Aggregate Effects of Firing Costs with Endogenous Firm Productivity Growth

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Abstract

This paper quantifies the aggregate effects of firing costs in a model of firm dynamics where firm-level productivity is determined by innovation. In the model, the productivity distribution is endogenous, and thus, potentially affected by policy changes, allowing the model to capture both the static (allocative efficiency) and dynamic effects (changes in the distribution of firms' productivity) of firing costs. The model is calibrated so as to match key features of firms' hiring and firing behavior using firm-level data from Spanish non-financial firms. I show that firing costs equivalent to 2.5 monthly wages produce a 4% loss in aggregate productivity relative to the frictionless economy. The aggregate productivity losses rise to more than 12% when firing cost are equivalent to one year's wage, which are substantially larger than those found in the literature. I show that the dynamic effects of firing costs are quantitatively relevant, explaining 43% of these productivity losses. Overall, the results suggest that ignoring the effects of frictions on the dynamics of firm productivity can substantially underestimate their aggregate effects.

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1 Introduction

There is a large body of research studying the productivity losses from firing costs. Following Hopenhayn and Rogerson (1993), most of this literature typically quantify the effects of firing cost by looking at the efficiency in the allocation of labor across firms given a productivity distribution. However, if firm growth is a risky process, firing costs would be a critical component of the cost of failure, affecting the incentives of firms to grow and potentially shaping the distribution of firms' productivity itself. By assuming an exogenous process for firm's productivity, previous literature cannot capture such dynamic effects, and thus, may underestimate the aggregate impact of firing costs. This paper fills the gap by quantifying the aggregate implications of firing costs in a model in which the dynamics of firms' productivity are endogenous.

I extend the standard firm dynamics model of Hopenhayn and Rogerson (1993) by incorporating an innovation technology that allows firms to have partial control over the probability of innovation —as in Atkeson and Burstein (2010)— and over the outcome of innovation itself. I model innovation building on the "control cost" approach borrowed from the game theory literature. In particular, firms in the model can choose, at a cost, the probability of innovation and, in case of innovation occurs, the distribution of next period's productivity. In models á la Atkeson and Burstein (2010) firms do not face the risk of a very negative shock—key for accounting for the effects of firing costs—unless the size of the productivity step is sufficiently large, which would generate unrealistic productivity dynamics.[1] My approach can generate sufficiently large downwards risk while keeping the dynamics of productivity realistic and allowing for a cleaner identification of the relevant parameters.

I estimate the parameters of the model by matching key moments regarding firm growth and firing and hiring behavior, using firm-level data from Spanish non-financial firms. The Spanish economy is of particular interest for this analysis. The Spanish labor market, considered as one of the most inefficient labor markets in Europe, is characterized by a high structural unemployment rate, a high volatility of employment, and an intensive

[1] These models assume that firms can invest resources in increasing the probability of a positive step in their productivity versus a negative one, but the size of this step is exogenously set. This implies that the level of risk firms face is limited by assumption. One could add an extreme shock to generate sufficient negative risk, but this would come at the cost of adding more parameters into the model.

use of temporary employment. Productivity in Spain is one of the lowest among developed countries. In 2010 Spanish TFP was 9% lower than it was in 1990, while for the US and Germany it was 20% higher. This paper connects the underperforming of Spanish productivity with the distortions of its labor market.

The model closely matches the targeted moments. In the baseline economy, small firms innovate more frequently, their innovations are more aggressive (as measured by the expected productivity growth) and more volatile (as measured by the standard deviation of productivity growth). These predictions imply that small firms grow faster and that their growth rates are more volatile. This is consistent with the empirical evidence.[2]

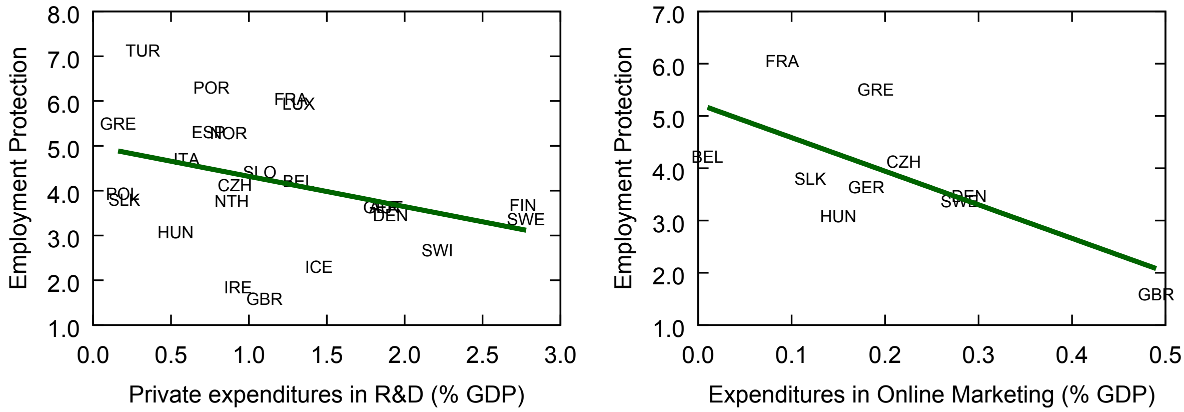
Using the calibrated model I ask, "What are the aggregate effects of firing costs?". In order to address this question, I compare the baseline economy, with firing cost equivalent to 2.5 monthly wages, with one in which firing cost are set to zero. I find that aggregate productivity is 4% lower in the baseline economy than in the frictionless one. This is a large effect compared to what has been found in the previous literature. For instance, Hopenhayn and Rogerson (1993) find a 2.5% drop in aggregate productivity when firing costs are equivalent to one year's wage. My model generates larger effects for a much lower level of firing costs. In fact, when I set the firing cost equivalent to one year's wage, the fall in aggregate productivity is of more than 12% relative to the frictionless economy, four times larger than in their paper. In a recent paper in which the dynamics of productivity depend on firm size (and thus, partially endogenous), Da-Rocha et al. (2019) find a 20% fall in aggregate productivity for a firing cost equivalent to 5 year's wage. My model generates half of this reduction with firing costs of *just* one year's wage.

The main reason behind this larger fall in aggregate productivity in my model is that productivity dynamics are endogenous. The firm dynamics literature typically assumes that firm productivity follows an exogenous process. In reality, however, firms have the option to undertake a large number of actions to improve their profits prospects, which I refer to as "innovation".[3] This means that, although partially stochastic, firm's growth is driven by firm's actions, which may be affected by economic conditions such as labor regulation. In particular, if innovation is costly and its outcome uncertain, firms incentives to make such investments will depend on the cost of failure, that is affected by the mag-

2. See for example Sutton (1997, 2002) or Klette and Kortum (2004).

3. Examples of these investments include product or process innovation but also demand-side investments such as marketing or sales campaigns.

Figure 1: Firing cost and firms' investments in growth-generating activities



Source: (i) *Employment Protection* refers to the sum of the OECD strictness of employment protection legislation indicators for permanent and temporary contracts; (ii) *Private expenditures in R&D* is taken from the OECD Main Science and Technology Indicators Database; (iii) *Expenditures in Online Marketing* is taken from Grece (2016).

nitude of firing costs. As a result, firms may optimally decide to invest less in innovation, reducing their productivity growth and the average firm productivity in the economy.

In figure 1 I plot some suggestive evidence on this negative relationship between firing costs and innovation. In particular, I plot the relationship between the strictness of employment protection legislation taken form the OECD, and two measures that fit well the broad definition of innovation in my paper: R&D expenditures (left panel) and firms spending on online marketing (right panel). In both cases, countries with high levels of firing frictions show lower spending on innovation.[4] This is what happens in the model. In the baseline economy, investment in innovation falls by 3.5% when firing costs equal the calibrated value of 2.5 monthly wages relative to the frictionless economy, making the average firm productivity to drop by almost 2%. When firing costs are of one year's wage, innovation expenses and average productivity fall by 12% and 6% respectively.

To quantify how much of the fall in aggregate productivity is accounted for the endogenous changes in the dynamics of firms' productivity I simulate an economy with positive firing costs but fixing the innovation choices from the frictionless economy. This makes the law of motion of productivity to be unaffected by changes in the firing cost. In this new economy, in which innovation is exogenous, the fall in aggregate productivity

4. Firing costs can also increase firms' incentives to make other types of investments, such as labor-saving technologies. However, this type of investments may have a larger impact on the production technology than on profitability (for given inputs), which is the focus of the paper.

is of 2.26%, substantially lower than in a model with endogenous innovation where firms can adjust their innovation choices. This means that 43% of the drop in aggregate productivity is explained by changes in the distribution of productivity. The remaining 57% is explained by the loss in allocative efficiency of labor across firms, *given* a productivity distribution. This finding suggests that models with exogenous productivity processes may largely underestimate the effects of frictions/policies such as firing cost.

The rest of the paper is organized as follows. Section 2 reviews the literature on firing cost and firm innovation. Section 3 presents the model economy. Section 4 explains the calibration procedure. Section 5 presents the results from changing firing cost. Finally, section 6 concludes.

