Monetary Policy Implications of State Dependent Prices and Wages

James Costain — ECB and Banco de España

Anton Nakov — ECB and CEPR

Borja Petit — CEMFI

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Abstract

We study the dynamic general equilibrium effects of monetary shocks in a model of state-dependent price and wage adjustment based on "control costs". Suppliers of retail goods and suppliers of labor are both monopolistic competitors that face idiosyncratic productivity shocks and nominal rigidities. Stickiness arises because precise choice is costly: decision-makers tolerate errors both in the timing of adjustments, and in the new level at which the price or wage is set, because making these choices with greater precision would not be cost-effective.

Our simulations are calibrated to microdata on the size and frequency of price and wage changes. Money shocks have less persistent real effects in our state-dependent model than they would in a time-dependent framework, but nonetheless we obtain sufficient monetary nonneutrality for consistency with macroeconomic evidence. Nonneutrality is primarily driven by wage rigidity, rather than price rigidity. State-dependent nominal rigidity implies a flatter Phillips curve as trend inflation declines, because nominal adjustments become less frequent, making short-run inflation less reactive to shocks.

Keywords: Sticky prices, sticky wages, state-dependent adjustment, logit equilibrium, near rationality, control costs

JEL Codes: E31, D81, C73

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1 Introduction

The nominal rigidity of prices and/or wages is a prominent assumption in monetary macroeconomics today. For reasons of analytical tractability, many studies are based on the Calvo (1983) framework, in which the probability of adjustment is constant. But several influential papers have claimed that if nominal stickiness is derived from rational decision-making, instead of being imposed in an *ad hoc* way, then the real macroeconomic effects of monetary policy are negligible (see for example the menu cost models of Caplin and Spulber 1987, and Golosov and Lucas Jr 2007). This finding motivates a wave of new research investigating how the conclusions of Calvo-style models and menu cost models hold up in a variety of state-dependent pricing frameworks that are closely calibrated to retail price microdata (*e.g.* Klenow and Kryvtsov 2008; Gagnon 2009; Matějka 2015; Midrigan 2011; Alvarez et al. 2011; Eichenbaum et al. 2011; Kehoe and Midrigan 2015; Dotsey et al. 2013; Alvarez et al. 2015; Costain and Nakov 2011, 2019).

Much of this new literature concludes, to quote Kehoe and Midrigan, that "prices are sticky after all". That is, while money is almost neutral in stripped-down menu cost models like Golosov and Lucas Jr (2007), related frameworks that fit retail microdata better show that price stickiness does matter at the aggregate level, delivering nontrivial real effects of monetary policy.[1] This apparent consensus represents an encouraging improvement in the link between microdata and modern macroeconomics, but it derives from studies where, for computational reasons, price stickiness was the only friction considered. This contrasts with the current generation of empirical DSGE models that rely not only on nominal rigidity of prices and wages, but also on many other frictions, such as consumption habits, investment adjustment costs, and labor matching frictions. Hence, to better assess the quantitative role of nominal rigidity for macroeconomic dynamics, it is still important to study models in which multiple frictions interact.

As a modest step forward, we analyze a model with one additional layer of state-dependent adjustment, allowing for wage stickiness as well as price stickiness. A natural point of departure for our analysis is Erceg et al. (2000), who study monopolistic price- and wage-setters, both operating under the Calvo framework. Following Erceg *et al.*, we set up the wage setters' problem so that it closely parallels price setting, but we allow for state dependence in both decisions. More precisely, we compare a framework in which both price- and wage-setters are constrained by the Calvo friction to one in which price- and wage-setters are both constrained by a state-dependent friction, and in addition we compare these with scenarios in which price- and/or wage-setting approaches perfect flexibility. We emphasize that our goal is to compare different specifications of price stickiness and wage stickiness while abstracting from any other frictions that might affect the labor market (or other markets). While the interaction of nominal rigidities with

1. The reason for nonneutrality is that the microdata seem to favor specifications in which the "selection effect" is weaker than Golosov and Lucas Jr (2007) found.

labor market matching is a major theme of the macro-labor literature, here we quantify the effects of state-dependent prices and wages by themselves, leaving their interaction with matching frictions for future work.

Our model of state-dependent adjustment is an extension of the "control cost" model of price stickiness proposed by Costain and Nakov (2019), henceforth CN19. Control costs are a modeling device from game theory intended to capture the idea that the costs of precise decision-making sometimes lead players to make some mistakes.[2] Under the control cost framework, a decision is regarded as a random variable defined over a set of feasible alternatives, and the decision-maker is assumed to face a cost function that increases with the precision of that random variable. Placing probability one on the optimal alternative is a very precise decision, so the decision-maker may instead economize on the costs of choice by tolerating some randomness (some errors) in the alternative chosen. CN19 models nominal rigidity by applying this framework *both* to the prices firms choose, and to firms' control of the *timing* of their adjustments. In equilibrium, managers of retail firms economize on the time devoted to decision-making by tolerating some low-cost errors in the prices they set, and some low-cost errors in the timing of their price adjustments.

There are a number of reasons why it is interesting to extend the CN19 framework to other frictions, beyond price stickiness. First, it describes adjustment costs in a sparsely parameterized way; the benchmark scenario in CN19 simultaneously fits many "puzzling" features of retail pricing by calibrating only two free parameters in the decision cost function. Second, these costs have an appealing interpretation: they represent time devoted by management to decision-making. The costs may plausibly be larger than the menu-type fixed costs associated with the physical act of changing the price, and may be compared, at least roughly, to case studies on time use. Third, the model is no harder to solve numerically than comparable menu cost models, but it is far more tractable than "rational inattention" in the tradition of Sims (2003). Fourth, the mathematical structure of the model —resetting a control variable at intermittent points of time— seems applicable to many decisions other than price adjustment, potentially allowing us to treat many margins of a general equilibrium model in a mutually consistent and comparable way. Finally, since the calibration strategy in the recent state-dependent pricing literature involves matching many moments of the distribution of individual price adjustments, it stretches credulity to abstract from errors. When matching (for example) the standard deviation of observed price adjustments, inferences about the standard deviation of the underlying shocks may differ greatly depending on whether or not we insist that each price adjustment represents a precisely optimal action.

2. See Stahl (1990), Mattsson and Weibull (2002), or Van Damme (2002), Ch. 4.

1.1 Related literature

Time-dependent price and wage rigidities frequently interact in contemporary DSGE models, such as Blanchard and Galí (2007) and Galí et al. (2012). One of the first papers to examine the interplay of these two rigidities, under the Calvo mechanism, was Erceg et al. (2000), which identified a tradeoff between stabilization of output, price inflation, and wage inflation. Huang and Liu (2002) studied the relative importance of price and wage rigidity in a time-dependent model, concluding that wage rigidity matters more for monetary non-neutrality; Christiano et al. (2005) concur. We revisit this question in a state-dependent model.

The literature that contrasts state-dependent pricing models to micro- and macrodata is extensive, as we discussed above; surveys include Klenow and Malin (2010) and Nakamura and Steinsson (2008). We know of only one previous study of state-dependent prices and wages in a DSGE model (Takahashi 2017). Takahashi's paper differs from ours in that it analyzes a stochastic menu cost model (following Dotsey et al. 1999) rather than a control cost model. More importantly, it also abstracts from idiosyncratic shocks, so it cannot be closely assessed relative to microdata on price and wage changes. Annual data relevant for analyzing the distribution of wage adjustments include those of the International Wage Flexibility Project (Dickens et al. 2007), which we will use here. Barattieri et al. (2014) analyze quarterly wage adjustments in SIPP data. Very recently, wage change data with higher frequency and higher coverage have also become available (Grigsby et al. 2018).

Since our framework abstracts from any frictions in labor mobility, it is not directly related to the search and matching literature. However, it can shed light on macro-labor issues such as the slope of the Phillips curve and the cyclicality of real wages and markups. Akerlof et al. (1996), Fahr and Smets (2010), Benigno and Ricci (2011), and Lindé and Trabandt (2018) have argued that downward nominal wage rigidity makes the Phillips curve flatter when inflation is low. We will show that the same result is obtained without downwardly asymmetric rigidity, if the adjustment hazard varies with inflation.

The cyclicality of the real wage has long been controversial (Huang et al. 2014; McCallum and Smets 2007; Smets and Wouters 2007). Christiano et al. (2016) report a small and insignificant procyclical response of the real wage to monetary shocks. Shimer (2012) argues that the "labor wedge", defined as the marginal product of labor minus workers' marginal rate of substitution, is strongly countercyclical. Equivalently, Gali et al. (2007) define an "efficiency gap" (marginal rate of substitution minus marginal product of labor) which they show is strongly procyclical. They further argue that the wedge (the negative of the gap) decomposes into two terms: a highly countercyclical markup of wages over the marginal rate of substitution, and a moderately countercyclical markup of prices over firms' marginal costs. The latter property is controversial: Nekarda and Ramey (2013) show that a wide variety of estimation procedures reject countercyclical markups of prices over firms' marginal costs, while Bils et al. (2018) argue using evidence on the self-employed support countercyclical price markups. This debate puts in question the central transmission mechanism of the simplest New Keynesian models, in which prices, but not wages, are rigid. Nonetheless, this leaves open the possibility that monetary nonneutrality may derive primarily from wage rigidity.

2 Model

We embed the near-rational nominal adjustment model of Costain and Nakov (2019) in a discrete-time New Keynesian general equilibrium framework that combines elements of Erceg et al. (2000) and of Golosov and Lucas Jr (2007). There is a continuum of retail firms and a continuum of workers; retail goods markets and labor markets are both monopolistically competitive. Each firm is the unique seller of a differentiated retail good, and resets his nominal price intermittently. Each worker is the unique seller of a differentiated type of labor, and resets its nominal wage intermittently. Price and wage adjustments are driven by idiosyncratic as well as aggregate shocks. Workers belong to a representative household; the budget constraint is defined at the household level. In addition, there is also a monetary authority that sets an exogenous growth process for the nominal money supply.

2.1 Monopolistic firms

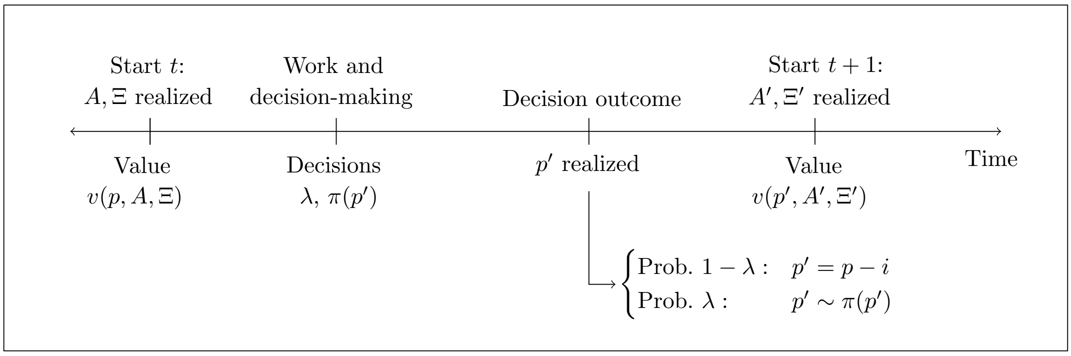
Each firm j produces output $Y_{j,t}$ under a constant returns technology $Y_{j,t} = A_{j,t}N_{j,t}$. Efficiency units of labor, denoted $N_{j,t}$, are the only input, and $A_{j,t}$ represents an idiosyncratic productivity process that follows a time-invariant Markov process on a bounded set, $A_{j,t} \in \Gamma^A \subseteq [\underline{A}, \overline{A}]$. Productivity innovations are *iid* across firms. Thus, $A_{j,t}$ is correlated with $A_{j,t-1}$, but it is uncorrelated with other firms' shocks. Firm j is a monopolistic competitor that sets a nominal price $P_{j,t}$. We write the model in real terms with $p_{j,t} = ln(P_{j,t}/P_t)$, where P_t is the aggregate price level. The firm faces the demand curve $Y_{j,t} = C_t e^{-\epsilon p_{j,t}}$, where C_t is aggregate real consumption.[3] We assume each firm must fulfill all demand at its chosen price. Since firms are infinitesimal, each firm ignores the effect of its own price on the aggregate price level. It hires labor at real wage rate $w_t = ln(W_t/P_t)$, generating real profits

$$e^{p_{j,t}}Y_{j,t} - e^{w_t}N_{j,t} = \left(e^{p_{j,t}} - \frac{e^{w_t}}{A_{j,t}}\right) C_t e^{-\epsilon p_{j,t}} \tag{1}$$

per period. Firms are owned by the household, so they discount real income between times t and $t+1$ at the household's stochastic discount rate $\Lambda_{t,t+1}$, defined below.

