DCS - Lab 2

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Problem 2

This problem asks us to see the reflection property of the parabola which is that all rays parallel to the axis of the parabola reflect in the parabola into concurrent rays through the focus of the parabola.

First of all, we consider the parabola $y = x^2/200$ and we represent the equation on the program.

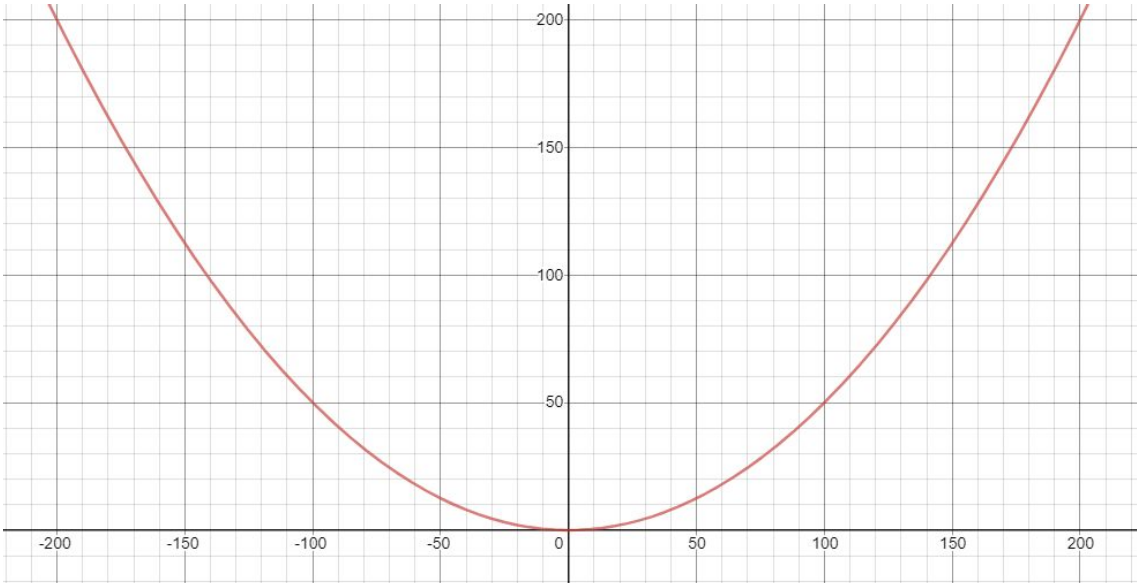

Figure 2.1. Representation of the parábola

Next, we define the diferent points which represent the start of the rays.

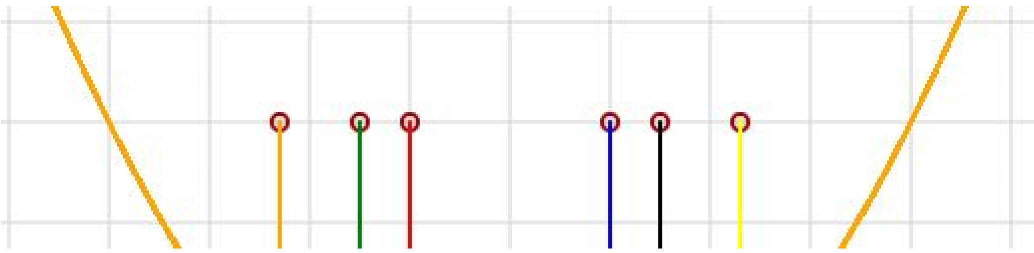

Figure 2.2. Points of the rays parallel to the parabola axis

The next thing to do is find the intersection between the rays and the parábola. We will do it taking the parabola equation and replace the X value with the X value of the point.

