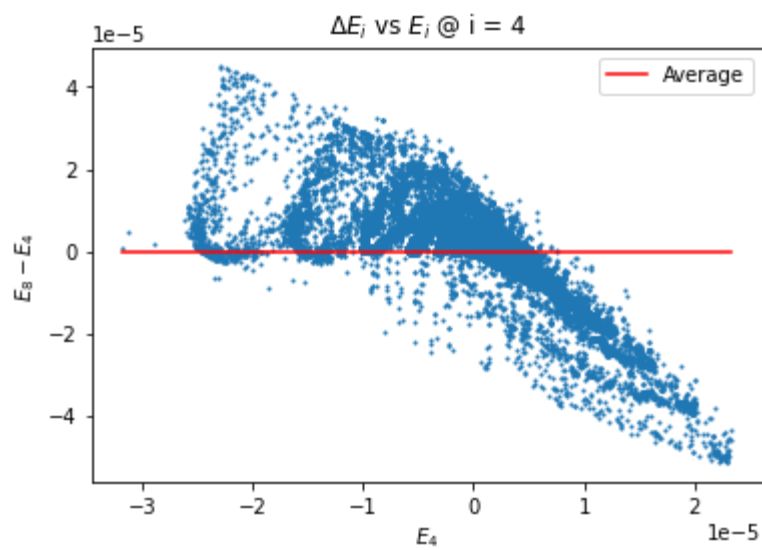
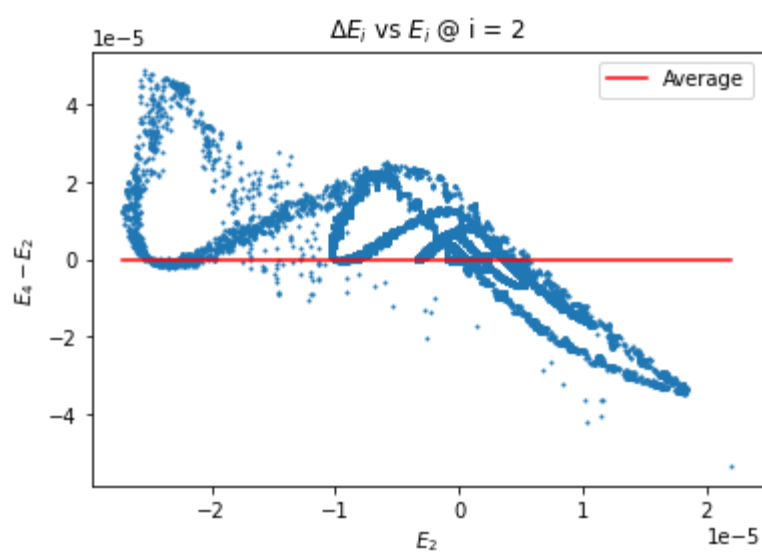
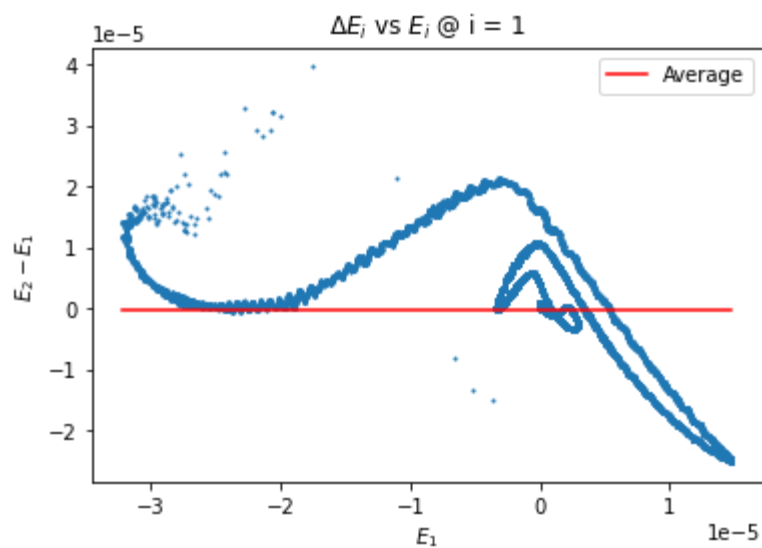


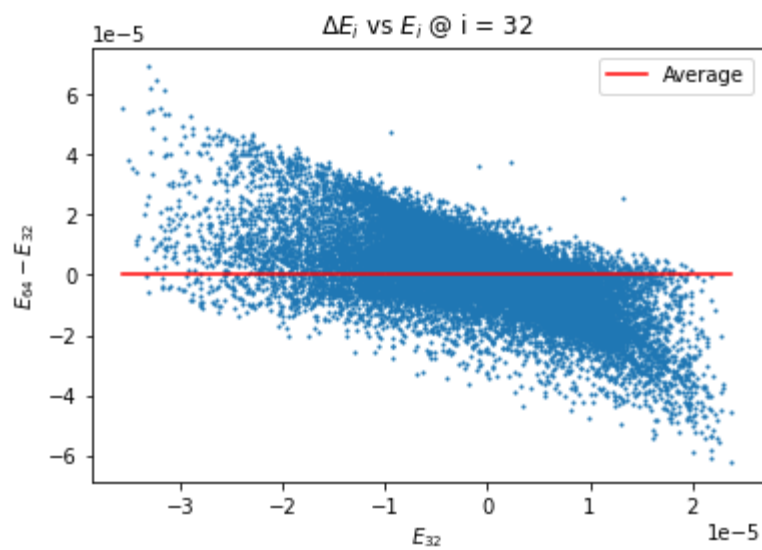
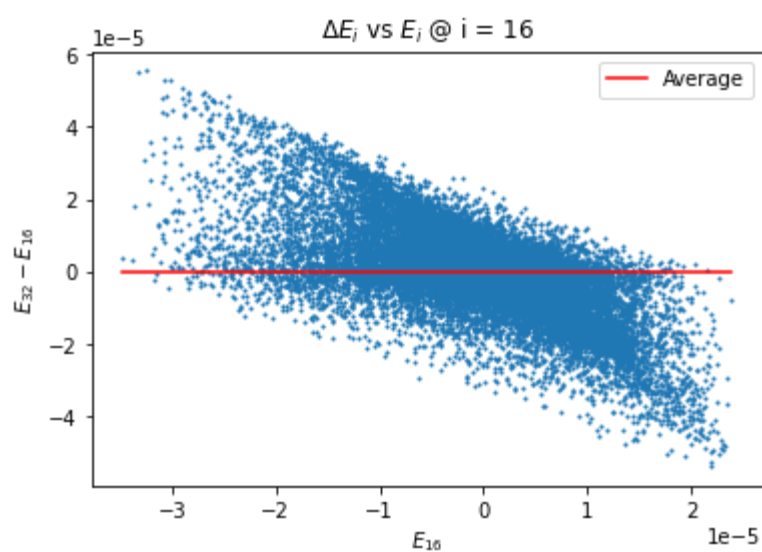
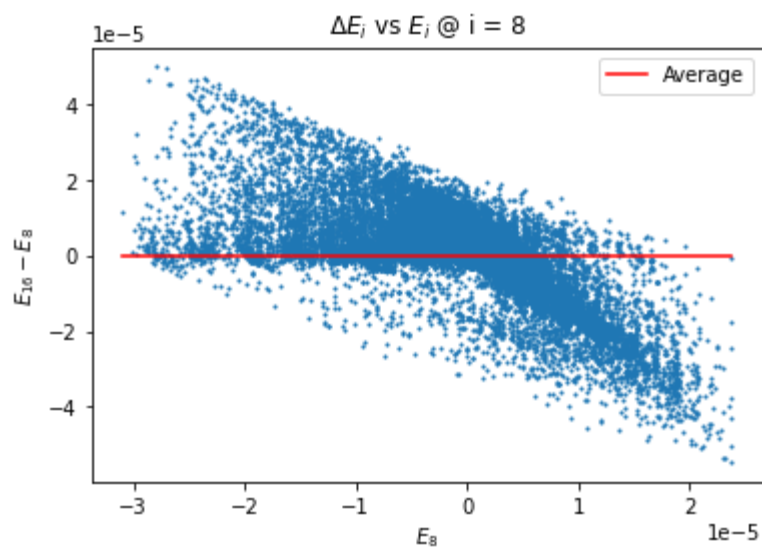
$v_{rms} = 0.5840606962146646$
 $z_{rms} = 0.1588461256252678$
 $K_{avg} = 0.5 * m * v_{rms}^2 = 0.17056344843137938 \quad (m=1)$
 $\Rightarrow 2 * K_{avg} = 0.34112689686275877$
 $W_{avg} = 15884.612562526778$