2 Literature Review

There is a large literature that evaluates the role of different policies in accounting for aggregate productivity differences across countries (Guner et al. 2008, Restuccia and Rogerson 2008, Hsieh and Klenow 2009, Bartelsman et al. 2013, Hsieh and Klenow 2014, García-Santana et al. 2016). While many of these papers use "wedges" to measure policy distortions, some others specify particular policies.[5] One of the policies that has attracted more attention is employment protection, starting with the analysis of firing costs of Hopenhayn and Rogerson (1993). The distortion introduced by firing taxes on firm hiring and firing decisions are well established in the literature, both empirically (Haltiwanger et al. 2014) and theoretically (Bentolila and Bertola 1990). These distortions prevent firms from operating at their optimal scale, worsening the allocation of labor across firms, and damaging aggregate productivity.

The literature studying the impact of firing cost on aggregate productivity typically finds moderate effects (Hopenhayn 2014). However, aggregate productivity losses may be larger when the firms' productivity distribution is endogenous. This is because firing cost may distort not only firing and hiring choices —and thus, the allocation of labor across firms— but also incentives of firms to invest in growth-generating activities, such as innovation, marketing campaigns, launching new products, etc. This is analyzed by Da-Rocha et al. (2019), who study the aggregate implications of firing cost in a continuous-

[5]. See Restuccia and Rogerson (2013) for a discussion on these different approaches.

time model in which the law of motion of firm's productivity is size-dependent. They find that aggregate productivity losses from firing cost are much larger than when the productivity distribution is completely exogenous, as it is typically assumed. My paper differs from theirs in two margins. First, I consider a model with a continuum of potential firm size, while they consider a model where firms can either be small or large. Second, in their paper, the law of motion of firm's productivity is size dependent, but the difference between large and small firms is exogenous. In my paper large and small firms will have different laws of motion for their productivities endogenously, as a result of different innovation choices.

More generally, my paper relates to a recent literature that argues that frictions may affect aggregate productivity not only thought the efficiency in the allocation of resources but also through a direct effect on the firm-level productivity distribution itself. For instance, López-Martín (2013) and Mukoyama and Osotimehin (2019) endogenizes the way in which frictions affect firm's productivity dynamics by including an innovation technology similar to the one in Grossman and Helpman (1991), Aghion and Howitt (1992) and Atkeson and Burstein (2010): firms invest resources in increasing the probability of innovation, and the outcome of innovation is defined as a fixed, exogenous increase in firm's productivity. Ranasinghe (2014) also look at the impact of frictions on the distribution of firms' productivity extending the Hopenhayn and Rogerson (1993) framework. He assumes firms can invest resources in innovation, which changes the parameters of a (flexible) parametric distribution driving next period's distribution. The main difference with respect to these papers is that, in my model, the distribution of firm-level productivity is entirely drive by firm choices, including the degree of uncertainty faced by firms.

An example in which the distribution is endogenous is Bhattacharya et al. (2013). They study the aggregate implications of size-dependent distortions, in the form of tax rates, in a context in which managers invest in their skills (equivalent to firms' productivity), and show that endogenous managerial investments substantially amplifies the effects of distortions. Their model, however, lacks uncertainty, which is key for the analysis of firing costs. Another difference is that none of this paper studies the impact of firing costs. A similar result is found in Gabler and Poschke (2013), who study the effects of firing costs, among other frictions, using a firm dynamics model in which firms can engage in experimentation and discard negative productivity shocks (unsuccessful experiments).

The effects of firing costs, however, mainly depend on downwards risk, which is limited in this paper.

Finally, my paper is related to the game theory literature from which I borrow the "control cost" approach used in my paper to model productivity dynamics. This modelilng device is used to model equilibria in which agents optimally make errors under the assumption that precision is costly. In this approach decisions are conceived as a random variable over a feasible set of alternatives —which in my setting are the different levels of productivity—, and the cost is given by the precision of this random variable. In Costain et al. (2019) we implement this idea to model price and wage adjustment decisions in an otherwise standard new-keynesian framework with heterogeneous agents. Turen (2018) model costly information acquisition in a price-setting problem using a "control cost" framework. To the best of my knowledge, my paper is the first that uses this approach to model the dynamics of firm-level productivity.

3 The Model

This section presents an extension of the workhorse model of Hopenhayn and Rogerson (1993) in which I introduce an innovation technology that allows firms to invest in both the probability and the outcome of innovation.

3.1 Overview

The economy is populated by a continuum of firms of unit mass, characterized by a profitability factor, denoted by d, and a number of workers hired in the past, n. The term $d \in \mathbb{D} \equiv \{d_1, d_2, \ldots, d_D\}$ is a factor that increases revenues for given inputs, so it captures both productivity (ie. technology) and demand factors (ie. tastes). For simplicity in the exposition, I will refer to d as firm's productivity throughout the rest of the paper.

Given an initial state (d, n), firms decide on hirings/firings, produce and collect profits. They are then hit by an exit shock. With probability $1 - \delta \in (0, 1)$, the firm continues in the market and make innovation decisions. With probability δ the firm exists and it is immediately replaced by a new firm. Entrants start with no workers and an initial productivity drawn from $\log(d_0) \sim N\left(\log(\mu_0) - \frac{1}{2}\sigma_0^2, \sigma_0^2\right)$, such that $E[d_0] = \mu_0$.

3.2 Firms

Firms produce a homogeneous good, and its price is normalized to 1. This good is used both to consume and to invest in innovation. It is produced using a decreasing returns to scale technology, $y(d,n) = d^{1-\gamma}n^{\gamma}$, where $\gamma \in (0,1)$ the degree of returns to scale. Firms profits are given by:

$$\Pi(d,n,n') \;=\; y(d,n') \;-\; wn' \;-\; \kappa_F w \max\{0, n-n'\}, \tag{1}$$

where w is the wage rate, and $\kappa_F w$ is the per-worker firing cost. Using this profits function, the value of a firm with productivity d and n workers is given by:

$$V(d,n) \;=\; \max_{n'} \;\; \Pi(d,n,n') \;+\; \beta(1-\delta)\mathcal{I}(d,n') \;+\; \beta\delta V_E(n'), \tag{2}$$

where $\beta \in (0,1)$ is the subjective discount factor, $\mathcal{I}(d,n)$ is the value of a firm with state (d,n) before the innovation stage, and $V_E(n)$ captures the value of exit for a firm with n workers. Since Spanish regulation imposes the obligation to pay dismissal costs in case of exit, I assume that $V_E(n) = -w\kappa_F n$.[6]

3.3 Productivity dynamics

The problem consists of choosing both the probability of innovation, $\lambda \in [0,1]$, and the outcome of innovation, given by the distribution of next period's productivity, π, satisfying:

$$\sum_{i=1}^{D} \pi(d_i|d,n) = 1. \tag{3}$$

We can think of the choice of λ as the extensive margin of innovation, and the choice of π as the intensive one. Another valid interpretation would be to think of λ as the probability of generating a new idea, and π as the implementation of such idea.

6. Despite firm owners being subject to limited liability, workers have priority at liquidation over the rest of debtors. Setting the exit value to 0, however, does not affect the quantitative results significantly. The reasons is that I consider a model with exogenous exit, and thus, firing costs do not have a selection effect (Poschke 2009).

Let $\mathcal{I}^I$ be the value of an innovating firm and $\mathcal{I}^N$ be the value of not innovating. The innovation problem reads as:

$$\mathcal{I}(d,n) = \max_{\lambda,\pi} \lambda \underbrace{\left(\max_{\pi} \sum_{i=1}^{D} \pi(d_i|d,n)V(d_i,n) - \mathcal{D}(\pi||\eta)\right)}_{\mathcal{I}^I(d,n)} + $$

$$+ (1-\lambda)\underbrace{\left(\sum_{i=1}^{D} \eta(d_i|d)V(d_i,n)\right)}_{\mathcal{I}^N(d,n)} - \mathcal{D}(\lambda||\bar{\lambda}) \quad (4)$$

subject to $\lambda \in [0,1]$ and equation (3). The cost of choosing λ is given by $\mathcal{D}(\lambda||\bar{\lambda})$ where $\bar{\lambda} \in (0,1)$ is a default probability of innovation. Similarly, the cost of choosing the distribution π is given by $\mathcal{D}(\pi||\eta)$ where η is a default distribution, satisfying:

$$\sum_{i=1}^{D} \eta(d_i|d)d_i = d(1-\mu).$$

The parameter $\mu > 0$ is the depreciation rate of productivity. This depreciation rate implies that non-innovative firms expect their productivity to fall, which increases the incentives to innovate. Another important consecuence is that productivity growth in this model only arises as the result of innovation, since the reverse-to-the-mean effect of the standard AR(1) productivity process used in the literature is not present.

The cost function $\mathcal{D}(x||z)$ is given by the Kullback-Leibler divergence measure, or relative entropy, between x and z. In particular,

$$\mathcal{D}(\lambda||\bar{\lambda}) = \frac{1}{\kappa_I}\left[\lambda \log\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\log\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right], \quad (5)$$

$$\mathcal{D}(\pi||\eta) = \frac{1}{\kappa_I}\left[\sum_{i=1}^{D} \pi(d_i|d,n)\log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|n)}\right)\right], \quad (6)$$

where κ_I is the innovation productivity given by $\kappa_I = \kappa_0 \exp(-\kappa_1 d)$, where $\kappa_0 > 0$ and $\kappa_1 \geq 0$. If $\kappa_1 > 0$ the productivity of innovation is lower for more productive firms, making it costlier for them to innovate, which is consistent with the lower growth rate of larger firms. The innovation productivity parameters is the same for both the extensive and the intensive margin. The reason to do so is that having equal productivity implies that the timing of choices does not affect the results.[7]

Note that equation (6) implies that setting a probability $\pi(d_i|d,n) < \eta(d_i|d)$ would reduce the cost $\mathcal{D}(\pi||\eta)$. However, recall that π is a proper probability distribution. Consequently, setting a low $\pi(d_i|d,n)$ would require setting a larger value somewhere else in the distribution π, increasing the total cost. In fact, it is easy to show that $\mathcal{D}(\pi||\eta) > 0$ for any distribution π different from η, and 0 if $\pi \equiv \eta$. The same reasoning applies to the choice of λ in equation (5).

