Aggregate Effects of Firing Costs with Endogenous Firm Productivity Growth

Borja Petit

CEMFI

March 2019

Abstract

This paper quantifies the aggregate effects of firing costs in a model of firm dynamics where firm-level productivity is determined by innovation. In the model, the productivity distribution is endogenous, and thus, potentially affected by policy changes, allowing the model to capture both the static (allocative efficiency) and dynamic effects (changes in the distribution of firms' productivity) of firing costs. The model is calibrated so as to match some key features of firms' hiring and firing behavior using firm-level data from Spanish non-financial firms. I show that firing costs equivalent to 2.5 monthly wages produce a 3% loss in aggregate productivity relative to the frictionless economy. The aggregate productivity losses rise to more than 10% when firing cost are equivalent to one year's wage. I show that the dynamic effects of firing costs are quantitatively relevant, explaining 45% of these productivity losses. Overall, the results suggest that ignoring the effects of frictions on the dynamics of firm productivity can substantially underestimate their aggregate effects.

JEL Codes: O1, O4, E1, E6

Key words: Firing cost, productivity, misallocation, firm innovation, firm growth

---

This project has largely benefited from discussion with Samuel Bentolila, Nezih Guner, Josep Pijoan-Mas and Diego Restuccia. All remaining errors are my sole responsibility. I thank Diego Astorga, Javier López-Segovia and Juan Carlos Ruiz-García for reading the first version of this paper and provide me with useful feedback.

Funding from Spain's Ministerio de Economía y Competitividad (Grants ECO2016-79848-P, BES-2017-082181 and María de Maeztu Programme for Units of Excellence in R&D, MDM-2016-0684) is gratefully acknowledge.

Email: bpetit@cemfi.edu.es. Mailing address: CEMFI, Calle Casado del Alisal 5, 28014 Madrid, Spain.

1 Introduction

There is a large body of research studying the productivity losses from firing costs. Following Hopenhayn and Rogerson (1993), most of this literature typically quantify the effects of firing cost by looking at the efficiency in the allocation of labor across firms, given a productivity distribution. However, if firm growth is a risky process, firing costs would be a critical component of the cost of failure, affect the incentives of firms to grow and shape the distribution of firms productivity itself. By assuming an exogenous process of firm growth, previous literature cannot capture such dynamic effects, and thus, may underestimate the aggregate effects of firing costs. This paper fills this gap by quantifying the aggregate implications of firing costs in a model in which the dynamics of firms' productivity are endogenous.

I extend the standard firm dynamics model of Hopenhayn and Rogerson (1993) by incorporating an innovation technology that allows firms to have partial control over the probability of innovation —as in Atkenson and Burstein (2010)— and over the outcome of innovation itself. I model innovation building on the "control cost" approach from game theory. In particular, firms in the model can choose the probability of innovation and, in case of innovation occurs, the distribution of next period's productivity. Both choices imply a cost, that is proportional to the relative entropy between the chosen and a benchmark probability/distribution.

I estimate the parameters of the model by matching some key moments regarding firm growth and firing and hiring behavior, using firm-level information from Spanish non-financial firms. The Spanish economy is of particular interest for this analysis. The Spanish labor market, considered as one of the most inefficient labor markets in Europe, is characterized by a high structural unemployment rate, a high volatility of employment, and an intensive use of temporary employment. Productivity in Spain is one of the lowest among developed countries. In 2010 Spanish TFP was 9% lower than it was in 1990, while for the US and Germany it was 20% higher. This paper connects the underperforming of Spanish productivity with the distortion of its labor market.

The model closely matches the targeted moments. In the baseline economy, small firms innovate more frequently, their innovations are more aggressive (as measured by the expected productivity growth) and more volatile (as measured by the standard deviation of productivity growth). These predictions imply that small firms grow faster and that their growth rates are more volatile. Both implications are consistent with the empirical evidence.[1]

Using the calibrated model I ask, "What are the aggregate effects of firing costs?". In order to address this question, I compare the baseline economy, with firing cost equivalent to 2.5 monthly wages, with one in which firing cost are set to zero. Aggregate TFP, measured as the average of

[1] See for example Sutton (1997, 2002) or Klette and Kortum (2004).

firm-level productivity weighted by the firm's labor share, is 3% lower in the baseline economy than in the frictionless one. This is a large effect compared to what has been found in previous literature. Hopenhayn and Rogerson (1993) find a 2.5% drop in aggregate productivity when firing costs are equivalent to one year's wage. My model generates larger effects for a much lower level of firing costs. In fact, when I set the firing cost equivalent to one year's wage, the fall in aggregate productivity is of more than 10% relative to the frictionless economy, four times larger than in their paper.

Firing cost makes it costlier to fire, which prevents firms from operating at their optimal size. In fact, job creation and destruction rates are 50% and 30% lower in the baseline economy than in the frictionless one. As a result, the efficiency in the allocation of labor across firms goes down, damaging aggregate productivity. However, firing cost does not only affect aggregate productivity by distorting firing and hiring choices. Firing cost also reduces the incentives of firms to grow since growing implies a larger potential cost of firing in the future. As a consequence, firms invest less in innovation, reducing their productivity growth and the average firm productivity in the economy. Investment in innovation falls by 3.5% when firing costs equal the calibrated value of 2.5 monthly wages relative to the frictionless economy, making the average firm productivity to drop by almost 2%. When firing cost is equivalent to one year's wage, innovation expenses, and average productivity fall by 12% and 6% respectively.

In my paper, productivity dynamics are endogenous, allowing me to decompose the effects of dismissal costs on TFP between static effects, i.e. allocative efficiency, and dynamic effects, i.e. changes in the distribution of firms productivity. In particular, I ask, "How much of the change in aggregate productivity is accounted for endogenous productivity dynamics?". To address this question I first decompose the change in aggregate productivity into changes in the average firm-level productivity and the covariance between productivity and labor share, as in Olley and Pakes (1996). Using this decomposition I show that 22% of the fall in aggregate productivity is explained by changes in the average firm productivity, while the remaining 78% is explained by changes in the allocative efficiency of labor across firms.

In the model firm innovation determines the whole distribution of next period's productivity. Hence, the effects of innovation on productivity go beyond changes in average productivity, and may also affect the allocation efficiency itself. For example, some firms optimally chose to face higher uncertainty than others, increasing the strength of the distortion induced by firing costs (Bentolila and Bertola 1990). Moreover, the shape of the distribution, and not only the average, may be affected by firing costs. To quantify the overall contribution of changes in the distribution of productivity I simulate the distorted economy fixing the innovation choices from the frictionless economy. Therefore, the law of motion of productivity in the distorted economy is exactly as in the frictionless one, and thus, neither the average nor the shape of the productivity distribution is affected by changes in the firing cost.

In this new economy, in which innovation is exogenous, the fall in aggregate productivity is of 1.8%, substantially lower than in a model with endogenous innovation where firms can adjust their innovation choices. This means that 45% of the drop in aggregate productivity is explained by changes in the distribution of productivity: 22% due to the change in the average productivity and 23% due to the change in the shape of the distribution. The remaining 55% is explained by the loss in allocative efficiency of labor across firms, *given* a productivity distribution.

The rest of the paper is organized as follows. Section 2 reviews the literature on firing cost and firm innovation. Section 3 presents the model economy. Section 4 explains the calibration procedure. Section 5 presents the results from changing firing cost. Finally, section 6 concludes.

2 Literature Review

There is a large literature that evaluates the role of different policies in accounting for aggregate productivity differences across countries (Guner et al. 2008, Restuccia and Rogerson 2008, Hsieh and Klenow 2009, Bartelsman et al. 2013, Hsieh and Klenow 2014, García-Santana et al. 2016). While many of the papers in this literature use "wedges" to measure policy distortions, some others specify particular policies.[2] One of the policies that has attracted more attention is employment protection, starting with the analysis of firing costs of Hopenhayn and Rogerson (1993). The distortion introduced by firing taxes on firm hiring and firing decisions are well established in the literature, both empirically (Haltiwanger et al. 2014) and theoretically (Bentolila and Bertola 1990). These distortions prevent firms from operating at their optimal scale, worsening the allocation of labor across firms, and damaging aggregate productivity.

The literature studying the impact of firing cost on aggregate productivity typically finds moderate effects (Hopenhayn 2014). However, aggregate productivity losses may be larger when the firms' productivity distribution is endogenous. This is because firing cost may distort not only firing and hiring choices —and thus, the allocation of labor across firms— but also incentives of firms to grow. This is analyzed by Da-Rocha et al. (2019), who study the aggregate implications of firing cost in a continuous-time model in which firms are either small or large, and in which the law of motion of firm's productivity is size-dependent. They find that aggregate productivity losses from firing cost are much larger than when the productivity distribution is completely exogenous, as it is typically assumed. My paper differs from theirs in two margins. First, I consider a model with a continuum of potential firm size. Second, in their paper, the law of motion of firm's productivity is size dependent, but the difference between large and small firms is exogenous. In my paper large and small firms will

[2] See Restuccia and Rogerson (2013) for a discussion on these different approaches.

have different laws of motion for their productivities endogenously, as a result of different innovation choices.

More generally, my paper relates to a recent literature that argues that frictions may affect aggregate productivity not only thought the efficiency in the allocation of resources but also through a direct effect on the firm-level productivity distribution itself. This is the case of Bhattacharya et al. (2013), who study the aggregate implications of size-dependent distortions, in the form of tax rates, in a context where managers invest in their skills, and show that endogenous managerial investments substantially amplifies the effects of distortions. The main difference with respect to my paper is that they consider a model without uncertainty in the outcome of managerial investments.