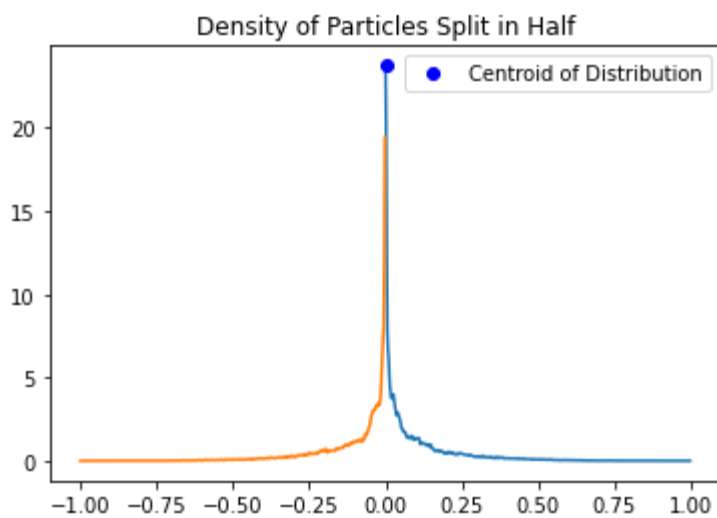
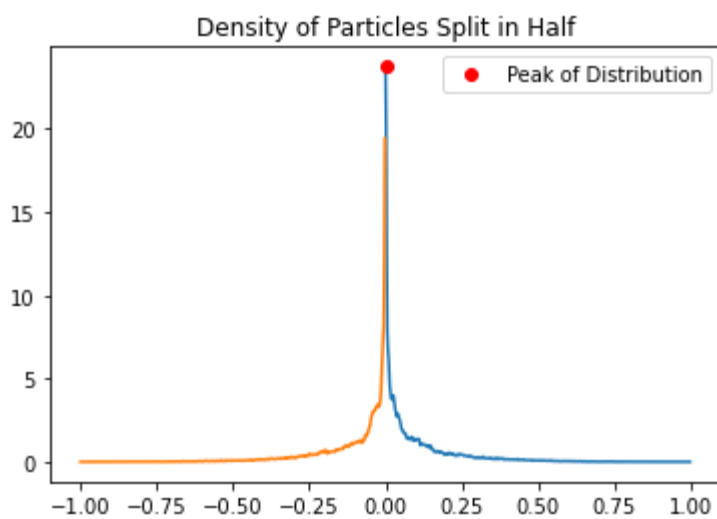
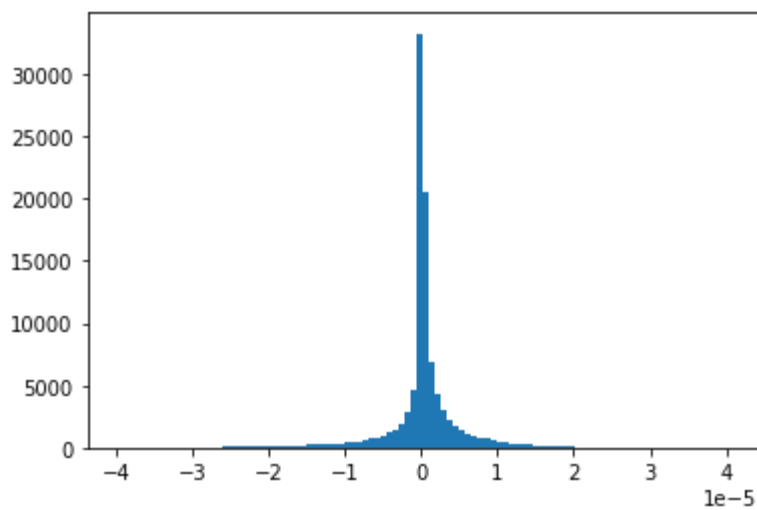
$K_{tot} = 0.17056344843137822$
 $K_{avg} = 1.7056344843137823e-06$
 $W_{tot} = -0.1534179521919812$
 $W_{avg} = -1.5341795219198121e-06$