One of the advantage of using the Kullback-Leibler divergence to measure the cost of firm choices is that it generates closed-form solutions for both the chosen probability λ and the chosen distribution π. In particular, taking the first order condition of (4) with respect to the probability $\pi(d_i|d,n)$ is given by:

$$V(d_i,n) = \frac{1}{\kappa_I}\left[1 + \log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|d)}\right)\right] + \xi, \tag{7}$$

where ξ is the multiplier on the constraint (3). The left-hand side captures the marginal gain from increasing $\pi(d_i|d,n)$, which equals the value of the firm with productivity d_i, while the right-hand side captures the marginal cost. The marginal cost is the sum of two terms: the "direct" innovation cost associated to the choice of $\pi(d_i|d,n)$ and the cost associated to the constraint. After using equation (7) in the constraint (3) and some rearrangement, one finds:

$$\pi(d_i|d,n) \;=\; \eta(d_i|d)\left[\frac{\exp\Big(\kappa_I V(d_i,n)\Big)}{\sum_{j=1}^{D}\eta(d_j|d)\exp\Big(\kappa_I V(d_j,n)\Big)}\right]. \tag{8}$$

That is: the chosen distribution takes a logit form. Furthermore, equation (8) implies that firms will deviate more from the default distribution at the two extremes of the range of d_i. On the one hand, very low values of d_i would imply a large fall in the value of the

[7]. In short, when κ_I is the same for both the extensive and the intensive margin choices, results are not affected by the order in which these two decisions are taken, as shown by Costain (2017). In fact, any combination of π and λ can be expressed as a distribution, so that one could solve the problem in one stage. Defining the innovation problem in two stages, however, allows for a cleaner interpretation of the paramters: $\bar{\lambda}$ is the innovation probability for a firms investing no resources in generating a new idea, while the parameters of η describe the distribution of the next period's productivity for a non-innovative firm.

firm, and thus, the firm optimally chooses to reduce the probability of such event. On the other, large values of d_i increase this value, so that firms will optimally choose to assign more probability to those. Note, however, that firms will not be able to assign a positive probability to very large values of d_i if $\eta(d_i|d) = 0$ as it would be infinitely costly. For the same reasons, setting $\pi(d_i|d, n) = 0$ for very low values of d_i is not feasible if $\eta(d_i) > 0$. Using equations (8) and (6), we can write the value of innovating as:

$$\mathcal{I}^I(d, n) \;=\; \frac{1}{\kappa_I} \log \left[\sum_{i=1}^{D} \eta(d_i|d) \exp\Big(\kappa_I V(d_i, n)\Big) \right] \tag{9}$$

Note that $E\left[\exp(x)\right] > \exp\left[E(x)\right]$, and thus, $\mathcal{I}^I(d, n) \geq \sum_{i=1}^{D} \eta(d_i|d) V(d_i, n) = \mathcal{I}^N(d, n)$.[8]

Finally, the first order condition of equation (4) with respect to the probability of innovation λ is:

$$\mathcal{I}^N(d, n) - \mathcal{I}^I(d, n) = \frac{1}{\kappa_I} \left[\log \lambda - \log \bar{\lambda} - \log(1 - \lambda) + \log(1 - \bar{\lambda})\right],$$

where the left-hand side are the gains from innovating, equal to the marginal product of λ, and the right-hand side is the marginal cost. Rearranging terms:

$$\lambda(d, n) \;=\; \frac{\bar{\lambda} \exp\Big(\kappa_I \mathcal{I}^I(d, n)\Big)}{\bar{\lambda} \exp\Big(\kappa_I \mathcal{I}^I(d, n)\Big) + (1 - \bar{\lambda}) \exp\Big(\kappa_I \mathcal{I}^N(d, n)\Big)}. \tag{10}$$

The probability of innovation $\lambda(d, n)$ is increasing in the difference between $\mathcal{I}^I$ and $\mathcal{I}^N$, which implies that $\lambda \geq \bar{\lambda}$, since $\mathcal{I}^I(d, n) \geq \mathcal{I}^N(d, n)$.

3.4 Households

The household problem follows Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2019). In particular, there is a homogeneous household with a continuum of members who own the firms, consume and supply labor. The problem reads:

$$U \;=\; \max_{C,L} \;\; \ln C \;-\; \theta L, \quad \text{s.t.} \;\; C \;=\; wL + F + \Pi \tag{11}$$

8. Appendix A derives this expression and explains how to implement the solution to this problem in the computer.

where C is household consumption, L is the total labor supply, F are the total firing taxes and Π are firms' profits. The parameter $\theta > 0$ captures the disutility of labor supply.

3.5 Stationary equilibrium

Let $x = (d, n)$ be the state vector, $\mathcal{X} \equiv \mathbb{D} \times \mathbb{R}_{\geq 0}$ be the state space and F be the distribution of firms over $\mathcal{X}$. For simplicity in the exposition, I consider a discretized state space so that $F(x)$ is the mass of firms with state x. The law of motion of the distribution of firms is

$$F'(x) \;=\; (1-\delta)\sum_{z\in\mathcal{X}}\Gamma(x|z)F(z) \;+\; \delta\Gamma^E(x)$$

where F' is the next period's distribution of firms, $\Gamma(x|z)$ is the incumbents' transition probability between states z and x, derived from firm choices, and Γ^E is the distribution of entrants that results from the discretization of the distribution of d_0.

The equilibrium of this economy is given by a wage rate, a distribution of firms over the state space, and a set of firm's policy functions (for n', λ and π) such that (i) policy functions solve firms' problem, (ii) the household first order condition is satisfied, (iii) labor market clears, and (iv) the distribution of firms over the state space $\mathcal{X}$ is invariant, $F'(x) = F(x)$, $\forall x \in \mathcal{X}$.

4 Calibration

The model is calibrated to the Spanish economy, using data from the Central de Balances dataset. This is a panel of non-financial Spanish firms, prepared by the Bank of Spain, including balance sheet information, income statement and some firm characteristics (sector, age, etc). The panel covers the years 1995 to 2015 and provides an excellent representation of the Spanish productive sector.[9] Since Spanish employment is highly volatile, I restrict the sample to years between 2005 and 2007 in order to avoid the Spanish boom (2000-2005) and the financial crisis of 2007. The model period is set to 1 year.

9. See Almunia et al. (2018) for an analysis of the Central de Balances dataset representativeness.

4.1 Exogenous parameters

I set the discount factor to $\beta = 0.95$.[10] I set the degree of returns to scale to $\gamma = 0.6$, somewhat lower than in Hopenhayn and Rogerson (1993), but within the standard values in the literature.[11] I normalize the equilibrium wage rate to 1 and make θ be such that the household first order condition is satisfied in the benchmark equilibrium. Finally, I set the exit probability parameter to 7.56% so that the average firm age in the model is 9.7 years, as in the data.

4.2 Endogenous parameters

The remaining parameters are internally calibrated using the model. In particular, I calibrate the mean and variance of the initial distribution of productivity, the firing cost parameter, the benchmark probability of innovation, the innovation cost parameters, and the benchmark distribution, η, which is modeled as:

$$\log(d') \;=\; \log(d) - \mu + \sigma\epsilon. \tag{12}$$

The parameter vector, $\Omega = (\mu_0, \sigma_0^2, \kappa_F, \bar{\lambda}, \kappa_0, \kappa_1, \mu, \sigma^2)$, is chosen such that the sum of squared differences between a set of model-generated moments and their empirical counterparts is minimized. In particular, $\hat{\Omega}$ solves:

$$\hat{\Omega} \;=\; \arg\min_{\Omega} \sum_{i=1}^{M} \omega_i \left(\frac{m_i(\Omega) - \bar{m}_i}{\bar{m}_i} \right)^2.$$

where M is the number of moments, ω_i the weight associated to moment i, and $m_i(\Omega)$ and $\bar{m}_i$ are the model-generated and empirical i-th moments respectively.

10. The average long-term government bond yields in Spain for the period 2005-2007 is 4% according to FRED data. I assume a risk premium of 1% and set the discount rate that corresponds to an annual interest rate of 5%.

11. Hopenhayn and Rogerson (1993) consider a degree of returns to scale of 0.64 for the US economy. Spain, however, is characterized for huge share of employment in small firms, so a value below 0.64 is a natural choice. Later I check how sensitive my results are to the value of this parameter.

Moment selection

My data lacks information on firms' innovation choices. Moreover, given the broad meaning of innovation in this paper, it is not clear what type of information one should use. However, the model establishes a clear link between productivity and size allowing me to discipline the innovation technology using employment data, as in Garcia-Macia et al. 2019. Note that hiring and firing choices in my model only depend on productivity, and thus, targeting the dynamics of employment would pin down the dynamics of productivity. For instance, given that productivity growth only emerges from innovation, the share of hiring firms and their growth rate are very informative about the share and growth rate of innovators. Thus, the model is calibrated to match the share of hiring firms and the hiring rate, defined as the ratio between hirings and previous employment, $\max\{0, n' - n\}/n$.