Also, Gabler and Poschke (2013) allow distortions to have a direct effect on the productivity distribution. They develop a firm dynamics model where firms have access to experimentation and use it to evaluate the implications of a number of frictions, including firing costs. My paper differs from theirs in three dimensions. First, firms in their paper can discard negative productivity shocks (unsuccessful experiments), while in mine they cannot. This difference is key in the analysis of firing cost as the source of risk that matters the most for firing decisions is downside risk. In fact, they find small effects of firing costs. A second difference between their paper and mine is that they consider a model in which firms can choose the risk of their experiments. In my paper, firms chose the whole distribution of the productivity shock.

López-Martín (2013) and Mukoyama and Osotimehin (2019) endogenizes the way in which frictions affect firm's productivity dynamics by including an innovation technology similar to the one in Grossman and Helpman (1991), Aghion and Howitt (1992) and Atkenson and Burstein (2010): firms invest resources in increasing the probability of innovation, and the outcome of innovation is defined as a fixed, exogenous increase in firm's productivity. My paper differs from theirs in that firms in my model will not only invest in increasing the innovation probability but also on its outcome. In short, the size of the productivity gain from a successful innovation is exogenous in their papers, but it arises endogenously in mine.

The main contribution of my paper is, therefore, the way in which I model innovation. To do so, I rely on the "control cost" approach from the game theory literature. This approach is a way of modeling equilibria in which agents make errors. The main assumption is that agents make mistakes because decisions are costly. In particular, decisions are conceived as a random variable over a feasible set of alternatives, and the cost is given by the precision of this random variable. Costain (2017) study the implications of using the "control cost" approach in contexts in which the cost is measured in terms of time. In Costain et al. (2019) we implement this idea to model price and wage adjustment decisions. Turen (2018) model costly information acquisition in a price-setting problem using the "control cost" approach. My paper is the first that uses the "control cost" approach to model innovation choices of firms.

3 The Model

I consider an extension of Hopenhayn and Rogerson (1993) in which I introduce a flexible innovation technology that allows firms to invest in both the probability and outcome of innovation.

3.1 Firms

The economy is populated by a continuum of firms of unit mass, characterized by a profitability factor, denoted by d, and a number of workers hired in the past, n. The term $d \in \{d_1, d_2, \ldots, d_D\}$ is a factor that increases revenues for given inputs, so it captures both productivity (ie. technology) and demand (ie. tastes) factors. For simplicity in the exposition, I will refer to d as firm's productivity throughout the rest of the paper.

Exit in the model is exogenous, and occurs with probability $\delta \in (0, 1)$. When a firm exits, it is immediately replaced by a new firm. Entrants start with no workers and an initial productivity is drawn from $\log(d_0) \sim N\left(-\frac{1}{2}\sigma_0^2, \sigma_0^2\right)$, such that $E[d_0] = 1$.

3.1.1 Profits

Firms produce a homogeneous good, and its price is normalized to 1. This good is used both to consume and to invest in innovation. It is produced using a decreasing returns to scale technology, $y(d, n) = Ad^{1-\gamma}n^{\gamma}$, where A is an aggregate productivity term and $\gamma \in (0, 1)$ the degree of returns to scale. Firms profits are given by:

$$\Pi(d, n, n') \;=\; y(d, n') \;-\; wn' \;-\; \kappa_F w \max\{0, n - n'\}, \tag{1}$$

where w is the wage rate, and $\kappa_F w$ is the per-worker firing cost.

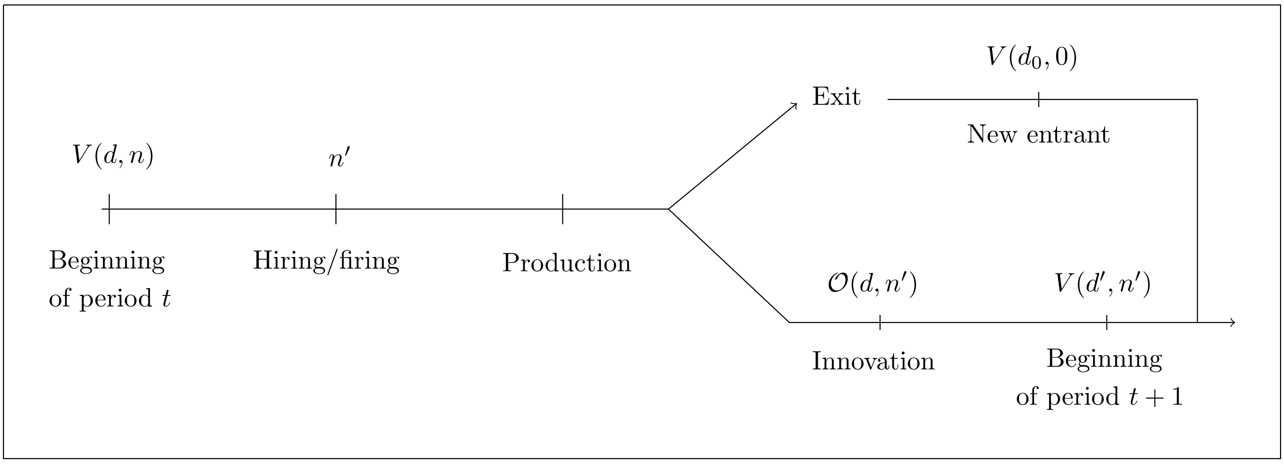
Figure 1 presents the timing of events in the problem of the firm. Given an initial state (d, n), firms decide on hirings/firings, produce and collect profits. Then they are hit by the exit shock. With probability $1 - \delta$, the firm continues in the market and enters the innovation stage, explained later.

The value of a firm with productivity d and n workers is given by:

$$V(d, n) \;=\; \max_{n'} \;\; \Pi(d, n, n') \;+\; \beta(1 - \delta)\mathcal{O}(d, n') \;+\; \beta\delta V_E(n'), \tag{2}$$

where $\beta \in (0, 1)$ is the subjective discount factor, $\mathcal{O}(d, n)$ is the value of a firm with state (d, n) before

Figure 1: Firms' timeline



the innovation stage, and $V_E(n)$ captures the value of exit for a firm with n workers. Since Spanish regulation imposes the obligation to pay dismissal costs in case of exit, I assume that $V_E(n) = -w\kappa_F n$.[3]

3.1.2 Productivity growth

The firm dynamic literature typically assumes that firm productivity follows an exogenous process. In reality, however, firms have the option to undertake a large number of actions to improve their profits prospects, from product or process innovation (supply-side innovation) to marketing campaigns (demand-side innovation). A number of papers model firm productivity growth as a result of innovation. Most of them assume that firms can increase the probability of an innovation that delivers a fixed, exogenous productivity gain (Atkenson and Burstein 2010; López-Martín 2013; Mukoyama and Osotimehin 2019). By doing this, these papers abstract from uncertainty in the outcome of the innovation. However, abstracting from such risk would reduce the role of firing cost in the economy, and thus, limit the model's ability to quantify their aggregate implications. This section develops an innovation technology that allows firms to control not only the uncertainty about whether to innovate but also the degree of uncertainty in the outcome of innovation they face.

I model innovation in a flexible way building on the so-called "control cost" approach from the game theory literature. The "control cost" approach is based on the assumption that deviations from the optimum occur because decisions are costly. Decisions are modeled as random variables distributed over a set of possible actions (which in my model are the different levels of next period's productivity) and the cost of the decision is assumed to increase with the precision of this random variable. I apply this approach to my setting by allowing firms to choose the innovation probability as well as the distribution of next period's productivity in case of innovation success. The cost of these choices is driven by the differences between the chosen probability/distribution and a default

[3]Despite firm owners being subject to limited liability, workers have priority at liquidation over the rest of debtors. The results do not change significantly if I assume $V_E(n) = 0$.

probability/distribution, that defines the law of motion of productivity for non-innovative firms. Thus, I think of "precision" (in the "control cost" terminology) as deviations from the default law of motion of productivity.

Innovation problem

The problem consists of choosing both the probability of innovation, λ, and the outcome of innovation, captured by the distribution of next period's productivity, π, that satisfies

$$\sum_{i=1}^{D} \pi(d_i|d,n) = 1. \tag{3}$$

We can think of λ as the probability of generating a new idea, and on π as the implementation of such idea.

Let $\mathcal{O}^I$ be the value of an innovating firm and $\mathcal{O}^N$ be the value of not innovating. Then, the innovation problem reads as:

$$\begin{aligned}\mathcal{O}(d,n) \quad = \quad \max_{\lambda,\pi}\ & \underbrace{\lambda\left(\sum_{i=1}^{D} \pi(d_i|d,n)V(d_i,n) - \mathcal{D}(\pi||\eta)\right)}_{\mathcal{O}^I(d,n)} + \\ & + \underbrace{(1-\lambda)\left(\sum_{i=1}^{D} \eta(d_i|d)V(d_i,n)\right)}_{\mathcal{O}^N(d,n)} - \mathcal{D}(\lambda||\bar{\lambda})\end{aligned} \tag{4}$$

subject to $\lambda \in [0,1]$ and equation (3). The cost of choosing λ is given by $\mathcal{D}(\lambda||\bar{\lambda})$ where $\bar{\lambda}$ is a default probability of innovation. Similarly, the cost of choosing the distribution π is given by $\mathcal{D}(\pi||\eta)$ where η is a default distribution, satisfying

$$\sum_{i=1}^{D} \eta(d_i|d)d_i = d_i(1-\mu),$$

where $\mu > 0$ is the depreciation rate of productivity.[4] This depreciation rate implies that non-innovative firms expect their productivity to fall, which increases the incentives to innovate.