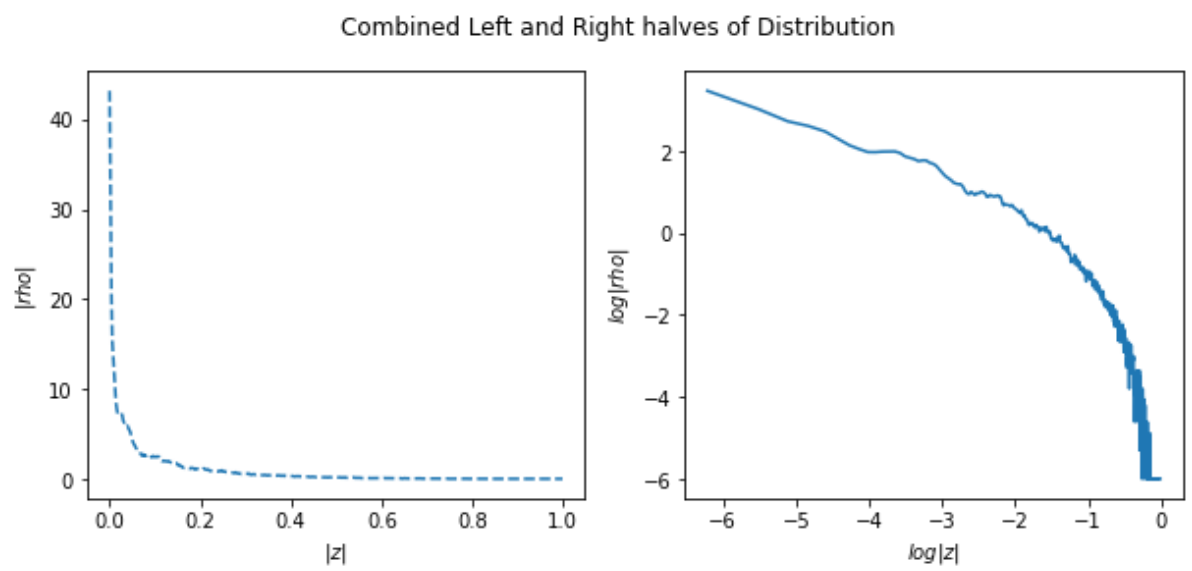




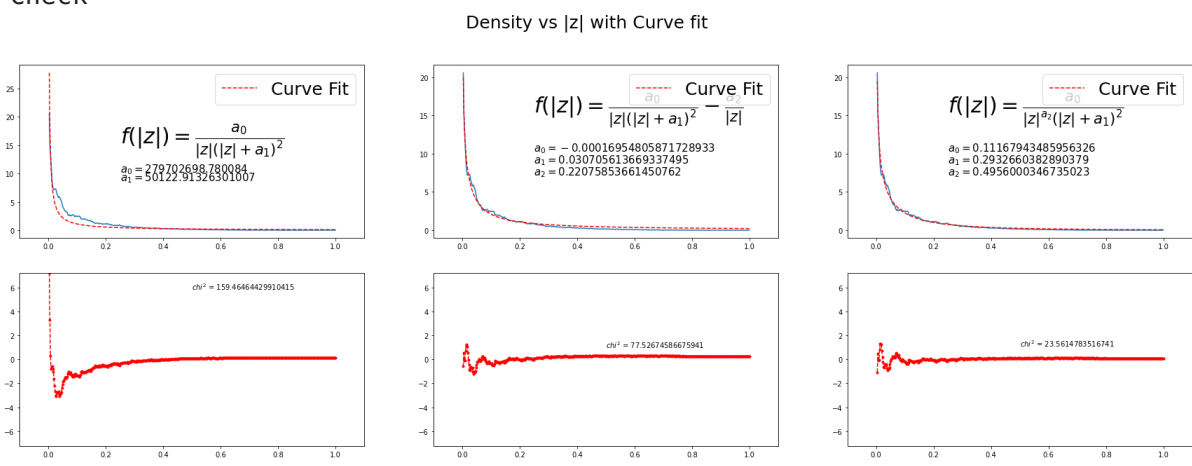


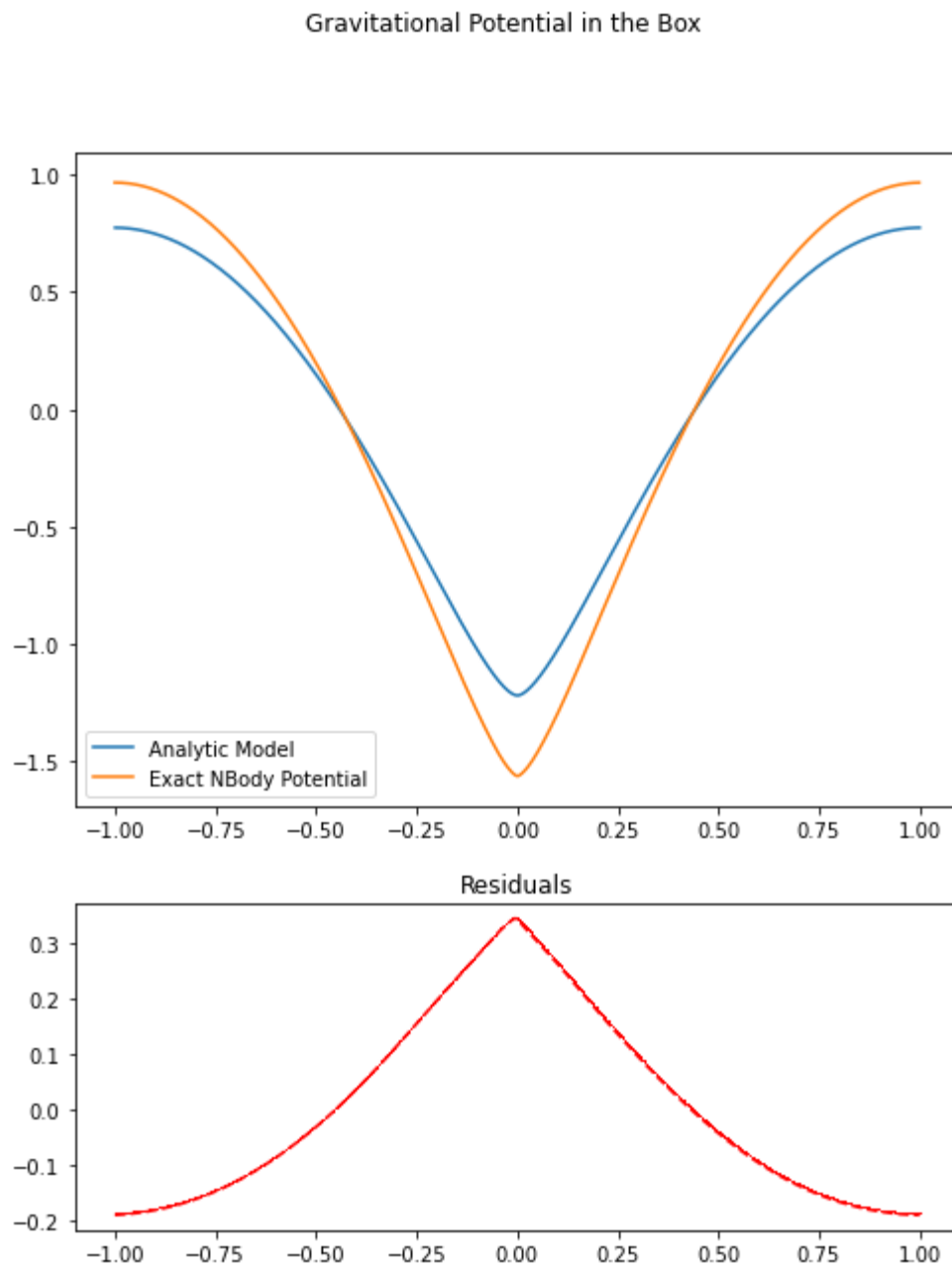






Check
Check
Check



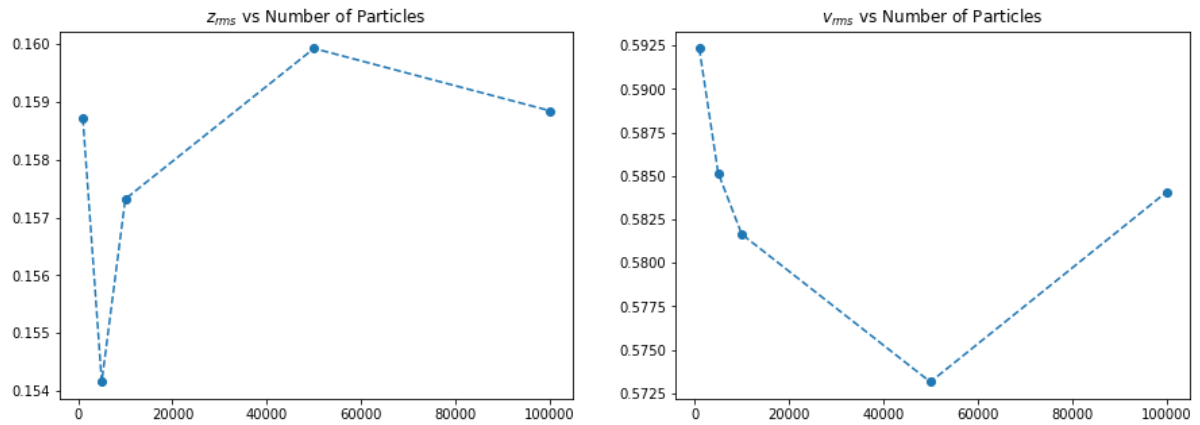


```
In [ ]: import matplotlib.pyplot as plt
v_rms_s = [0.5923277051957598,0.5851737606787921,0.581649317774426,0.5731593
z_rms_s = [0.15871675883738892,0.1541549084183391,0.15731951911022882,0.159
Num_p_s = [1000,5000,10000,50000,100000]

fig,ax=plt.subplots(1,2,figsize= (15,5))
ax[0].plot(Num_p_s,z_rms_s,'o--')
ax[0].set_title("$z_{rms}$ vs Number of Particles")
#ax[0].set_ylim(0,0.5)

ax[1].plot(Num_p_s,v_rms_s,'o--')
ax[1].set_title("$v_{rms}$ vs Number of Particles")

plt.show()
```



```
In [ ]: import numpy as np
import matplotlib.pyplot as plt

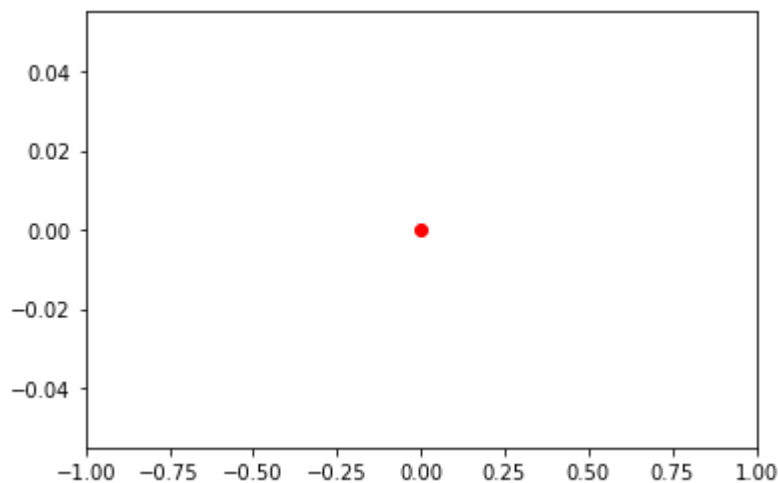
