

# Black-Box Optimization Benchmarking Template for Noiseless Function Testbed

PSO algorithm

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## ACM Reference format:

Borja Arroyo. 2019. Black-Box Optimization Benchmarking Template for Noiseless Function Testbed. In *Proceedings of the Genetic and Evolutionary Computation Conference 2019, Prague, Czech Republic, July 13–17, 2019 (GECCO '19)*, 5 pages.

DOI: 10.1145/123\_4

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- [6] N. Hansen, T. Tušar, O. Mersmann, A. Auger, and D. Brockhoff. 2016. COCO: The Experimental Procedure. *ArXiv e-prints* arXiv:1603.08776 (2016).

## 1 PARAMETER TUNING

This PSO was executed with  $w = 0.5$ ,  $c1 = 2$ ,  $c2 = 1$ , which are, in order, the weight assigned to the previous step velocity or inertia; the weight assigned to the best individual position; and the weight assigned to the best swarm position. Furthermore, this experiment was developed throughout a hundred iterations over a swarm formed by as well a hundred individuals.

## 2 CPU TIMING

In order to evaluate the CPU timing of the algorithm, we have run the PSO on the bbbob test suite [4] with restarts for a maximum budget equal to  $1000D$  function evaluations according to [6]. The Python code was run on a Windows Intel(R) Core(TM) i5-4460 CPU @ 3.20GHz with 1 processor and 4 cores. The time per function evaluation for dimensions 2, 3, 5, 10, 20, 40 equals  $1.1e-05$ ,  $1.4e-05$ ,  $2.0e-05$ ,  $3.6e-05$ ,  $6.4e-05$ , and  $1.2e-04$  seconds respectively.

## 3 RESULTS

Results of PSO2 from experiments according to [6] and [2] on the benchmark functions given in [1, 5] are presented in Figures 1, 2, 3, and 4 and in Tables 1 and 2. The experiments were performed with COCO [3], version 2.3.1, the plots were produced with version 2.3.1.

## REFERENCES

- [1] S. Finck, N. Hansen, R. Ros, and A. Auger. 2009. *Real-Parameter Black-Box Optimization Benchmarking 2009: Presentation of the Noiseless Functions*. Technical Report 2009/20. Research Center PPE. <http://coco.lri.fr/downloads/download15.03/bbobdocfunctions.pdf> Updated February 2010.
- [2] N. Hansen, A. Auger, D. Brockhoff, D. Tušar, and T. Tušar. 2016. COCO: Performance Assessment. *ArXiv e-prints* arXiv:1605.03560 (2016).
- [3] N. Hansen, A. Auger, O. Mersmann, T. Tušar, and D. Brockhoff. 2016. COCO: A Platform for Comparing Continuous Optimizers in a Black-Box Setting. *ArXiv e-prints* arXiv:1603.08785 (2016).
- [4] N. Hansen, S. Finck, R. Ros, and A. Auger. 2009. *Real-Parameter Black-Box Optimization Benchmarking 2009: Noiseless Functions Definitions*. Technical Report RR-6829. INRIA. <http://hal.inria.fr/inria-00362633/en/>

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GECCO '19, Prague, Czech Republic