Innovation productivity decreases with firm productivity, which makes it costlier to innovate for high productivity firms. In order to control for the strength of this effect, I target the firm size distribution. Note that if innovation is equally costly for high and low productivity firms, high-productivity firms would grow faster than low-productivity ones, generating a bimodal firm size distribution. Given the focus of this paper on firing cost, firing behavior is particularly relevant for the analysis. I match the share of firing firms and the firing rate, defined analogously to the hiring rate. Finally, given that innovation is particularly flexible, it is important to control for the shape of the resulting distribution of next period's productivity. To do so I match the average and the coefficient of variation of firm size, both for the whole population of firms and for entrants.

Identification and model fit

Although all moments are affected by all the parameters, some relationship between specific parameters and moments can be postulated. The arguments that follow do not prove identification, but ease the interpretation of the parameter values.

The average productivity of entrants, μ_0, is particularly relevant to match the average size of entrants. The variance of the initial productivity draw, σ_0^2, drives the dispersion in firm size among entrants, and therefore, the coefficient of variation in firm size among entrants. The variance of the benchmark distribution σ^2 limits the dispersion of the chosen distribution among innovators, and thus, drives the overall dispersion in firm size. The parameter κ_0 controls how much innovative firms can grow and, as argued

Table 1: Calibration. Model fit

Moment	Model	Data
Average size of entrants	3.53	3.40
Coefficient of variation of firm size	1.21	1.19
Coefficient of variation of firm size among entrants	1.39	1.36
Share of firing firms	0.26	0.27
Share of hiring firms	0.35	0.34
Firing rate among firing firms	0.19	0.20
Hiring rate among hiring firms	0.44	0.44
Share of firms with 0-5 workers	0.63	0.60
Share of firms with 6-10 workers	0.21	0.20
Share of firms with 11-15 workers	0.07	0.08
Share of firms with 16-20 workers	0.04	0.04
Share of firms with 21-25 workers	0.02	0.02
Share of firms with 25+ workers	0.04	0.05

before, is informative to match the hiring rate observed in the data. The parameter κ_1 controls the rate at which the cost of innovation increases with firm's productivity, and thus, the ability to grow among high-productivity firms, driving the firm size distribution. Since productivity growth only emerges from innovation, the share of innovators is very informative about the share of hiring firms. The default probability of innovation $\bar{\lambda}$ limits precisely the probability of innovation and thus, is very informative about the share of hiring firms. Among those firms not innovating, the parameter μ drives the size in the productivity fall, and therefore, it is very informative about the firing rate. In fact, matching the firing rate is key to control the magnitude of downwards risk, which is a key driver of the effects of firing costs. Finally, the firing cost parameter κ_F drives the share of firms firing workers.

Table 2 collects the estimated parameters and table 1 the model fit. The model closely matches the moments concerning firing and hiring behavior, as well as the firm size distribution. The latter is particularly relevant since it provides support for the innovation technology used in the paper. Moreover, the model generates a distribution of firm size that matches, not just the average firm size, but also the dispersion in firm size, which provides further support to the innovation technology. In the next section, I discuss the main predictions generated by my innovation technology and show that those predictions are consistent with the existing empirical evidence on firm growth.

The firing cost parameter is calibrated to 0.20. This means that the cost of firing one worker equals 2.5 monthly wages. According to Spanish labor regulation, a dismissed

Table 2: Calibration. Parameter values

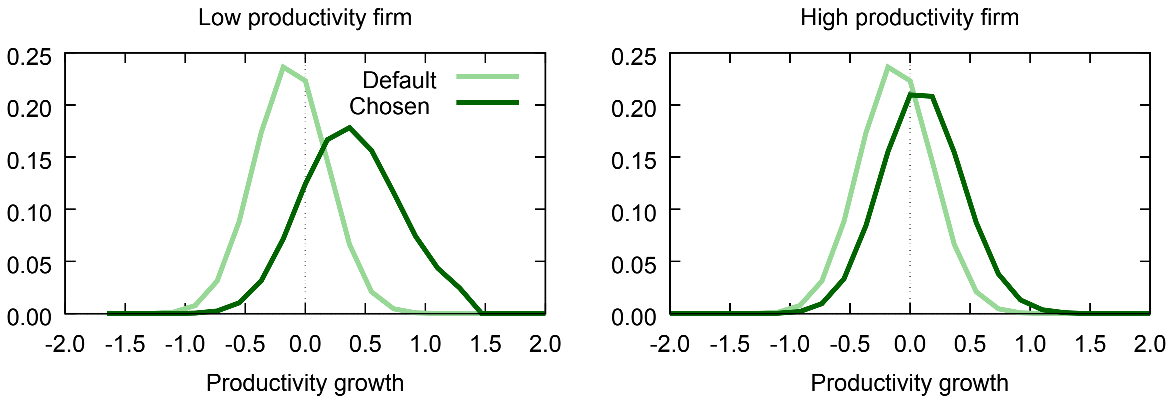
Parameter			Description
μ_0	$=$	2.95	Average productivity of entrants
σ_0	$=$	1.10	Standard deviation of initial productivity draw
μ	$=$	0.07	Depreciation of productivity (default distribution)
σ	$=$	0.30	Standard deviation of shocks (default distribution)
κ_0	$=$	0.14	Cost of innovation, level parameter
κ_1	$=$	1.25	Cost of innovation, shape parameter
$\bar{\lambda}$	$=$	0.47	Default probability of innovation
κ_F	$=$	0.20	Firing cost

worker has the right to received 40 days of wages per year worked in the firm. Note, however, that the Spanish economy is characterized by the heavy use of temporary workers, whose firing cost are either zero or very small. Thus, κ_F should be interpreted as an average firing cost for both temporary and permanent workers. The depreciation rate of productivity is calibrated to 0.07. Thus, a firm investing no resources in innovation expects to loss 7% of its current productivity next period. The productivity of innovation is decreasing in firm's productivity, which contributes to the good fit of the firm size distribution. The magnitude of κ_0 and κ_1 do not have a clear interpretation. However, they imply that firms in the baseline economy spend 16% of total output in innovation.[12] Although this may be too high for innovation expenses, it should be noticed that innovation in this model includes all sort of firm actions aimed at increasing profitability prospects, and not only product or process innovation as typically assumed in innovation papers.

The default probability of innovation is 0.47, which is 9 p.p. lower than the average innovation probability in the baseline economy. Given the structure of the innovation problem, most innovation investments are devoted to the choice of the next period's productivity. This is because the cost of choosing a distribution π is incorporated in the value of innovating, lowering gains for innovation, as shown in equation (4). As a result, higher investments in the distribution π lowers the incentives to invest in the innovation probability.

12. According to OECD Spanish firms spend around 1% of turnover on innovation. The data is available in the following link: http://dx.doi.org/10.1787/835838585236. However, this data only includes technological innovation (supply-side innovation).

Figure 2: Productivity growth. Next period's productivity distribution



Notes: The x-axis refers to the difference in log productivity $\Delta \log d$. The dark line is the chosen next period's distribution π for a low and a high productivity firm. The light line represents the default distribution, η, given by equation (12), which is the same for low and high-productivity firms by assumption.

5 Results

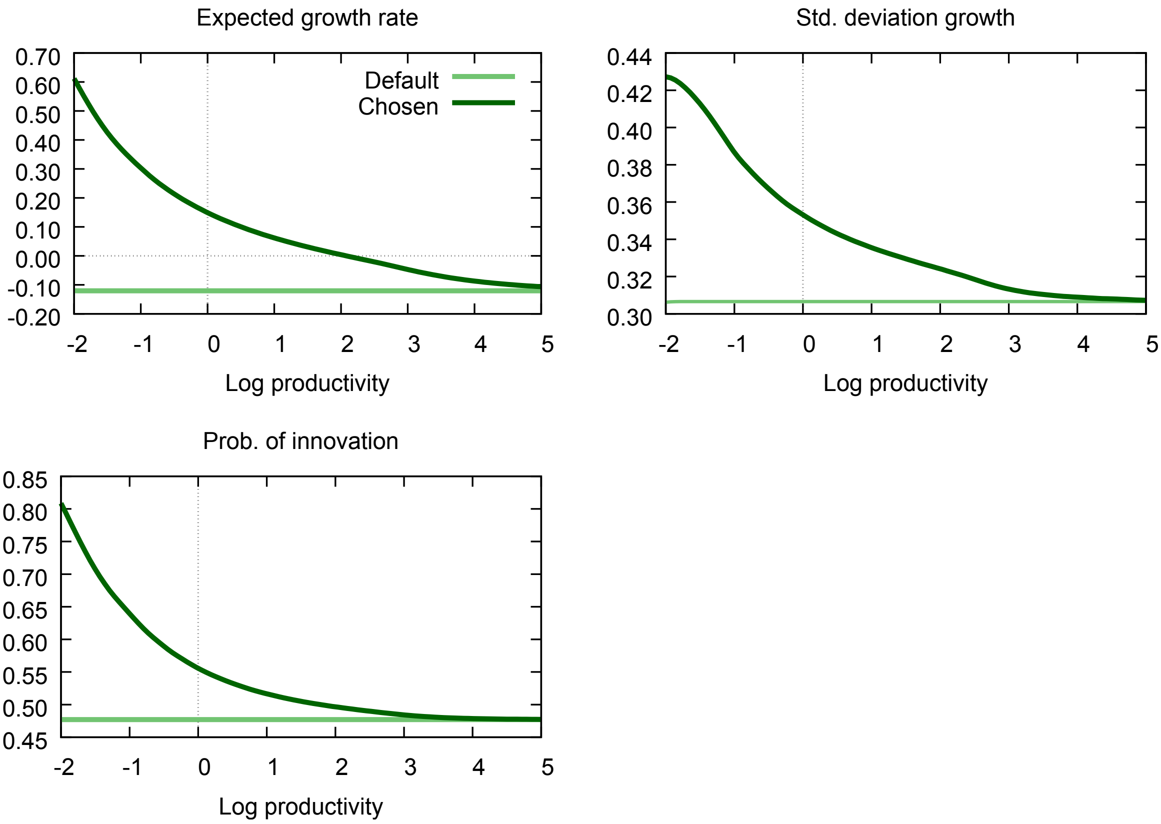
Before analyzing the effects of firing cost, it is worth describing firms' innovation behavior in the baseline equilibrium, to illustrate how the my approach to model firm innovaiton can generate realistic productivity dynamics. Then, sections 5.2 to 5.4 presents the aggregate impact of changing the value of κ_F, and its decomposition.