My_Package_PATH = "/home/boris/Documents/Research/Coding"
import sys
sys.path.insert(1, My_Package_PATH)
import OneD.NBody as NB

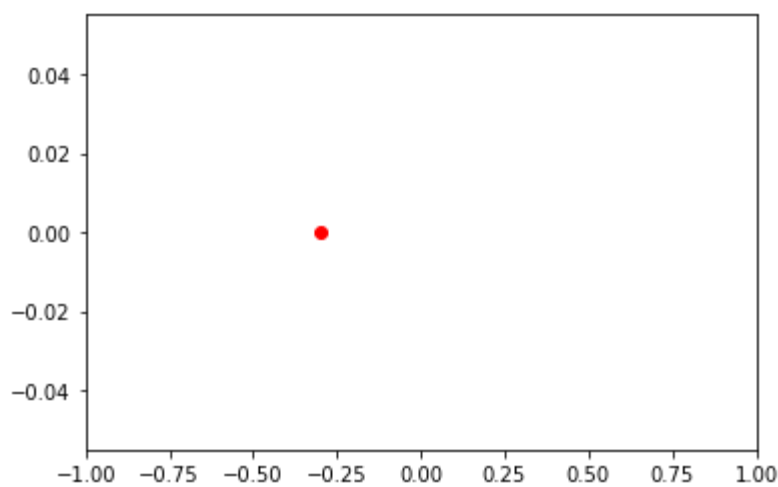
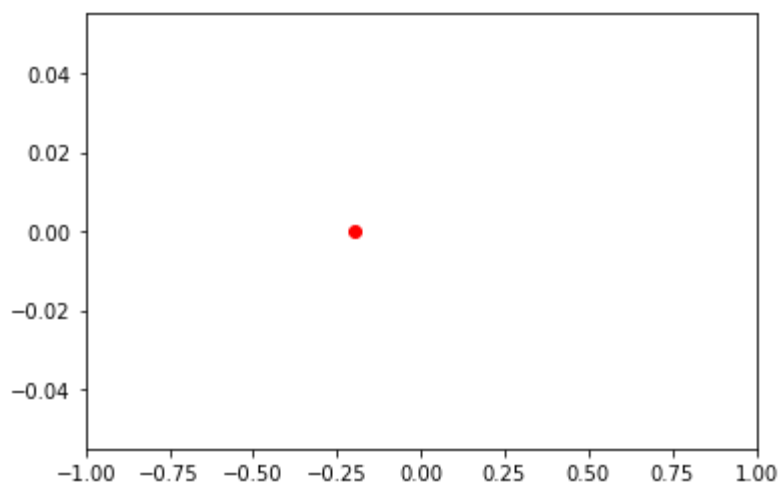
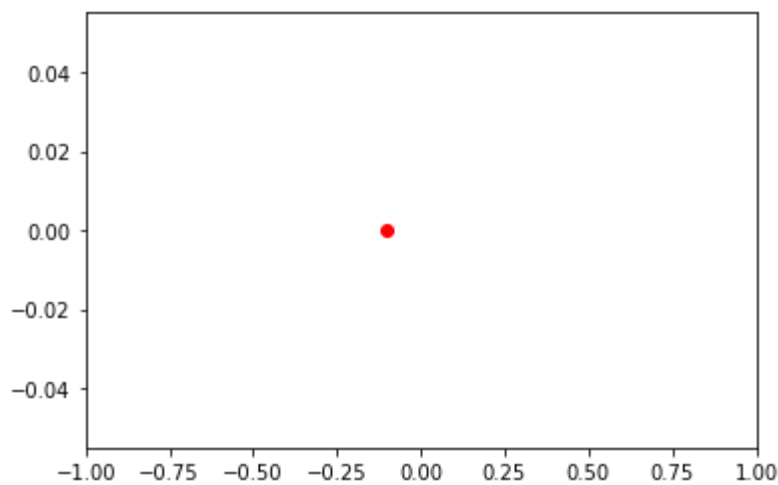
z = np.linspace(-1,1)

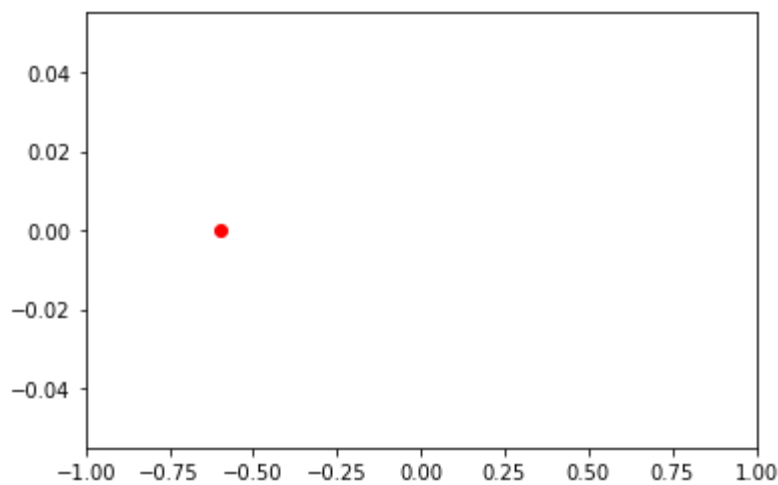
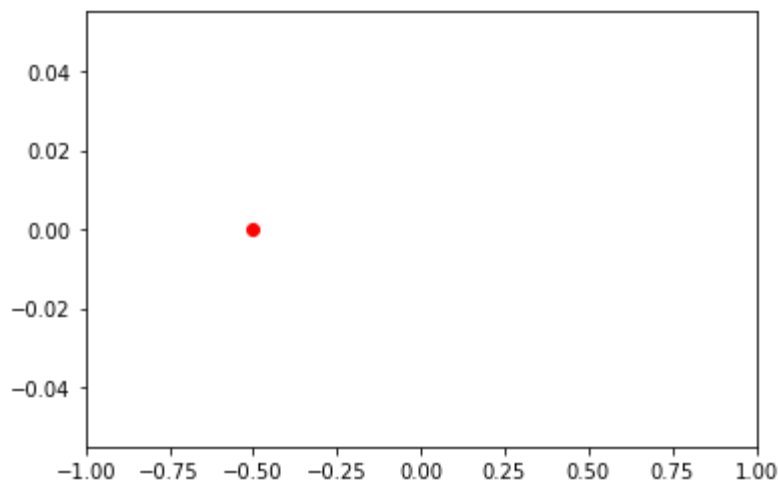
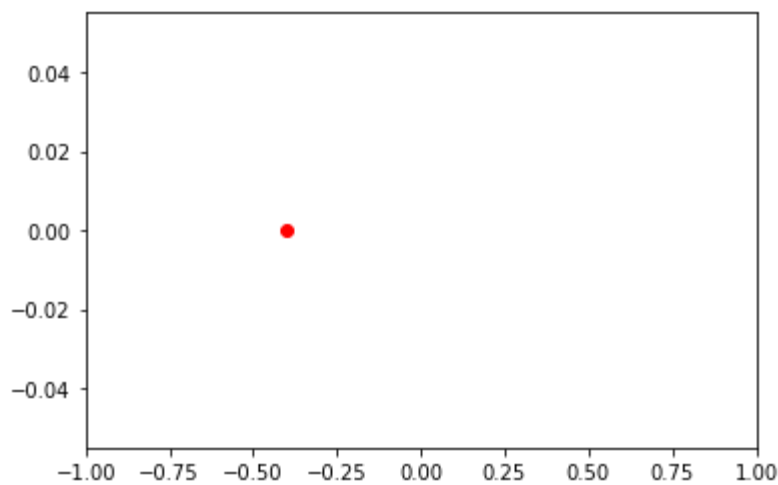
x = 0
v = 1
star = NB.star(0,1,x,v)

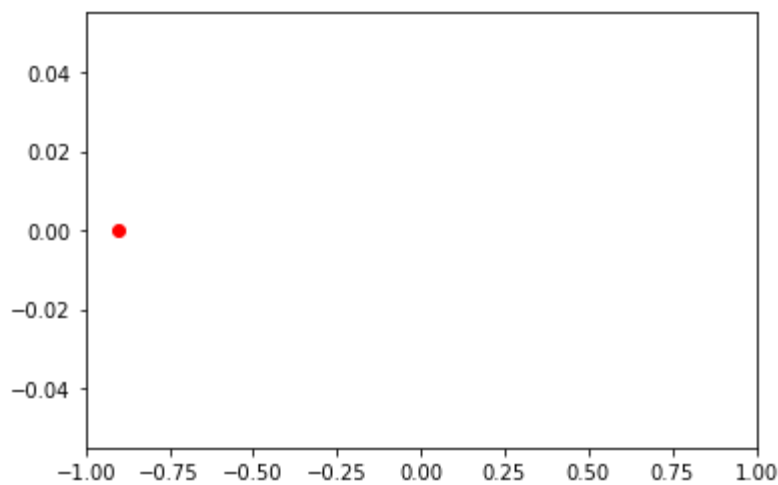
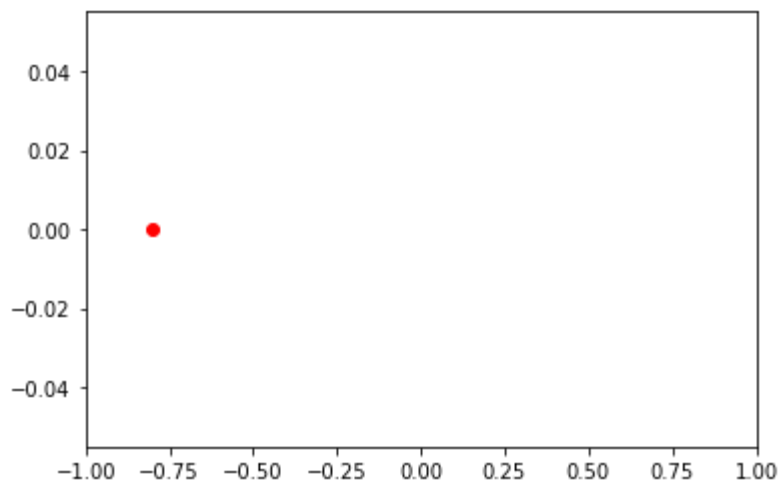
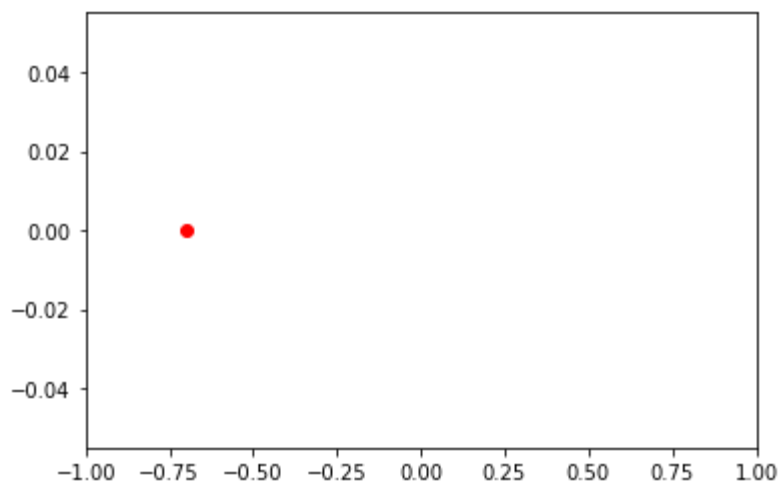
dt = 0.1
t = 0
i = 0
while t < 2:
    plt.plot(star.x,0,'ro')
    plt.xlim(-1,1)
    plt.show()

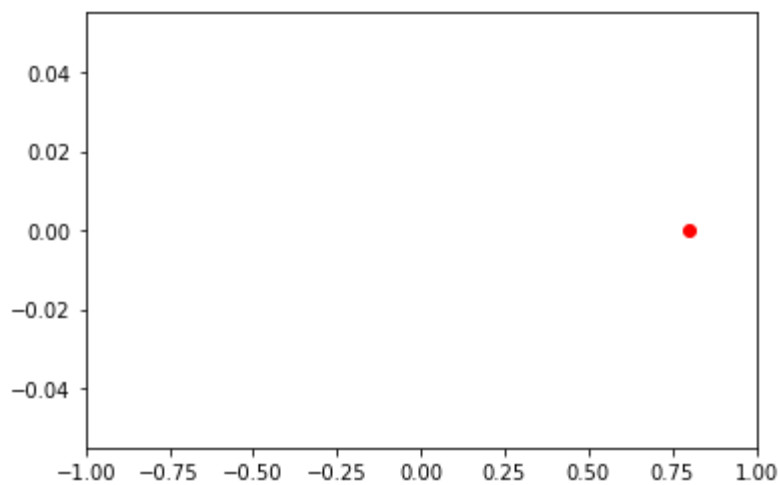
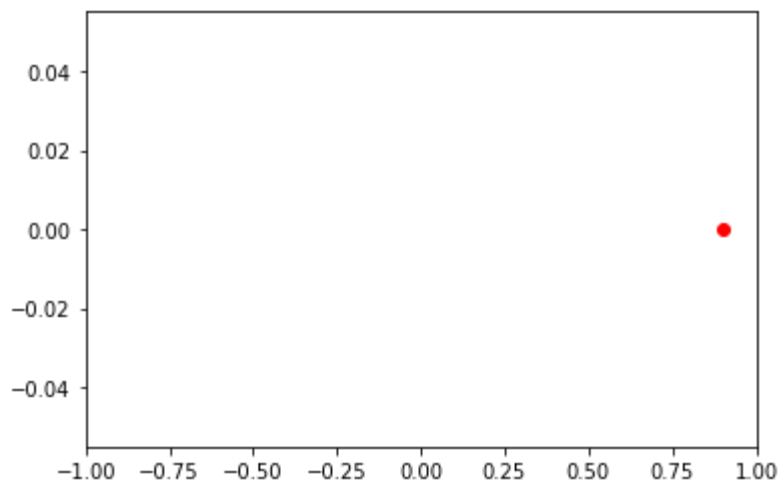
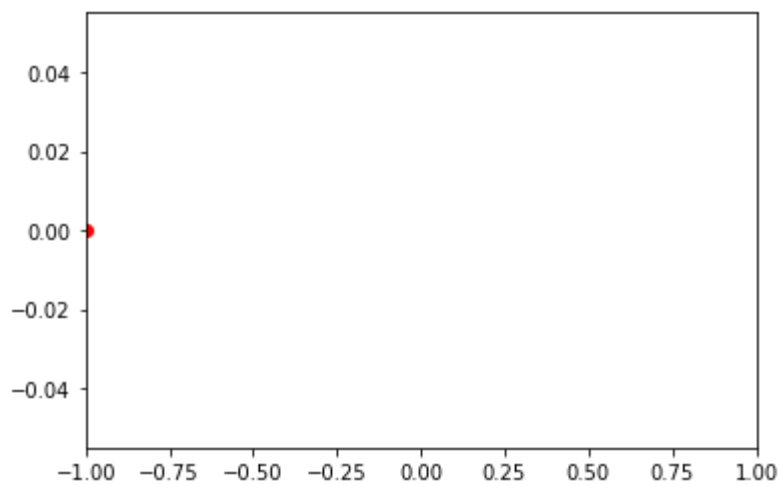
    star.x -= v*dt
    star.reposition(2)
    t += dt
```