3. We use succinct notation where time subscripts represent dependence on the aggregate state of the economy, Ξ_t. Time-subscripts on aggregate variables represent functions of Ξ_t: $w_t \equiv w(\Xi_t)$ is the aggregate wage, and $C_t \equiv C(\Xi_t)$ is aggregate consumption demand. Likewise, time subscripts on value and policy functions represent dependence on Ξ_t: $v_t(p, A) \equiv v(p, A, \Xi_t)$, $o_t(p, A) \equiv o(p, A, \Xi_t)$, and so forth.

Figure 1: Firms' timeline



To clarify the structure of decision-making, it helps to distinguish value functions at several different points in time. First, let $v_t(p, A)$ be the value of a firm that begins period t with real price p and productivity A, prior to any time-t decisions, and prior to time-t output. Figure 1 presents the firms' timeline. We assume that choices take time, so if the firm decides in period t to adjust its price, the new price only becomes effective at time $t+1$.[4] Next, let $o_t(p, A)$ be the firm's continuation value, net of current profits, when it still has the option to adjust prices. That is,

$$v_t(p, A) \;=\; \left(e^p - \frac{e^{w_t}}{A}\right) C_t e^{-\epsilon p} + o_t(p, A) \tag{2}$$

The function $o_t(p, A)$ incorporates the value of the firm's two possible time-t decisions: whether to adjust its price, and if so, which new price to set for period $t+1$. The firm may make errors in either of these choices. We discuss these two decisions in turn, beginning with the latter.

2.1.1 Choosing a new price

Our model formalizes the idea that nominal rigidities may derive primarily from the costs of decision-making. While one might assume that by paying a fixed cost, the firm can make the optimal choice, this would implicitly impose a corner solution with perfect precision. We find it more appealing and realistic to assume that firms can devote more or less time and resources to decision-making, thus choosing more or less precisely. In equilibrium in our framework firms will typically prefer to make choices with an interior degree of precision. Thus their chosen action will not always be the one that would have been optimal in the absence of decision costs; instead, most choices will involve some degree of "error".

Consistent with this general description, we adopt the "control cost" approach from game theory (see Van Damme 2002, Chapter 4). A key feature of this approach is that we model the price decision indirectly: the firm's problem is written "as if" it chooses a

4. A one-period lag would be unrealistic if the time period were very long. But when we calibrate the model, we will impose a monthly time period, so that a one-period lag is not excessively restrictive.

probability distribution over prices, rather than choosing the price *per se*.[5] The problem incorporates a cost function that increases with precision: concentrating greater probability on a smaller range of prices increases costs. Many measures of precision could be used to define decision costs; we choose a definition based on relative entropy, also known as Kullback-Leibler divergence, which is a measure of the difference between two probability distributions. For two possible distributions $\pi_1(x)$ and $\pi_2(x)$ of some random variable x with support on set $\mathcal{X}$, the Kullback-Leibler divergence $\mathcal{D}(\pi_1||\pi_2)$ of π_1 relative to π_2 is defined by[6]

$$\mathcal{D}(\pi_1||\pi_2) = \int_{\mathcal{X}} \pi_1(x) \ln \left(\frac{\pi_1(x)}{\pi_2(x)} \right) dx. \tag{3}$$

Following Stahl (1990) and Mattsson and Weibull (2002), we assume that the decision cost is proportional to the Kullback-Leibler divergence of the chosen distribution ($\pi_1(x)$ above) relative to an exogenous benchmark distribution ($\pi_2(x)$ above). Thus, if no decision costs are paid, the action x is distributed according to the benchmark distribution, $\pi_2(x)$. But by putting more effort into the decision process, the decision-maker can shrink the distribution of the action towards the most desirable alternatives.

We assume that decision costs are denominated in units of time. The only control variable that the firm must manage is its nominal price. We regard each adjustment of the nominal price as a costly decision; hence when the firm sets a new nominal price $\tilde{P}$, this remains constant in nominal terms until the firm again chooses to make an adjustment. We benchmark the cost of the decision process against an exogenous benchmark distribution $\eta(\widetilde{p})$ with support Γ^p.[7]

Assumption 1 *The time cost of choosing a distribution $\pi(\widetilde{p})$ over real prices $\widetilde{p} \in \Gamma^p$ is $\kappa_f \mathcal{D}(\pi||\eta)$, where $\kappa_f > 0$ is a constant, and $\eta(\widetilde{p})$ is an exogenously-given benchmark distribution with support Γ^p.*

Here κ_f represents the marginal cost of entropy reduction, in units of labor time. The cost function described in Assumption 1 is nonnegative and convex.[8] The upper bound on the cost function is associated with a distribution that places all probability on a single price $\widetilde{p}$ (concretely, costs are maximized when all probability is placed on one price that minimizes the benchmark probability $\eta(\widetilde{p})$). The lower bound on this cost function is zero, associated with choosing the distribution $\pi(\widetilde{p})$ equal to the benchmark distribution $\eta(\widetilde{p})$.

5. Luce (1959) and Machina (1985) are early advocates of analyzing decisions in terms of a probability distribution over alternatives; this approach is also adopted by Sims (2003). See Chapter 2 of Anderson et al. (1992) for discussion.

6. While we write (3) with an integral, we can be agnostic at this point about whether $\mathcal{X}$ is a discrete or continuous set. If it is a continuous set, then π_1 and π_2 should be interpreted as density functions. If it is a discrete set, then π_1 and π_2 should be interpreted as vectors of probabilities, and the integral in (3) should be interpreted as a sum.

7. Our setup imposes a control cost function with an exogenously-given default distribution. Steiner et al. (2017) show that a general dynamic rational inattention problem is equivalent to a control cost problem with an optimally-chosen default distribution. Fixing the default distribution exogenously greatly improves the numerical tractability of our framework, but still yields a form of stickiness similar to that obtained from rational inattention.

8. Cover and Thomas (2006), Theorem 2.7.2.

Now consider the pricing decision under this cost function. If the firm sets a new price at time t, this new price only becomes effective at $t+1$, and its value is:

$$v_t^e(\widetilde{p}, A) \equiv E_t\left[\Lambda_{t,t+1} v_{t+1}(\widetilde{p} - i_{t+1}, A') \middle| A\right], \tag{4}$$

where $i_{t+1} = ln(P_{t+1}/P_t)$ is inflation between periods t and $t+1$, $\Lambda_{t,t+1}$ is the household stochastic discount factor, and $E_t[\bullet|A]$ represents an expectation over the time $t+1$ variables $\Xi' \equiv \Xi_{t+1}$ and $A' \equiv A_{j,t+1}$ conditional on the time t aggregate state Ξ_t and firm j's time t productivity $A_{j,t} = A$. Following the control costs methodology, we assume the firm maximizes its value by allocating probability across possible prices $\widetilde{p}$, taking account of decision costs, as follows:

$$\tilde{v}_t(A) = \max_{\pi(\widetilde{p})} \int \pi(\widetilde{p}) v_t^e(\widetilde{p}, A) d\widetilde{p} - e^{w_t} \kappa_f \mathcal{D}(\pi||\eta) \tag{5}$$

$$\text{s.t.} \quad \int \pi(p) dp = 1 \tag{6}$$

Note that the decision costs in (5) are converted to comparable units by multiplying by the wage rate. We write the real value of the pricing decision as $\tilde{v}_t(A)$, where $A \equiv A_{j,t}$ is the firm's current productivity.

The first-order condition for $\pi(p)$ in problem (5) is[9]

$$v_t^e(p, A) - \kappa_f e^{w_t}\left[1 + \ln\left(\frac{\pi(p)}{\eta(p)}\right)\right] - \mu = 0,$$

where μ is the multiplier on the constraint (6). Some rearrangement yields a weighted multinomial logit formula:

$$\pi_t(\widetilde{p}|A) \equiv \frac{\eta(\widetilde{p}) \exp\left(\frac{v_t^e(\widetilde{p}, A)}{\kappa_f e^{w_t}}\right)}{\int \eta(p) \exp\left(\frac{v_t^e(p, A)}{\kappa_f e^{w_t}}\right) dp} \tag{7}$$

The parameter κ_f in the logit function can be interpreted as the degree of noise in the decision process; in the limit as $\kappa_f \to 0$, equation (7) converges to the policy function under full rationality, so that the optimal price is chosen with probability one. Plugging the logarithm of π_t into the objective, we can also derive an analytical formula for the value function:

$$\frac{\tilde{v}_t(A)}{\kappa_f e^{w_t}} = \ln\left(\int \eta(\widetilde{p}) \exp\left(\frac{v_t^e(\widetilde{p}, A)}{\kappa_f e^{w_t}}\right) d\widetilde{p}\right). \tag{8}$$

This formula gives the firm's real value when adjusting its current price, net of decision costs.

Some interpretive comments may be helpful at this point. First, although we write

9. Note that if we take future values $v_t^e(\widetilde{p}, A)$, as given, problem (5) maximizes a concave objective subject to a linear constraint. Therefore a unique maximum exists for any given backwards induction step.

the firm's problem "as if" it chooses a probability distribution over prices, this should not be taken literally— actually choosing a distribution would be a complex, costly diversion from the true task of choosing the price itself. Rather, we define the decision as a choice of a mixed strategy because this is a way to incorporate errors into the model. And we describe it as an optimization problem because this disciplines the errors; it amounts to assuming that the firm devotes time and effort to avoiding especially costly mistakes. Aspects of the model that we do take seriously include (a) making decisions is costly in terms of time and other resources; (b) therefore decision-makers do not always take the action that would otherwise be optimal; (c) *ceteris paribus*, more valuable actions are more probable; (d) in a retail pricing context, these considerations apply to the timing of price adjustment, in addition to the actual price chosen, as we will see in the next subsection.

Second, the problem is written conditional on the true expected discounted values $v_t^e(\widetilde{p}, A)$ of the possible real prices $\widetilde{p}$, instead of conditioning on a prior, as a "rational inattention" model would. This reflects the fact that we are *not* assuming imperfect information. But this is different from saying that the firm "knows" the true values $v_t^e(\widetilde{p}, A)$. Instead, our interpretation is that the firm has sufficient information to calculate $v_t^e(\widetilde{p}, A)$ in principle. Even so, drawing correct conclusions from that information, and acting accordingly, may be costly.[10]

2.1.2 Choosing the timing of price adjustment

We next analyze, in an analogous manner, the decision whether or not to adjust at time t. As in section 2.1.1, we define costs relative to a benchmark probability distribution over possible actions. But for this choice, at any t, there are only two options: adjust now, or not. Since the probabilities of these two alternatives must sum to one, effectively the relevant benchmark is just a single number, which we can interpret as an exogenous default hazard rate.

We suppose the time period is sufficiently short so that we can ignore multiple adjustments within a single period. If the firm chooses not to adjust its current price, then its nominal price next period will be unchanged: $P' = P$; the expected value of this unchanged price, from the point of view of period t, is $v_t^e(p, A)$, given by (4). If instead the firm adjusts its price at t, then its expected value is $\tilde{v}_t(A)$, given by (8). Now suppose it adjusts its price with (endogenous) probability λ. We measure the cost of this adjustment hazard in terms of Kullback-Leibler divergence, relative to some arbitrary Poisson process with arrival rate $\bar{\lambda}$:

Assumption 2 *The time cost incurred in period t by setting the price adjustment hazard*

10. Since economists are accustomed to models of perfect rationality, they often equate observing a given information set with knowing all quantities that can be calculated from that information set. But when rationality is less than perfect, we cannot equate these two assumptions. Here, we assume firms can observe all relevant shocks and state variables, but we do not equate this with actually knowing $v_t^e(\widetilde{p}, A)$ or knowing the optimal action, and therefore we do not equate it with implementing the optimal action with probability one.