5.1 Endogenous productivity dynamics

Many papers in the literature of firm growth document the negative relationships between firm size and growth and between firm size and volatility of growth.[13] The model is consistent with these facts. Figure 2 presents the default and chosen distributions of productivity growth for a low- and high-productivity firm. The average productivity growth in case of innovation (thus, taking the chosen distribution, π) is as high as 0.22 for low productivity firms and 0 for high productivity firms, who just offset the negative productivity trend. At the same time, the standard deviation of productivity growth is of 0.45 for low productivity firms, and of 0.35 for high productivity ones. Key for this result is the fact that the productivity of innovation is assumed to be decreasing in firm's productivity.

13. See for example Sutton (1997), Sutton (2002) or Klette and Kortum (2004). See figures B.5 and B.6 for the corresponding relationships in my data.

Figure 3: Innovation choices, by firm productivity



Notes: I compute the expected productivity growth rate and standard deviation of productivity growth for each point in the discretized state space using the corresponding distribution of next period's productivity, π (chosen) or η (default), and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d, where the default probability is $\bar{\lambda}$ and the chosen one is given by $\lambda(d, n)$. Figure B.7 replicates these graphs by number of employees.

This can be seen more generally in figure 3, where I plot the expected productivity growth rate, the standard deviation of firm productivity growth and the probability of innovation by firm productivity in the baseline economy in which $\kappa_F = 0.20$. Later we will discuss how these figures change when we increase/decrease the firing cost. Three main predictions arise from the model: (i) low productivity firms innovate more frequently, (ii) they undertake more aggressive innovations and (iii) their innovations are riskier, as measured by the expected productivity growth and the standard deviation of expected firm productivity growth, respectively. As a result, low productivity (small) firms in the model grow faster and face higher uncertainty.

Figure 3 highlights the importance of allowing firms to have (partial) control over the whole distribution of next period's productivity. Models based on Atkeson and Burstein (2010) allow firms to affect the probability of innovation while keeping fixed the "size" of the innovation. Alternatively, once could fix the probability of innovation and allow firms to invest in the average productivity growth. However, in both cases, the volatility of productivity growth is constant across firms, and unaffected by the distortion. In this model, firms endogenously face different degrees of uncertainty, which is key to account for the effects of firing costs (Bentolila and Bertola 1990).

5.2 Aggregate effects of firing costs

The main goal of this paper is to better understand the aggregate consequences of firing costs. To facilitate the exposition and the comparison with previous literature, I simulate the frictionless economy, in which $\kappa_F = 0$, and compare it with an economy with positive firing costs. But before going over the results, we first need to define the main object of interest: aggregate productivity. I define aggregate productivity as:

$$\text{aggregate productivity} \;=\; \left(\int_{x \in \mathcal{X}} d(x)^{1-\gamma} s(x) d\mu(x) \right)^{\frac{1}{1-\gamma}} \tag{13}$$

where $x = (d, n)$ is the firm's state vector, $s(x) = n^{\gamma}(x) / \left(\int_{x \in \mathcal{X}} n(x)^{\gamma} d\mu(d, x) \right)$, and $\mu(x)$ is the stationary mass of firms with state x, satisfying $\int_x d\mu(x) = 1$.[14]

Table 3 collects the results of this experiment. Table entries represent the percentage (negative) change in the corresponding variable relative to the frictionless economy. In the first column, I compare the frictionless economy with the one that arises from the calibration exercise presented in section 4, in which the firing cost is $\kappa_F = 0.20$. The second column collects the results from simulating an economy in which I set the firing cost to $\kappa_F = 0.40$, twice as large as the calibrated value. Finally, for comparison with the literature, I simulate an economy in which firing costs are equivalent to one year's wage. Figure B.10 plots the percentage change in aggregate productivity and in average productivity for different values of κ_F, ranging from zero to 0.40, both for the general equilibrium solution and for the partial equilibrium one.

In line with the findings of previous literature, I find that firing costs damage aggregate productivity significantly. In particular, a firing cost equivalent to 2.5 monthly wages generates a 4% fall in aggregate productivity relative to the frictionless economy. This

14. This definition of aggregate productivity closely follows the one used in Da-Rocha et al. (2019), adapted to the production function presented in section 3.

Table 3: Aggregate effects of firing cost
(% fall relative to frictionless economy)

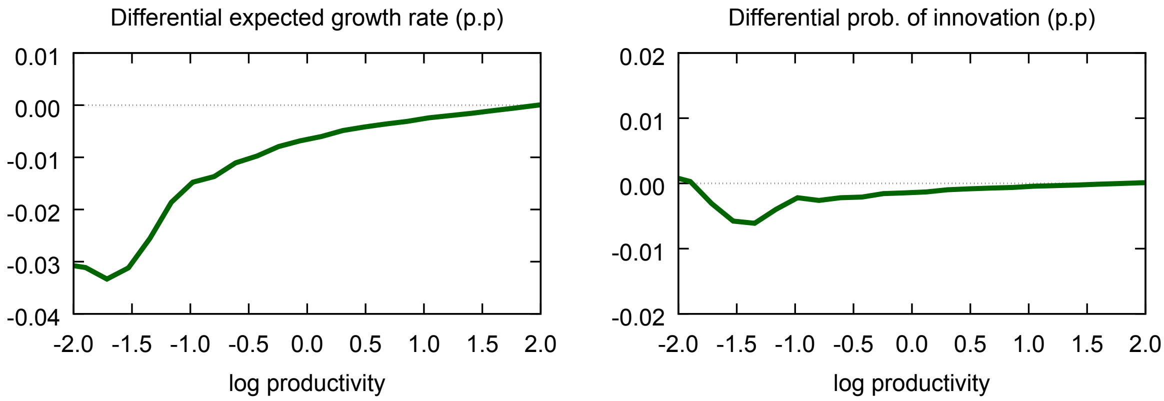
	$\kappa_F = 0.20$	$\kappa_F = 0.40$	$\kappa_F = 1.00$
Aggregate productivity	4.05	6.69	12.7
Output	2.50	4.54	9.46
Average productivity	1.82	3.10	6.54
Average firm size	2.55	4.67	9.67
Innovation expenses	3.47	5.86	11.8
Job destruction rate	52.5	68.6	85.7
Job creation rate	30.8	40.3	50.3

is a large number compared to the literature. In Hopenhayn and Rogerson (1993) they find a 2.1% decrease in productivity from a firing cost equivalent to one year's wage. In my model, the fall in aggregate productivity from a firing cost of this size is larger than 12%. Da-Rocha et al. (2019) find a 20% fall in aggregate productivity from a firing cost equivalent to 5 year's wage. My model generates more than half of the fall found by Da-Rocha et al. (2019), with a firing cost of just 1 year's wage.

These comparisons, however, must be taken with caution. Both Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2019) are calibrated to the US. Moreover, Hopenhayn and Rogerson (1993) consider a model with endogenous exit and mass of firms, while Da-Rocha et al. (2019) and I assumes a constant mass of firms and exogenous exit. These differences make the comparison not perfect. However, it is still useful to compare my results to those found by these two papers to put the magnitude of my findings into some context. The conclusion that arises from this comparison is that the effects of firing cost on aggregate productivity are significantly larger than previously thought when the productivity distribution is endogenous.

The main additional channel compare to previous papers is that firing costs in my model affect the whole productivity distribution by changing firms' incentives to innovate. When firing costs are introduced, growing larger implies a higher potential cost of firing in the future, increasing the overall (expected) cost of innovation. To quantitatively see how firing costs shift firms' innovation decisions, I plot the differential expected productivity growth rate and the differential probability of innovation between the frictionless economy and one in which firing cost are set to $\kappa_F = 0.2$ in figure 4. Figures B.8 and B.9 plot the same results when the distorted economy has a level of firing cost 0.4 and of one year's wage respectively.