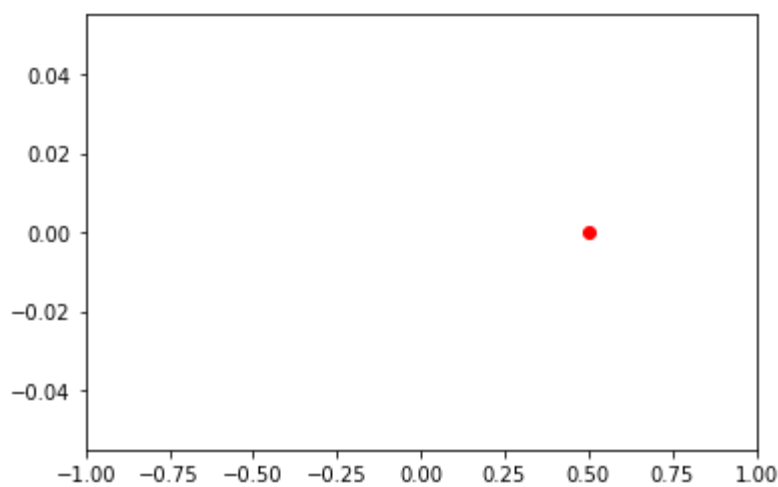
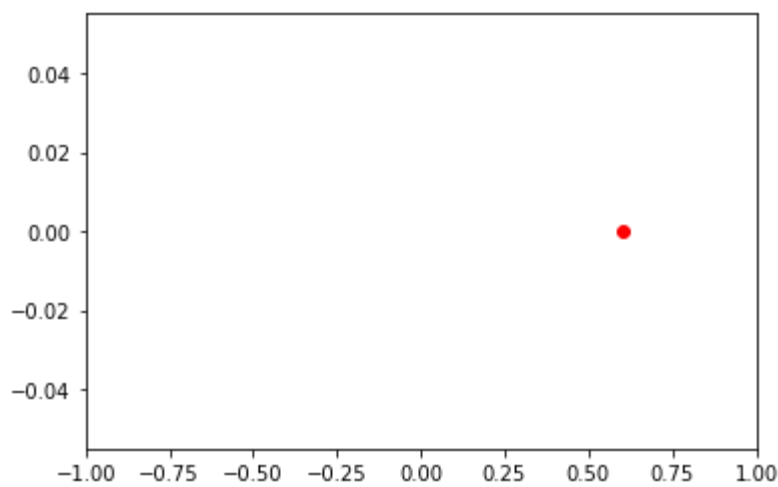
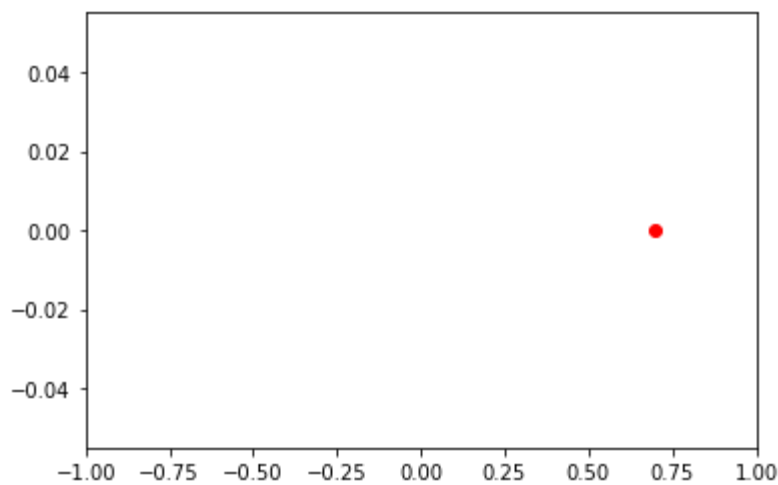