The cost function $\mathcal{D}(x||z)$ is given by the Kullback-Leibler divergence measure, or relative entropy,

[4] In the quantitative analysis, I will approximate η by discretizing a random walk process with a negative trend. In particular,

$$d' \ = \ d - \mu + \sigma\epsilon$$

where $\sigma > 0$ is the standard deviation of productivity shocks.

between x and z. In particular,

$$\mathcal{D}(\lambda||\bar{\lambda}) \quad = \quad \frac{1}{\kappa_I}\left[\lambda \log\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\log\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right], \tag{5}$$

$$\mathcal{D}(\pi||\eta) \quad = \quad \frac{1}{\kappa_I}\left[\sum_{i=1}^{D}\pi(d_i|d,n)\log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|n)}\right)\right], \tag{6}$$

where κ_I be innovation productivity given by $\kappa_I = \kappa_0 \exp(-\kappa_1 d)$, where $\kappa_0 > 0$ and $\kappa_1 \geq 0$. If $\kappa_1 > 0$ the productivity of innovation is lower for more productive firms, making it costlier for them to innovate, which is consistent with the lower growth rate of large firms.

Note that equation (6) implies that setting a probability $\pi(d_i|d,n) < \eta(d_i|d)$ would reduce the cost $\mathcal{D}(\pi||\eta)$. However, recall that π is a proper probability distribution, and thus $\sum_{i=1}^{D}\pi(d_i;n) = 1$. Consequently, setting a low $\pi(d_i|d,n)$ would require setting a larger value somewhere else in the distribution π, increasing the total cost.

One of the advantage of using the Kullback-Leibler divergence to measure the cost of firm choices is that it allows for closed-form solutions for both the chosen probability λ and the chosen distribution π. In particular, the first order condition with respect to the probability $\pi(d_i|d,n)$ is given by:

$$V(d_i,n) = \frac{1}{\kappa_I}\left[1 + \log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|d)}\right)\right] + \xi, \tag{7}$$

where ξ is the multiplier on the constraint (3). The left-hand side captures the marginal gain from increasing $\pi(d_i|d,n)$, which equals the value fo the firm with productivity d_i, while the right-hand side captures the marginal cost. The marginal cost is the sum of two terms: the "direct" innovation cost associated to the choice of $\pi(d_i|d,n)$ and the cost associated to the constraint. Note that, as previously explained, increasing $\pi(d_i|d,n)$ requires lowering the probability somewhere else in the distribution π. After some rearrangement in equation (7):

$$\pi(d_i|d,n) \quad = \quad \eta(d_i|d)\left[\frac{\exp\Big(\kappa_I V(d_i,n)\Big)}{\sum_{j=1}^{D}\eta(d_j|d)\exp\Big(\kappa_I V(d_j,n)\Big)}\right], \tag{8}$$

so that the chosen distribution is proportional to the default distribution η. Equation (8) implies that firms will deviate more from the default distribution at the two extremes of the range of d_i. On the one hand, very low values of d_i would imply a large fall in the value of the firm, and thus, the firm optimally chooses to reduce the probability of such event. On the other, large values of d_i increase this value, so that firms will optimally choose to assign more probability to those. Note, however, that firms will not be able to assign probability to very large values of d_i if $\eta(d_i|d) = 0$ as it would be infinitely costly. Using equations (8) and (6), we can rewrite the value of innovating as:

$$\mathcal{O}^I(d,n) \;=\; \frac{1}{\kappa_I}\log\left[\sum_{i=1}^{D}\eta(d_i|d)\exp\Big(\kappa_I V(d_i,n)\Big)\right] \tag{9}$$

Note that $E\left[\exp(x)\right] > \exp\left[E(x)\right]$, and thus, $\mathcal{O}^I(d,n) \geq \mathcal{O}^N(d,n)$.

The first order condition of (4) with respect to the probability of innovation λ is:

$$\mathcal{O}^N(d,n) - \mathcal{O}^I(d,n) = \frac{1}{\kappa_I}\left[\log\lambda - \log\bar{\lambda} - \log(1-\lambda) + \log(1-\bar{\lambda})\right],$$

where the left-hand side are the gains from innovating, equal to the marginal product of λ, and the right-hand side is the marginal cost. Rearranging terms:

$$\lambda(d,n) \;=\; \frac{\bar{\lambda}\exp\Big(\kappa_I\mathcal{O}^I(d,n)\Big)}{\bar{\lambda}\exp\Big(\kappa_I\mathcal{O}^I(d,n)\Big) + (1-\bar{\lambda})\exp\Big(\kappa_I\mathcal{O}^N(d,n)\Big)}. \tag{10}$$

As in the case of π, the chosen probability of innovation is proportional to the default probability $\bar{\lambda}$. Moreover, the probability of innovation $\lambda(d,n)$ is increasing in the difference between $\mathcal{O}^I$ and $\mathcal{O}^N$. Moreover, the chosen probability will always be larger than $\bar{\lambda}$, since $\mathcal{O}^I(d,n) \geq \mathcal{O}^N(d,n)$.

3.2 Households

The household problem follows Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2019). In particular, there is a a homogeneous household with a continuum of members who own the firms, consume and supply labor. The problem reads:

$$U \;=\; \max_{C,L}\;\; \ln C \;-\; \theta L, \quad \text{s.t.} \;\; C \;=\; wL + F + \Pi \tag{11}$$

where C is household consumption, L is the total labor supply, F are the total firing taxes and Π are firms profits. The parameter $\theta > 0$ captures the disutility of labor supply.

3.3 Stationary equilibrium

Let $x = (d, n)$ be the state vector, $\mathcal{X}$ be the state space and F be the distribution of firms over the state space. For simplicity in the exposition, I consider a discretized state space so that $F(x)$ is the mass of firms with state x. The law of motion of the distribution of firms is

$$F'(x) \; = \; (1-\delta) \sum_{z \in \mathcal{X}} \Gamma(x|z) F(z) \; + \; \delta \Gamma^E(x)$$

where F' is the next period's distribution of firms, $\Gamma(x|z)$ is the incumbents' transition probability between states z and x, and Γ^E is the distribution of entrants, satisfying $\sum_x \Gamma^E(x) = 1$.

The equilibrium of this economy is given by a wage rate, a distribution of firms over the state space, and a set of firm's policy function such that policy functions solve firms' problem (2), the household first order condition is satisfied, and labor market clears, and the distribution of firms over the state space $\mathcal{X}$ is invariant, $F'(x) = F(x)$, $\forall x \in \mathcal{X}$.

4 Calibration

The model is calibrated to the Spanish economy, using data from the Central de Balances dataset. This is a panel of non-financial Spanish firms, prepared by the Bank of Spain, including balance sheet information, income statement and some firm characteristics (sector, age, etc). The panel covers the years 1995 to 2015. Since Spanish employment is highly volatile, I restrict the sample to years between 2005 and 2007 in order to avoid the Spanish boom (2000-2005) and the financial crisis of 2007.

4.1 Exogenous parameters

I set the discount factor to $\beta = 0.95$.[5] I set the degree of returns to scale to $\gamma = 0.6$, somewhat lower than in Hopenhayn and Rogerson (1993), but within the standard values in the literature. I normalize the equilibrium wage rate to 1 and make θ be such that the household first order condition is satisfied. Finally, I set the exit probability to 7.56% so that the average firm age in the model is 9.7 years, as in the data.

[5] The average long-term government bond yields in Spain for the period 2005-2007 is 4% according to FRED data. I assume a discount rate that corresponds to a 5% annual interest rate.

4.2 Endogenous parameters

The remaining eight parameters, $\Omega = (A, \sigma_0^2, \kappa_F, \bar{\lambda}, \kappa_0, \kappa_1, \mu, \sigma^2)$, are internally calibrated using the model so as to match a number of empirical moments form the data. In particular, I chose the vector of parameters, $\hat{\Omega}$, such that the sum of squared differences between a set of model-generated moments and their empirical counterparts is minimized. Then $\hat{\Omega}$ solves:

$$\hat{\Omega} \;=\; \arg\min_{\Omega} \sum_{i=1}^{M} \omega_i \left(\frac{m_i(\Omega) - \bar{m}_i}{\bar{m}_i} \right)^2 .$$

where M is the number of moments, ω_i the weight associated to moment i, and $m_i(\Omega)$ and $\bar{m}_i$ are the model-generated and empirical i-th moment respectively.

Moment selection

My data lacks information on firm innovation choices. Moreover, given the broad meaning of innovation in this paper, it is not clear what type of information one should use. However, the model establishes a clear link between productivity and size allowing me to discipline the innovation technology using employment data (Garcia-Macia et al. 2019). For instance, given that productivity growth only emerges from innovation, the share of hiring firms and their growth rate are very informative about the share and growth rate of innovators. Thus, the model is calibrated to match the share of hiring firms and the hiring rate, defined as the ratio between hirings and previous employment, $\max\{0, n' - n\}/n$.