$\lambda \in [0,1]$ *in period* t *is* $\kappa_f \mathcal{D}(\lambda||\bar{\lambda})$*, where* $\kappa_f > 0$ *and* $\bar{\lambda} \in [0,1]$ *are exogenous parameters.*

The optimal adjustment probability at time t solves the following Bellman equation:

$$o_t(p,A) \;=\; \max_{\lambda \in [0,1]} \; (1-\lambda)v_t^e(p,A) \;+\; \lambda \tilde{v}_t(A) \;-\; e^{w_t}\kappa_f \mathcal{D}(\lambda||\bar{\lambda})). \tag{9}$$

Recall that $o_t(p,A)$ represents the continuation value of the firm, net of decision costs, when it still has the option to adjust, or not to do so. The first order condition from (9) is

$$v_t^e(p,A) - \tilde{v}_t(A) = \kappa_f e^{w_t}\left[\ln\lambda - \ln\bar{\lambda} - \ln(1-\lambda) + \ln(1-\bar{\lambda})\right]. \tag{10}$$

Rearranging, we can solve (10) to obtain

$$\lambda_t(p,A) \;=\; \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda})\exp\left(\frac{-v_t(p,A)}{\kappa_f e^{w_t}}\right)}, \tag{11}$$

where $d_t(p,A)$ is the expected gain from adjustment:

$$d_t(p,A) \;\equiv\; \tilde{v}_t(A) - v_t^e(p,A).$$

The weighted binary logit hazard (11) was also derived by Woodford (2009) from a model with a Shannon constraint. The free parameter $\bar{\lambda}$ measures the rate of decision making; concretely, the probability of adjustment in one discrete time period is $\bar{\lambda}$ when the firm is indifferent between adjusting and not adjusting (*i.e.* when $d_t(p,A) = 0$).

2.1.3 Deriving the Bellman equation

Next, to calculate the value function $v_t(p,A)$, we concatenate the two decision steps described in sections 2.1.1 and 2.1.2. If the firm starts period t with real price p, then its value $v_t(p,A) \equiv v_t(p,A,\Xi_t)$ at the beginning of t satisfies:

$$\begin{aligned} v_t(p,A) \;=\; \max_{\lambda,\pi(\widetilde{p})} & \left(e^p - \frac{e^{w_t}}{A}\right) C_t e^{-\epsilon p} + (1-\lambda)v_t^e(p,A) - e^{w_t}\kappa_f \mathcal{D}(\lambda||\bar{\lambda}) + \\ & + \lambda\left[\int \pi(\widetilde{p}) v_t^e(\widetilde{p},A)d\widetilde{p} - e^{w_t}\kappa_f \lceil(\pi||\eta)\right] \\ \text{s.t.} \quad & \int \pi(p)dp = 1. \end{aligned} \tag{12}$$

This Bellman equation subtracts off the two cost functions seen in the previous subsections.[11] There is a time cost associated with monitoring whether or not a price adjustment

11. For expositional transparency, we described pricing and timing above as two separate decisions, each with its own entropy costs. However, these two steps can equivalently be rewritten as a single decision, subject to a single entropy-based cost function, encompassing the alternatives of non-adjustment or of adjustment to any $\widetilde{p} \in \Gamma_t^p$. For details, see CN19, Sec. 2.2. We will see below that the worker's problem must generally be written as a single combined decision, except in the special case of linear labor disutility.

is required, which we will call

$$\mu_t(p, A) \;\equiv\; \kappa_f \left[\lambda \ln\left(\frac{\lambda}{\bar{\lambda}}\right) + (1-\lambda)\ln\left(\frac{1-\lambda}{1-\bar{\lambda}}\right)\right]. \tag{13}$$

The time cost of choosing which new price to set is

$$\tau_t(p, A) \;\equiv\; \lambda\kappa_f \int \pi(\widetilde{p}) \ln\left(\frac{\pi(\widetilde{p})}{\eta(\widetilde{p})}\right) d\widetilde{p}. \tag{14}$$

Finally, the time devoted to the actual production of goods will be written as

$$N_t(p, A) \;\equiv\; \frac{C_t}{A} e^{-\epsilon p}. \tag{15}$$

Hence, the firm's total demand for labor hours is $N_t(p, A) + \mu_t(p, A) + \tau_t(p, A)$.

2.2 Labor market

We next construct a model of nominal wage rigidity analogous to our treatment of nominal price rigidity. We suppose each worker i is the monopolistic supplier of a specific type of labor $H_{i,t}$, sold at wage $w_{i,t}$ per unit of time. The productivity of worker i's labor $H_{i,t}$ is shifted by a shock process $Z_{i,t}$, which follows a time-invariant Markov process on a bounded set, $Z_{i,t} \in \Gamma^Z \subset [\underline{Z}, \overline{Z}]$. We will define $N_{i,t} = Z_{i,t}H_{i,t}$ as the "effective labor" of worker i. By this definition, we can say that the price of effective labor is $\frac{e^{w_{i,t}}}{Z_{i,t}}$. The idiosyncratic shock process $Z_{i,t}$ represents worker-specific productivity dynamics, which may include various forms of human capital accumulation.

Firm j's labor input into goods production, $N_{j,t}$, is defined as a CES aggregate across varieties of effective labor i, with elasticity of substitution ϵ_n. That is,

$$N_{j,t} = \left\{ \int_0^1 N_{ijt}^{\frac{\epsilon_n - 1}{\epsilon_n}} di \right\}^{\frac{\epsilon_n}{\epsilon_n - 1}}. \tag{16}$$

Under this demand structure, the firm's optimal hiring satisfies

$$H_{ijt} \;\equiv\; \frac{N_{ijt}}{Z_{i,t}} \;=\; Z_{i,t}^{\epsilon_n - 1}\left(e^{-\epsilon_n w_{i,t}}\right) N_{j,t}, \tag{17}$$

Firm j's real wage bill for goods production is then

$$\int_0^1 e^{w_{i,t}} H_{ijt} di \;=\; e^{w_t} N_{j,t}. \tag{18}$$

We assume that firms use the same CES mix of labor for decision making that they use for goods production. Then (17) implies that total demand for worker i's time is $H_{i,t} = H_t(w_{i,t}, Z_{i,t})$, defined by

$$H_{i,t} \;=\; Z_{i,t}^{\epsilon_n - 1}\left(e^{-\epsilon_n w_{i,t}}\right) N_t \;\equiv\; H_t(w_{i,t}, Z_{i,t}), \tag{19}$$

where N_t represents aggregate labor demand by all firms. N_t includes labor demand for goods production, given by (15), and labor demand for decision making, given by (13)-(14).

The worker adjusts her nominal wage $W_{i,t}$ intermittently to maximize the value of labor income net of labor disutility. She faces control costs, both on her timing decision, and on the choice of which wage to set. We assume workers act in the interest of the households of which they form part, and that their consumption is fully insured by the household; hence they discount future income at the same rate $\Lambda_{t,t+1}$ that applies to the household and firms. Now let $l_t(w, Z)$ be the real value of a worker with wage $w = ln(W/P)$ and productivity Z at the beginning of period t, before supplying labor, and before making any decisions. As in the case of price decisions, we assume that a wage adjustment in period t becomes effective in period $t+1$. Therefore the value of setting the wage to an arbitrary new value $\widetilde{w}$ is

$$l_t^e(\widetilde{w}, Z) \equiv E_t \left[\Lambda_{t,t+1} l_{t+1}(\widetilde{w}, Z') \middle| Z \right].$$

We make two assumptions about workers' decision costs that are analogous to our assumptions about firms.

Assumption 3 *The time cost of choosing a distribution $\pi^w(w)$ over real wages $w \in \Gamma^w$ is $\kappa_w \mathcal{D}(\pi^w || \eta^w)$, where $\kappa_w > 0$ is a constant, and $\eta^w(w)$ is an exogenously-given benchmark distribution with support Γ^w.*

Assumption 4 *The time cost incurred in period t by setting the wage adjustment hazard $\rho \in [0,1]$ in period t is $\kappa_w \mathcal{D}(\rho || \bar{\rho})$, where $\kappa_w > 0$ and $\bar{\rho} \in [0,1]$ are exogenous parameters.*

Now, let the disutility of labor be given by

$$X(H) = \chi \frac{H^{1+\zeta}}{1+\zeta}, \tag{20}$$

where H includes both the time a worker provides to the firms and the time devoted to the worker's own decisions, and $\zeta > 0$ is the inverse of the Frisch elasticity of labor supply. Adopting this convex disutility function implies that we will not be able to separate the choice of the wage from the choice of the *timing* of wage adjustment, as we did when we described the price-setting process. Nonetheless, the wage-setting problem takes a form closely analogous to the pricing problem (12):

$$\begin{aligned}
l_t(w,Z) \quad &= \max_{\tau^w,\mu^w,\rho,\pi^w(w)} \; wH_t(w,Z) - (u'(C_t))^{-1} \cdot X\left(H_t(w,Z)+\mu^w+\tau^w\right) \; + \\
&\qquad\qquad + (1-\rho)l_t^e(w,Z) + \rho \int \pi^w(\widetilde{w}) l_t^e(\widetilde{w},Z) d\widetilde{w} \\
\text{s.t.} \quad & \int \pi^w(w)dw = 1, \\
& \mu^w = \kappa_w \left[\rho \ln\left(\frac{\rho}{\bar{\rho}}\right) + (1-\rho)\ln\left(\frac{1-\rho}{1-\bar{\rho}}\right)\right], \\
& \tau^w = \rho\kappa_w \int \pi^w(\widetilde{w}) \ln\left(\frac{\pi^w(\widetilde{w})}{\eta^w(\widetilde{w})}\right) d\widetilde{w}.
\end{aligned} \tag{21}$$

We scale labor disutility $X(H)$ by the factor $(u'(C_t))^{-1}$ to express it in the same units. In (21), $\mu^w \equiv \kappa_w \mathcal{D}(\rho||\bar{\rho})$ represents time dedicated to monitoring whether or not it is a good moment to reset the wage. Since the probability of resetting the wage at a given time t is ρ, Assumption 4 implies that the (expected) time devoted to choosing a new wage in period t is $\tau^w \equiv \rho\kappa_w \mathcal{D}(\pi^w||\eta^w)$.

To clarify, recall that we stated the firm's decision in two separate steps, (9) and (5), representing the decision of whether or not to adjust prices, and the decision of what price to set conditional on adjustment, respectively. This decomposition was possible because we assumed the firm could hire any quantity of labor at the (aggregate) wage rate w_t, making its labor costs a linear function of its labor demand. But imposing a linear cost function for a worker's time use would be highly restrictive, implying an infinite elasticity of labor supply. In the absence of decision costs, a worker would set the real wage as a function of the marginal utility of consumption only; productivity shocks would cause variation in hours worked without any change in the wage.[12] To avoid this restrictive assumption, we adopt a convex disutility specification. But therefore we cannot simply condition on a given, constant marginal cost of labor; time supplied to firms affects the cost of time on each decision margin, so the both margins are analyzed simultaneously in the wage setting problem (21).

Nonetheless, the policy functions for wage setting and wage adjustment timing resemble the policy functions from the firm's problem. Following our previous calculations, we find that if the worker adjusts, she chooses the following density over real wages $\widetilde{w}$:[13]

$$\pi_t^w(\widetilde{w}|w,Z) \;\equiv\; \frac{\eta_t^w(\widetilde{w})\exp\left(\frac{l_t^e(\widetilde{w},Z)}{\kappa_w x_t(w,Z)}\right)}{\int \eta_t^w(w)\exp\left(\frac{l_t^e(w,Z)}{\kappa_w x_t(w,Z)}\right)dw}, \tag{22}$$

12. A specification with linear labor disutility is analyzed in our working paper, CITE HERE.
13. There is also an analytical formula for the worker's value function, analogous to (8); see the working paper CITE HERE.

where $x_t(w,Z)$ denotes the marginal disutility of time in period t:

$$x_t(w,Z) \;\equiv\; \frac{1}{u'(C_t)} X'(H_t(w,Z) + \tau^w + \mu^w)\,. \tag{23}$$

Likewise, if the worker's beginning-of-period wage and productivity are w and Z, her optimal adjustment probability must satisfy:

$$\rho_t(w,Z) \quad = \quad \frac{\bar{\rho}}{\bar{\rho} + (1-\bar{\rho})\exp\left(\frac{-d_t^w(w,Z)}{\kappa_w x_t(w,Z)}\right)}\,, \tag{24}$$

where

$$d_t^w(w,Z) \;\equiv \tilde{l}_t(w,Z) - l_t^e(w,Z)$$

represents the gain in value from adjusting rather than leaving the wage unchanged.