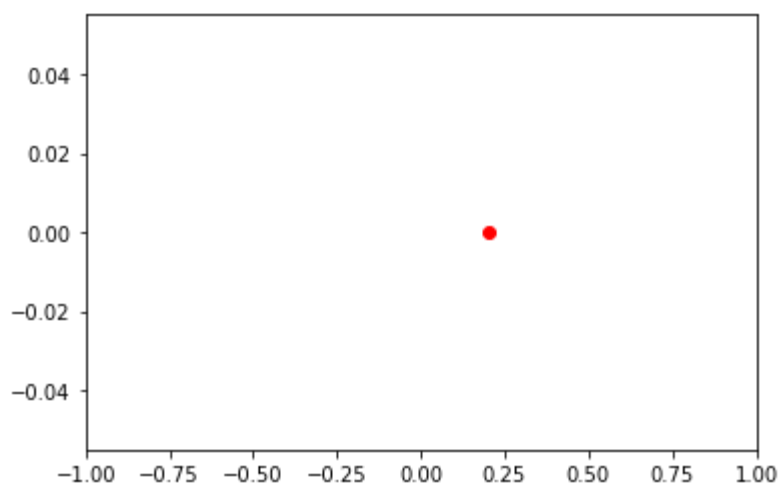
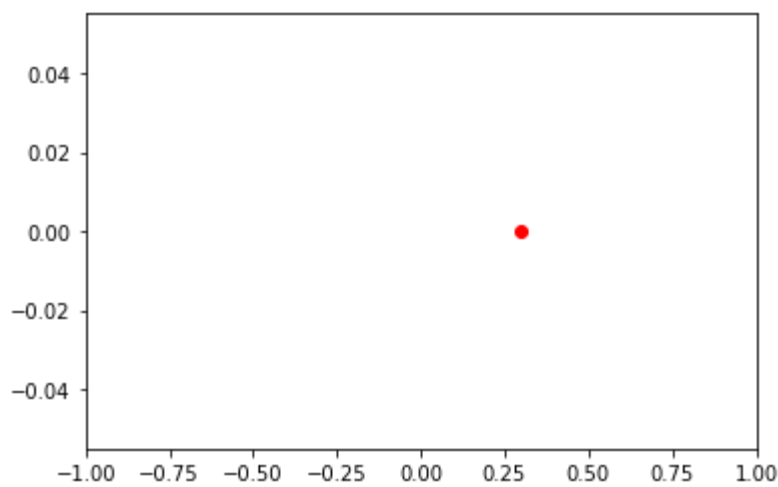
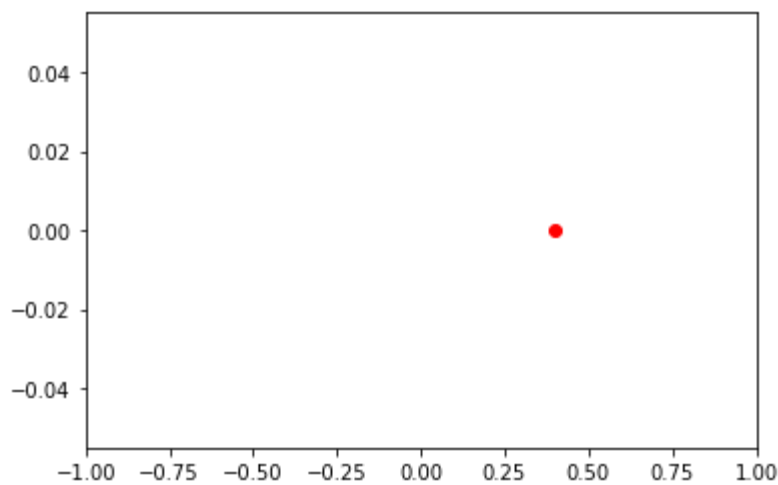


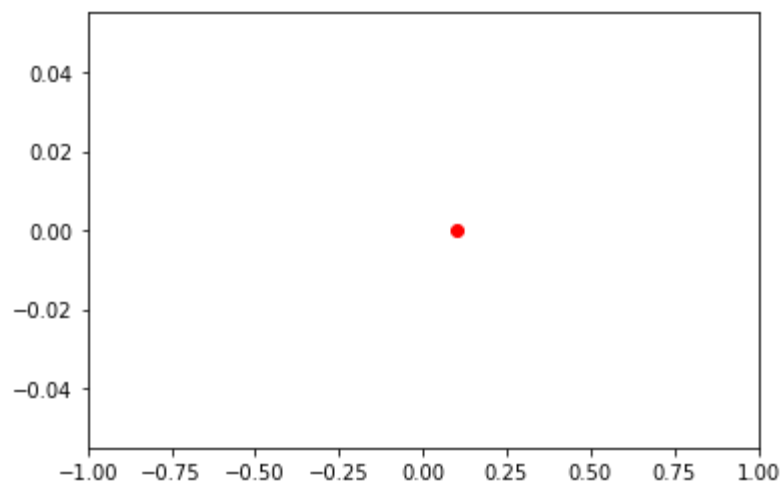












```

In [ ]: import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm
from matplotlib.colors import LogNorm, Normalize
import os
import subprocess
import cv2
from PIL import Image
import scipy.optimize as opt

#Import My Library
My_Package_PATH = "/home/boris/Documents/Research/Coding"
import sys
sys.path.insert(1, My_Package_PATH)
import OneD.WaveNonDim as ND
import OneD.NBody as NB
import OneD.GlobalFuncs as GF

#Set up Directory for saving files/images/videos
# Will not rename this again
dirExtension = "1D_Codes/Non-Dim/Analysis"
Directory = os.getcwd()#+"/"+dirExtension #os.curdir() #"/home/boris/Documen
print(Directory)

r,m,Num_bosons,sigma,Num_stars = [0.5,1.0,0,1,10000]

mu = m #M_scale = 1

L = 2
N = 10**3
z = np.linspace(-L/2,L/2,N)
dz = z[1]-z[0]

folder = "ParticlesOnly_Snapshots"
stars_x = np.loadtxt(folder+"/"+f"StarsOnly_Pos.csv", dtype = float, delimit
stars_v = np.loadtxt(folder+"/"+f"StarsOnly_Vel.csv", dtype = float, delimit
Energies = np.loadtxt(folder+"/"+f"Energies.csv", dtype = float,delimiter = "
#chi = np.loadtxt(folder+"/"+f"Chi.csv", dtype = complex, delimiter=",")
chi = np.zeros_like(z)
centroids = np.loadtxt(folder+"/"+f"Centroids.csv",dtype = float, delimiter='

stars = [NB.star(i,sigma,stars_x[i],stars_v[i]) for i in range(len(stars_x))

grid_counts = NB.grid_count(stars,L,z)

rho = (grid_counts/dz)*sigma

i = 0
max_bool = False
while max_bool == False:
    for j in range(len(rho)):
        if rho[j] > rho[i]: #if you come across an index j that points to a
            #then set i equal to j
            i = j
            #break
        else:
            max_index = i
            max_bool = True

```

```

        max_bool = True
max_rho = rho[max_index]

#Other method to accumulate left and right sides:
for star in stars:
    star.x = star.x - z[max_index] #shift
    star.reposition(L) #reposition

grid_counts = NB.grid_count(stars,L,z)
rho_part = (grid_counts/dz)*sigma
#Add the density from the FDM
rho_FDM = mu*np.absolute(chi)**2
rho = rho_FDM + rho_part

centroid_z = 0
for j in range(len(grid_counts)):
    centroid_z += z[j]*grid_counts[j]
centroid_z = centroid_z / Num_stars

stars_x = [star.x for star in stars]

std = np.std(stars_v)
mean_x = np.mean(stars_x)

R = 0
while True:
    R += dz
    mass_enclosed = 0
    star_collection = []
    for star in stars:
        if np.abs(star.x-mean_x) <= R:
            mass_enclosed += 1
            star_collection.append(star)
    print(R,mass_enclosed)
    if mass_enclosed >= 0.5*Num_stars:
        break

print(R)
plt.figure()
plt.scatter(stars_x,stars_v,s=1)
xx = np.linspace(-R,R,100)
plt.plot(xx,np.sqrt(R-xx**2))
plt.plot(xx,-np.sqrt(R-xx**2))
plt.scatter([star.x for star in star_collection],[star.v for star in star_collection])
plt.show()

Sigma = std**2 / R
print(Sigma)