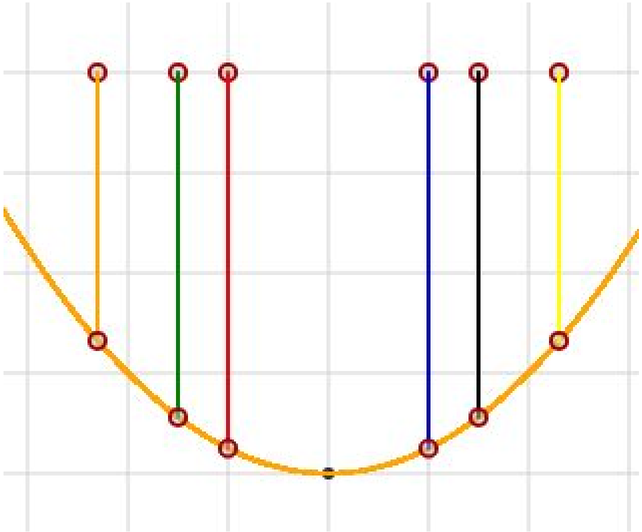

Figure 2.3. Intersection of the rays and the parabola

With the following theorical explanation, we will find the ray reflected in the parabola. Seeing the next picture we want to find the red ray starting from the yellow ray.
We know that the angle between the yellow ray and the tangent (black line) are the same angle between the ray reflected(red line) in the parabola and the tangent. But we can also appreciate that the red ray is the negation of the reflection of the yellow ray on the tangent (green ray).

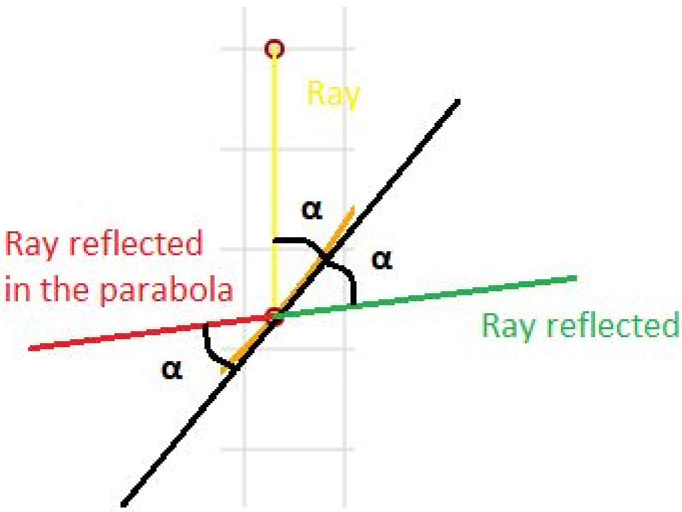

Figure 2.4. Representation of the parabola reflection and tangent reflection

Next step is to find the reflected rays. To find them, we can do it from the tangent. Tangent acts as a reflection line. So we reflect the rays by the tangent (used the code from lab 1).