The key to solving the worker's equations is to calculate the marginal disutility of time, $x_t(w,Z)$. Suppose we know the aggregate variables w_t, C_t, and N_t; hence the labor demand function $H_t(w,Z)$ is known from (19). In a context of backwards induction, where the function $L_t^e(w,Z)$ is known, we can then use a fixed-point calculation to find $x_t(w,Z)$. By guessing $x_t(w,Z)$ at a given pair (w,Z), we can construct the probabilities and the hazard rate from (22) and (24), and then calculate the decision time components $\tau_t^w(w,Z)$ and $\mu_t^w(w,Z)$ from the constraints on (21). Summing $H_t(w,Z)+\tau_t^w(w,Z)+\mu_t^w(w,Z)$, we can then update $x_t(w,Z)$ using (23).[14]

2.3 Household

The household is formed by a continuum of heterogeneous workers of unit mass, who aggregate their resources to decide on household consumption C_t, real bond $b_t = ln(B_t/P_t)$ and real money holdings $m_t = ln(M_t/P_t)$. Utility is discounted by factor $\beta \equiv \beta_I \beta_D$ per period, where β_I represents the effect of pure impatience, and β_D reflects the possibility of death (each individual worker dies and is replaced by a new individual with probability $1-\beta_D$ per period). Household consumption C_t is a CES aggregate of differentiated products $C_{j,t}$:

$$C_t = \left\{ \int_0^1 C_{j,t}^{\frac{\epsilon-1}{\epsilon}}\, dj \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{25}$$

where ϵ is the elasticity of substitution across varieties.

In addition to the wage setting decisions already discussed, the household's problem

14. As we argued (???) earlier for the worker's problem, problem (21) can be rewritten in terms of a single entropy cost term (a convex function) and a linear objective function. Since labor disutility is also convex, a unique well-defined solution exists for the maximization problem at any backwards induction step. Hence we conclude that the algorithm described here to calculate $x_t(w,Z)$ has a unique fixed point, which characterizes the marginal value of time in problem (21).

consists of choosing $\{C_{j,t}, b_t, m_t\}_{t=0}^{\infty}$ to maximize expected discounted utility:

$$\mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{C_\tau^{1-\gamma} - 1}{1-\gamma} \; - \; \int_0^1 X(H_{i\tau}^{tot}) di \; + \; \nu \ln (m_\tau) \right) \right] \tag{26}$$

subject to a per-period budget constraint:

$$\int_0^1 e^{p_{j,t}} C_{j,t} dj + e^{m_t} + R_t^{-1} e^{b_t} = \int_0^1 e^{w_{i,t}} H_{i,t} di + \frac{e^{m_{t-1}}}{\Pi_t} + \frac{e^{b_{t-1}}}{\Pi_t} + T_t^M + T_t^D. \tag{27}$$

Here $\int_0^1 e^{p_{j,t}} C_{j,t} dj$ is total real consumption, $\Pi_t = P_t/P_{t-1}$ is gross inflation, T_t^M is a lump sum transfer from the central bank, and T_t^D is a dividend payment from the firms. $\int_0^1 e^{w_{i,t}} H_{i,t} di$ is total labor compensation received from supplying the differentiated labor varieties $H_{i,t}$, and $H_{i,t}^{tot} = H_{i,t} + \tau_{i,t}^w + \mu_{i,t}^w$ is the total labor effort, including decision-making, of worker i. Each worker's labor and decision-making will vary with their current state (w, Z) as discussed previously.

Optimal consumption across the differentiated goods implies

$$C_{j,t} \;=\; e^{-\epsilon p_{j,t}} C_t, \tag{28}$$

so real spending can be written as $C_t = \int_0^1 e^{p_{j,t}} C_{j,t} dj$ under the aggregate price identity

$$1 \;\equiv\; \int_0^1 e^{(1-\epsilon) p_{j,t}} dj. \tag{29}$$

The first-order conditions for total consumption and for money use are

$$R_t^{-1} \;= E_t \left\{ \Pi_{t+1} \left(1 - \frac{v'(m_t)}{u'(C_t)} \right) \right\} \;=\; E_t[\Lambda_{t,t+1}], \tag{30}$$

where the household's stochastic discount factor is given by:

$$\Lambda_{t,t+1} \;\equiv\; \frac{u'(C_{t+1})}{u'(C_t)}. \tag{31}$$

2.4 Monetary policy and aggregate state vector

We consider a monetary authority that generates an exogenous process for nominal money growth. We assume the nominal money supply is affected by an AR(1) shock g,[15]

$$g_t = \phi_g g_{t-1} + \epsilon_t^g, \tag{32}$$

15. In related work (Costain and Nakov 2011) we have studied state-dependent pricing when the monetary authority follows a Taylor rule. Our conclusions about the degree of state-dependence, microeconomic stylized facts, and the real effects of monetary policy were not greatly affected by the type of monetary policy rule considered. Therefore we focus here on the simple, transparent case of a money growth rule.

where $0 \leq \phi_g < 1$ and $\epsilon_t^g \sim i.i.d.N(0, \sigma_g^2)$. Here g_t represents the time t rate of money growth:

$$M_t/M_{t-1} \equiv \mu_t = \mu^* \exp(g_t). \tag{33}$$

Seigniorage revenues are paid to the household as a lump sum transfer T_t^M, and the government budget is balanced each period.

To describe the aggregate state of the economy, we must take into account aggregate shocks and the distribution of idiosyncratic states. The Appendix to our working paper version shows that the model is homogeneous of degree one in nominal variables, and hence the corresponding real state variable would be:

$$\Xi_t \equiv (g_t, \Psi_t, \Psi_t^w). \tag{34}$$

where Ψ_t is the distribution of firms on real prices and productivities, and Ψ_t^w is the distribution of workers on real wages and productivities. We show that this is a valid state variable for the economy by constructing an equilibrium in terms of Ξ. Appendix A contains the derivations of the conditions for aggregate consistency, and appendix B contains detailed derivations of the distributional dynamics.

3 Calibration

We simulate the model at monthly frequency on a discrete grid, and discipline the key parameters of our model using microdata on price and wage adjustments.

Data sources As in CN19, our pricing data come from the Dominick's supermarket dataset documented by Midrigan (2011).[16] These data represent weekly regular price changes, excluding temporary sales, and are displayed (in logs) as a blue-shaded histogram in the left panel of Figure 2. We aggregate weekly adjustment rates to monthly rates for comparability with most related studies. We exclude sales because recent literature has shown that monetary nonneutrality depends primarily on the frequency of "regular" or "non-sale" price changes (e.g. Eichenbaum et al. 2011; Guimaraes and Sheedy 2011; or Kehoe and Midrigan 2015).

Our wage change data are from the International Wage Flexibility Project (IWFP), seen as a blue histogram in the right panel of Fig. 2; these data are taken from Fig. 2a of Dickens et al. (2007). The histogram aggregates data on wage adjustments across multiple countries. While most of the underlying national data are drawn from surveys of firms, they refer to annual nominal wage changes of individual workers who remain employed by the same firm. The IWFP focused on annual changes because it observed a widespread tendency for wages to change once a year in many countries, which in turn means that

16. We are grateful to Virgiliu Midrigan for making his price data available to us, and to the James M. Kilts Center at the Univ. of Chicago GSB, which is the original source of those data.

Table 1: Exogenous parameters

Parameter	Description	Value	Source
$1-\beta$	Discount rate (monthly)	0.0033	4% annual discount rate
$1-\beta_D$	Death probability (monthly)	0.0021	(Economic) Life span of 40 years
z_0	Log productivity at birth	-0.6	Normalization
ζ	Inverse Frisch elasticity	0.5	Standard value
ν	Coefficient on utility of money	1	Standard value
γ	Intertemporal elasticity of subs.	2	Golosov and Lucas Jr (2007)
χ	Coefficient on disutility of labor	6	Golosov and Lucas Jr (2007)
ϵ, ϵ_N	Elasticities of subs. across varieties	7	Golosov and Lucas Jr (2007)
μ^*	Long-run money growth rate	0.02	Annual inflation in Dominicks' data

much of the available survey data addresses annual changes. Clearly this makes our data on wage changes less than perfect for comparison with our price change data, which are at weekly frequency. Nonetheless, to get a quantitative benchmark for our theoretical model, we will take the IWFP data at face value.[17] Therefore, we assume that the monthly frequency of nominal wage adjustment is 1/12=0.083, and calculate nominal wage change statistics directly from the IWFP histogram.

Exogenous parameters Some parameters are taken either from related papers or from standard values in the literature. We set the (inverse) Frisch elasticity to $\zeta = 0.5$, and $\nu = 1$. Following Golosov and Lucas Jr (2007), we set $\gamma = 2$, $\chi = 6$, and $\epsilon = 7$, and we set the same the elasticity of substitution across varieties of labor as that across goods: $\epsilon_N = 7$. The discount factor is set to $\beta = 0.9967$, which correspond to a four percent annual discount rate. The monthly death probability is $1 - \beta_D = 0.0021$, implying an expected working life of forty years. The log productivity of newborn workers is set to $z_0 = -0.6$, so that workers expect a 60% productivity gain over their life cycles. We assume two percent annual money growth in steady state, consistent with our retail pricing data. Table 1 collects these parameters.

Calibrated parameters We then calibrate internally the parameters underlying the price- and wage-setting problems and productivity processes of firms and workers. These processes are assumed to follow discretized approximations of the following AR(1) dynam-

17. Grigsby et al. (2018) study wage adjustment using higher-frequency data more comparable to those from the retail price adjustment literature. In U.S. data from a large payroll data processing firm, they find a wage adjustment probability of 26.0% quarterly and 72.7% annually; the mean absolute wage change, conditional on adjustment, is 10.7%. Considering job stayers only, they find a 66.3% annual wage change probability, with a mean absolute change of 6.34%. While their data imply somewhat higher wage variability than the IWFP data in our graphs, nonetheless, for job stayers, the order of magnitude is similar.

Table 2: Calibrated parameters

Parameter			Value
Firms	Default hazard (monthly)	$\bar{\lambda}$	0.2707
	Adjustment cost	κ_f	0.0177
	Productivity persistence	ρ_a	0.6441
	Standard deviation productivity shocks	σ_a	0.0703
Workers	Default hazard (monthly)	$\bar{\rho}$	0.2317
	Adjustment cost	κ_w	0.0275
	Productivity persistence	ρ_z	0.9700
	Standard deviation productivity shocks	σ_z	0.0574

ics:

$$
\begin{aligned}
a_{j,t} = ln(A_{j,t}) &= \rho_a a_{jt-1} + \sigma_a \epsilon_t^a, \\
z_{i,t} = ln(Z_{j,t}) &= \rho_z z_{it-1} + \sigma_z \epsilon_t^z,
\end{aligned}
$$

where ϵ_t^a and ϵ_t^z are *i.i.d.* normal shocks with mean zero and unit variance.