Innovation productivity decreases with firm productivity. This makes it costlier to innovate for high productivity firms. In order to control for the strength of this effect, I target the firm size distribution. Note that if innovation is equally costly for high and low productivity firms, high-productivity firms would grow faster than low-productivity ones, generating a bimodal firm size distribution. Given the focus of this paper on firing cost, firing behavior is particularly relevant for the analysis. I match the share of firing firms and the firing rate, defined analogously to the hiring rate. Finally, given that innovation is particularly flexible, it is important to control for the shape of the resulting distribution of next period's productivity. To do so I match the average and the coefficient of variation of firm size, both for the whole population of firms and for entrants.

Identification and model fit

Although all moments are affected by all the parameters, some relationship between specific parameters and moments can be postulated. The arguments that follow do not prove identification, but ease the interpretation of the parameter values.

Since the average productivity of entrants is normalized to 1, the aggregate productivity term, A,

Table 1: Calibration Model fit

Moment	Model	Data
Average firm size	7.23	6.92
Average size of entrants	3.53	3.40
Coefficient of variation of firm size	1.21	1.19
Coefficient of variation of firm size among entrants	1.39	1.36
Share of firing firms	0.26	0.27
Share of hiring firms	0.35	0.34
Firing rate among firing firms	0.19	0.20
Hiring rate among hiring firms	0.44	0.44
Share of firms with 0-5 workers	0.63	0.60
Share of firms with 6-10 workers	0.21	0.20
Share of firms with 11-15 workers	0.07	0.08
Share of firms with 16-20 workers	0.04	0.04
Share of firms with 21-25 workers	0.02	0.02
Share of firms with 25+ workers	0.04	0.05

equals the productivity of entrants, so it is particularly relevant to match the average size of entrants. The variance of the initial productivity draw, σ_0^2, drives the dispersion in firm size among entrants, and therefore, the coefficient of variation in firm size among entrants.

The variance of the benchmark distribution σ^2 limits the dispersion of the chosen distribution among innovators, and thus, drives the overall dispersion in firm size. The parameter κ_0 controls how much innovative firms can grow and it is particularly relevant to match the hiring rate. The parameter κ_1 controls the rate at which the cost of innovation increases with firm's productivity, and thus, the ability to grow among high-productivity firms, driving the firm size distribution. Since productivity growth only emerges from innovation, the share of innovators is very informative about the share of hiring firms. The default probability of innovation $\bar{\lambda}$ limits precisely the probability of innovation and thus, drives the share of hiring firms. Among those firms not innovating, the parameter μ drives the size in the productivity fall, and therefore, it is very informative about the firing rate. Finally, the firing cost parameter κ_F drives the share of firms firing workers.

Table 2 collects the estimated parameters and table 1 the model fit. Despite its simple structure, the model closely matches the moments concerning firing and hiring behavior, as well as the firm size distribution. This is particularly relevant since it provides support for the innovation technology used in the paper. Moreover, the model generates a distribution of firm size that matches, not just the average firm size, but also the dispersion in firm size, which provides further support to the innovation technology. In the next section, I discuss the main predictions that the innovation technology delivers and show that those predictions are consistent with the existing empirical evidence on firm growth.

The firing cost parameter is calibrated to 0.20. This means that the cost of firing one worker equals 2.5 monthly wages. According to Spanish labor regulation, a dismissed worker has the right to

Table 2: Calibration Parameter values

Parameter			Description
A	$=$	2.95	Aggregate productivity term
σ_0	$=$	1.10	Standard deviation of initial productivity draw
μ	$=$	0.07	Depreciation of productivity (default distribution)
σ	$=$	0.30	Standard deviation of shocks (default distribution)
κ_0	$=$	0.14	Cost of innovation, level parameter
κ_1	$=$	1.25	Cost of innovation, shape parameter
$\bar{\lambda}$	$=$	0.47	Default probability of innovation
κ_F	$=$	0.20	Firing cost

received 40 days of wages per year worked in the firm. Note, however, that the Spanish economy is characterized by the heavy use of temporary workers, whose firing cost are either zero or very small. Thus, κ_F should be interpreted as an average firing cost for both temporary and permanent workers.

The depreciation rate of productivity is calibrated to 0.07. Thus, a firm investing no resources in innovation expects to loss 7% of its current productivity next period. The productivity of innovation is decreasing in firm's productivity, which contributes to the fit of the firm size distribution. The magnitude of κ_0 and κ_1 do not have a clear interpretation. However, they imply that firms in the baseline economy spend 16% of total output in innovation.[6] Although this may be too high for innovation expenses, it should be noticed that innovation in this model includes all sort of firm actions aimed at increasing profitability prospects, and not only product or process innovation.

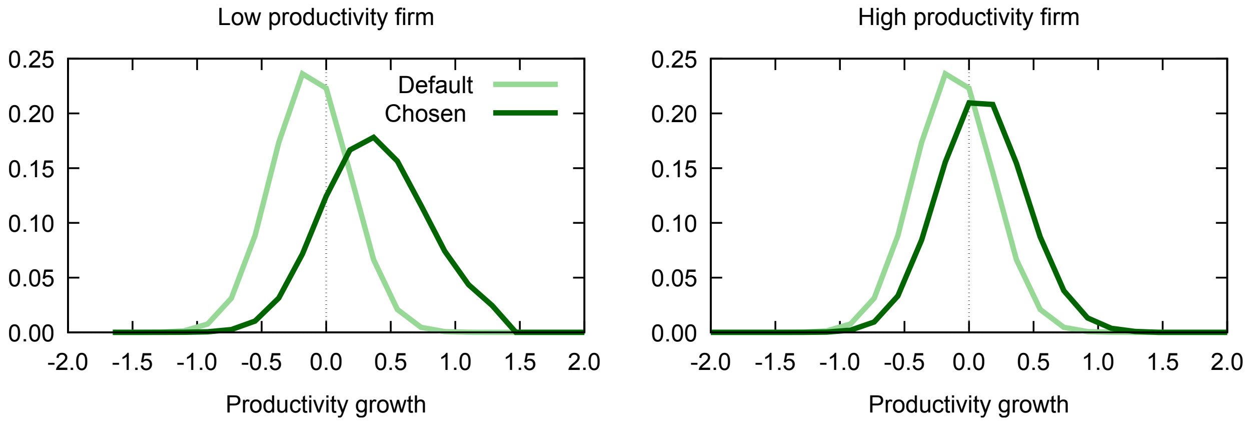
The default probability of innovation is 0.47, which is 9 p.p. lower than the average innovation probability in the baseline economy. Given the structure of the innovation problem, most innovation investments are devoted to the choice of the next period's productivity. This is because the cost of choosing a distribution π is incorporated in the value of innovating, lowering gains for innovation, as shown in equation (4). As a result, higher investments in the distribution π lowers the incentives to invest in the innovation probability.

5 Results

This is the first paper using the "control cost" approach to model firm productivity growth. Moreover, the only source of growth in the model is innovation. In models in which productivity follows an AR(1) process, firms with negative log productivity grow because of a "reverse-to-the-mean" effect. This is not present in my model because, for non-innovators, $E[d'|d] = d(1-\mu) < d$, for any value of d. For this reason, it worth describing firms' innovation behavior in the baseline equilibrium. We then

[6] According to OECD Spanish firms spend around 1% of turnover on innovation. The data is available in the following link: http://dx.doi.org/10.1787/835838585236. However, this data only includes technological innovation (supply-side innovation).

Figure 2: Productivity growth. Next period's productivity distribution



Notes: The x-axis refers to the difference in log productivity $\Delta \log d$. The light line represents the default distribution, η, and the dark one represents the chosen distribution π.

analyze the aggregate effects of firing costs.