Figure 4: Innovation choices. Experiment, $\kappa_F = 0.2$ *vs.* $\kappa_F = 0$



Notes: To compute the differences in expected productivity growth between non-innovative and innovative firms, I average expected growth over firm size for each productivity d using the corresponding distribution of next period's productivity (π for innovative firms and η for non-innovative firms) as in figure 3. I do the same for the probability of innovation.

Firms invest less in both the probability of innovation and in the outcome of such innovation, as measured by the expected productivity growth. In particular, the aggregate innovation expenses fall by 3.5%, inducing a 1.82% reduction in the (unweighted) average productivity in the economy. This change is mainly due to adjustments in the amount of resources invested in the next period's productivity. The drop in the expected productivity growth is substantial: up to 3 p.p. lower productivity growth rate for low productivity firms. The fall is up to 12 p.p. when setting the firing cost to $\kappa_F = 1$. The probability of innovation is almost unaffected by changes in κ_F. The reason is that both the value of innovating and the value of not innovating fall when firing costs increase and thus, gains from innovation are roughly equal to those in the frictionless economy. Despite being unaffected by changes in the firing cost, the innovation probability is still an important margin in the analysis. This is because the probability of innovation is (endogenously) different for low and high productivity firms, crucially affecting their incentives to fire and hire workers.

The distorted economy also exhibits lower job destruction and creation rates (defined as total firings/hirings over total employment). In particular, the share of newly hired workers in the economy falls by 30%, while the share of fired workers drops by more than 52%. Since firms find it costlier to fire workers now than before, they decide to keep workers even if their size is larger than the optimal one. At the same time, firms below their optimal size decide not to hire due to precautionary motives. Since there

Table 4: Sensitivity analysis

(% fall in aggregate productivity relative to frictionless economy)

	Parameter (benchmark % fall in aggregate productivity = 4.05)	Shock	
		+5%	−5%
μ_0	Average productivity of entrants	3.63	4.08
σ_0	Standard deviation of initial productivity draw	4.04	4.07
μ	Depreciation of productivity (default distribution)	4.12	3.96
σ	Standard deviation of shocks (default distribution)	4.09	3.93
κ_0	Cost of innovation, level parameter	4.06	4.02
κ_1	Cost of innovation, shape parameter	4.09	3.94
$\bar{\lambda}$	Default probability of innovation	3.99	4.10

is uncertainty about future productivity, the firms know that they may need to fire in the future, which prevents them from hiring in the first place. Note that the change in job creation is less pronounced than that in job destruction, as in Bentolila and Bertola (1990). These two distortions give rise to inefficiencies in the allocation of labor, which further damages aggregate productivity.

5.2.1 Sensitivity analysis

In this section, I check how sensitive the results presented in table 3 are to changes in the calibrated parameter values. In particular, I compare the aggregate productivity losses from a firing cost of 2.5 monthly wages shocking each calibrated parameter at a time, first increasing it by 5%, and then lowering it by 5%. To ensure comparability, I recompute the disutility of labor supply, θ, so that the equilibrium wage is equal to 1 for each alternative calibration.

The results, collected in table 4, suggest that the evaluation of the fall in aggregate productivity from firing costs of 2.5 monthly wages ($\kappa_F = 0.20$) is very robust to changes in the calibrated parameters. Table B.1 collects the results from this sensitivity analysis including all the relevant variables presented in table 3.

Another important parameter of the model is the degree of returns to scale, γ. In order to check how sensible my results are to the value of γ, I set a $\gamma = 0.66$ (a 10% increase relative to its baseline value), recalibrate the rest of the parameters, and then compute the losses in aggregate productivity associated with firing costs. I find that aggregate productivity falls by 4.94%, 8.67% and 15.3% for a level of firing cost equivalent to 2.5 monthly wages ($\kappa_F = 0.2$), 5 monthly wages ($\kappa_F = 0.40$) and one year wages ($\kappa_F = 1$)

respectively. These numbers are slightly higher than the results presented in the first row of table 3, suggesting that my choice of γ is conservative.

5.3 What is the role of endogenous productivity dynamics?

In order to clearly identify the role of endogenous firm productivity in accounting for the fall in aggregate productivity, I repeat the experiments shown in section 5.2 fixing the innovation behavior from the frictionless economy. In short, I simulate a distorted economy in which I impose a law of motion for firm productivity given by

$$d' \sim \begin{cases} \pi(d, n|\kappa_F = 0) & \text{w.p.} \quad \lambda(d, n|\kappa_F = 0) \\ \eta(d) & \text{w.p.} \quad 1 - \lambda(d, n|\kappa_F = 0) \end{cases}$$

where $\lambda(d, n|\kappa_F = 0)$ and $\pi(d, n|\kappa_F = 0)$ are the resulting innovation probabilities and distributions from the frictionless economy in which firing costs are set to zero. To make the two economies comparable, I also keep fixed the cost of innovation which is now added as a fixed cost to the value of the firm. Results are collected in table 5. The first two columns collect the results from the exercise in section 5.2, in which innovation is endogenous, and thus reacts to changes in κ_F. The two last columns collect the results from changing the firing cost in an economy with exogenous innovation, in which I fixed the innovation behavior that arises the frictionless economy.

In the model with exogenous innovation, a firing cost of $\kappa_F = 0.2$ implies a fall in aggregate productivity of 2.3% which is significantly lower than in a model with endogenous productivity dynamics. In particular, changes in firms' innovation choices account for around 43% of the aggregate productivity losses associated to a firing cost of $\kappa_F = 0.20$, more than 44% and 46% when I set $\kappa_F = 0.40$ and $\kappa_F = 1$ respectively. In Da-Rocha et al. (2019) they find that 80% of the overall drop in aggregate productivity is accounted by changes in the distribution of firms. This is much larger than in my model. The reason is that they do not allow firms to adjust the dynamics of productivity when the firing cost parameter changes. In their model, the dynamics of firm productivity are size-dependent, but the differences between large and small are exogenous are fixed. Thus, conditional on firm size, the law of motion of firm productivity is unchanged when the firing cost parameter changes.

Table 5: Aggregate effects of firing cost. Exogenous innovation
(% fall relative to frictionless economy)

	Endogenous Inn.			Exogenous Inn.		
Firing cost, κ_F	0.20	0.40	1.00	0.20	0.40	1.00
Aggregate productivity	4.05	6.69	12.7	2.26	3.72	6.84
Output	2.50	4.54	9.46	1.74	3.38	7.05
Average productivity	1.82	3.10	6.45	0.00	0.00	0.00
Average firm size	2.55	4.67	9.67	2.52	4.75	9.72
Innovation expenses	3.47	5.86	11.8	0.00	0.00	0.00

Endogenous firm productivity dynamics are also important in accounting for the changes in aggregate output. In particular, the fall in aggregate output with exogenous innovation is equal to 2.5%, 2.4% and 7% when firing cost is 0.2, 0.4 and 1 respectively. This represents a 35% to 25% of the overall fall in aggregate output. The effects of firing costs on the average firm size are similar both with endogenous and with exogenous innovation. The reason is the different response of the wage rate in equilibrium. When innovation is endogenous, the wage rate falls by 1.7% when firing costs are of 2.5 monthly wages, and by more than 5% when they are of one year's wage. When innovation is exogenous, these numbers are 0.9% and 3%.

6 Conclusions

This paper presented a firm dynamics model with endogenous productivity growth to analyze the aggregate effects of firing cost. Making the dynamics of productivity endogenous allows the model to capture both the static effects of firing taxes —allocative efficiency— as well as the dynamic effects of such friction —changes in the distribution of firms' productivity. It is the first model that introduces an innovation technology that allows firms to control not only the probability of innovation but also the outcome. The model parameters are calibrated so as to match the firm size distribution and the hiring and firing behavior of Spanish firms. I show that my flexible innovation technology is able to generate a distribution of firm size that is very close to that in the data, both in terms of size and in terms of dispersion. Moreover, the model is also able to generate larger and more volatile growth among low productivity firms.

I use the calibrated model to quantitatively asses the aggregate effects of firing cost.

I show that a firing cost equivalent to 2.5 monthly wages (the calibrated value) generates a 3% drop in aggregate productivity relative to the frictionless economy. When firing cost is equivalent to one year's wage, the fall in productivity is of more than 10%, substantially larger than found in previous literature. I then decompose the fall in aggregate productivity between losses in allocative efficiency and changes in the distribution of firm productivity, by fixing the law of motion of firm-level productivity to the one that arises endogenously from the frictionless economy. I show that 55% of the aggregate productivity losses are explained a worse allocation of labor across firms, while the remaining 45% is accounted for changes in the distribution of productivities.

This result suggests that researchers should take the effects of frictions on the dynamics of productivity into account when evaluating their aggregate effects. This paper applies this idea to firing cost, but it can be extended to any other frictions, such as distortionary corporate taxation or credit constraints.

My paper focuses on the effects of firing costs on firms. However, the literature has shown that firing costs may generate important welfare gains once we incorporate risk-averse workers into the model. An interesting avenue for future research would be to compute a welfare analysis of firing costs, incorporating heterogeneous risk-averse workers and hiring frictions into the model. It would also be interesting to see how employment protection can be redefined to overcome its negative impact of firms' incentives to grow. An example would be to make firing costs to depend on firm age, such that firing costs do not prevent young firms to invest in growth generating activities. I leave these questions for future research.