Overall, we calibrate 8 parameters, collected in the vector $\mathcal{P} = (\bar{\lambda}, \kappa_f, \rho_a, \sigma_a, \bar{\rho}, \kappa_w, \rho_z, \sigma_z)$. We select $\mathcal{P}$ by minimizing:

$$
\mathcal{F}(\mathcal{P}) = (\mathcal{M}(\mathcal{P}) - \bar{\mathcal{M}})' \mathcal{W} (\mathcal{M}(\mathcal{P}) - \bar{\mathcal{M}}),
$$

where $\mathcal{M}(\mathcal{P})$ is a vector of model-generated moments when the parameter vector is $\mathcal{P}$, $\bar{\mathcal{M}}$ is a vector of the corresponding moments computed from our price and wage data, and $\mathcal{W}$ is a square matrix whose main diagonal contains the weights of each moment in the calibration. The vector $\mathcal{M}(\mathcal{P})$ contains the adjustment hazards for prices and wages, as well as the histogram of (discretized) nonzero log price and log wage changes.

Model evaluation We are able to perfectly match the adjustment probabilities of prices and wages (10.2% for prices and 8.3% for wages). In contrast, perfectly fitting the adjustment histograms is not possible. Fig. 2 presents the model-generated histogram of price and wage changes, as well as those in the data. The model generates distributions of price and wage adjustment that are close to those in the data, although somewhat smoother. In our model, the histogram of wage adjustments has a rather complex shape, with heavy tails. Likewise, the IWFP histogram (blue shaded area) has a lot of weight in the tails. Most of the mass is concentrated on small positive wage adjustments, but there is a fat right tail and a long, thin left tail, and there is "missing mass" of small negative adjustments. This pattern is often taken to indicate downward nominal wage rigidity. It is interesting that our model, in which rigidities are entirely symmetric, also seems to show some "missing mass" just below zero, although this effect is weaker than it is in the IWFP data. While downward adjustments are no more costly than upward adjustments in our model, workers have little incentive to make small negative changes because they expect their productivity to grow as they age, and because nominal prices have an inflationary trend. Thus, while workers have an incentive to set a higher wage when they become more productive, they can react to small negative productivity shocks by waiting for price inflation to reduce their real wage.

Figure 2: Distribution of nonzero price and wage changes

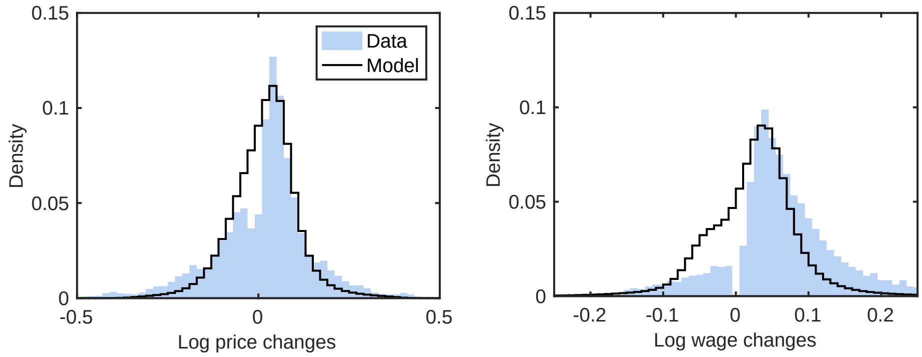


Fig. 3 shows the logit probabilities governing price resets and wage resets (left panels) and firms' and workers' adjustment hazards (right panels). For firms (top row), the probabilities are shown as functions of the lagged price and costs (inverse productivity); for workers, the probabilities are shown as functions of the lagged wage and productivity. Firms prefer higher prices when costs are higher, and the probability of adjustment rises smoothly as firms deviate from the prices they prefer (conditional on costs). In contrast to the linear labor disutility case (not shown), workers set substantially higher wages as their productivity rises.[18] The preferred wage varies by roughly $\pm 30\%$ as worker productivity varies between its maximum and minimum values in the Tauchen (1986) grid approximation, which differ by $\pm 45\%$.

4 Results

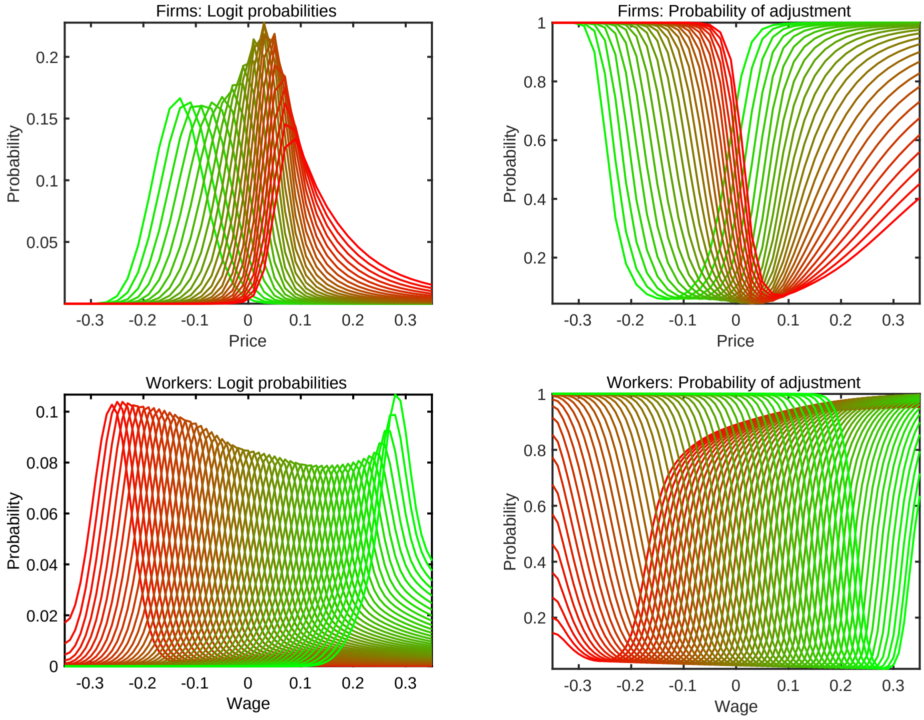
We first investigate the implications of nominal rigidity by simulating and comparing several calibrations of the model with varying degrees of noise in the pricing and wage-setting processes, both in steady state and in response to money shocks. We then exploit the state-dependency of price and wage adjustment choices to study the effects of long-run inflation and the potential non-linearities in the effects of monetary policy.

4.1 Price and wage stickiness: steady state implications

In this section we study how decision costs affect the frequency and the distribution of price and wage adjustments. To do so, we simulate five counterfactual economies with different degrees of friction in the price and/or wage adjustment decision. Table 3 presents the five counterfactuals, as compared with the baseline estimated version, which we will label V1.

18. ADD FOOTNOTE DISCUSSING RESULTS OF LINEAR CASE, FROM WP.

Figure 3: Adjustment behavior in the benchmark model



Notes: Distribution of price and wage adjustments (left panel) and price and wage adjustment probabilities (right panel). For firms, green lines represents low cost (high a). For workers, green lines represents high productivity (high z).

We define semi-flexible prices (wages) by dividing the baseline estimated value of κ_f (κ_w) by 10, and flexible prices (wages) by dividing κ_f (κ_w) by 100. To ease the exposition we label the counterfactual versions as V2-V6.

Table 3: Adjustment parameters — Counterfactual exercises

	Baseline	Decreasing P stickiness		Decreasing W stickiness		Flexible P and W
	V1	V2	V3	V4	V5	V6
Firms (κ_f)	$\kappa_f^0 = 0.0177$	$\kappa_f^0/10$	$\kappa_f^0/100$	κ_f^0	κ_f^0	$\kappa_f^0/100$
Workers (κ_w)	$\kappa_w^0 = 0.0275$	κ_w^0	κ_w^0	$\kappa_w^0/10$	$\kappa_w^0/100$	$\kappa_w^0/100$

We first look at the steady-state results of reducing price and/or wage stickiness. Table 4 reports steady-state statistics, comparing versions with different combinations of noise parameters (V1-V6). Decreased noise in price setting or wage setting makes adjustment more frequent. Crucially, the rise in the frequency of monthly wage adjustment is very large, from 8.34% in version V1 to 30.8% in version V6. Relatedly, lower noise implies smaller absolute price and wage changes, a lower standard deviation and kurtosis of price and wage changes, and more of the smallest changes (less than 5% or less than 2.5%). Wage adjustment is more costly than price adjustment, both because of the estimated noise parameters and because convex labor disutility means that some wage adjustments are particularly costly.

Table 4: Evaluating the model with different values of κ_f and κ_w

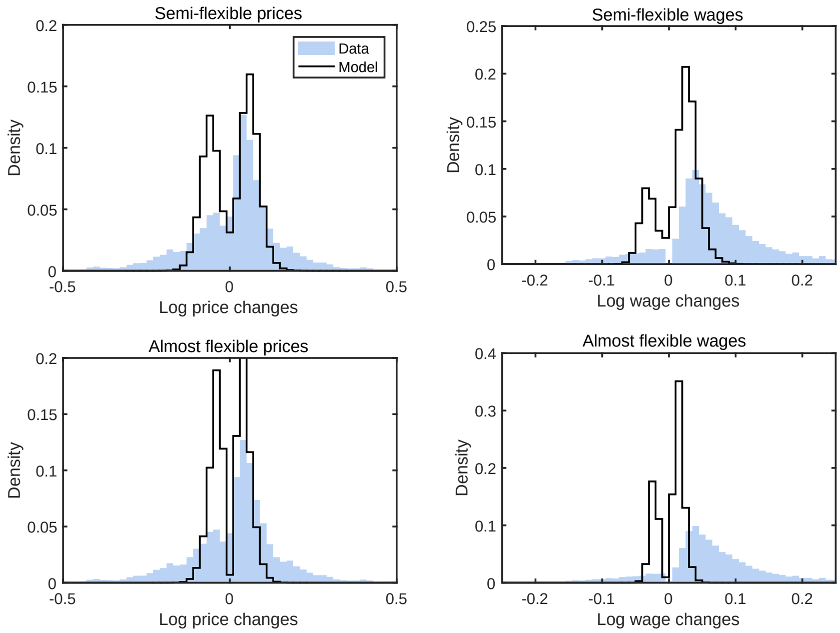
	Prices						
	Data	V1	V2	V3	V4	V5	V6
Frequency of price change (%)	10.2	10.2	24.8	59.5	10.2	10.2	59.7
Mean price change (%)	1.60	1.67	0.69	0.29	1.67	1.67	0.29
Mean absolute price changes (%)	9.90	6.94	6.06	4.53	6.92	6.92	4.52
Standard deviation of price changes (%)	13.2	8.96	6.70	5.03	8.94	8.93	5.03
Skewness of price changes	-0.42	-0.12	-0.15	-0.06	-0.12	-0.12	-0.06
Kurtosis of price changes	4.81	4.60	1.85	2.01	4.60	4.60	2.01
% share of price changes > 0%	65.1	56.5	53.9	52.4	56.5	56.5	52.4
% share of abs price changes < 5%	35.5	45.0	36.4	65.2	45.0	45.0	65.3
% share of abs price changes < 2.5%	12.0	27.3	13.8	25.7	27.3	27.3	25.8
Resetting cost, % revenues	–	0.50	0.20	0.07	0.49	0.49	0.07
Timing cost, % revenues	–	0.48	0.10	0.03	0.48	0.48	0.03
Loss relative to flexible prices[b], % revenues	–	2.49	1.39	1.01	2.48	2.48	1.01

	Wages						
	Data	V1	V2	V3	V4	V5	V6
Frequency of wage change (%)	8.30	8.34	8.33	8.33	13.4	30.8	30.7
Mean wage change (%)	5.10	3.00	3.01	3.01	1.86	0.81	0.81
Mean absolute wage changes (%)	6.47	5.50	5.50	5.50	3.17	1.95	1.96
Standard deviation of wage changes (%)	6.52	4.74	6.73	6.72	2.99	1.95	1.95
Skewness of wage changes	0.35	0.43	0.17	0.17	-0.56	-0.46	-0.46
Kurtosis of wage changes	4.39	11.9	11.8	11.7	2.56	2.00	2.00
% share of wage changes > 0%	86.5	70.6	70.6	70.6	73.2	66.8	66.8
% share of abs wage changes < 5%	43.0	60.8	60.8	60.8	93.4	100	99.9
% share of abs wage changes < 2.5%	11.8	25.2	25.2	25.2	33.2	80.2	80.0
Resetting cost, % labor income	–	0.50	0.20	0.07	0.49	0.49	0.07
Timing cost, % labor income	–	0.48	0.10	0.03	0.48	0.48	0.03
Loss relative to flexible wages[a], % labor income	–	2.49	1.39	1.01	2.48	2.48	1.01

[a] *Note*: Gain accruing to a single firm or worker not constrained by decision costs ($\kappa_f = 0$ or $\kappa_w = 0$).