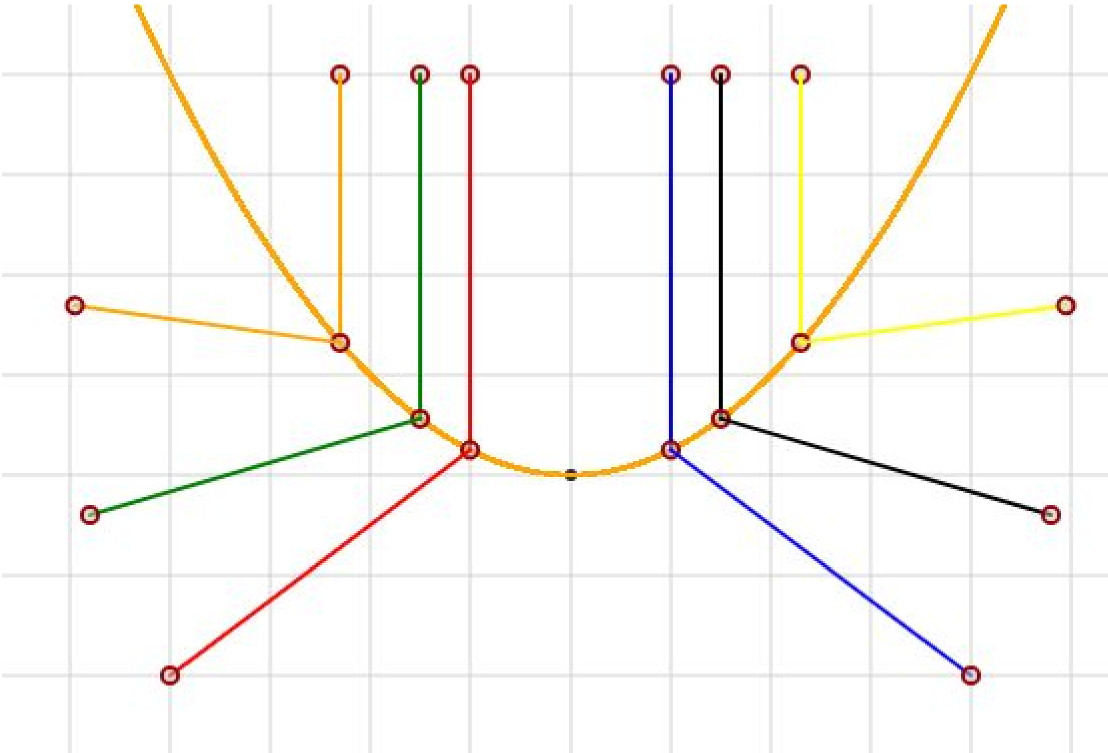

Figure 2.5. Rays reflected by the tangent

And to get the right rays reflected we only have to negate that result. And we get:

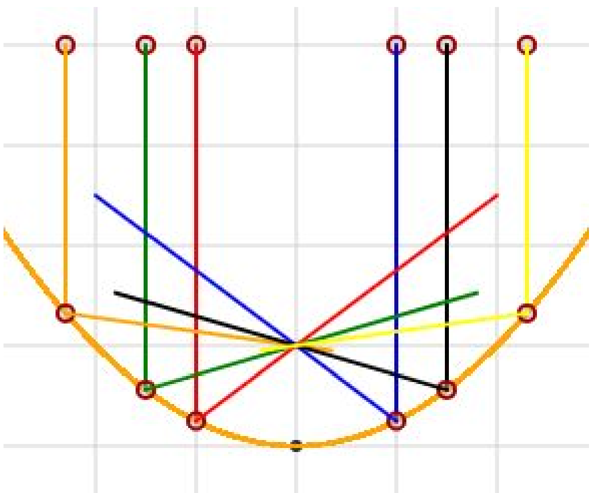

Figure 2.6. Rays reflected in the parabola

And the last step, is get two vectors of the reflected rays and find the intersection to get the focus.
The focus of this parabola is the point (0,50).

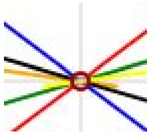

Figure 2.7. Focus

Problem 3

In this problem, it was requested to track a point of a circle that starts from the lowest point of the circle with angle 0 on the line y = 0.
We must to create a parametric equation for this exercise, next steps have been followed to get the equation:

1) Find the parametric equation of the center of the circle.
2) Find the vector that starts from the center of the circle to the starting point.
3) Apply the rotation of the circle(clockwise rotation) on the vector and obtain the parametric equation.
4) Obtain the result by adding the parametric equations of the circle plus the rotation of the vector.

1. Find the parametric equation of the center of the circle
   Knowing that we are working on X axis and we have the radius of the circle as the input parameter. We know that the circle will rotate on the X axis, so we obtain the next parametric equation for all the points in the center of the circle:

$$C(t) = \left(2\pi r \cdot \frac{t}{2\pi}, r\right)$$

$$C(t) = (r \cdot t, r)$$

t is an angle in radians and the horizontal displacement will be equal to the length of the circle segment with the respect to *t.* Vertically, there isn't move. So, we will stay in r.