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DOI: 10.1145/123\_4

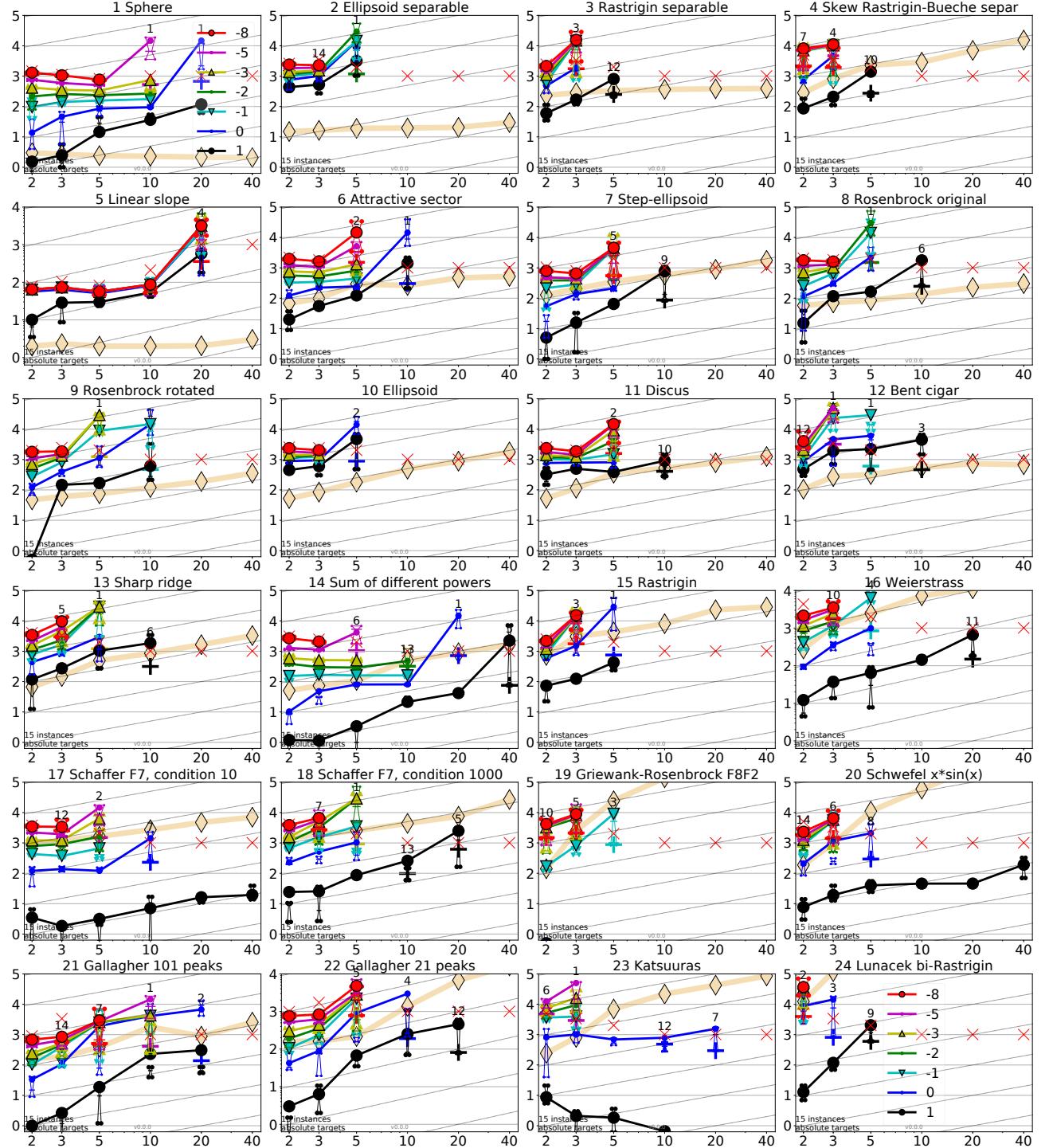
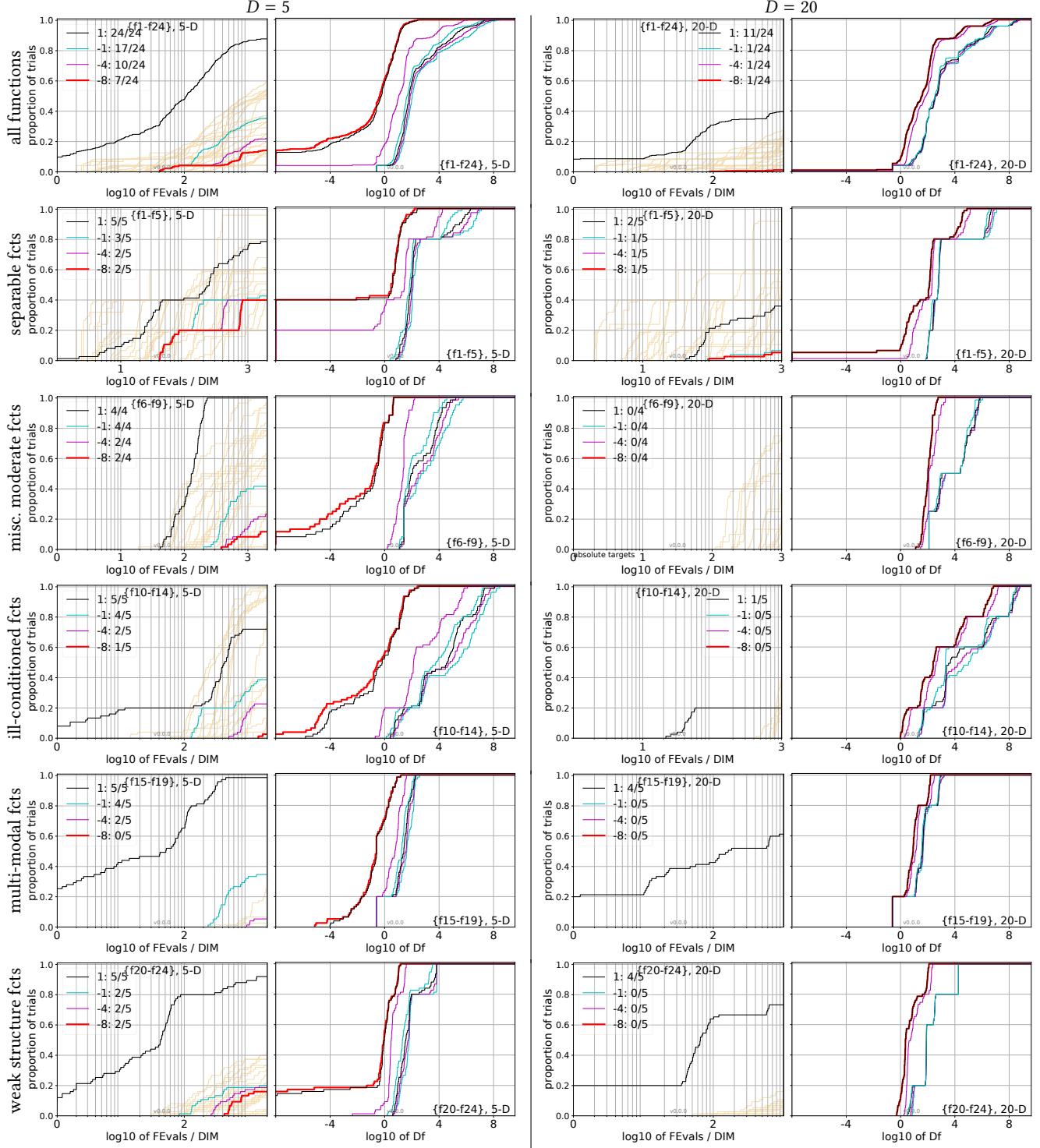