The histograms of wage and price changes in the data and in models V2 to V5 are shown in Fig. 4; these may be compared to the benchmark specification V1 in Fig. 2. When prices and wages are sticky, both histograms are smooth and display rather fat tails; price adjustments are mildly left-skewed while wage adjustments are mildly right-skewed. As prices (wages) become more flexible, the price (wage) adjustment histogram becomes sharply bimodal. Thus, as decision noise decreases, price adjustments increasingly resemble the familiar (S, s) behavior associated with a menu cost model. Errors in pricing and timing smooth out the distribution of changes under calibration V1, but as noise is reduced, the preponderance of price changes occur around two upper and lower thresholds. Very small changes are rare, because it is not worth paying the cost of changing the price when it is already near its target value. The chosen decision cost $\kappa_f \mathcal{D}(\pi||\eta)$ decreases as κ_f declines; this is why the distance between the two peaks of the price change histogram

Figure 4: Distribution of nonzero price and wage changes: varying stickiness



Notes: left panel shows the effect of decreasing price stickiness on the distribution of nonzero price adjustments keeping wages sticky (versions V2 and V3). Right panel shows the effects of decreasing wage stickiness on the distribution of nonzero wage adjustments keeping prices sticky (versions V4 and V5).

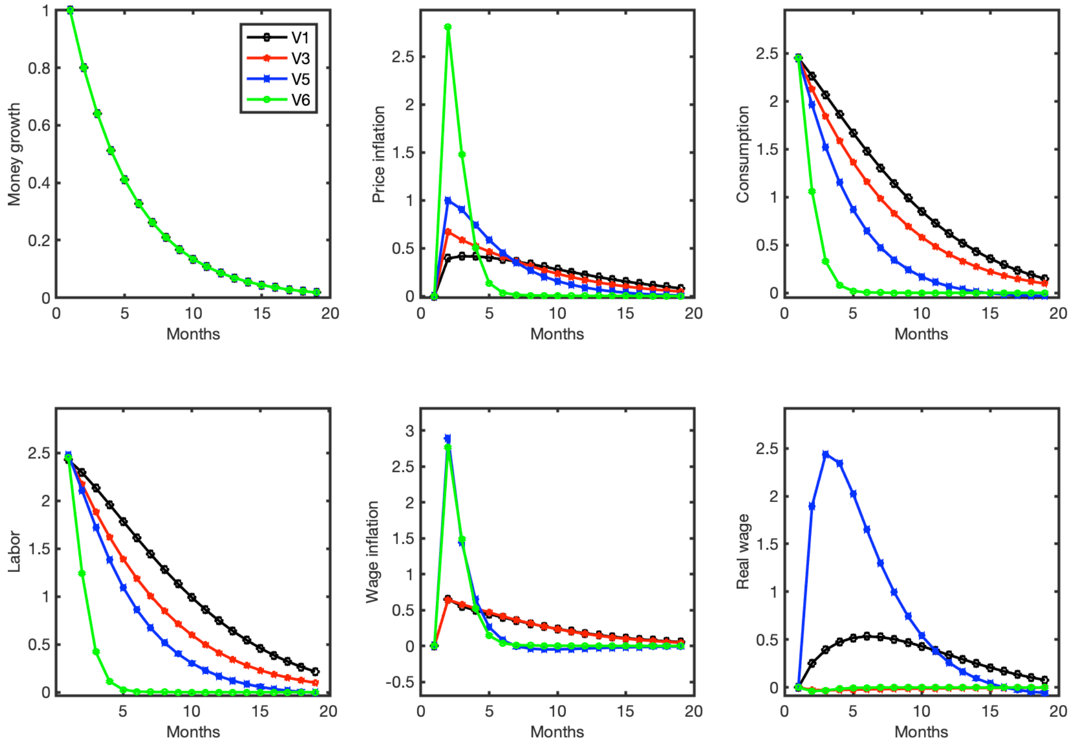
decreases as we move down the left panels of Fig. 4.

4.2 Price and wage stickiness: dynamic implications

We next analyze how price- and wage-setting frictions affect the non-neutrality of monetary shocks. Fig. 5 shows the effects of an autocorrelated money growth shock with monthly persistence 0.8. The figure compares the responses of price and wage inflation, consumption, hours and the real wage as price and wage stickiness vary, across models V1, V3, V5 and V6. The sticky-price, sticky-wage specification implies substantial real effects: consumption and labor rise 2.5% on impact, with a half-life of seven months. The version with reduced wage stickiness (V5) has similar real effects on impact, but much lower persistence, because it implies a large and persistent increase in real wages that offsets firms' incentive to demand more labor. As expected the smallest real impact comes from version V6, which has very low persistence, as in the Golosov and Lucas Jr (2007) menu cost model. We thus find that the real effects of money shocks are large as long as wages are sticky. Version V3 (sticky wages and flexible prices) has almost the same consumption response as V1, and lies substantially above V5 (flexible wages and sticky prices).

The reason wage stickiness is crucial is that it keeps firms' marginal costs from adjusting rapidly, so even though prices are much more flexible in version V3 than V1, the impulse

Figure 5: Money growth shock: effects of nominal rigidity



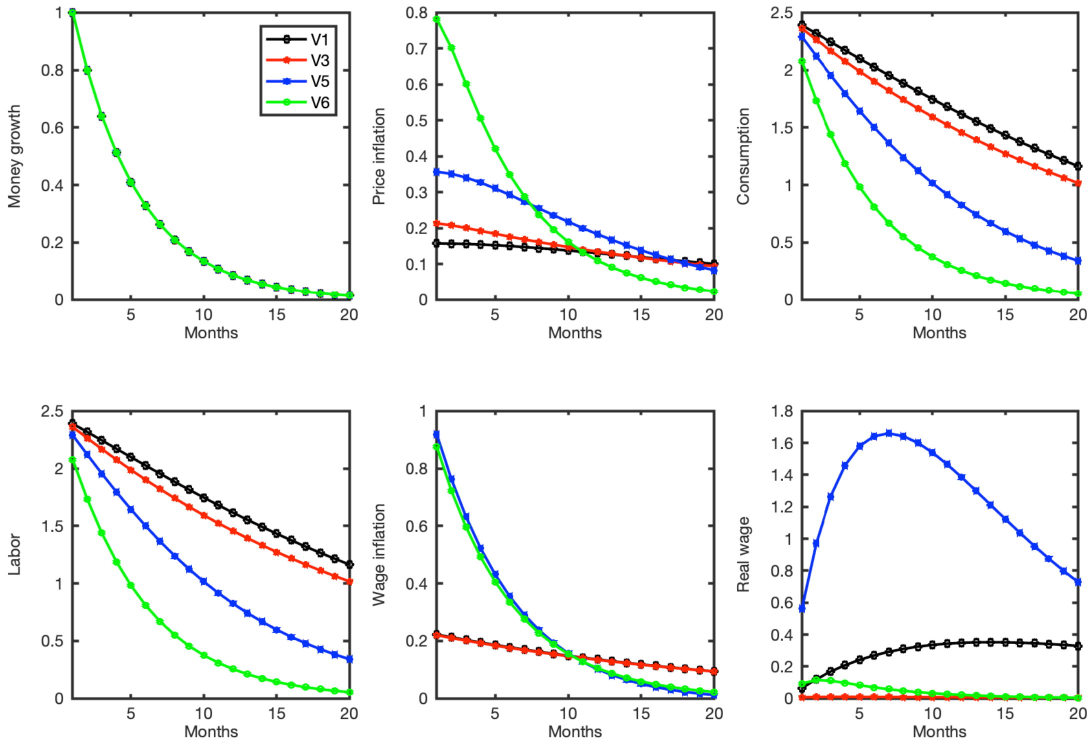
Notes: Impulse responses of inflation and consumption to money growth shock. Black: Benchmark model (V1), with both prices and wages sticky. Red: V3, flexible prices and sticky wages. Blue: V5, sticky prices and flexible wages. Green: V6: both prices and wages flexible.

response of price inflation is quite similar in both cases. Both wages and prices adjust gradually in version V3, giving a real impact on consumption and output that is almost as large and persistent as that seen in case V1. The key takeaway is that wage rigidity matters more than price rigidity for the overall degree of monetary nonneutrality in this model.

State-dependent model vs Calvo frictions Qualitatively similar results are found under a Calvo specification. Note that our state-dependent model generates substantially different adjustment hazards across model versions, with the wage adjustment hazard rising above 30% in versions V5 and V6. Fig. 5 shows impulse responses under Calvo specifications in which we change the adjustment hazards to reflect the hazards obtained from the state-dependent model versions shown in Fig. 4. The big difference between the state-dependent simulations and the Calvo simulations is that the latter have substantially greater persistence: the half-life of the consumption response is more than twice as long under Calvo frictions as it is in the benchmark case of our estimated state-dependent model.

Summarizing, wage stickiness is substantially more important for monetary non-neutrality than price stickiness alone. Sticky wages imply that firms' marginal costs only adjust slowly in response to the shock, which slows down firms' price adjustments even if prices

Figure 6: Money growth shock: effects of nominal rigidity. Calvo pricing.



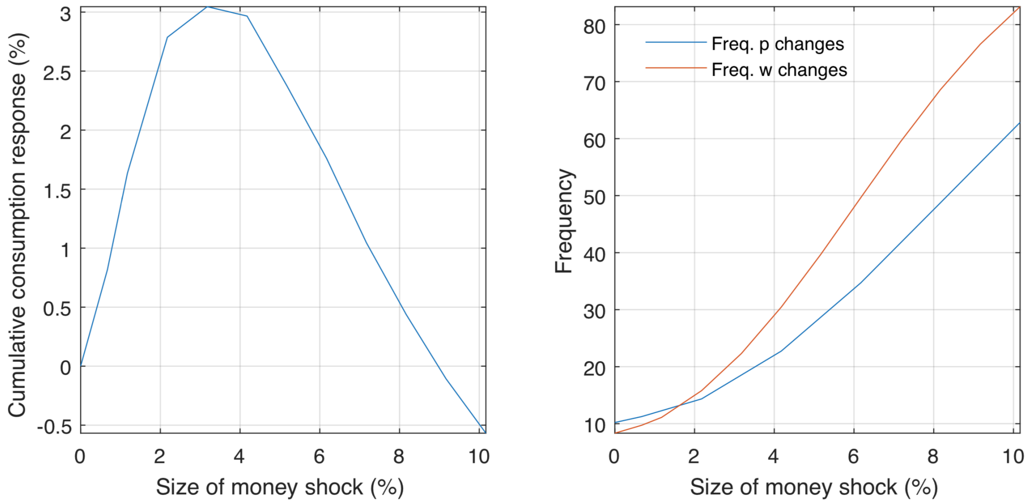
Notes: Impulse responses of inflation and consumption to money growth shock, under Calvo adjustment. Black: Benchmark Calvo specification (V1C), with both prices and wages sticky. Red: V3C, flexible prices and sticky wages. Blue: V5C, sticky prices and flexible wages. Green: V6C: both prices and wages flexible.

are relatively flexible. The importance of wage rigidity for propagation of nominal shocks to real variables provides support for New Keynesian mechanisms in the light of empirical evidence against procyclical markups of price over marginal cost (Nekarda and Ramey 2013). On the other hand, these findings do not offer any strong macroeconomic reason to favor the benchmark specification V1, with both rigidities, versus version V3, where only wages are rigid. Empirical studies rarely find a significantly nonzero response of the real wage to monetary policy shocks (see for example Christiano et al. 2005; McCallum and Smets 2007; Olivei and Tenreyro 2007; Christiano et al. 2016). Thus it is easy to reject the strongly procyclical real wage (and countercyclical profits) of specification V5, but both versions V1 (with a mildly positive real wage response) and V3 (with a mildly negative response) lie within the range of behavior consistent with macroeconomic evidence.