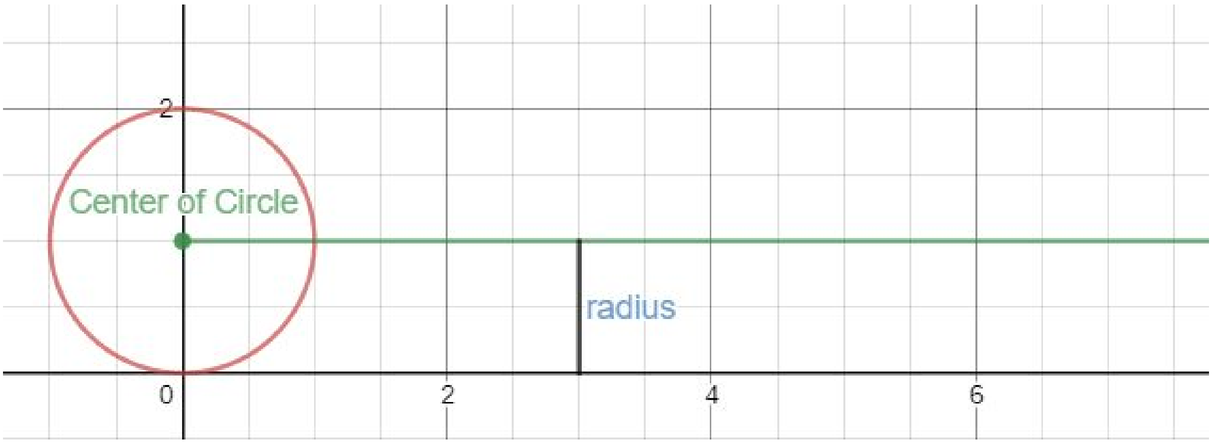

Figure 3.2. Shows the representation of the parametric equation of the center of the circle (green line).

2. Find the vector that starts from the center of the circle to the starting point
   We want to obtain the vector from the center of the circle in the initial position to the initial point of the final curve. The exercise specifies that, with angle 0, the first position of the curve is the lowest point of the circle.
   Therefore, we can deduce that the vector will be vertical with down direction. The equation of this vector would be:

$$v = (0, -r)$$

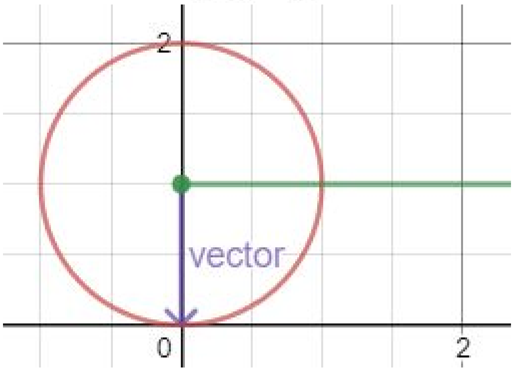

Figure 3.3. Image of the vector

3. Apply rotation to the vector
   We know how the circle will rotate (clockwise). So we multiply the vector by the rotation matrix to get:

$$\begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} x \begin{pmatrix} 0 \\ -r \end{pmatrix} = \begin{pmatrix} r \cdot \sin t \\ -r \cdot \cos t \end{pmatrix}$$

4. <u>Obtain the result by adding the parametric equations of the circle plus the rotation of the vector</u>
   We move the points of the center of the circle to where the vector is pointing to find the point of the solution:

$$
\begin{aligned}
R(t) &= (r \cdot t, r) \quad + \quad (r \cdot \sin t, -r \cdot \cos t) \\
&= (r \cdot t + r \cdot \sin t, r - r \cdot \cos t) \\
&= \big(r(t + \sin t), r(1 - \cos t)\big)
\end{aligned}
$$

   Note: The representation of the positive values on canvas is down. Therefore, to get the same curve like the exercise we only have to negate the second value of the parametric equation

$$\big(r(t + \sin t), -r(1 - \cos t)\big)$$