Figure 1: Scaling of runtime with dimension to reach certain target values  $\Delta f$ . Lines: average runtime (aRT); Cross (+): median runtime of successful runs to reach the most difficult target that was reached at least once (but not always); Cross (x): maximum number of  $f$ -evaluations in any trial. Notched boxes: interquartile range with median of simulated runs; All values are divided by dimension and plotted as  $\log_{10}$  values versus dimension. Shown is the aRT for fixed values of  $\Delta f = 10^k$  with  $k$  given in the legend. Numbers above aRT-symbols (if appearing) indicate the number of trials reaching the respective target. The light thick line with diamonds indicates the best algorithm from BBOB 2009 for the most difficult target. Horizontal lines mean linear scaling, slanted grid lines depict quadratic scaling.



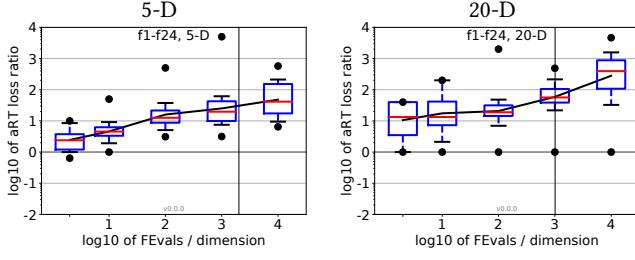
**Figure 2:** Empirical cumulative distribution functions (ECDF), plotting the fraction of trials with an outcome not larger than the respective value on the  $x$ -axis. Left subplots: ECDF of the number of function evaluations (FEEvals) divided by search space dimension  $D$ , to fall below  $f_{\text{opt}} + \Delta f$  with  $\Delta f = 10^k$ , where  $k$  is the first value in the legend. The thick red line represents the most difficult target value  $f_{\text{opt}} + 10^{-8}$ . Legends indicate for each target the number of functions that were solved in at least one trial within the displayed budget. Right subplots: ECDF of the best achieved  $\Delta f$  for running times of  $0.5D, 1.2D, 3D, 10D, 100D, 1000D, \dots$  function evaluations (from right to left cycling cyan-magenta-black...) and final  $\Delta f$ -value (red), where  $\Delta f$  and  $Df$  denote the difference to the optimal function value. Light brown lines in the background show ECDFs for the most difficult target of all algorithms benchmarked during BBOB-2009.

$\Delta f$	1e+1	1e+0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	$\Delta f$	1e+1	1e+0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f<sub>1</sub></b>	11 6.6(9)	12 35(9)	12 66(13)	12 98(8)	12 139(11)	12 209(15)	12 285(11)	15/15	<b>f<sub>13</sub></b>	132 39(39)	195 76(11)	250 583(424)	319 459(420)	1310 113(81)	1752 214(7)	2255 314(748)	15/15