4.3 Nonlinearities in the effects of monetary policy

Our model is able to capture how adjustment behavior of firms and workers change with the environment. In this section we study how the effects of monetary policy vary with the size of the shock and the long-run inflation target of the monetary authority.

Figure 7: Comparing small and large money supply shocks in the benchmark model.



Left: cumulative impulse response of consumption to one-time increase in the money supply.
Right: change in adjustment frequency, on impact, for wages (orange) and prices (blue).

4.3.1 Large vs. small shocks

Fig. 7 shows that as money supply shocks become larger, their impact falls proportionally more on inflation and less on the real economy. The figure compares the impact of one-time, permanent, uncorrelated shocks to the money supply varying from two to sixteen percentage points. A two-percent jump in the money supply causes a small, persistent rise in inflation, and a persistent increase in consumption, peaking at 0.8% on impact. The impact effect on consumption increases to 1.4% (1.8%) for a four (six) percent jump in the money supply; but the persistence of the real effects drops rapidly with the size of the shock, so the cumulative real change is actually smaller for a six-percent money shock than it is for a four-percent shock. The reason is that larger shocks give firms and workers ever stronger incentives to adjust prices and wages immediately (a stronger selection effect). Thus, most of the nominal reaction occurs immediately, making the real effects smaller. Indeed, for money supply shocks larger than 8%, the real stimulus on impact shrinks, and the brief initial rise is followed by a prolonged slump in consumption and labor due to inflationary distortions.

4.3.2 Effects of long-run inflation target

In the context of the current prolonged episode of low inflation, it is interesting to ask how our model's behavior changes with trend inflation. Fig. 8 and Table 5 document some of the differences across annual trend inflation rates from -1% to 10%. The figure compares the impulse responses of our estimated benchmark model V1 to a 1% money supply shock (with monthly autocorrelation 0.8, as before) as trend inflation varies. The largest real effects are obtained when trend inflation is zero (orange); they are slightly smaller at either

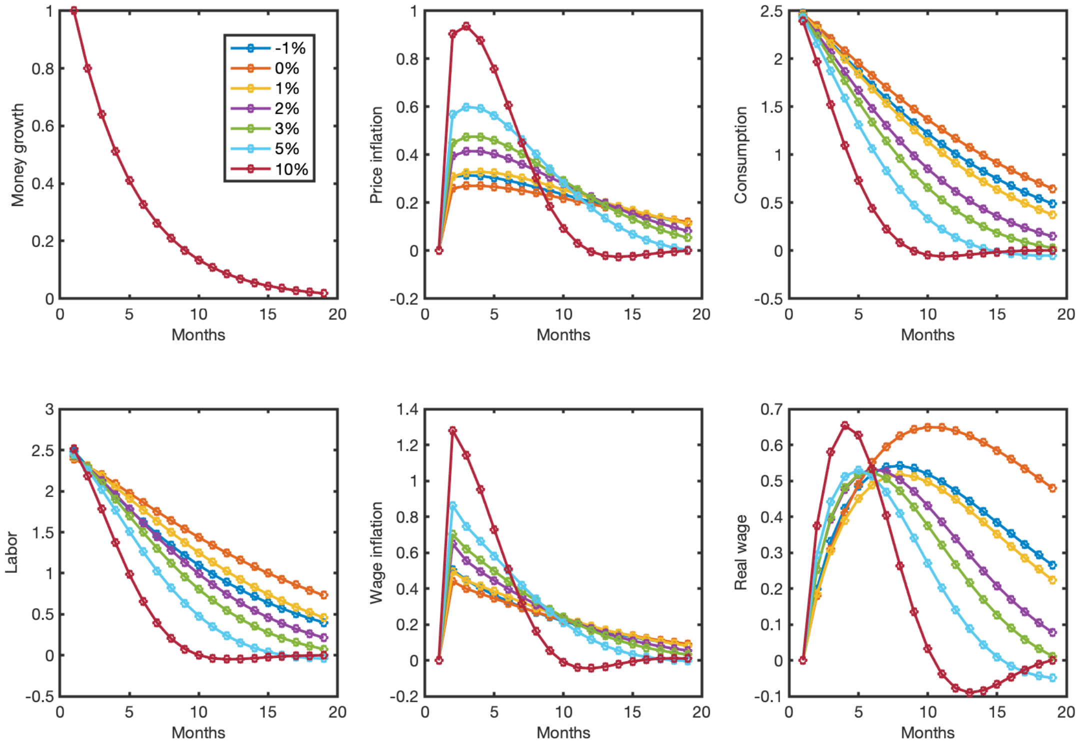
Table 5: Evaluating the benchmark model (V1), at different trend inflation rates

	Prices					
Trend inflation rate	-1%	0%	1%	2%	3%	5%
Frequency of price change (%)	9.04	7.53	9.05	10.2	11.0	12.5
Mean price change (%)	-0.93	0.00	0.92	1.67	2.23	3.25
Mean absolute price changes (%)	6.50	4.18	6.59	6.94	7.21	7.72
Standard deviation of price changes (%)	8.65	8.54	8.80	8.96	9.05	9.19
Skewness of price changes	0.24	0.10	-0.03	-0.12	-0.19	-0.30
Kurtosis of price changes	4.88	5.12	4.80	4.60	4.49	4.36
% share of price changes > 0%	37.9	43.5	51.4	56.5	59.7	64.8
% share of abs price changes < 5%	49.4	53.7	49.1	45.0	42.0	37.3
% share of abs price changes < 2.5%	30.8	35.7	30.7	27.3	25.1	21.9
Resetting cost, % revenues	0.45	0.38	0.45	0.50	0.53	0.59
Timing cost, % revenues	0.50	0.56	0.51	0.48	0.47	0.45
Loss relative to flexible prices[a], % revenues	2.44	2.40	2.44	2.49	2.52	2.59
Phillips multiplier[b], impact	0.12	0.11	0.13	0.16	0.18	0.23
Phillips multiplier[b], long run	0.17	0.15	0.20	0.25	0.30	0.39

	Wages					
Trend inflation rate	-1%	0%	1%	2%	3%	5%
Frequency of wage change (%)	7.28	6.95	7.53	8.34	9.11	10.8
Mean wage change (%)	-0.05	1.10	2.17	3.00	3.58	4.53
Mean absolute wage changes (%)	4.91	4.93	5.20	5.50	5.76	6.27
Standard deviation of wage changes (%)	6.77	6.82	6.82	4.74	6.67	6.55
Skewness of wage changes	0.82	0.59	0.35	0.43	0.06	-0.11
Kurtosis of wage changes	12.1	12.3	12.0	11.9	12.0	12.0
% share of wage changes > 0%	42.7	53.7	63.4	70.6	75.0	80.8
% share of abs wage changes < 5%	68.2	69.0	65.3	60.8	57.0	49.8
% share of abs wage changes < 2.5%	29.1	30.5	28.0	25.2	22.9	17.1
Resetting cost, % labor income	1.00	0.95	1.00	1.09	1.17	1.33
Timing cost, % labor income	0.97	0.99	0.97	0.94	0.91	0.87
Loss relative to flexible wages[a], % labor income	2.56	2.52	2.63	2.77	2.89	3.16
Phillips multiplier[c], impact	0.20	0.18	0.21	0.27	0.29	0.35
Phillips multiplier[c], long run	0.19	0.14	0.18	0.23	0.27	0.34

[a] *Note*: Gain accruing to a single firm or worker not constrained by decision costs ($\kappa_f = 0$ or $\kappa_w = 0$).
[b] *Note*: Phillips multiplier defined as the ratio of the change in price inflation to the change in log employment, either on impact on in the long run.
[c] *Note*: Phillips multiplier defined as the ratio of the change in wage inflation to the change in log employment, either on impact on in the long run.

Figure 8: Impulse responses at varying trend inflation rates in the benchmark model



Notes: Impulse responses to a 1 percentage point money supply shock (autocorrelation 0.8), starting from annual trend inflation rates of -1% (dark blue), 0% (red), 1% (yellow), 2% (baseline case, purple), 3% (green), 5% (light blue), and 10% (dark red).

plus or minus one percent trend inflation (yellow and light blue respectively). While there is little difference in the contemporaneous impact of money on consumption across trend inflation rates, higher trend inflation rapidly lowers the persistence of the real effects. The half-life of the consumption response falls from 10 months at 0% trend inflation to seven months in the baseline simulation (purple), which features a 2% trend inflation rate, and the half-life of consumption falls to four months at a ten percent trend inflation rate. On the other hand, these moderate changes in trend inflation have a big impact on the response of inflation to a monetary shock: inflation rises more than twice as much on impact, starting from 5% trend inflation, as it does after the same shock in the absence of a nominal trend.

Stated differently, if we define the "Phillips multiplier" as the ratio of the change in inflation to the change in log employment on impact, then Table 5 shows that this multiplier is more than doubled, from 0.108 to 0.232, as trend inflation rises from 0% to 5%. Alternatively, we could define this multiplier as the ratio of the area under the inflation impulse response to the area under the log employment impulse response; this is reported in the table as the "Long-run Phillips multiplier",[19] which rises from 0.147 at zero trend inflation to 0.394 at 5% annual inflation.

19. Barnichon and Mesters (2018) propose directly estimating this multiplier to measure the tradeoff between inflation and unemployment.

These results suggest that our model may help explain the notably flat Phillips curve that has been observed in recent years. For example, Blanchard (2016) estimates that the slope of the Phillips curve, controlling for expected inflation, decreased to roughly 0.2 (in absolute value) for the Great Moderation period, consistent with our Phillips multipliers for trend inflation near 2%. This finding is particularly interesting because many papers have argued that downward nominal wage rigidity decreases the slope of the Phillips curve at low inflation (Benigno and Ricci 2011; Lindé and Trabandt 2018). But our framework does not feature any asymmetry between the costs of upward and downward adjustments of wages or prices. Instead, the flattening of the Phillips curve is a result of state-dependent adjustment. At low inflation, the frequencies of wage and price adjustments both decrease, falling from 10.8% and 12.5% per month at 5% trend inflation to 6.95% and 7.53% when trend inflation is zero. Likewise, the adjustments get smaller: the mean absolute wage and price changes are 6.27% and 7.72% at 5% inflation, falling to 4.93% and 6.18% at zero trend inflation. Since workers and firms are less reactive to shocks at low inflation, the overall price level also becomes less reactive (causing the real economy to become more reactive). Hence, the Phillips curve becomes substantially flatter.

5 Conclusions

We have developed a DSGE model with state-dependent price and wage rigidity, combining monopolistic competition in goods and labor (as in Erceg et al. 2000) with nominal rigidity due to costly decision-making (as in Costain and Nakov 2019). Our heterogeneous-agents approach, with idiosyncratic shocks both to firms and to workers, allows us to fit our model to microdata on price and wage adjustments, but also permits us to calculate the dynamic effects of monetary policy shocks. Our model assumes that labor can be costlessly reallocated across firms at any time, so our study should be understood as documenting the interactions of nominal price stickiness with nominal wage stickiness, abstracting from matching frictions or any other forms of labor specificity. We assume convex disutility of labor, so that workers prefer to vary their wages, as well as their hours, in response to shocks.

At a microeconomic level, we compare different calibrations to see how nominal rigidities affect price and wage adjustment behavior. We estimate the decision cost parameters and productivity processes to match hazard rates and adjustment histograms from price and wage microdata; our estimation is numerically feasible since it only requires computing the model's steady state. Our estimates match the frequency of adjustment from microdata, and produce a histogram somewhat smoother than that observed in the data. Firms in our estimated model spend less than one percent of revenues on decisions related to price setting, while workers devote approximately two percent of their time to decisions about wage setting. Allowing for a trend in idiosyncratic productivity over the life cycle implies that small negative wage changes are relatively infrequent; this helps explain a pattern which is often interpreted as evidence of downward nominal wage rigidity, in spite of the fact that there is no inherent downward rigidity in our framework.