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Online appendix

A Computation

In this section, I briefly describe how to solve the model numerically. First, I discretize the state space is $\#_d \times \#_n$ points, where $\#_d = 60$ is the number of points in the grid for productivity and $\#_n = 50$ is the number of points in the grid for employment.[15]

The problem in (2) is solved by value function iteration. For each point in the state space, (d, n), I find the optimal employment choice, n', using the Golden Search algorithm. This algorithm does not ensure finding a global maxima when the objective function is not well-behaved. To make sure I pick the optimal employment choice, I use the algorithm to solve for the optimal employment choice conditional on $n' > n$ and $n' < n$ separately, and then compare the two solutions with $n' = n$. Given the optimal choice of n', I compute the distribution of next period's productivity using equation (8). I repeat this algorithm until the value function converges.

The exponential term in equation (8) can easily go to infinity, depending on the maximum real number the computer can manage. To avoid this computational problem, one can redefine the value function and define equation (8) as:

$$\pi(d_i|d,n) = \frac{\eta(d_i|d)\exp\Big(\kappa_I \tilde{V}(d_i,n)\Big)}{\sum_{j=1}^{D}\eta(d_j|d)\exp\Big(\kappa_I \tilde{V}(d_j,n)\Big)} \tag{14}$$

where $\tilde{V}(d,n) = V(d,n) - \mathbb{C}$ and $\mathbb{C} = \max\{V(\cdot,n)\}$. Note that this normalization does not alter the value of $\pi(d'|d,n)$, but ensures that the exponential term is never larger than

15. The grid sizes are such that increasing them does not alter the results.

one. Using this normalization, the cost of innovation becomes:

$$\begin{aligned}
\mathcal{D}(\pi||\eta) &= \frac{1}{\kappa_I}\left[\sum_{i=1}^{D}\pi(d_i|d,n)\log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|n)}\right)\right] = \\
&= \sum_{i=1}^{D}\pi(d_i|d,n)\tilde{V}(d_i,n)dx - \frac{1}{\kappa_I}\log\left[\sum_{i=1}^{D}\eta(d_i|d)\exp\left(\kappa_I\tilde{V}(d_i,n)\right)\right] = \\
&= \sum_{i=1}^{D}\pi(d_i|d,n)V(d_i,n)dx - \mathbb{C} + \frac{1}{\kappa_I}\kappa_I\mathbb{C} - \frac{1}{\kappa_I}\log\left[\sum_{i=1}^{D}\eta(d_i|d)\exp\left(\kappa_I V(d_i,n)\right)\right] = \\
&= \sum_{i=1}^{D}\pi(d_i|d,n)V(d_i,n)dx - \frac{1}{\kappa_I}\log\left[\sum_{i=1}^{D}\eta(d_i|d)\exp\left(\kappa_I V(d_i,n)\right)\right]
\end{aligned}$$

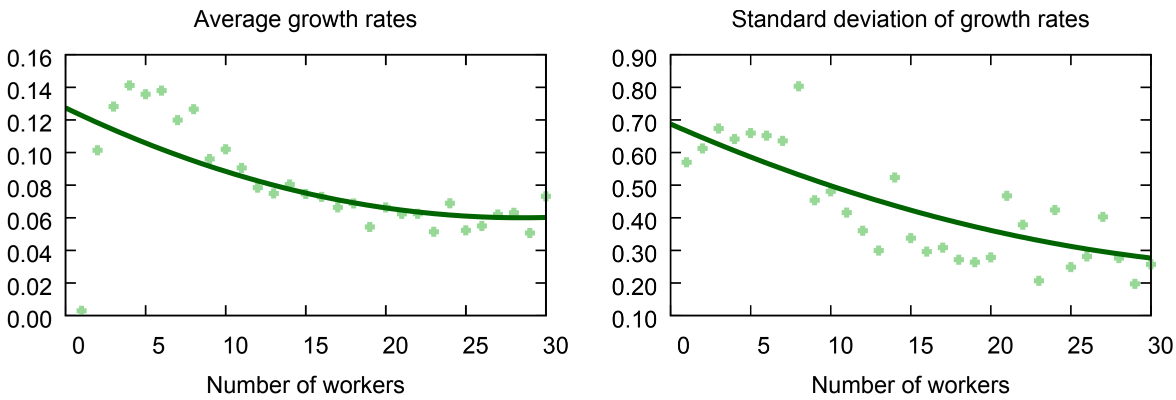
and the value function at the innovation stage:

$$\mathcal{I}^I(d,n) = \sum_{i=1}^{D}\pi(d_i|d,n)V(d_i,n) - \mathcal{D}(\pi||\eta) = \frac{1}{\kappa_I}\log\left[\sum_{i=1}^{D}\eta(d_i|d)\exp\left(\kappa_I V(d_i,n)\right)\right]$$

which equals the expression derived in section 3.3.

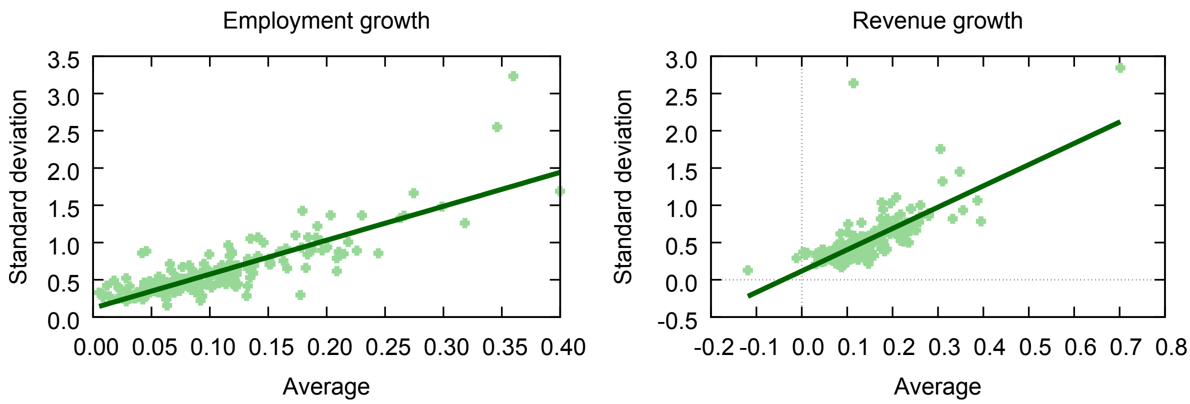
B Additional figures and tables

Figure B.5: Firm growth and growth volatility by firm size



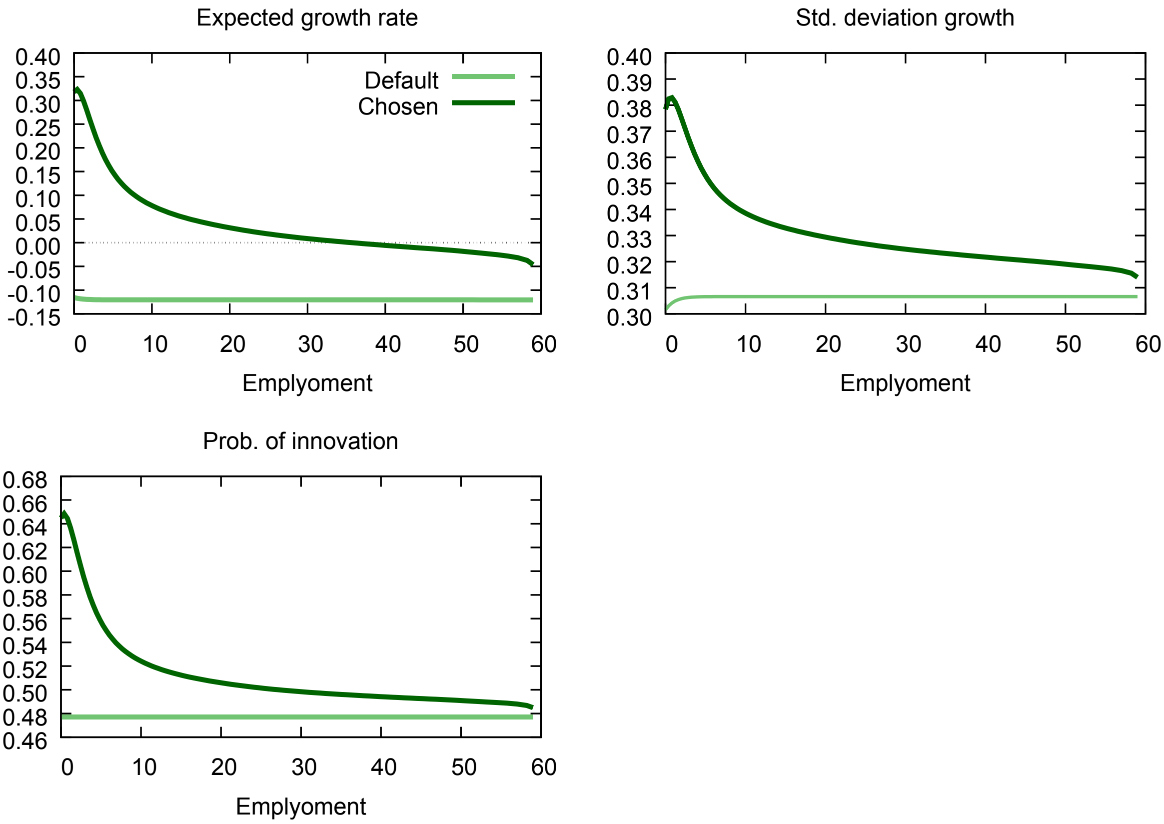
Notes: Dots represent size-specific average and standard deviation of employment growth rates, and the dark line is a quadratic fit. *Source*: Central de Balances dataset, 2005-2007.

Figure B.6: Firm growth and growth volatility across sectors



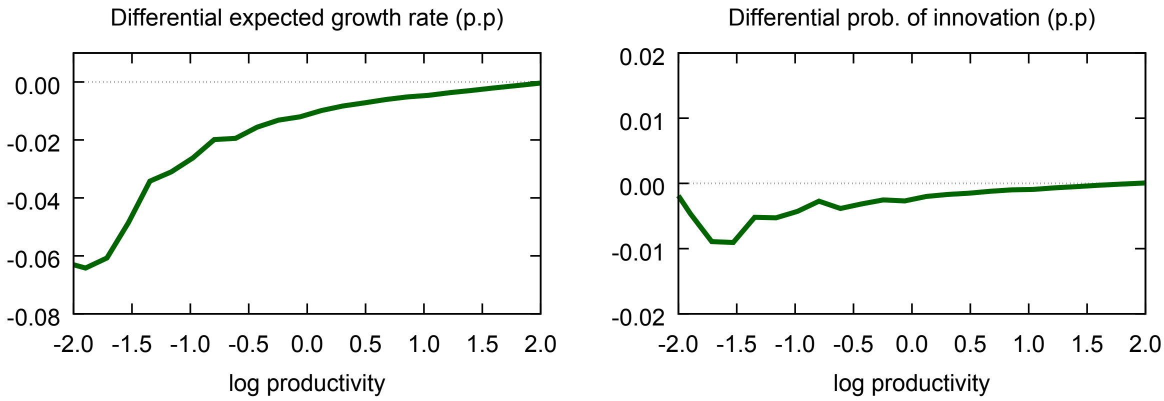
Notes: Dots represent sector-specific average and standard deviation of employment and revenues growth rates, and the dark line is a linear fit. *Source*: Central de Balances dataset, 2005-2007.

Figure B.7: Innovation choices, by firm size



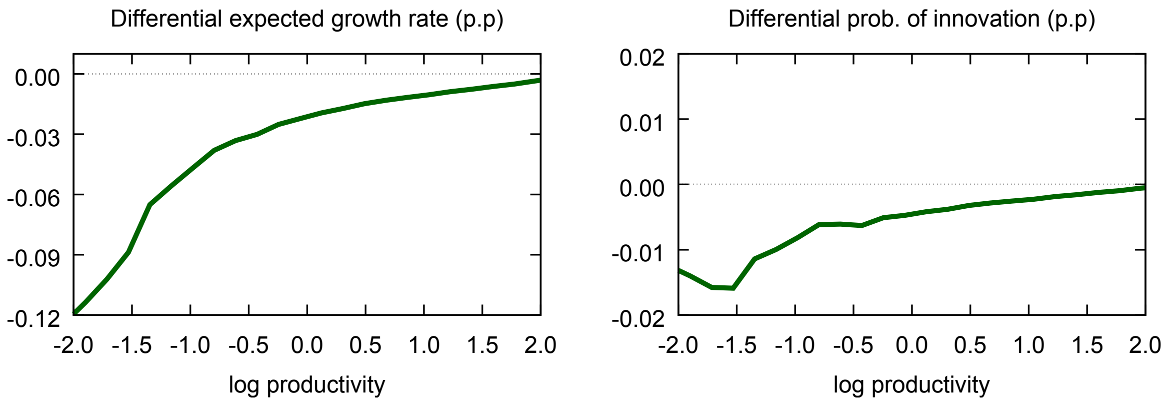
Notes: I compute the expected productivity growth rate and standard deviation of productivity growth for each point in the discretized state space using the corresponding distribution of next period's productivity, π or η, and then average across productivity for each value of n. The probability of innovation is also averaged across productivity for every value of n.

Figure B.8: Innovation choices. Experiment, $\kappa_F = 0.4$ *vs.* $\kappa_F = 0$



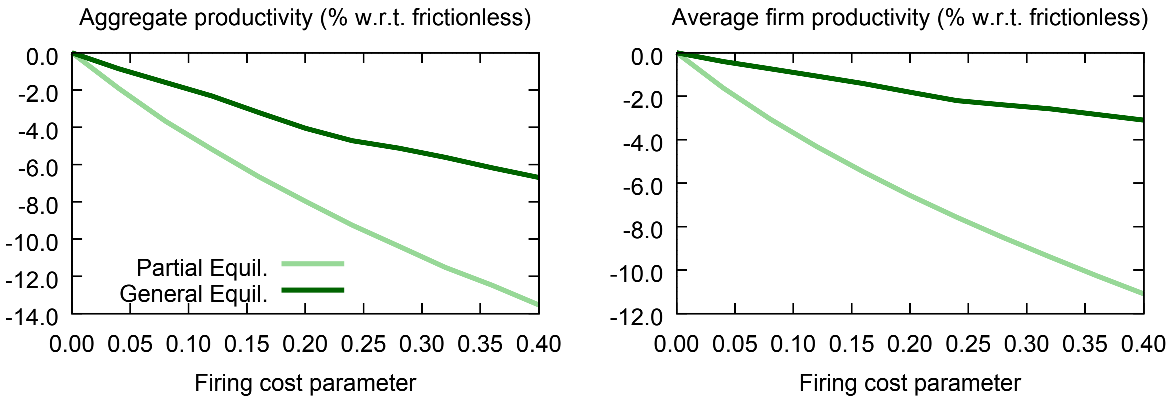
Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.9: Innovation choices. Experiment, $\kappa_F = 1$ *vs.* $\kappa_F = 0$



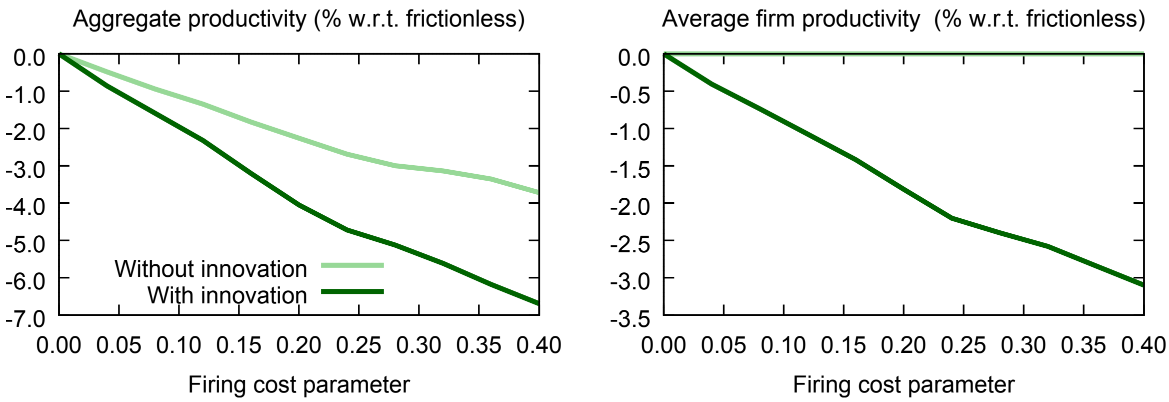
Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.10: Aggregate effects of firing costs. General *vs.* Partial equilibrium



Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The light line represents the partial equilibrium results, where the wage rate is not adjusted. The dark line represents the general equilibrium results that emerge from adjusting the wage rate.

Figure B.11: Aggregate effects of firing costs. Exogenous *vs.* Endogenous innovation



Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The dark line represents the results when innovation is endogenous, and thus, firms' innovation choices react to changes in the firing cost. The light line represents the results when innovation is exogenous so that innovation choices are unaffected by changes in the firing cost.

Table B.1: Sensitivity Analysis – More results

(% fall relative to frictionless economy)

		Aggregate productivity	Average productivity	Innovation expenses	Aggregate output	Aggregate employment	Job Destruction	Job Creation
Benchmark		4.05	1.82	3.47	2.50	2.55	52.48	30.83
+5%	μ_0	3.63	1.86	3.23	2.48	2.51	52.55	29.87
	σ_0	4.04	1.82	3.48	2.50	2.55	52.45	30.80
	μ	4.12	1.78	3.49	2.51	2.59	52.08	30.91
	σ_z	4.09	1.95	3.47	2.54	2.53	52.02	30.62
	κ_0	4.06	1.73	3.48	2.49	2.59	52.06	30.86
	κ_1	4.09	1.71	3.45	2.49	2.60	52.33	31.07
	$\bar{\lambda}$	3.99	1.83	3.42	2.48	2.52	52.86	30.83
-5%	μ_0	4.08	1.51	3.31	2.47	2.67	51.85	31.24
	σ_0	4.07	1.81	3.46	2.50	2.56	52.52	30.87
	μ	3.96	1.85	3.45	2.48	2.51	52.88	30.75
	σ_z	3.93	1.62	3.41	2.42	2.55	52.99	31.09
	κ_0	4.02	1.89	3.44	2.50	2.51	52.98	30.82
	κ_1	3.94	1.94	3.48	2.49	2.47	52.76	30.53
	$\bar{\lambda}$	4.10	1.80	3.52	2.51	2.58	52.11	30.84