<b>f<sub>2</sub></b>	83 190(120)	87 812(924)	88 817(832)	90 1655(1788)	90 $\infty$	92 $\infty$	94 $\infty$	0/15	<b>f<sub>14</sub></b>	10 1.7(3)	41 10(2)	58 14(2)	90 16(2)	139 18(4)	251 83(81)	476 314(748)	0/15
<b>f<sub>3</sub></b>	716 5.6(7)	1622 $\infty$	1637 $\infty$	1642 $\infty$	1646 $\infty$	1650 $\infty$	1654 $\infty$	15/15	<b>f<sub>15</sub></b>	511 4.2(11)	9310 16(20)	19369 $\infty$	19743 $\infty$	20073 $\infty$	20769 $\infty$	21359 $\infty$	14/15
<b>f<sub>4</sub></b>	809 8.7(22)	1633 $\infty$	1688 $\infty$	1758 $\infty$	1817 $\infty$	1886 $\infty$	1903 $\infty$	15/15	<b>f<sub>16</sub></b>	120 2.7(2)	612 8.1(13)	2662 12(23)	10163 $\infty$	10449 $\infty$	11644 $\infty$	12095 $\infty$	15/15
<b>f<sub>5</sub></b>	10 15(3)	10 25(5)	10 28(10)	10 28(6)	10 28(6)	10 28(10)	10 28(9)	15/15	<b>f<sub>17</sub></b>	5.0 3.1(5)	215 2.8(1)	89 3.7(6)	2861 2.8(2)	3669 8.7(12)	6351 12(9)	7934 $\infty$	15/15
<b>f<sub>6</sub></b>	114 5.4(2)	214 5.7(0.9)	281 7.9(3)	404 10(3)	580 11(14)	1038 25(18)	1332 54(87)	2/15	<b>f<sub>18</sub></b>	103 4.2(1)	378 14(15)	3968 4.4(3)	8451 17(10)	9280 16(25)	10905 $\infty$	12469 $\infty$	15/15
<b>f<sub>7</sub></b>	24 13(3)	324 3.3(2)	1171 18(41)	1451 15(24)	1572 14(18)	1597 14(19)	1597 14(19)	15/15	<b>f<sub>19</sub></b>	1 1	1 1	242 188(52)	1.0e5 $\infty$	1.2e5 $\infty$	1.2e5 $\infty$	1.2e5 $\infty$	15/15
<b>f<sub>8</sub></b>	73 11(3)	273 42(19)	336 210(316)	372 400(455)	391 $\infty$	410 $\infty$	422 $\infty$	15/15	<b>f<sub>20</sub></b>	16 13(7)	851 12(24)	3811 $\infty$	51362 $\infty$	54470 $\infty$	54861 $\infty$	55313 $\infty$	14/15
<b>f<sub>9</sub></b>	35 24(4)	127 44(30)	214 207(97)	263 556(883)	300 492(682)	335 $\infty$	369 $\infty$	15/15	<b>f<sub>21</sub></b>	41 2.3(3)	1157 8.3(11)	1674 7.5(12)	1692 7.6(18)	1705 7.7(12)	1705 7.9(9)	1757 8.1(5)	14/15
<b>f<sub>10</sub></b>	349 68(80)	500 140(197)	574 $\infty$	607 $\infty$	626 $\infty$	829 $\infty$	880 $\infty$	15/15	<b>f<sub>22</sub></b>	71 4.7(1)	386 12(19)	938 12(18)	980 14(23)	1008 14(24)	1040 15(24)	1068 18(18)	14/15
<b>f<sub>11</sub></b>	143 14(7)	202 20(17)	763 10(7)	977 21(22)	1177 39(65)	1467 49(43)	1673 44(71)	2/15	<b>f<sub>23</sub></b>	3.0 3.1(2)	518 6.7(6)	12429 $\infty$	27890 $\infty$	31654 $\infty$	33030 $\infty$	34256 $\infty$	15/15
<b>f<sub>12</sub></b>	108 103(49)	268 113(122)	371 389(524)	413 $\infty$	461 $\infty$	1303 $\infty$	1494 $\infty$	15/15	<b>f<sub>24</sub></b>	1622 6.3(13)	2.2e5 $\infty$	6.4e6 $\infty$	9.6e6 $\infty$	1.3e7 $\infty$	1.3e7 $\infty$	3/15	