Our model implies a policy-relevant degree of monetary nonneutrality. Compared with the time-dependent framework of Calvo (1983), our state-dependent model implies similar real effects of money growth shocks on impact, but only half the persistence. We find that wage stickiness is a stronger source of monetary nonneutrality than price stickiness; a version of our model with wage stickiness only produces almost as much non-neutrality as versions with wage and price stickiness together. This accords with the consensus from time-dependent models of nominal rigidity (Huang and Liu, 2002; Christiano et al., 2005); our study is the first to demonstrate this result in a state-dependent model. In contrast, the version of our model with price stickiness only has much reduced real effects of money shocks, and implies a strong, counterfactual rise in the real wage after a monetary stimulus.

Monetary policy has a number of highly nonlinear effects in our framework. Larger money shocks cause adjustment hazards to rise, so inflation responds more quickly and real effects are proportionally smaller. Indeed, the absolute size of the cumulative real impact is maximized by a rise of roughly 4% in the money supply; money shocks substantially larger than this have a predominantly negative impact on real variables. Decreasing the trend inflation rate causes adjustment hazards to fall, both for prices and wages. This alters the slope of the Phillips curve, as lower responsiveness of price-setting and wage-setting makes inflation less responsive to macro shocks too. The real effects of a money shock are largest at zero trend inflation, and decrease as the inflation trend becomes negative or positive. The effects on the slope of the Phillips curve are quantitatively significant: its slope more than doubles as trend inflation rises from 0% to 5% annually. A flatter Phillips curve at low trend inflation rates has often been explained by appealing to downward nominal wage rigiditiy, but in our context it is caused by state-dependent changes in adjustment frequencies, not by any downward asymmetry in the costs of adjustment.

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Online Appendix

A Aggregate consistency

When supply equals demand for each good j, total supply and demand of effective labor satisfy

$$N_t - \mu_t - \tau_t = \int_0^1 \frac{C_{j,t}}{A_{j,t}} dj \;=\; C_t \int\int \psi_t(p,a) e^{-\epsilon p - a} da\, dp \equiv \Delta_t C_t. \tag{35}$$

Here μ_t is total time devoted to deciding whether to adjust prices, and τ_t is total time devoted to choosing which price to set by firms that adjust:

$$\mu_t \;=\; \int\int \psi_t(p,a)\mu_t(p,a)dadp \tag{36}$$

$$\tau_t \;=\; \int\int \psi_t(p,a)\tau_t(p,a)dadp \tag{37}$$

where firm-specific decision times are given by (13)-(14). Equation (35) also defines a measure of price dispersion, $\Delta_t \equiv \int_0^1 e^{-\epsilon p_{j,t}} A_{j,t}^{-1} dj$, weighted to allow for heterogeneous productivity. As in Yun (2005), an increase in Δ_t decreases the goods produced per unit of labor, effectively acting like a negative aggregate productivity shock.

In nominal terms, the price level and wage level are given as follows

$$\int\int P^{1-\epsilon}\phi_t(P,A)dA\, dP \;=\; P(\Omega_t)^{1-\epsilon}, \tag{38}$$

$$\int\int \left(\frac{W}{Z}\right)^{1-\epsilon_N} \phi_t^W(W,Z)dZ\, dW \;=\; W(\Omega_t)^{1-\epsilon_N}. \tag{39}$$

Given (38), the real price level is one by definition:

$$\int\int e^{(1-\epsilon)p}\psi_t(p,a)da\, dp \;=\; 1. \tag{40}$$

The real wage level satisfies

$$\int\int e^{(1-\epsilon_N)(w-z)}\psi_t^W(w,z)dz\, dw \;=\; e^{(1-\epsilon_N)w(\Xi_t)}. \tag{41}$$

B Distributional dynamics

The distribution of firms' prices and productivities, and likewise that of workers' wages and productivities, evolves over time as firms and workers respond to idiosyncratic and aggregate shocks. We first state the equations governing the dynamics of the distribution across firms.

We continue to use the notation $P_{j,t}$ to refer to the nominal price at which firm j produces in period t, prior to adjustment. This may of course differ from its price $\widetilde{P}_{j,t}$ at the end of t, when price adjustments are realized. Therefore we will distinguish the beginning-of-period distribution of prices and log productivities, $\Phi_t(P_{j,t}, a_{j,t})$, from the distribution of prices and log productivities at the end of t, $\widetilde{\Phi}_t(\widetilde{P}_{j,t}, a_{j,t})$. But instead of tracking nominal prices $P_{j,t}$, it is simpler to focus on log real prices $p_{j,t}$. Therefore, in analogy to the nominal distributions, we define $\Psi_t(p_{j,t}, a_{j,t})$ as the real distribution at the beginning of t, when production takes place, and $\widetilde{\Psi}_t(\widetilde{p}_{j,t}, a_{j,t})$ as the real distribution at the end of t. Finally, we also use lower-case letters to represent the joint densities associated with these distributions, which we write as $\phi_t(P_{j,t}, a_{j,t})$, $\widetilde{\phi}_t(\widetilde{P}_{j,t}, a_{j,t})$, $\psi_t(p_{j,t}, a_{j,t})$, and $\widetilde{\psi}_t(\widetilde{p}_{j,t}, a_{j,t})$, respectively.[20]

Two stochastic processes drive the dynamics of the distribution. First, there is the Markov process for firm-specific log productivity, which we can write in terms of the following *c.d.f.*:

$$S(a'|a) \;=\; prob(a_{j,t} \leq a'|a_{j,t-1} = a), \tag{42}$$

or in terms of the corresponding density function:

$$s(a'|a) \;=\; \frac{\partial}{\partial a'} S(a'|a). \tag{43}$$

Thus, suppose that the density of nominal prices and log productivities at the end of period $t-1$ is $\tilde{\phi}_{t-1}(\tilde{P}, a)$. The density at the beginning of t, after productivity shocks, will therefore be

$$\phi_t(\tilde{P}, a') \;=\; \int s(a'|a)\widetilde{\phi}_{t-1}(\tilde{P}, a)da. \tag{44}$$

But this equation conditions on a given nominal price $\tilde{P}$. Holding fixed a firm's nominal price, its real log price is changed by inflation, from $\widetilde{p}_{i,t-1}$ to $p_{i,t} \equiv \widetilde{p}_{i,t-1} - i_t$. Therefore the density of real log prices and log productivities at the beginning of t is given by

$$\psi_t\left(\widetilde{p} - i_t, a'\right) \;=\; \int s(a'|a)\widetilde{\psi}_{t-1}(\widetilde{p}, a)da, \tag{45}$$

and hence the cumulative distribution at the beginning of t, in real terms, is

$$\Psi_t(p, a') \;=\; \int^{p}\int^{a'}\left(\int s(b|a)\widetilde{\psi}_{t-1}\left(q + i_t, a\right)da\right)db\,dq. \tag{46}$$

The second stochastic process that determines the dynamics is the process of real price updates, which we have defined in terms of a conditional density of logit form in (7). A firm with real log price p and log productivity a at the beginning of period t adjusts its price with probability $\lambda\left(\frac{d_t(p,a)}{\kappa_f w_t}\right)$, where

$$d_t(p, a) \;\equiv\; \widetilde{v}_t(a) - v_t^e(p, a).$$

20. Our notation in this section assumes that all densities are well-defined on a continuous support, but we do not actually impose this assumption on the model. With slightly more sophisticated notation we could allow explicitly for distributions with mass points, or with discrete support.

Upon adjustment, its new real log price is distributed according to $\pi_t(\tilde{p}|a)$. Therefore, if the density of firms at the beginning of t is $\psi_t(p,a)$, the density at the end of t is given by

$$\widetilde{\psi}_t(\widetilde{p},a) \;=\; \left(1-\lambda\left(\frac{d_t(\widetilde{p},a)}{\kappa_f w_t}\right)\right)\psi_t(\widetilde{p},a)+\int\lambda\left(\frac{d_t(p,a)}{\kappa_f w_t}\right)\pi_t(\widetilde{p}|a)\psi_t(p,a)dp.$$

The cumulative distribution at the end of t is simply given by integrating up this density:

$$\widetilde{\Psi}_t(p,a) \;=\; \int^{\widetilde{p}}\int^{a}\widetilde{\psi}_t(q,b)db\,dq.$$

The dynamics of wages and worker productivities is analogous, except that an individual worker may die and be replaced by a new worker with probability $1-\beta_D$ per period. It suffices to go directly to the real log dynamics, without developing notation for the nominal dynamics. Let $\Psi^w_t(w_{i,t},z_{i,t})$ be the distribution of real log prices and log worker productivities at the beginning of the period, when production takes place, and let $\widetilde{\Psi}^s_t(\widetilde{w}_{i,t},z_{i,t})$ be the corresponding distribution of *surviving* workers at the end of the period. We write the densities associated with these distributions as $\psi^w_t(w_{i,t},z_{i,t})$ and $\widetilde{\psi}^s_t(\widetilde{w}_{i,t},z_{i,t})$, respectively.

Now, consider a worker with real log wage w and log productivity z at the beginning of period t; she adjusts her wage with probability $\rho\left(\frac{d^w_t(w,z)}{\kappa_w\xi_t(w,z)}\right)$, where

$$d^w_t(w,z) \;\equiv\; \widetilde{l}_t(w,z)-l^e_t(w,z).$$

Upon adjustment, her new real log wage is distributed according to $\pi^w_t(\tilde{w}|w,z)$. Therefore, if the density of workers at the beginning of t is $\psi^w_t(w,z)$, the density at the end of t is given by

$$\widetilde{\psi}^w_t(\widetilde{w},z) \;=\; \left(1-\rho\left(\frac{d^w_t(\widetilde{w},z)}{\kappa_w\xi_t(\widetilde{w},z)}\right)\right)\psi^w_t(\widetilde{w},z)+\int\rho\left(\frac{d^w_t(w,z)}{\kappa_w\xi_t(w,z)}\right)\pi^w_t(\widetilde{w}|w,z)\psi^w_t(w,z)dw.$$

The cumulative distribution at the end of t integrates up this density:

$$\widetilde{\Psi}^w_t(\widetilde{w},z) \;=\; \int^{\tilde{w}}\int^{z}\psi_t(q,b)db\,dq.$$

A worker alive in period t survives to period $t+1$ with probability β_D. The worker's productivity, conditional on survival, is driven by the Markov process S^z:

$$S^z(z'|z) \;=\; prob(z_{i,t+1}\le z'|z_{i,t}=z), \tag{47}$$

with the following density function:

$$s^z(z'|z) \;=\; \frac{\partial}{\partial z'}S(z'|z).$$

Meanwhile, holding fixed a worker's nominal wage, her real log wage is changed by inflation, from $\widetilde{w}_{i,t}$ at the end of t, to $w_{i,t+1} \equiv \widetilde{w}_{i,t} - i_{t+1}$. Therefore the density of real log wages and log worker productivities among surviving workers at the beginning of $t+1$ is given by

$$\psi^s_{t+1}\left(\widetilde{w} - i_{t+1}, z'\right) \;=\; \int s^z(z'|z)\tilde{\psi}^w_t(\widetilde{w}, z)dz. \tag{48}$$

Hence the cumulative distribution at the beginning of t integrates up the density in (48) and adds on the component of new-born workers, who have distribution Ψ^0_t:

$$\Psi^w_{t+1}(w,z) \;=\; \beta_D \int^w \int^z \left(\int s^z(b|y)\tilde{\psi}^w_t\left(q + i_{t+1}, y\right) dy \right) db\, dq + (1-\beta_D)\Psi^0_{t+1}(w,z).$$

Taking account of birth and death matters here because it allows us to impose a productivity process that has an upward trend over the course of an individual's working life: a worker typically ends her career at a wage higher than the one she started with. We find that this upward trend is important for matching the distribution of wage adjustments. We denote the distribution of wages and productivity for newborn workers at time t by Ψ^0_t.

For simplicity, we assume that the wage of a newborn worker is the wage that she would set, conditional on her productivity, if her wage were costlessly flexible at all times. We make this simplifying assumption to avoid modeling an initial decision-making state prior to beginning life as a worker. Since our analysis only addresses the properties of wage changes, ignoring the level of the initial wage, this simplifying assumption has a negligible impact on the empirical properties we will document here.