```

/home/boris/Documents/Research/Coding/1D_Codes/Non-Dim/Analysis

```

-----
OSError                                Traceback (most recent call last)
/home/boris/Documents/Research/Coding/1D_Codes/Non-Dim/Analysis/CurveFitting
.ipynb Cell 5' in <cell line: 37>()
      <a href='vscode-notebook-cell:/home/boris/Documents/Research/Coding/1D
_Codes/Non-Dim/Analysis/CurveFitting.ipynb#ch0000004?line=34'>35</a> stars_
x = np.loadtxt(folder+"/"+f"StarsOnly_Pos.csv", dtype = float, delimiter
="," )
      <a href='vscode-notebook-cell:/home/boris/Documents/Research/Coding/1D
_Codes/Non-Dim/Analysis/CurveFitting.ipynb#ch0000004?line=35'>36</a> stars_
v = np.loadtxt(folder+"/"+f"StarsOnly_Vel.csv", dtype = float, delimiter
="," )
--> <a href='vscode-notebook-cell:/home/boris/Documents/Research/Coding/1D
_Codes/Non-Dim/Analysis/CurveFitting.ipynb#ch0000004?line=36'>37</a> Energi
es = np.loadtxt(folder+"/"+f"Energies.csv", dtype = float, delimiter = ",")
      <a href='vscode-notebook-cell:/home/boris/Documents/Research/Coding/1D
_Codes/Non-Dim/Analysis/CurveFitting.ipynb#ch0000004?line=37'>38</a> #chi =
np.loadtxt(folder+"/"+f"Chi.csv", dtype = complex, delimiter=",")
      <a href='vscode-notebook-cell:/home/boris/Documents/Research/Coding/1D
_Codes/Non-Dim/Analysis/CurveFitting.ipynb#ch0000004?line=38'>39</a> chi =
np.zeros_like(z)

File /usr/lib/python3/dist-packages/numpy/lib/npio.py:1067, in loadtxt(fna
me, dtype, comments, delimiter, converters, skiprows, usecols, unpack, ndmi
n, encoding, max_rows, like)
    1065     fname = os_fspath(fname)
    1066     if _is_string_like(fname):
--> 1067         fh = np.lib._datasource.open(fname, 'rt', encoding=encoding)
    1068         fencoding = getattr(fh, 'encoding', 'latin1')
    1069         fh = iter(fh)

File /usr/lib/python3/dist-packages/numpy/lib/_datasource.py:193, in open(p
ath, mode, destpath, encoding, newline)
    156     """
    157     Open `path` with `mode` and return the file object.
    158
    159     (...)
    189
    190     """
    192     ds = DataSource(destpath)
--> 193     return ds.open(path, mode, encoding=encoding, newline=newline)

File /usr/lib/python3/dist-packages/numpy/lib/_datasource.py:533, in DataSo
urce.open(self, path, mode, encoding, newline)
    530         return _file_openers[ext](found, mode=mode,
    531                                     encoding=encoding, newline=newline)
    532     else:
--> 533         raise IOError("%s not found." % path)

OSError: ParticlesOnly_Snapshots/Energies.csv not found.

```

```

In [ ]: G = 6.67E-11
print(R)
print("-----")
print("")

Sigma = std**2 / (np.pi* R**(3/2))
print(Sigma)

print(10000/R)
print(std**2)
print(10000/(np.pi*R**2))

print(std**2 * R)

print("-----")
print("")
new_std = np.std([star.v for star in star_collection])
Sigma = new_std**2 / (np.pi* R**(3/2))
print(f"Sigma = {Sigma}" )

print(10000/R)
print(new_std**2)
print(10000/(np.pi*R**2))

print(new_std**2 * R)

```

0.048048048048046965

100502.33917739333

208125.00000000047

3325.3676414862753

1378791.600352967

159.77742421555317

Sigma = 61907.311466298816

208125.00000000047

2048.3560084912815

1378791.600352967

98.41950791549479

```

In [ ]: v_rms = np.sqrt(np.mean([star.v**2 for star in stars]))
print(v_rms)

```

57.66620191650019


```
In [ ]: v_mean = np.mean([star.v for star in stars])
std = np.sqrt(np.sum([(star.v - v_mean)**2 for star in stars])/(len(stars)-1))
print(std)

Sigma = std**2 / (np.pi * R**(3/2))
print(Sigma)

print(10000/R)
print(std**2)
print(10000/(np.pi*R**2))

print(std**2 * R)
```

```
57.66888425752163
100512.39041643498
208125.00000000047
3325.7002115074265
1378791.600352967
159.79340355590878
```

```

In [ ]: phi_part = GF.fourier_potentialV2(rho_part,L)
phi_part = phi_part - np.mean(phi_part)
print(np.mean(phi_part))

phi_part = phi_part - np.max(phi_part)

# Compute Chandrasekhar's potential energy tensor:
a_part = NB.acceleration(phi_part,L)
W = 0
for i in range(len(z)):
    dW = rho_part[i]*z[i]*a_part[i]
    W += dW
print(W)

a_part = NB.acceleration(phi_part,L)
W = 0
for i in range(len(z)):
    dW = -0.5*rho_part[i]*phi_part[i]
    W += dW
print(W)

# Compute only for the stars that exist:
a_part = NB.acceleration(phi_part,L)
W = 0
for star in stars:
    g = NB.g(star,a_part,dz)

    dW = - star.x*g
    W += dW / Num_stars
print(W)

# phi_part = GF.fourier_potentialV2(rho_part,L)
# a_part = NB.acceleration(phi_part,L)
# W = 0
# for i in range(len(z)):
#     dW = - dz*a_part[i]**2 / (8*np.pi)
#     W += dW
# print(W)
#W = np.sum(phi_part)
#print(W)

# Compute only for the stars that exist:
W = 0
for star in stars:
    #g = NB.g(star,a_part,dz)
    i = int(star.x//dz)
    rem = star.x % dz

    if i != len(phi_part)-1:
        value = phi_part[i] + rem*(phi_part[i+1]-phi_part[i])/dz
    elif i == len(phi_part)-1:
        # then i+1 ==> 0
        value = phi_part[i] + rem*(phi_part[0]-phi_part[i])/dz

    phi_star = value
    dW = phi_star
    W += dW
print(W)

```

```

0.0
-16785655529.943058
55851483242.48459
-1500.1026550328993
-7621006.55368937

```

Compute Total KE and Total Potential Energy of Stars

```

In [ ]: # Compute total KE of stars:
K = 0
for star in stars:
    dK = 0.5*sigma*star.v**2
    K += dK
print(K)
#average KE:
print(K/Num_stars)

# #Compute Total Potential
# W = 0
# for star in stars:
#     #g = NB.g(star,a_part,dz)
#     i = int(star.x//dz)
#     rem = star.x % dz

#     if i != len(phi_part)-1:
#         value = phi_part[i] + rem*(phi_part[i+1]-phi_part[i])/dz
#     elif i == len(phi_part)-1:
#         # then i+1 ==> 0
#         value = phi_part[i] + rem*(phi_part[0]-phi_part[i])/dz

#     phi_star = value
#     dW = phi_star
#     W += dW
# print(W)
# #average W:
# print(W/Num_stars)

# Compute only for the stars that exist:
a_part = NB.acceleration(phi_part,L)
W = 0
for star in stars:
    g = NB.g(star,a_part,dz)

    dW = - sigma*star.x*g
    W += dW
print(W)
print(W/Num_stars)

```

```

16626954.217372933
1662.6954217372934
-15001026.55032902
-1500.102655032902