**Table 1:** Average running time (aRT in number of function evaluations) divided by the aRT of the best algorithm from BBOB 2009 in dimension 5. This aRT ratio and, in braces as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best aRT (preceded by the target  $\Delta f$ -value in *italics*) in the first. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm from BBOB 2009, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k > 1$  is following the ↓ symbol, with Bonferroni correction by the number of functions (24). Data produced with COCO v0.0.0

$\Delta f$	1e+1	1e+0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	$\Delta f$	1e+1	1e+0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
<b>f<sub>1</sub></b>	43 55(27)	43 6891(5050)	43 $\infty$	43 $\infty$	43 $\infty$	43 $\infty$	43 $\infty$	15/15	<b>f<sub>13</sub></b>	652 10(1)	2021 239	2751 304	3507 451	18749 932	24455 1648	30201 15661	15/15
<b>f<sub>2</sub></b>	385 $\infty$	386 $\infty$	387 $\infty$	388 $\infty$	390 $\infty$	391 $\infty$	393 $\infty$	15/15	<b>f<sub>14</sub></b>	75 11(3)	239 1243(2430)	304 $\infty$	451 $\infty$	932 $\infty$	1648 $\infty$	15661 $\infty$	15/15
<b>f<sub>3</sub></b>	5066 $\infty$	7626 $\infty$	7635 $\infty$	7637 $\infty$	7643 $\infty$	7646 $\infty$	7651 $\infty$	15/15	<b>f<sub>15</sub></b>	30378 $\infty$	1.5e5 $\infty$	3.1e5 $\infty$	3.2e5 $\infty$	3.2e5 $\infty$	4.5e5 $\infty$	4.6e5 $\infty$	15/15
<b>f<sub>4</sub></b>	4722 $\infty$	7628 $\infty$	7666 $\infty$	7686 $\infty$	7700 $\infty$	7758 $\infty$	1.4e5 $\infty$	9/15	<b>f<sub>16</sub></b>	1384 10(15)	27265 $\infty$	77015 $\infty$	1.4e5 $\infty$	1.9e5 $\infty$	2.0e5 $\infty$	2.2e5 $\infty$	15/15
<b>f<sub>5</sub></b>	41 285(266)	41 1140(1549)	41 1152(855)	41 1542(1990)	41 1542(1621)	41 1542(1703)	41 1542(2230)	4/15	<b>f<sub>17</sub></b>	63 5.2(4)	1030 $\infty$	4005 $\infty$	12242 $\infty$	30677 $\infty$	56288 $\infty$	80472 $\infty$	15/15
<b>f<sub>6</sub></b>	1296 $\infty$	2343 $\infty$	3413 $\infty$	4255 $\infty$	5220 $\infty$	6728 $\infty$	8409 $\infty$	15/15	<b>f<sub>18</sub></b>	621 81(106)	3972 $\infty$	19561 $\infty$	28555 $\infty$	67569 $\infty$	1.3e5 $\infty$	1.5e5 $\infty$	15/15
<b>f<sub>7</sub></b>	1351 $\infty$	4274 $\infty$	9503 $\infty$	16523 $\infty$	16524 $\infty$	16524 $\infty$	16969 $\infty$	15/15	<b>f<sub>19</sub></b>	1 1	1 3.4e5	1 4.7e6	1 6.2e6	1 7.6e6	1 7.6e6	1 7.6e6	15/15
<b>f<sub>8</sub></b>	2039 $\infty$	3871 $\infty$	4040 $\infty$	4148 $\infty$	4219 $\infty$	4371 $\infty$	4484 $\infty$	15/15	<b>f<sub>20</sub></b>	82 11(2)	46150 $\infty$	3.1e6 $\infty$	5.5e6 $\infty$	5.5e6 $\infty$	5.6e6 $\infty$	5.6e6 $\infty$	14/15
<b>f<sub>9</sub></b>	1716 $\infty$	3102 $\infty$	3277 $\infty$	3379 $\infty$	3455 $\infty$	3594 $\infty$	3727 $\infty$	15/15	<b>f<sub>21</sub></b>	561 11(14)	6541 20(20)	14103 $\infty$	14318 $\infty$	14643 $\infty$	15567 $\infty$	17589 $\infty$	15/15
<b>f<sub>10</sub></b>	7413 $\infty$	8661 $\infty$	10735 $\infty$	13641 $\infty$	14920 $\infty$	17073 $\infty$	17476 $\infty$	15/15	<b>f<sub>22</sub></b>	467 20(44)	5580 $\infty$	23491 $\infty$	24163 $\infty$	24948 $\infty$	26847 $\infty$	1.3e5 $\infty$	12/15
<b>f<sub>11</sub></b>	1002 $\infty$	2228 $\infty$	6278 $\infty$	8586 $\infty$	9762 $\infty$	12285 $\infty$	14831 $\infty$	15/15	<b>f<sub>23</sub></b>	3.0 2.0(2)	1614 19(26)	67457 $\infty$	3.7e5 $\infty$	4.9e5 $\infty$	8.1e5 $\infty$	8.4e5 $\infty$	15/15
<b>f<sub>12</sub></b>	1042 $\infty$	1938 $\infty$	2740 $\infty$	3156 $\infty$	4140 $\infty$	12407 $\infty$	13827 $\infty$	15/15	<b>f<sub>24</sub></b>	1.3e6 $\infty$	7.5e6 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	5.2e7 $\infty$	3/15

**Table 2:** Average running time (aRT in number of function evaluations) divided by the aRT of the best algorithm from BBOB 2009 in dimension 20. This aRT ratio and, in braces as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear in the second row of each cell, the best aRT (preceded by the target  $\Delta f$ -value in *italics*) in the first. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Bold entries are statistically significantly better (according to the rank-sum test) compared to the best algorithm from BBOB 2009, with  $p = 0.05$  or  $p = 10^{-k}$  when the number  $k > 1$  is following the ↓ symbol, with Bonferroni correction by the number of functions (24). Data produced with COCO v0.0.0



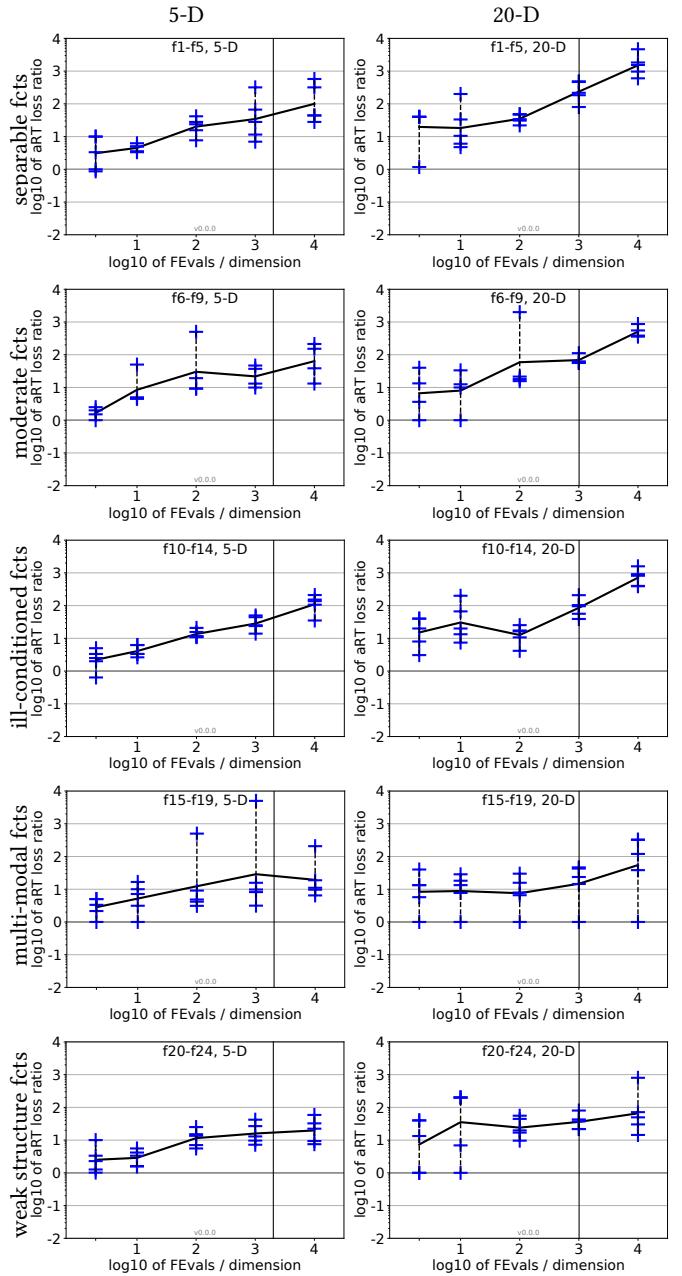
$f_1-f_{24}$ in 5-D, maxFE/D=2020						
#FEs/D	best	10%	25%	<b>med</b>	75%	90%
2	0.64	0.99	1.1	2.4	4.2	10
10	1.0	1.6	3.3	4.5	6.2	11
100	3.1	4.8	8.4	13	22	88
1e3	3.2	7.2	9.9	20	43	92
1e4	6.5	9.2	16	41	1.5e2	2.2e2
RLUS/D	2e3	2e3	2e3	2e3	2e3	2e3

$f_1-f_{24}$ in 20-D, maxFE/D=1010						
#FEs/D	best	10%	25%	<b>med</b>	75%	90%
2	1.0	1.0	3.4	13	40	40
10	1.0	1.0	7.2	13	50	2.0e2
100	1.0	6.3	13	19	32	49
1e3	1.0	21	38	56	1.1e2	2.4e2
1e4	1.0	29	96	4.0e2	8.9e2	1.6e3
RLUS/D	1e3	1e3	1e3	1e3	1e3	1e3

**Figure 3:** aRT loss ratio versus the budget in number of  $f$ -evaluations divided by dimension. For each given budget FEvals, the target value  $f_t$  is computed as the best target  $f$ -value reached within the budget by the given algorithm. Shown is then the aRT to reach  $f_t$  for the given algorithm or the budget, if the best algorithm from BBOB 2009 reached a better target within the budget, divided by the aRT of the best algorithm from BBOB 2009 to reach  $f_t$ . Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum aRT loss ratio (points). The vertical line gives the maximal number of function evaluations in a single trial in this function subset. See also Figure 4 for results on each function subgroup.

Data produced with COCO v0.0.0



**Figure 4:** aRT loss ratios (see Figure 3 for details). Each cross (+) represents a single function, the line is the geometric mean.