5.1 Endogenous productivity dynamics

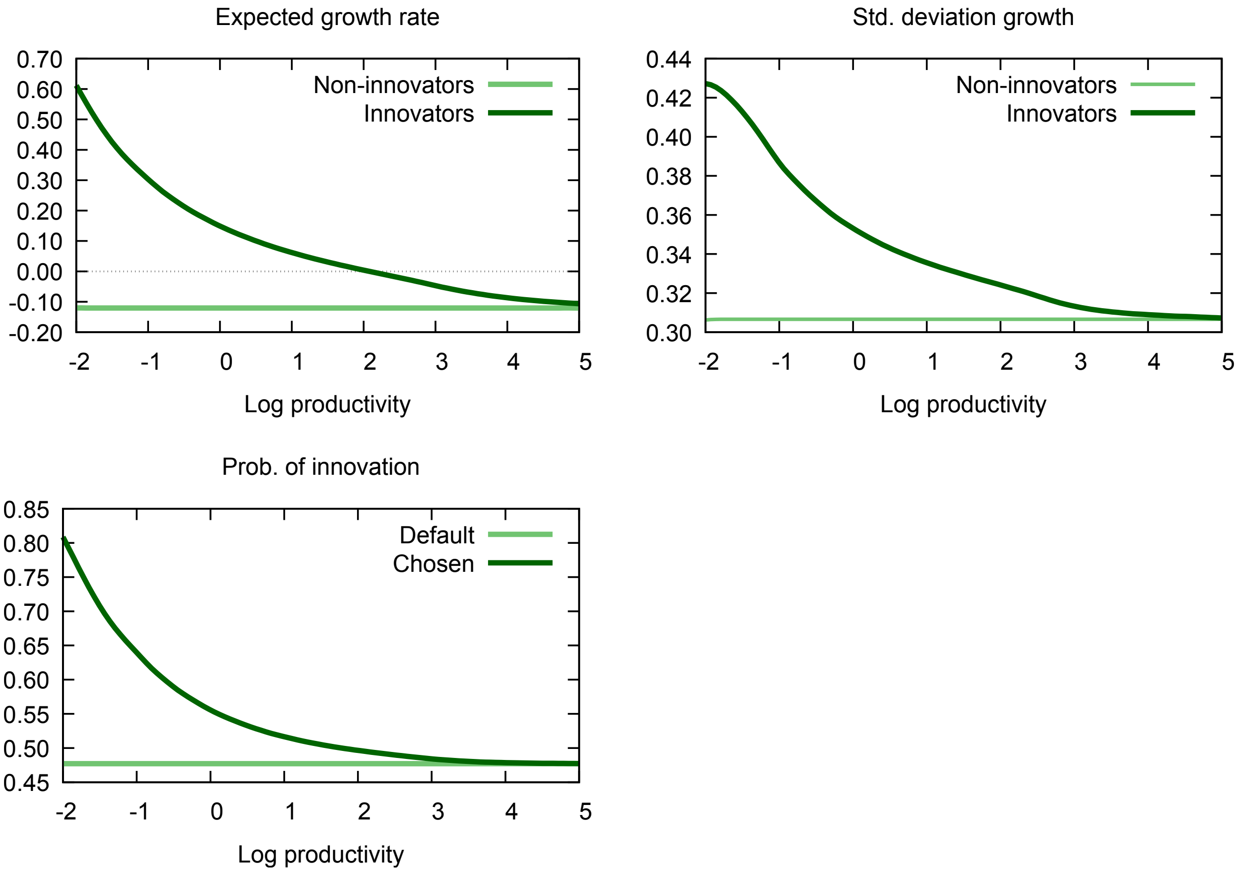
Many papers in the literature of firm growth document the negative relationships between firm size and growth and between firm size and volatility of growth.[7] The model is consistent with these facts. Figure 2 presents the default and chosen distributions of productivity growth for a low- and high-productivity firm. The average productivity growth in case of innovation (thus, taking the chosen distribution, π) is as high as 0.22 for low productivity firms and 0 for high productivity firms, who just offset the negative productivity trend. At the same time, the standard deviation of productivity growth is of 0.45 for low productivity firms, and of 0.35 for high productivity ones. Key for this result is the fact that the productivity of innovation is assumed to be decreasing in firm's productivity.

This can be seen more generally in figure 3, where I plot the expected productivity growth rate, the standard deviation of firm productivity growth and the probability of innovation by firm productivity for innovators and non-innovators, in the baseline economy in which $\kappa_F = 0.20$. We will later discuss how these figures change when we increase/decrease the firing cost. Three main predictions arise from the model: (i) low productivity firms innovate more frequently, (ii) they undertake more aggressive and (iii) their innovations are riskier, as measured by the expected productivity growth and the standard deviation of expected firm productivity growth, respectively. As a result, low productivity (small) firms in the model grow faster and face higher uncertainty.

Figure 3 highlights the importance of allowing firms to have (partial) control over the whole distribution of next period's productivity. Some models of firm innovation allow firms to affect the

[7] See for example Sutton (1997), Sutton (2002) or Klette and Kortum (2004). See figure B.6 for the corresponding relationships in my data.

Figure 3: Innovation choices



Notes: I compute the expected productivity growth rate and standard deviation of productivity growth for each point in the discretized state space using the corresponding distribution of next period's productivity, π or η, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

probability of innovation while keeping fixed the "size" of the innovation. Some others fix the probability of innovation and allow firms to invest in the average productivity growth. However, in both cases, the volatility of productivity growth is constant across firms, and unaffected by the distortion. In this model, however, firms can affect the whole distribution of next period's productivity, along with the probability of innovation, which endogenously generates heterogeneity in the average and volatility of productivity growth. The fact that firms face different degrees of uncertainty is particularly relevant when analyzing the effects of firing costs (Bentolila and Bertola 1990). In my paper, this happens endogenously.

5.2 Aggregate effects of firing costs

The main goal of this paper is to better understand the aggregate consequences of firing costs. To facilitate the exposition and the comparison with previous literature, I simulate the frictionless economy, in which $\kappa_F = 0$, and compare it with an economy with positive firing costs. But before going over

Table 3: Aggregate effects of firing cost
(% fall relative to frictionless economy)

	$\kappa_I = 0.20$	$\kappa_I = 0.40$	$\kappa_I = 1.00$
TFP	3.01	5.08	10.6
Average productivity	1.82	3.10	6.54
Average productivity growth	2.22	3.76	7.19
Innovation expenses	3.47	5.86	11.8
Output	2.50	4.54	9.46
Employment	2.55	4.67	9.67
Job destruction rate	52.5	68.6	85.7
Job creation rate	30.8	40.3	50.3

the results, we first need to define the main object of interest: aggregate productivity. In particular, aggregate TFP in this economy is defined as

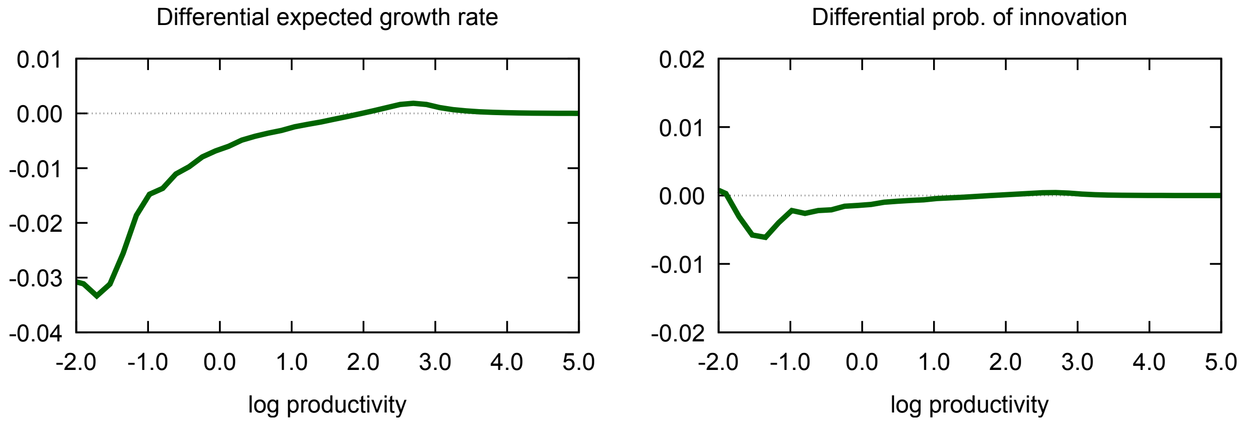
$$\text{TFP} = \int_{x\in\mathcal{X}} d(x)s(x)d\mu(x) \tag{12}$$

where $x = (d, n)$ is the firm's state vector, $s(x) = n'(x)/N$ is the firm's labor share and $\mu(x)$ is the stationary distribution of firms, satisfying $\int_x d\mu(x) = 1$. The (qualitative) results presented in this section still hold when the weights $s(x)$ are given by the firm's output share.

Table 3 collects the results of this experiment. Table entries represent the percentage change in the corresponding variable relative to the frictionless economy. In the first column, I compare the frictionless economy with the one that arises from our calibration exercise, in which the firing cost is $\kappa_F = 0.20$. The second column collects the results from simulating an economy in which I set the firing cost to $\kappa_F = 0.40$, twice as large as the calibrated value. Finally, for comparison with the literature, I simulate an economy in which firing costs are equivalent to one year's wage. Figure B.7 plots the percentage change in average productivity and aggregate TFP, output, and employment for different values of κ_F, from zero to 0.40, both for the general equilibrium solution and for the partial equilibrium one, in which I do not allow the wage rate to adjust.

The main conclusion from this experiment is that a firing cost damage aggregate productivity significantly. A firing cost equivalent to 2.5 monthly wages generates a 3% fall in aggregate productivity relative to the frictionless economy. This is a large number compared to the literature. In Hopenhayn and Rogerson (1993) they find a 2.1% decrease in productivity from a firing cost equivalent to one year's wage. In my model, the fall in aggregate productivity from a firing cost of this size is larger than 10%. Da-Rocha et al. (2019) find a 20% fall in aggregate productivity from a firing cost equivalent to 5 year's wage. My model generates as much as half of the fall Da-Rocha et al. (2019) find, with a firing cost of just 1 year's wage.

Figure 4: Innovation choices. Experiment, $\kappa_F = 0.2$ *vs.* $\kappa_F = 0$



Notes: To compute the differences in expected growth between non-innovative and innovative firms, I average expected growth over firm size for each productivity d using the corresponding distribution of next period's productivity (π for innovative firms and η for non-innovative firms) as in figure 3. I do the same for the probability of innovation.

The drop in aggregate productivity is due to both a worse allocation of labor across firms —static effects— and a shift of the whole productivity distribution —dynamic effects. When firing costs are introduced, growing larger implies a higher potential cost of firing in the future, lowering the incentives to invest in innovation. In figure 4 I plot the differential expected productivity growth rate and the differential probability of innovation between the frictionless economy with one in which firing cost are set to $\kappa_F = 0.2$. Figure B.8 plots the same results when the distorted economy has a level of firing cost of one year's wage.

Firms invest less in both the probability of innovation and in the outcome of such innovation, as measured by the expected productivity growth. In particular, the aggregate innovation expenses fall by 3.5%. This change is mainly due to adjustments in the amount of resources invested in the next period's productivity. The drop in the expected productivity growth is substantial: up to 3 p.p. lower productivity growth rate for low productivity firms. The fall is up to 12 p.p. when setting the firing cost to $\kappa_F = 1$. In the aggregate, productivity growth rate falls by 2.2%, 3.8% and 7.2% when firing cost is set to 0.2, 0.4 and one year's wage respectively. The probability of innovation is almost unaffected by changes in κ_F. The reason is that both the value of innovation and the value of not innovating fall when firing costs increase and thus, gains from innovation are roughly equal to those in the frictionless economy. Despite being unaffected by changes in the firing cost, the innovation probability is still an important margin in the analysis. The reason is that the probability of innovation is (endogenously) different for low and high productivity firms, crucially affecting their incentives to fire and hire workers.

The distorted economy also exhibits lower job destruction and creation rates (defined as total firings/hirings over total employment). In particular, the share of newly hired workers in the economy falls by 31%, while the share of fired workers drops by more than 50%. Since firms find it costlier to fire workers now than before, they decide to keep workers even if their size is larger than the optimal one. At the same time, firms below their optimal size decide not to hire due to precautionary motives. Since there is uncertainty about future productivity, the firms know that they may need to fire in the future, which prevents them from hiring in the first place. Note that the change in job creation is less pronounced than that in job destruction, as in Bentolila and Bertola (1990). These two distortions give rise to inefficiencies in the allocation of labor, which further damages aggregate productivity.

5.3 Allocative efficiency *versus* average productivity

Results from table 3 show that firing costs reduce aggregate productivity through two channels: by distorting hiring and firing choices and by lowering average firm productivity. To disentangle the relative importance of these two channels, I apply the productivity decomposition presented in Olley and Pakes (1996). In particular, equation (12) can be rewritten as:

$$\text{TFP} \;=\; \bar{d} \;+ \int_{x \in \mathcal{X}} \tilde{d}(x)\tilde{s}(x) d\mu(x) \;=\; \bar{d} \;+\; C(d,n) \tag{13}$$

where $\mu(x)$ is the distribution of firms, $\bar{a} = \int a(x)d\mu(x)$ and $\tilde{a}(x) = a(x) - \bar{a}$, for $a = \{d, s\}$. The first term measures the average firm productivity, while the second is the covariance between firm's productivity and firm's employment share, which captures the efficiency in the allocation of labor across firms. In the frictionless economy this covariance is maximized. When firing and hiring choices are distorted —in our case by firing cost— the covariance between productivity and labor share falls, lowering TFP. We can then decompose aggregate productivity losses as:

$$\underbrace{\frac{\Delta\text{TFP}}{\text{TFP}}}_{\text{TFP gains}} = \underbrace{\frac{\Delta\bar{d}}{\text{TFP}}}_{\text{Productivity gains}} + \underbrace{\frac{\Delta C(d,n)}{\text{TFP}}}_{\text{Efficiency gains}}$$

Setting a firing cost of $\kappa_F = 0.2$ generates a drop in aggregate productivity of 3%, as shown in table 3. Out of this drop in TFP, 22% is explained by changes in the average firm productivity, while the remaining 78% is accounted for by the efficiency losses in the allocation of labor. This decomposition is roughly unchanged for larger values of the firing cost parameter. These results indicate that models without firm productivity growth, endogenous or exogenous, would miss a quantitatively important part of the potential aggregate productivity losses from firing costs.

5.4 What is the role of endogenous productivity dynamics?

The exercise presented in section 5.3 allows us to decompose the fall in aggregate productivity into changes in average firm productivity and changes in the covariance between firm productivity and labor share. However, the innovation technology presented in this paper allows firms to affect not only the expected productivity growth but the whole distribution of productivity growth. For instance, innovation choices determine the degree of uncertainty in the law of motion of productivity, which crucially affects the strength of the distortion induced by the firing cost. Moreover, as seen in figure 4, the effects of firing costs on productivity growth are heterogeneous. Thus, innovation choices may also affect the covariance between firm's productivity and labor share. As a result, the decomposition of Olley and Pakes (1996) can not completely quantify the role of endogenous productivity dynamics in accounting for the fall in aggregate productivity shown in section 5.2.

In order to clearly identify the role of endogenous firm productivity I repeat the experiments of section 5.2 fixing the innovation behavior from the frictionless economy. In short, I simulate a distorted economy in which I impose a law of motion for firm productivity given by

$$d' \sim \begin{cases} \pi(d,n|\kappa_F = 0) & \text{w.p.} \quad \lambda(d,n|\kappa_F = 0) \\ \eta(d) & \text{w.p.} \quad 1 - \lambda(d,n|\kappa_F = 0) \end{cases}$$

where $\pi(d,n|\kappa_F = 0)$ and $\lambda(d,n|\kappa_F = 0)$ are the resulting innovation probabilities and distributions from the frictionless economy in which firing costs are set to zero. To make the two economies comparable, I also keep fixed the cost of innovation which is now added as a fixed cost to the value of the firm. Results are collected in table 4. The first two columns collect the results from the exercise in section 5.2, in which innovation is endogenous, and thus reacts to changes in κ_F. The two last columns collect the results from changing the firing cost in an economy with exogenous innovation, in which I fixed the innovation behavior that arises the frictionless economy.

In the model with exogenous innovation, a firing cost of $\kappa_F = 0.2$ implies a fall in aggregate productivity loss of 1.7% which is significantly lower than in a model with endogenous productivity dynamics. In particular, the possibility of adjusting the innovation choices to changes in firing costs accounts for around 45% of the aggregate productivity losses associated to a firing cost of $\kappa_F = 0.20$, and up to 46% and 48% when I set $\kappa_F = 0.40$ and $\kappa_F = 1$ respectively. In Da-Rocha et al. (2019) they find that 80% of the overall drop in aggregate productivity is accounted by changes in the distribution of firms. This is much larger than in my model. The reason is that they do not allow firms to adjust the dynamics of productivity when the firing cost parameter changes. In their model, the dynamics of firm productivity are size-dependent, but the differences between large and small are exogenous are

Table 4: Aggregate effects of firing cost. Exogenous innovation
(% fall relative to frictionless economy)

	Endogenous Inn.			Exogenous Inn.		
Firing cost, κ_F	0.20	0.40	1.00	0.20	0.40	1.00
TFP	3.01	5.08	10.6	1.68	2.73	5.52
Average productivity	1.82	3.10	6.45	0.00	0.00	0.00
Output	2.50	4.54	9.46	1.74	3.38	7.05
Employment	2.55	4.67	9.67	2.52	4.75	9.72
Innovation expenses	3.47	5.86	11.8	0.00	0.00	0.00

fixed. Thus, conditional on firm size, the law of motion of firm productivity is unchanged when the firing cost parameter changes.

One of the consequences of fixing innovation choices is that the average firm productivity is not affected. Therefore, the results presented in table 4 embody those from the decomposition exercise of section 5.3. To the effect of changes in the average productivity, here we also control for changes in growth rates across firms, as well as changes in the volatility of next period's productivity. In short, by fixing the innovation behavior of firms, we abstract from both changes in the average firm productivity and from changes in the shape of the productivity distribution. By comparing the three economies (endogenous innovation, fixed average productivity, and exogenous innovation) we can further decompose the drop in aggregate productivity as:

- 22% due to the change in the average firm productivity
- 23% due to the change in the shape of the productivity distribution
- 55% due to the distortion of hiring and firing choices

Endogenous firm productivity dynamics are also important in accounting for the changes in aggregate output. In particular, the fall in aggregate output with exogenous innovation is of 2.5%, 2.4% and 7% when firing cost is 0.2, 0.4 and 1 respectively. This represents a 35% to 25% of the overall fall in aggregate output. Employment losses are, however, similar both with endogenous and with exogenous innovation. The reason is the different response of the wage rate in equilibrium. When innovation is endogenous, the wage rate falls by 1.7% when firing costs are of 2.5 monthly wages, and by more than 5% when they are of one year's wage. When innovation is exogenous, these numbers are 0.9% and 3%.

6 Conclusions

This paper presented a firm dynamics model with endogenous productivity growth to analyze the aggregate effects of firing cost. Making the dynamics of productivity endogenous allows the model to capture both the static effects of firing taxes —allocative efficiency— as well as the dynamic effects of such friction —changes in the distribution of firms' productivity. It is the first model that introduces an innovation technology that allows firms to control not only the probability of innovation but also the outcome. The model parameters are calibrated so as to match the firm size distribution and the hiring and firing behavior of Spanish firms. I show that my flexible innovation technology is able to generate a distribution of firm size that is very close to that in the data, both in terms of size and in terms of dispersion. Moreover, the model is also able to generate larger and more volatile growth among low productivity firms.

I use the calibrated model to quantitatively asses the aggregate effects of firing cost. I show that a firing cost equivalent to 2.5 monthly wages (the calibrated value) generates a 3% drop in aggregate productivity relative to the frictionless economy. When firing cost is equivalent to one year's wage, the fall in productivity is of more than 10%, substantially larger than found in previous literature. This fall combines a lower firm productivity growth (that falls by 2% and 7%) and a worse allocation of labor across firms. In fact, job creation and destruction rates are 50% and 30% with a firing cost of 2.5 monthly wages that they are in the frictionless economy. Aggregate output and employment also fall. Setting the firing cost at its calibrated value generates a 2.5% drop in output and employment, relative to the frictionless economy. A firing cost of one year's wage generates a 9% fall in employment and output.

I then decompose the fall in TFP between losses in allocative efficiency and changes in the distribution of firm productivity, by fixing the law of motion of firm-level productivity to the one that arises endogenously from the frictionless economy. I show that 55% of the TFP losses are explained a worse allocation of labor across firms, while the remaining 45% is accounted for changes in the distribution of productivities. I further decompose the contribution of changes in the distribution of productivity into changes in the average firm-level productivity and the changes in the shape of the distribution using the decomposition proposed in Olley and Pakes (1996). I find that 22% of the aggregate productivity losses are due to a reduction in the average firm productivity, while the remaining 23% is explained by changes in the shape of the distribution of firms' productivity.

This result suggests that, when evaluating the aggregate effects of frictions/policies, one should take their effects on the dynamics of productivity into account. This paper applies this idea to firing cost. I leave the analysis of other frictions and policies, such as corporate taxation, for future research.

References

Aghion, P. and P. Howitt (1992). "A Model of Growth through Creative Destruction". *Econometrica 60*(2), 323–351.

Atkenson, A. and A. Burstein (2010). "Innovation, Firm Dynamics and International Trade". *Journal of Political Economy 118*(3), 433–484.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013). "Cross-Country Differences in Productivity: The Role of Allocation and Selection". *American Economic Review 103*(1), 305–334.

Bentolila, S. and G. Bertola (1990). "Firing Costs and Labour Demand: How Bad is Eurosclerosis". *The Review of Economic Studies 57*, 381–402.

Bhattacharya, D., N. Guner, and G. Ventura (2013). "Distortions, Endogenous Managerial Skills and Productivity Differences". *Review of Economic Dynamics 16*(1), 11 – 25.

Costain, J. (2017). "Costly Decisions and Sequential Bargaining". Bank of Spain WP 1729.

Costain, J., A. Nakov, and B. Petit (2019). "Monetary Policy Implications of State-dependent Prices and Wages". CEPR DP 13398.

Da-Rocha, J.-M., D. Restuccia, and M. M. Tavares (2019). "Firing Costs, Misallocation, and Aggregate Productivity". *Journal of Economic Dynamics and Control 98*, 60–81.

Gabler, A. and M. Poschke (2013). "Experimentation by firms, distortions, and aggregate productivity". *Review of Economic Dynamics 16*(1), 26 – 38.

Garcia-Macia, D., C.-T. Hsieh, and P. J. Klenow (2019). "How Destructive is Innovation?". Unpublished manuscript.

García-Santana, M., E. Moral-Benito, J. Pijoan-Mas, and R. Ramos (2016). "Growing Like Spain: 1995-2007". CEPR Discussion Paper No. DP11144.

Grossman, G. M. and E. Helpman (1991). "Quality Ladders in the Theory of Growth". *The Review of Economic Studies 58*(1), 43–61.

Guner, N., G. Ventura, and Y. Xu (2008). "Macroeconomic Implications of Size-dependent Policies". *Review of Economic Dynamics 11*(4), 721–744.

Haltiwanger, J., S. Scarpetta, and H. Schweiger (2014). "Cross Country Differences in Job Reallocation: The Role of Industry, Firm Size and Regulations". *Labour Economics 26*, 11 – 25.

Hopenhayn, H. (2014). "Firms, Misallocation, and Aggregate Productivity: A Review". *Annual Review of Economics 6*, 735–770.

Hopenhayn, H. and R. Rogerson (1993). "Job Turnover and Policy Evaluation: A General Equilibrium Analysis". *Journal of Political Economy 101*(5), 915–938.

Hsieh, C.-T. and P. J. Klenow (2009). "Misallocation and manufacturing TFP in China and India". *The Quarterly Journal of Economics 124*(4), 1403–1448.

Hsieh, C.-T. and P. J. Klenow (2014). "The Life Cycle of Plants in India and Mexico". *The Quarterly Journal of Economics 129*(3), 1035–1084.

Klette, T. J. and S. Kortum (2004). "Innovating Firms and Aggregate Innovation". *Journal of Political Economy 112*(5), 986–1018.

López-Martín, B. (2013). "From Firm Productivity Dynamics to Aggregate Efficiency". Unpublished manuscript.

Mukoyama, T. and S. Osotimehin (2019). "Barriers to Reallocation and Economic Growth: the Effects of Firing Costs". *American Economic Journal: Macroeconomics*, forthcoming.

Olley, G. S. and A. Pakes (1996). "The Dynamics of Productivity in the Telecommunications Equipment Industry". *Econometrica 64*(6), 1263–1297.

Restuccia, D. and R. Rogerson (2008). "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments". *Review of Economics Dynamics 11*, 707 – 730.

Restuccia, D. and R. Rogerson (2013). "Misallocation and Productivity". *Review of Economic Dynamics 16*(1), 1 – 10.

Sutton, J. (1997). "Gibrat's Legacy". *Journal of Economic Literature* (35), 40–59.

Sutton, J. (2002). "The Variance of Firm Growth Rates: the 'Scaling' Puzzle". *Physica A 312*(4), 577–590.

Turen, J. (2018). "Rational Inattention-driven Dispersion with Volatility Shocks". Unpublished manuscript.

Online Appendix

A Computation

In this section, I briefly describe how to solve the model numerically. First I discretize the state space is $\#_d \times \#_n$ points, where $\#_d$ is the number of points in the grid for productivity and $\#_n$ is the number of points in the grid for employment.

The problem in (2) is solved by value function iteration. For each point in the state space, (d, n), I find the optimal employment choice, n', using the Golden Search algorithm. Then, given the choice of n', I compute the distribution of next period's productivity using equation (8). The exponential term in equation (8) can easily go to infinity, depending on the maximum real number the computer can use. To avoid this, I redefine equation (8) as:

$$\pi(d'|d,n) = \frac{\eta(d'|d)\exp\left(\kappa_I \tilde{V}(d',n)\right)}{\int \eta(x|d)\exp\left(\kappa_I \tilde{V}(x,n)\right)dx} \tag{14}$$

where $\tilde{V}(d,n) = V(d,n) - \mathbb{C}$ and $\mathbb{C} = \max\{V(d,n)\}$. Note that this normalization does not alter the value of $\pi(d'|d,n)$. The fact that $\mathbb{C} = \max\{V(d,n)\}$ ensures that the exponential term is never larger than 1. Then, the cost of innovation becomes

$$\begin{aligned}
\mathcal{D}(\pi||\eta) &= \int \pi(x)\tilde{V}(x,n)dx - \frac{1}{\kappa_I}\log\left[\int \eta(x|d)\exp\left(\kappa_I \tilde{V}(x,n)\right)dx\right] = \\
&= \int \pi(x)\tilde{V}(x,n)dx - \mathbb{C} - \frac{1}{\kappa_I}\log\left[\int \eta(x|d)\exp\left(\kappa_I \tilde{V}(x,n)\right)dx\right]
\end{aligned}$$

and the value function at the innovation stage:

$$\begin{aligned}
\mathcal{O}^I(d,n) &= \int \pi(x|d,n)V(x,n)dx - \mathcal{D}(\pi||\eta) = \\
&= \mathbb{C} + \frac{1}{\kappa_I}\log\left[\int \eta(x|d)\exp\left(\kappa_I V(x,n)\right)\exp\left(-\kappa_I \mathbb{C}\right)dx\right] = \\
&= \frac{1}{\kappa_I}\log\left[\int \eta(x|d)\exp\left(\kappa_I V(x,n)\right)dx\right]
\end{aligned}$$

which equals the expression derived in section 3.1.2.

B Additional figures

Figure B.5: Distribution of firms. Baseline economy.

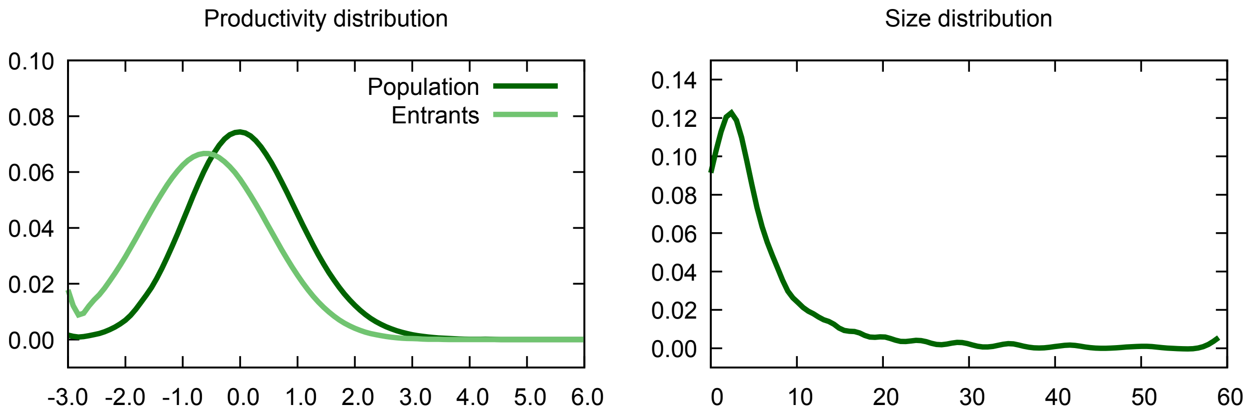
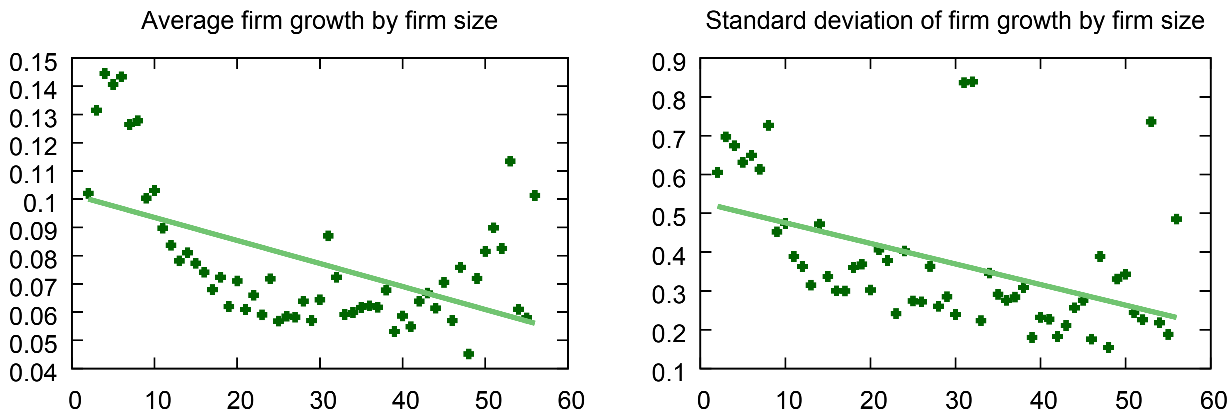
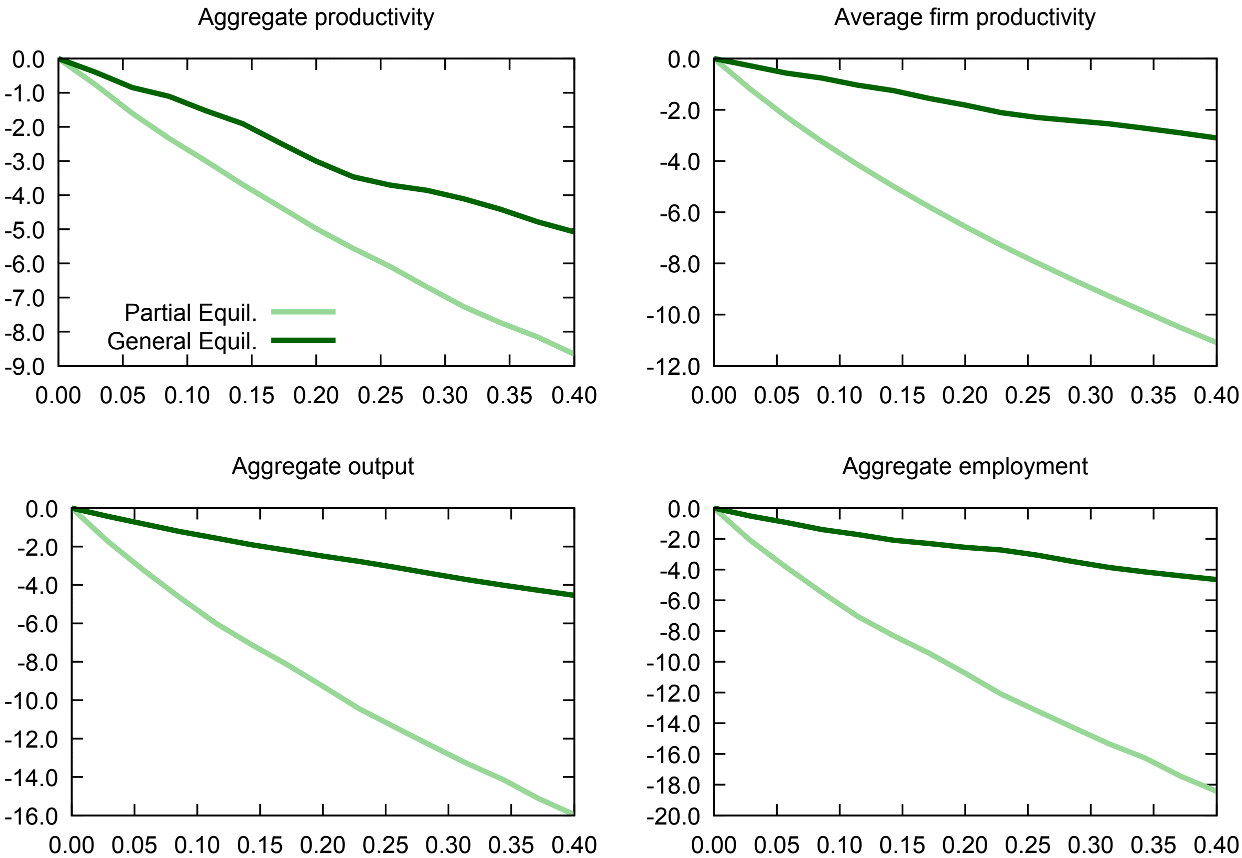


Figure B.6: Firm growth in the data



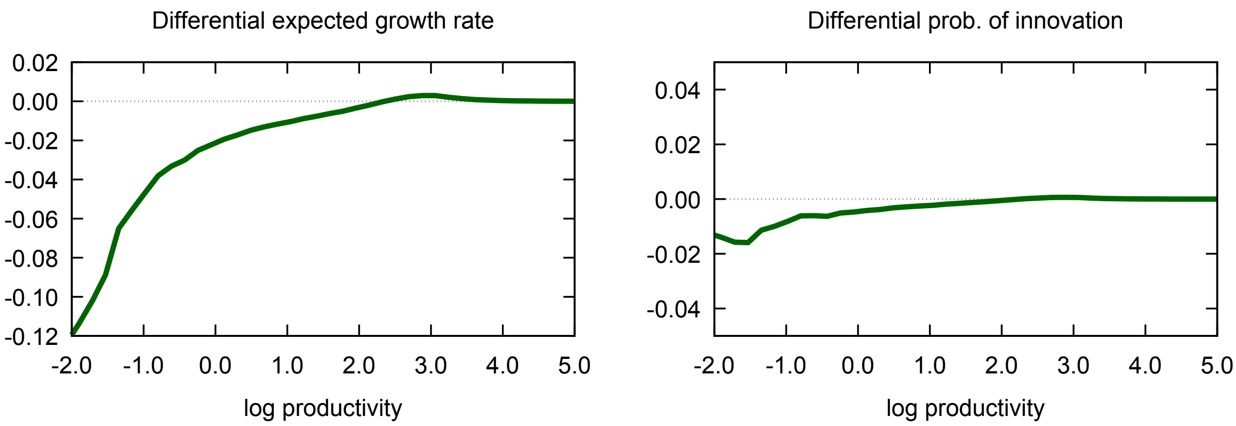
Notes: Dots represent size-specific average and standard deviation of firm growth rates, and the light line is a linear fit.

Figure B.7: Aggregate effects of firing costs



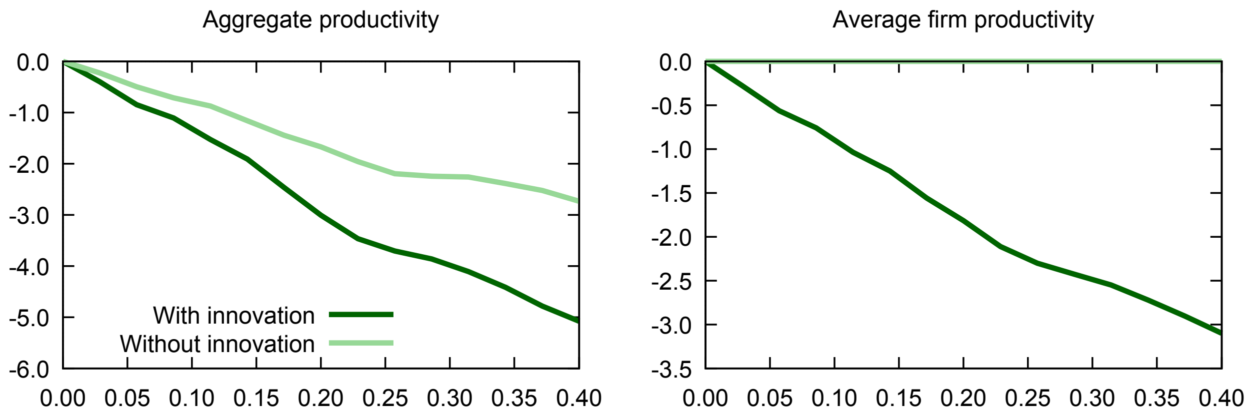
Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The light line represents the partial equilibrium results, where the wage rate is not adjusted. The dark line represents the general equilibrium results that emerge from adjusting the wage rate.

Figure B.8: Innovation choices Experiment, $\kappa_F = 1$ *vs.* $\kappa_F = 0$



Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.9: Aggregate effects of firing costs Exogenous innovation



Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The dark line represents the results when innovation is endogenous, and thus, firms' innovation choices react to changes in the firing cost. The light line represents the results when innovation is exogenous so that innovation choices are unaffected by change sin the firing cost.