```

Calculate v_{rms} and R_{syst}

Want to verify

$$\langle v^2 \rangle = \frac{GM}{R_{syst}}$$

```
In [ ]: v_rms = np.sqrt(np.mean([star.v**2 for star in stars]))
z_rms = np.sqrt(np.mean([star.x**2 for star in stars]))
print(f"v_rms = {v_rms}")
print(z_rms)
#v_rms = np.sqrt(np.sum([star.v**2 for star in stars])/Num_stars)

K = 0.5 * v_rms**2
print(f"K_avg = 0.5*m*v_rms^2 = {K} (m=1)")
print(f"=> 2*K_avg = {2*K}")

print(z_rms*Num_stars)

print("-----")

R_syst = Num_stars / v_rms**2
print(R_syst)

rho_0 = np.mean(rho_part)
print(4*rho_0*z_rms)

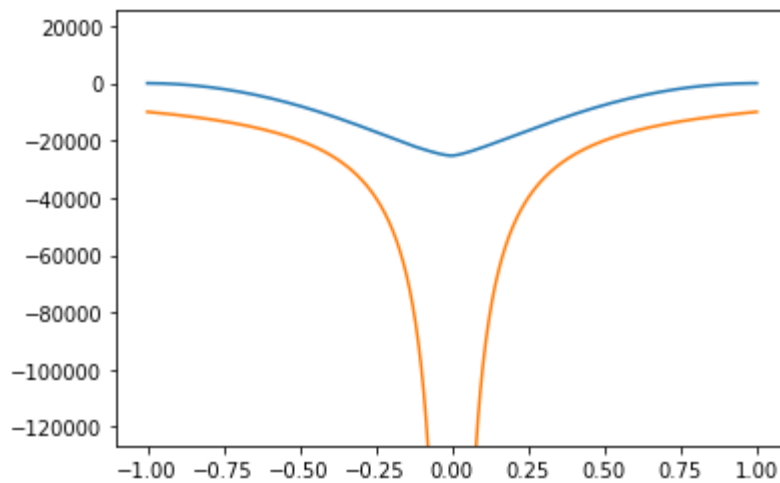
print(v_rms**2 / (2*np.pi*z_rms))

print(16*np.pi*rho_0**2*z_rms**3 / Num_stars)

v_rms = 57.66620191650019
0.1567842541769929
K_avg = 0.5*m*v_rms^2 = 1662.6954217372852 (m=1)
=> 2*K_avg = 3325.3908434745704
1567.842541769929
-----
3.0071653140030277
3132.549398456389
3375.673107160588
483.3349205244898
```

```
In [ ]: plt.plot(z, phi_part)
plt.plot(z, -Num_stars/np.abs(z))
plt.ylim(5*np.min(phi_part), -np.min(phi_part))
```

```
Out[ ]: (-126523.1647523825, 25304.632950476498)
```



```
In [ ]: phi_part = phi_part - (np.max(phi_part)-np.max(-Num_stars/np.abs(z)))

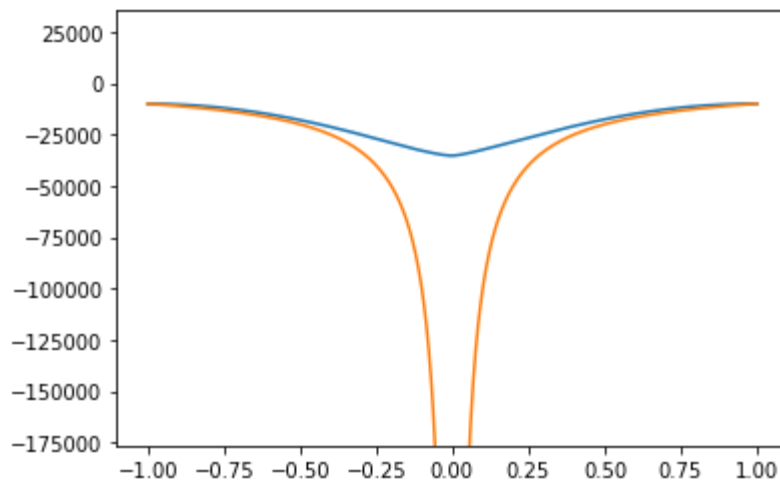
plt.plot(z,phi_part)
plt.plot(z,-Num_stars/np.abs(z))
plt.ylim(5*np.min(phi_part),-np.min(phi_part))
plt.show()

# Compute total KE of stars:
K = 0
for star in stars:
    dK = 0.5*star.v**2
    K += dK
print(K)
#average KE:
print(K/Num_stars)

#Compute Total Potential
W = 0
for star in stars:
    #g = NB.g(star,a_part,dz)
    i = int(star.x//dz)
    rem = star.x % dz

    if i != len(phi_part)-1:
        value = phi_part[i] + rem*(phi_part[i+1]-phi_part[i])/dz
    elif i == len(phi_part)-1:
        # then i+1 <=> 0
        value = phi_part[i] + rem*(phi_part[0]-phi_part[i])/dz

    phi_star = value
    dW = phi_star
    W += dW
print(W)
#average W:
print(W/Num_stars)
```



```
16626954.217372933
1662.6954217372934
-107621006.55368945
-10762.100655368946
```

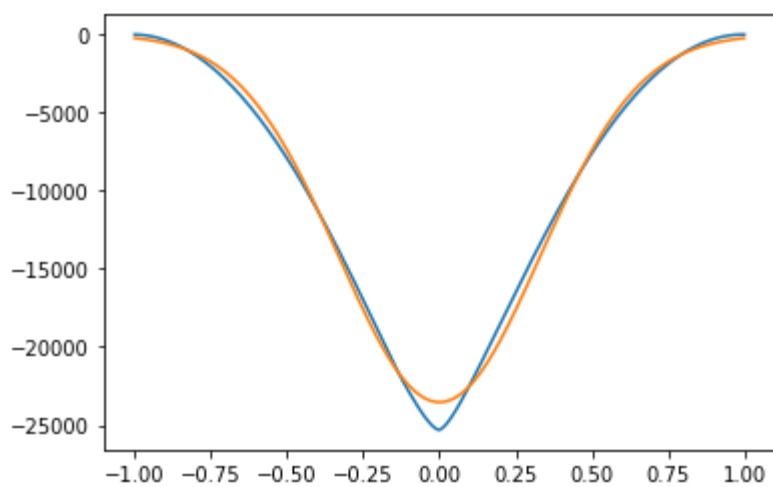
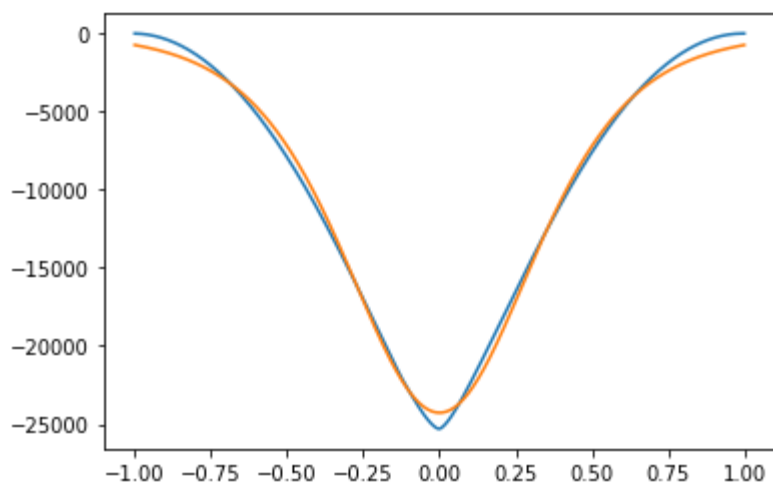
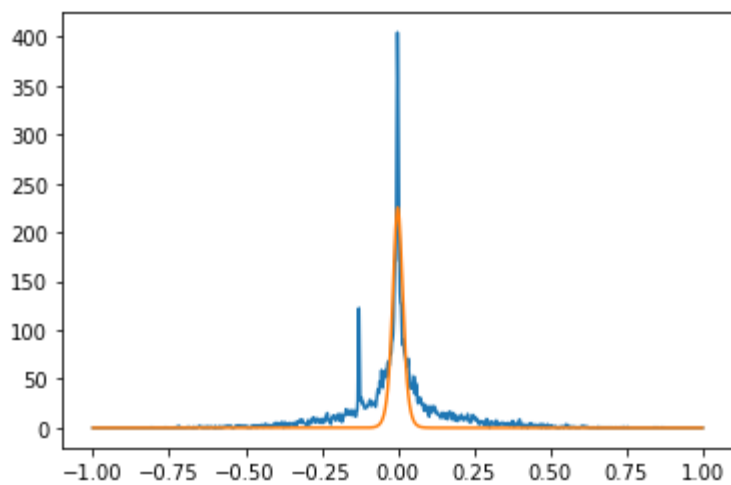
```
In [ ]: def f(z,*p):
    u_0 = p[0]
    z_0 = p[1]
    return u_0 / np.cosh(0.5*z/z_0)**2

guess = [rho_0,z_0]
popt,pcov = opt.curve_fit(f,z,grid_counts,p0 = guess)
plt.plot(z,grid_counts)
plt.plot(z,f(z,*popt))
plt.show()

guess = [rho_0,z_0]
popt,pcov = opt.curve_fit(f,z,phi_part,p0 = guess)
plt.plot(z,phi_part)
plt.plot(z,f(z,*popt))
plt.show()

def g(z,*p):
    return p[0]*np.exp(-z**2 / p[1])

guess = [-rho_0,z_0]
popt,pcov = opt.curve_fit(g,z,phi_part,p0 = guess)
plt.plot(z,phi_part)
plt.plot(z,g(z,*popt))
plt.show()
```



In []: