Aggregate Effects of Firing Costs with Endogenous Firm Productivity Growth

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Abstract

This paper quantifies the aggregate effects of firing costs in a model of firm dynamics where firm-level productivity is determined by innovation. In the model, the productivity distribution is endogenous, and thus, potentially affected by policy changes, allowing the model to capture both the static (allocative efficiency) and dynamic effects (changes in the distribution of firms' productivity) of firing costs. The model is calibrated so as to match key features of firms' hiring and firing behavior using firm-level data from Spanish non-financial firms. I show that firing costs equivalent to 2.5 monthly wages produce a 4% loss in aggregate productivity relative to the frictionless economy. The aggregate productivity losses rise to more than 10% when firing costs are equivalent to one year's wage. I show that the dynamic effects of firing costs are quantitatively relevant, explaining 35% of these productivity losses. Overall, the results suggest that ignoring the effects of frictions on the dynamics of firm productivity can substantially underestimate their aggregate effects.

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1 Introduction

There is a large body of research studying the productivity losses from firing costs. Following Hopenhayn and Rogerson (1993), most of this literature typically quantify the effects of firing costs by looking at the efficiency in the allocation of labor across firms given a productivity distribution. However, if firm growth is a risky process, firing costs would be a critical component of the cost of failure, affecting the incentives of firms to grow and potentially shaping the distribution of firms' productivity itself. By assuming an exogenous process for firm's productivity, previous literature cannot capture such dynamic effects, and thus, may underestimate the aggregate impact of firing costs. This paper fills the gap by quantifying the aggregate implications of firing costs in a model in which the dynamics of firms' productivity are endogenous.

I extend the standard firm dynamics model of Hopenhayn and Rogerson (1993) by incorporating an innovation technology that allows firms to have partial control over the probability of innovation—as in Atkeson and Burstein (2010)—and over the outcome of innovation itself. I model innovation building on the "control cost" approach borrowed from the game theory literature. In particular, firms in the model can choose, at a cost, the probability of innovation and, in case innovation occurs, the distribution of next period's productivity. In models á la Atkeson and Burstein (2010) firms do not face the risk of a very negative shock—key to account for the effects of firing costs—unless the size of the productivity step is sufficiently large, which would generate unrealistic productivity dynamics.[1] My approach can generate sufficiently large downwards risk while keeping the dynamics of productivity realistic and allowing for a cleaner identification of the relevant parameters.

I estimate the parameters of the model by matching key moments regarding firm growth and firing and hiring behavior, using firm-level data from Spanish non-financial firms. The Spanish economy is of particular interest for this analysis. The Spanish labor market, considered as one of the most inefficient labor markets in Europe, is characterized by a high structural unemployment rate, a high volatility of employment, and an intensive

[1] These models assume that firms can invest resources in increasing the probability of a positive step in their productivity versus a negative one, but the size of this step is exogenously set. This implies that the level of risk firms face is limited by assumption. One could add an extreme shock to generate sufficient negative risk, but this would come at the cost of adding more parameters into the model.

use of temporary employment. Productivity in Spain is one of the lowest among developed countries. In 2010 Spanish TFP was 9% lower than it was in 1990, while for the US and Germany it was 20% higher. This paper connects the underperforming of Spanish productivity with the distortions of its labor market.

The model closely matches the targeted moments. In the baseline economy, small firms innovate more frequently, their innovations are more aggressive (as measured by the expected productivity growth) and more volatile (as measured by the standard deviation of productivity growth). These predictions imply that small firms grow faster and that their growth rates are more volatile. This is consistent with the empirical evidence.[2]

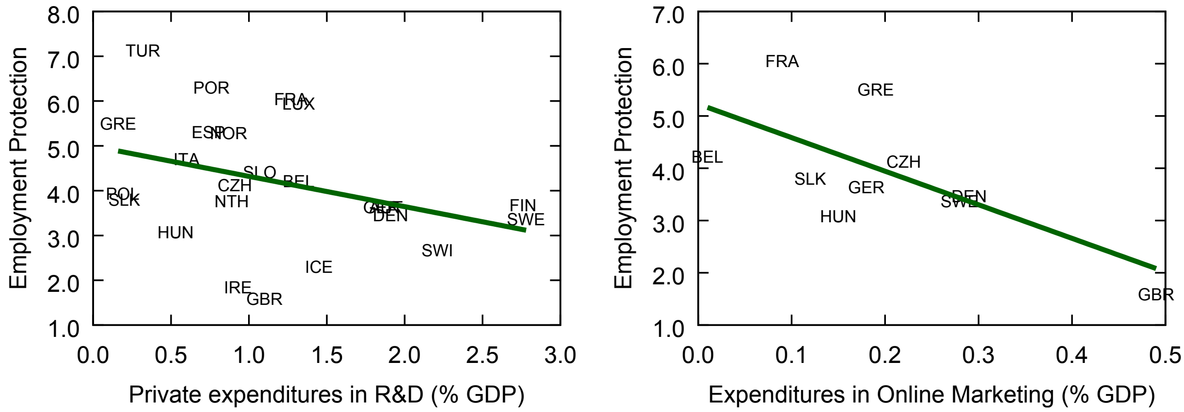
Using the calibrated model I ask, "What are the aggregate effects of firing costs?". In order to address this question, I compare the baseline economy, with a firing cost parameter equivalent to its calibrated value of 2.5 monthly wages, with one in which firing costs are set to zero. I find that aggregate productivity is 3.7% lower in the baseline economy than in the frictionless one. This is a large effect compared to what has been found in the previous literature. For instance, Hopenhayn and Rogerson (1993) find a 2.5% drop in aggregate productivity when firing costs are equivalent to one year's wage. Using a model in which the dynamics of productivity are affected by firing costs, Da-Rocha et al. (2019) find a 4.2% fall in aggregate productivity for a level of firing costs equivalent to one year's wage. When I set the firing costs to one year's wage, the fall in aggregate productivity is of more than 10% relative to the frictionless economy

The main reason behind this larger fall in aggregate productivity in my model is that productivity dynamics are endogenous. The firm dynamics literature typically assumes that firm productivity follows an exogenous process. In reality, however, firms have the option to undertake a large number of actions to improve their profits prospects, which I refer to as "innovation".[3] This means that, although partially stochastic, firm's growth is driven by firm's actions, which may be affected by economic conditions such as labor regulation. In particular, if innovation is costly and its outcome uncertain, firms incentives to make such investments will depend on the cost of failure, that is affected by the mag-

2. See for example Sutton (1997, 2002) or Klette and Kortum (2004). See figure B.1 in Appendix B for the corresponding relationships in the Spanish data. Haltiwanger et al. (2013) show that the size-growth correlation disappears when firm age is controlled for. In the model presented in this paper, however, firm age does not have any economic interpretation.

3. Examples of these investments include product or process innovation but also demand-side investments such as marketing or sales campaigns.

Figure 1: Firing costs and firms' investments in growth-generating activities



Source: (i) *Employment Protection* refers to the sum of the OECD strictness of employment protection legislation indicators for permanent and temporary contracts; (ii) *Private expenditures in R&D* is taken from the OECD Main Science and Technology Indicators Database; (iii) *Expenditures in Online Marketing* is taken from Grece (2016).

nitude of firing costs. As a result, firms may optimally decide to invest less in innovation, reducing their productivity growth and the average firm productivity in the economy.

In figure 1 I plot some suggestive evidence on this negative relationship between firing costs and innovation. In particular, I plot the relationship between the strictness of employment protection legislation taken form the OECD, and two measures that fit well the broad definition of innovation in my paper: R&D expenditures (left panel) and firms spending on online marketing (right panel). In both cases, countries with high levels of firing costs show lower spending on innovation.[4] This is what happens in the model. In the baseline economy, investment in innovation falls by 2.7% when firing costs equal the calibrated value of 2.5 monthly wages relative to the frictionless economy, making the average firm productivity to drop by 1.4%. When firing costs are of one year's wage, innovation expenses and average productivity fall by 8% and 4.5% respectively.

To quantify how much of the fall in aggregate productivity is accounted for the endogenous changes in the dynamics of firms' productivity I simulate an economy with positive firing costs but fixing the innovation choices from the frictionless economy. This makes the law of motion of productivity to be unaffected by changes in the firing cost. In this new economy, in which innovation is exogenous, the fall in aggregate productivity

4. Firing costs can also increase firms' incentives to make other types of investments, such as labor-saving technologies. However, this type of investments may have a larger impact on the production technology than on profitability (for given inputs), which is the focus of the paper.

is of 2.4%, substantially lower than in a model with endogenous innovation where firms can adjust their innovation choices. This means that 35% of the drop in aggregate productivity is explained by changes in the distribution of productivity. The remaining 65% is explained by the loss in allocative efficiency of labor across firms, *given* a productivity distribution. This finding suggests that models with exogenous productivity processes may largely underestimate the effects of frictions/policies such as firing cost.

The rest of the paper is organized as follows. Section 2 reviews the literature on firing costs and firm innovation. Section 3 presents the model economy. Section 4 explains the calibration procedure. Section 5 presents the results from changing firing cost. Finally, section 6 concludes.

2 Literature Review

There is a large literature that evaluates the role of different policies and frictions in accounting for aggregate productivity differences across countries (Guner et al. 2008, Restuccia and Rogerson 2008, Hsieh and Klenow 2009, Bartelsman et al. 2013, Hsieh and Klenow 2014, García-Santana et al. 2016). While many of these papers use "wedges" to measure policy distortions, some others specify particular policies.[5] One of the policies that has attracted more attention is employment protection, starting with the analysis of firing costs of Hopenhayn and Rogerson (1993). The distortion introduced by firing taxes on firm hiring and firing decisions are well established in the literature, both empirically (Haltiwanger et al. 2014) and theoretically (Bentolila and Bertola 1990). These distortions prevent firms from operating at their optimal scale, worsening the allocation of labor across firms, and damaging aggregate productivity.

The literature studying the impact of firing costs on aggregate productivity typically finds moderate effects (Hopenhayn 2014). However, aggregate productivity losses may be larger when the firms' productivity distribution is endogenous as firing costs may also distort incentives of firms to invest in growth-generating activities, such as innovation, marketing campaigns, launching new products, etc. This is first explored by Da-Rocha et al. (2019), who study the aggregate implications of firing costs in a continuous-time

5. See Restuccia and Rogerson (2013) for a discussion on these different approaches.

model in which the law of motion of firm's productivity is size-dependent. My paper differs from theirs in two margins. First, I consider a model with a continuum of potential firm size, while they consider a model where firms can either be small or large. Second, in their paper, the law of motion of firm's productivity is size dependent, but the difference between large and small firms is exogenous. In my paper large and small firms will have different laws of motion for their productivities endogenously, as a result of different innovation choices which are affected by the introduction of firing costs.

More generally, my paper relates to a recent literature that explores how frictions affect aggregate productivity not only thought the efficiency in the allocation of resources, but also through a direct effect on the firm-level productivity distribution itself. For instance, López-Martín (2013) and Mukoyama and Osotimehin (2019) endogenizes the way in which frictions affect firm's productivity dynamics by including an innovation technology similar to the one in Grossman and Helpman (1991), Aghion and Howitt (1992) and Atkeson and Burstein (2010). Ranasinghe (2014) extends the Hopenhayn and Rogerson (1993) framework by allowing firms to invest in innovation, which changes the parameters of a flexible parametric distribution driving next period's distribution. Gabler and Poschke (2013) study the effects of firing costs, among other frictions, using a firm dynamics model in which firms can engage in experimentation and discard negative productivity shocks.

My paper contributes to this literature by making the distribution of firm-level productivity entirely driven by firm choices, including the degree of uncertainty faced by firms. This have two main implications. On the one hand, it allows my model to generate a sufficiently large downwards risk while keeping the firms size distribution realistic. This is particularly relevant for the analysis of firing costs since this friction introduces an asymmetric adjustment costs. On the other hand, the parameters governing the innovation process have a clear interpretation and can be identified using moments on firm size and hiring and firing choices. The innovation process used in this paper is also computationally convenient as it generates closed-form solutions for innovation choices. Thus, the model can be used to study the effects of different policies and friction accounting for their effects on the firm-productivity distribution without adding model complexity.

Finally, my paper is related to the game theory literature from which I borrow the "control cost" approach used to model productivity dynamics. This modeling device is used to model equilibria in which agents optimally make errors under the assumption that precision is costly. In this approach, decisions are conceived as a random variable over a feasible set of alternatives—which in my setting are the different levels of productivity—, and the cost is given by the precision of this random variable. In Costain et al. (2019) we implement this idea to model price and wage adjustment decisions in an otherwise standard new-keynesian framework with heterogeneous agents. Turen (2018) model costly information acquisition in a price-setting problem using a "control cost" framework. To the best of my knowledge, my paper is the first that uses this approach to model the dynamics of firm-level productivity.

3 The Model

This section presents an extension of the workhorse general equilibrium model of Hopenhayn and Rogerson (1993) in which I introduce an innovation technology that allows firms to invest in both the probability and the outcome of innovation.

3.1 Overview

The economy is populated by a continuum of firms of unit mass, characterized by a profitability factor, denoted by d, and a number of workers hired in the past, n. The term $d \in \mathbb{D} \equiv \{d_1, d_2, \dots, d_D\}$ is a factor that increases revenues for given inputs, so it captures both productivity (ie. technology) and demand factors (ie. tastes). For simplicity in the exposition, I will refer to d as firm's productivity throughout the rest of the paper.

Given an initial state (d, n), firms decide on hirings/firings, produce and collect profits. Then, they are hit by an exit shock. With probability $1 - \delta \in (0, 1)$, the firm continues in the market and make innovation decisions. With probability δ the firm exists and it is immediately replaced by a new firm. Entrants start with no workers and an initial productivity drawn from $\log(d_0) \sim \eta^0$ with $E[d_0] = \mu_0$.

3.2 Firms

Firms produce a homogeneous good, and its price is normalized to 1. This good is used both to consume and to invest in innovation. It is produced using a decreasing returns to scale technology, $y(d, n) = d^{1-\gamma} n^{\gamma}$, where $\gamma \in (0, 1)$ the degree of returns to scale. Firms' operating profits are given by:

$$\Pi(d,n,n') \;=\; y(d,n') \;-\; wn' \;-\; \kappa_F w \max\{0, n-n'\}, \tag{1}$$

where w is the wage rate, and $\kappa_F w$ is the per-worker firing cost. Using this profits function, the value of a firm with productivity d and n workers is given by:

$$V(d,n) \;=\; \max_{n'} \;\; \Pi(d,n,n') \;+\; \beta(1-\delta)\mathcal{I}(d,n') \;+\; \beta\delta V_E(n'), \tag{2}$$

where $\beta \in (0,1)$ is the subjective discount factor, $\mathcal{I}(d,n)$ is the value of a firm with state (d,n) before the innovation stage, and $V_E(n)$ captures the value of exit for a firm with n workers. Since Spanish regulation imposes the obligation to pay dismissal costs in case of exit, I assume that $V_E(n) = -w\kappa_F n$.[6]

3.3 Productivity dynamics

The problem consists of choosing both the probability of innovation, $\lambda \in [0,1]$, and the outcome of innovation, given by the distribution of next period's productivity, $\pi = (\pi_1, \pi_2, \ldots, \pi_D)$, satisfying:

$$\sum_{i=1}^{D} \pi_i = 1. \tag{3}$$

We can think of the choice of λ as the extensive margin of innovation, and the choice of π as the intensive one. Another valid interpretation would be to think of λ as the probability of generating a new idea, and π as the implementation of such idea.

Let $\mathcal{I}^I$ be the value of an innovating firm and $\mathcal{I}^N$ be the value of not innovating. The

6. Despite firm owners being subject to limited liability, workers have priority at liquidation over the rest of debtors. Setting the exit value to 0, however, does not affect the quantitative results significantly. The reasons is that I consider a model with exogenous exit, and thus, firing costs do not have a selection effect (Poschke 2009).

innovation problem reads as:

$$
\begin{aligned}
\mathcal{I}(d,n) \quad = \quad \max_{\lambda} \quad \lambda \underbrace{\left(\max_{\{\pi_i\}_{i=1}^{D}} \sum_{i=1}^{D} \pi_i V(d_i,n) - \mathcal{D}(\pi||\eta) \right)}_{\mathcal{I}^I(d,n)} + \\
+ (1-\lambda) \underbrace{\left(\sum_{i=1}^{D} \eta(d_i|d) V(d_i,n) \right)}_{\mathcal{I}^N(d,n)} - \mathcal{D}(\lambda||\bar{\lambda}) \qquad (4)
\end{aligned}
$$

subject to $\lambda \in [0,1]$ and equation (3). The result of this maximization problem is a probability of innovation $\lambda(d,n)$ and a distribution of next period's productivity $\pi(d'|d,n) = (\pi_1, \pi_2, \ldots, \pi_D)$, where $\pi_i = \pi(d_i|d,n)$ is the probability of getting a productivity d_i next period for an innovative firm with state (d,n). The cost of choosing $\lambda(d,n)$ is given by $\mathcal{D}(\lambda||\bar{\lambda})$ where $\bar{\lambda} \in (0,1)$ is a default probability of innovation. Similarly, the cost of choosing the distribution $\pi(d'|d,n)$ is given by $\mathcal{D}(\pi||\eta)$ where η is a default distribution that depends on firm's current productivity, satisfying:

$$
\sum_{i=1}^{D} \eta(d_i|d) d_i = d(1-\mu).
$$

The parameter $\mu > 0$ is the depreciation rate of productivity. This depreciation rate implies that non-innovative firms expect their productivity to fall, which increases the incentives to innovate. Another important implication is that productivity growth in this model only arises as the result of innovation, since the reverse-to-the-mean effect of the standard AR(1) productivity process used in the literature is not present.

The cost function $\mathcal{D}(x||z)$ is given by the Kullback-Leibler divergence measure, or relative entropy, between distributions x and z. In particular,

$$
\mathcal{D}(\lambda||\bar{\lambda}) \quad = \quad \kappa_I \left[\lambda(d,n) \log\left(\frac{\lambda(d,n)}{\bar{\lambda}} \right) + (1-\lambda(d,n)) \log\left(\frac{1-\lambda(d,n)}{1-\bar{\lambda}} \right) \right], \qquad (5)
$$

$$
\mathcal{D}(\pi||\eta) \quad = \quad \kappa_I \left[\sum_{i=1}^{D} \pi(d_i|d,n) \log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|n)} \right) \right], \qquad (6)
$$

where κ_I is the innovation cost given by $\kappa_I = \kappa_0 d^{\kappa_1}$, with $\kappa_0 > 0$ and $\kappa_1 \geq 0$. If $\kappa_1 > 0$

the cost of innovation is higher for high productivity firms, which is consistent with the lower growth rate of larger firms.[7] The intuition behind this assumption is that high productivity firms find it more difficult to generate ideas that improve their productivity beyond their already high level. The innovation cost parameters is the same for both the extensive and the intensive margin. The main implication of this assumption is that the timing of choices does not affect the results (Costain 2017).[8] Moreover, assuming equal costs implies that any combination of π and λ can be expressed as a distribution, so that one could solve the problem in one stage.[9]

Note that equation (6) implies that setting a probability $\pi(d_i|d,n) < \eta(d_i|d)$ would reduce the cost $\mathcal{D}(\pi||\eta)$. However, recall that π is a proper probability distribution. Consequently, setting a low $\pi(d_i|d,n)$ would require setting a larger value somewhere else in the distribution π, increasing the total cost. In fact, it is easy to show that $\mathcal{D}(\pi||\eta) > 0$ for any distribution π different from η, and 0 if $\pi \equiv \eta$. The same reasoning applies to the choice of λ in equation (5).[10]

One of the advantage of using the Kullback-Leibler divergence to measure the cost of firm choices is that it generates closed-form solutions for both the chosen probability λ and the chosen distribution π. To solve the innovation problem, we first solve the choice of the next period's productivity distribution for innovating firms.

Choice of next period's productivity distribution, π

The choice of the next period's distribution for innovative firms consists of choosing D probabilities, where D is the number of points in the grid of productivity. The first

7. Figure B.1 shows the relationship between firm growth and firm size in the Spanish data,

8. In short, when κ_I is the same for both the extensive and the intensive margin choices, results are not affected by the order in which these two decisions are taken.

9. Defining the innovation problem in two stages, however, allows for a cleaner interpretation of the parameters: $\bar{\lambda}$ is the innovation probability for a firms investing no resources in generating a new idea, while the parameters of η describe the distribution of the next period's productivity for a non-innovative firm.

10. Overall, firms' expected profits conditional on a choice of λ and π are given by:

$$\hat{\Pi}(d,n,n') \;=\; \Pi(d,n,n') - \beta(1-\delta)\left[\mathcal{D}(\lambda||\bar{\lambda}) + \lambda\mathcal{D}(\pi||\eta)\right]$$

where $\Pi(d,n,n')$ are firm's operating profits defined in equation (1).

compute the first order condition of (4) with respect to the probability $\pi(d_i|d,n)$ is:

$$V(d_i,n) = \kappa_I \left[1+\log\left(\frac{\pi(d_i|d,n)}{\eta(d_i|d)}\right)\right]+\xi, \tag{7}$$

where ξ is the multiplier on the constraint (3). The left-hand side in equation (7) is the marginal gain from increasing $\pi(d_i|d,n)$, which equals the value of the firm with productivity d_i, while the right-hand side is the marginal cost. The marginal cost is the sum of two terms: the "direct" innovation cost associated to the choice of $\pi(d_i|d,n)$ and the cost associated to the constraint. Using the D first order conditions described by equation (7) in the constraint (3), and after some rearrangement, one finds:

$$\pi(d_i|d,n) = \eta(d_i|d)\left[\frac{\exp\left(V(d_i,n)/\kappa_I\right)}{\sum_{j=1}^{D}\eta(d_j|d)\exp\left(V(d_j,n)/\kappa_I\right)}\right]. \tag{8}$$

The problem is such that the chosen distribution takes a logit form, which is a well known implication of using the Kullback-Leibler divergence to measure the choice cost.[11] Note that equation (8) implies that the ability of firms to skip from very low productivity shocks is limited, allowing the model to generate sufficiently large downwards risk. In particular, firms will not be able to set a zero (positive) probability of getting d_i if $\eta(d_i|d)$ is positive (zero). The reasons is that setting a positive (zero) probability $\pi(d_i|d,n)$ if the default probability $\eta(d_i|d)$ is zero (positive) would be infinitely costly, as shown in equation (6). Finally, using equations (8) and (6), we can write $\mathcal{I}^I(d,n)$ as:

$$\mathcal{I}^I(d,n) = \kappa_I \log\left[\sum_{i=1}^{D}\eta(d_i|d)\exp\left(V(d_i,n)/\kappa_I\right)\right] \tag{9}$$

Note that Jensen's inequality implies that $E[\exp(x)] > \exp[E(x)]$ for any non-degenerate random variable, and therefore, $\mathcal{I}^I(d,n) > \sum_{i=1}^{D}\eta(d_i|d)V(d_i,n) = \mathcal{I}^N(d,n)$, so that firms will always prefer to innovate.[12]

11. See for instance Costain et al. (2019) or Turen (2018).

12. Appendix A derives this expression and explains how to implement the solution to this problem in the computer.

Choice of the innovation probability, λ

The choice of the innovation probability consists of setting the probability λ with which the firm can choose the next period's productivity distribution. The first order condition of equation (4) with respect to the probability of innovation λ is:

$$\mathcal{I}^I(d,n) - \mathcal{I}^N(d,n) = \kappa_I \left[\log\lambda(d,n) - \log\bar{\lambda} - \log(1-\lambda(d,n)) + \log(1-\bar{\lambda})\right],$$

where the left-hand side are the gains from innovation, equal to the marginal product of $\lambda(d,n)$, and the right-hand side is the marginal cost. Rearranging terms:

$$\lambda(d,n) \;=\; \frac{\bar{\lambda}\exp\left(\mathcal{I}^I(d,n)/\kappa_I\right)}{\bar{\lambda}\exp\left(\mathcal{I}^I(d,n)/\kappa_I\right) + (1-\bar{\lambda})\exp\left(\mathcal{I}^N(d,n)/\kappa_I\right)}. \tag{10}$$

The probability of innovation $\lambda(d,n)$ is increasing in the difference between $\mathcal{I}^I$ and $\mathcal{I}^N$, which implies that $\lambda(d,n) > \bar{\lambda}$, since $\mathcal{I}^I(d,n) > \mathcal{I}^N(d,n)$.

3.4 Households

The household problem closely follows Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2019). In particular, there is a homogeneous household with a continuum of members who own the firms, consume and supply labor. The problem reads:

$$\begin{aligned} U \;&=\; \max_{C,L} \;\; \ln C \;-\; \theta L \\ \text{s.t.} \quad C \;&=\; wL + F + \Pi \end{aligned}$$

where C is household consumption, L is the total labor supply, F are the total firing taxes and Π are firms' profits. The parameter $\theta > 0$ captures the disutility of labor supply. The optimal labor choice is characterized by $w = \theta C$.

3.5 Stationary equilibrium

Let $x = (d,n)$ be the state vector, $\mathcal{X} \equiv \mathbb{D} \times \mathbb{R}_{\geq 0}$ be the state space and F be the distribution of firms over $\mathcal{X}$. For simplicity in the exposition, I consider a discretized state space so that $F(x)$ is the mass of firms with state x. The law of motion of the distribution of firms is

$$F'(x) \;=\; (1-\delta)\sum_{z\in\mathcal{X}}\Gamma(x|z)F(z) \;+\; \delta\Gamma^E(x)$$

where F' is the next period's distribution of firms, $\Gamma(x|z)$ is the incumbents' transition probability between states z and x, derived from firm choices, and Γ^E is the distribution of entrants that results from the discretization of the distribution of d_0.

The equilibrium of this economy is given by a wage rate, a distribution of firms over the state space, and a set of firm's policy functions (for n', λ and π) such that (i) policy functions solve firms' problem, (ii) the household first order condition is satisfied, (iii) labor market clears, and (iv) the distribution of firms over the state space $\mathcal{X}$ is invariant, $F'(x) = F(x)$, $\forall x \in \mathcal{X}$.

4 Calibration

The model is calibrated to the Spanish economy, using data from the Central de Balances dataset. This is a panel of non-financial Spanish firms, prepared by the Bank of Spain, including balance sheet information, income statement and some firm characteristics (sector, age, etc). The panel covers the years 1995 to 2015 and provides an excellent representation of the Spanish productive sector.[13] Since Spanish employment is highly volatile, I restrict the sample to years between 2005 and 2007 in order to avoid the Spanish boom (2000-2005) and the financial crisis of 2007. The model period is set to 1 year.

4.1 Exogenous parameters

I set the discount factor to $\beta = 0.95$.[14] I set the degree of returns to scale to $\gamma = 0.6$, somewhat lower than in Hopenhayn and Rogerson (1993), but within the standard values

13. See Almunia et al. (2018) for an analysis of the Central de Balances dataset representativeness.

14. The average long-term government bond yields in Spain for the period 2005-2007 is 4% according to FRED data. I assume a risk premium of 1% and set the discount rate that corresponds to an annual interest rate of 5%.

in the literature.[15] I normalize the equilibrium wage rate to 1 and make θ be such that the household first order condition is satisfied in the benchmark equilibrium. Finally, I set the exit probability parameter to 7.56% so that the average firm age in the model is 9.7 years, as in the data.

4.2 Endogenous parameters

The remaining parameters are internally calibrated using the model. In particular, I calibrate the parameters of the initial distribution of productivity, the firing cost parameter, the benchmark probability of innovation, the innovation cost parameters, and the benchmark distribution, η. The latter is modeled as a discretized unit root process:

$$\log(d') \;=\; \log(d) - \mu + \sigma\epsilon. \tag{11}$$

The initial productivity distribution is defined as a discretized normal distribution with parameters μ_0 and σ_0^2, such that $\log(d_0) \sim N\left(\log(\mu_0) - \frac{1}{2}\sigma_0^2, \sigma_0^2\right)$.

The parameter vector, $\Omega = (\mu_0, \sigma_0^2, \kappa_F, \bar{\lambda}, \kappa_0, \kappa_1, \mu, \sigma^2)$, is chosen such that the sum of squared differences between a set of model-generated moments and their empirical counterparts is minimized. In particular, $\hat{\Omega}$ solves:

$$\hat{\Omega} \;=\; \arg\min_{\Omega} \sum_{i=1}^{M} \omega_i \left(\frac{m_i(\Omega) - \bar{m}_i}{\bar{m}_i} \right)^2 .$$

where M is the number of moments, ω_i the weight associated to moment i, and $m_i(\Omega)$ and $\bar{m}_i$ are the model-generated and empirical i-th moments respectively.

Moment selection and identification

My data lacks information on firms' innovation choices. Moreover, given the broad meaning of innovation in this paper, it is not clear what type of information one should use. However, the model establishes a clear link between productivity and size allowing me to discipline the innovation technology using employment data, as in Garcia-Macia et al.

15. Hopenhayn and Rogerson (1993) consider a degree of returns to scale of 0.64 for the US economy. Spain, however, is characterized for huge share of employment in small firms, so a value below 0.64 is a natural choice. Later I check how sensitive my results are to the value of this parameter.

(2019). Note that hiring and firing choices in my model only depend on productivity, and thus, targeting the dynamics of employment would pin down the dynamics of productivity. For instance, given that productivity growth only emerges from innovation, the share of hiring firms and their growth rate are very informative about the share and growth rate of innovators. Thus, the model is calibrated to match the share of hiring firms and the hiring rate, defined as the ratio between hirings and previous employment, $\max\{0, n' - n\}/n$.

Innovation cost is assumed to be increasing in firm productivity. In order to control for the strength of this effect, I target the firm size distribution. Note that, if innovation is equally costly for high and low productivity firms, high-productivity firms would grow faster than low-productivity ones, generating a bimodal firm size distribution. Given the focus of this paper on firing cost, firing behavior is particularly relevant for the analysis. I match the share of firing firms and the firing rate, defined analogously to the hiring rate. Finally, given that innovation is particularly flexible, it is important to control for the shape of the resulting distribution of next period's productivity. To do so I match the average and the coefficient of variation of firm size, both for the whole population of firms and for entrants.

Although all moments are affected by all the parameters, some relationship between specific parameters and moments can be postulated.[16] The average productivity of entrants, μ_0, is particularly relevant to match the average size of entrants. The variance of the initial productivity draw, σ_0^2, drives the dispersion in firm size among entrants, and therefore, their coefficient of variation in firm size. The variance of the benchmark distribution, σ^2, controls the dispersion of the chosen distribution among innovators, and thus, drives the overall dispersion in firm size. The parameter κ_0 limits how much innovative firms can grow and, as argued before, is informative to match the hiring rate observed in the data. The parameter κ_1 controls the rate at which the cost of innovation increases with firm's productivity, and thus, the ability to grow among high-productivity firms, driving the firm size distribution. Since productivity growth only emerges from innovation, the share of innovators is very informative about the share of hiring firms. The default probability of innovation, $\bar{\lambda}$, limits precisely the probability of innovation and thus, is very informative about the share of hiring firms. Among those firms not innovat-

[16]. These arguments do not prove identification, but ease the interpretation of the parameter values.

Table 1: Calibration. Model fit

Moment	Model	Data
Average size of entrants	3.53	3.40
Coefficient of variation of firm size	1.21	1.19
Coefficient of variation of firm size among entrants	1.39	1.36
Share of firing firms	0.26	0.27
Share of hiring firms	0.35	0.34
Firing rate among firing firms	0.19	0.20
Hiring rate among hiring firms	0.44	0.44
Share of firms with 0-5 workers	0.63	0.60
Share of firms with 6-10 workers	0.21	0.20
Share of firms with 11-15 workers	0.07	0.08
Share of firms with 16-20 workers	0.04	0.04
Share of firms with 21-25 workers	0.02	0.02
Share of firms with 25+ workers	0.04	0.05

ing, the productivity depreciation parameter, μ, drives the size in the productivity fall, and therefore, it is very informative about the firing rate, which is key to control the magnitude of downwards risk. Finally, the firing cost parameter, κ_F, drives the share of firms firing workers.

Parameter values and model fit

Table 1 presents the model fit and table 2 collects the estimated parameters. The model closely matches the moments concerning firing and hiring behavior. Also the firm size distribution is closely matched.[17] This is particularly relevant since it provides support for the innovation technology used in the paper. Moreover, the model generates a distribution of firm size that matches, not just the average firm size, but also the dispersion in firm size, which provides further support to the innovation technology. In the next section, I discuss the main predictions generated by my innovation technology and show that those predictions are consistent with the existing empirical evidence on firm growth.

The firing cost parameter is calibrated to 0.20. This means that the cost of firing one worker equals 2.5 monthly wages. According to Spanish labor regulation, a dismissed worker has the right to received 40 days of wages per year worked in the firm. Note, however, that the Spanish economy is characterized by the heavy use of temporary workers,

[17]. For firms with more than 25 workers, the model also generates a distribution that is in line with that in the data. For instance, the median size of firms with more than 25 workers is 35.5 workers in the data and 35 in the model.

Table 2: Calibration. Parameter values

Parameter			Description
μ_0	$=$	2.95	Average productivity of entrants
σ_0	$=$	1.10	Standard deviation of initial productivity draw
μ	$=$	0.07	Depreciation of productivity (default distribution)
σ	$=$	0.30	Standard deviation of shocks (default distribution)
κ_0	$=$	7.22	Cost of innovation, level parameter
κ_1	$=$	1.25	Cost of innovation, shape parameter
$\bar{\lambda}$	$=$	0.47	Default probability of innovation
κ_F	$=$	0.20	Firing cost

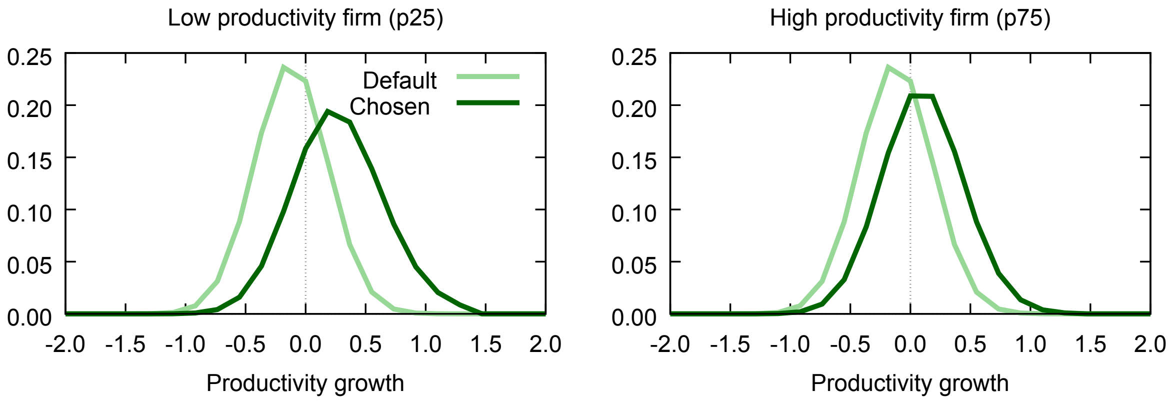
whose firing costs are either zero or very small. Thus, κ_F should be interpreted as an average firing cost for both temporary and permanent workers. The depreciation rate of productivity, μ, is calibrated to 0.07, meaning that a firm investing no resources in innovation expects to loss 7% of its current productivity next period. The standard deviation of productivity under the default distribution is 0.3, similar to Poschke (2009) who also assumes a unit root process for firm's productivity.

The magnitude of κ_0 and κ_1 do not have a clear interpretation. However, they imply that firms in the baseline economy spend 16% of total output in innovation.[18] Although this may be too high for innovation expenses, it should be noticed that innovation in this model includes all sort of firm actions aimed at increasing profitability prospects, and not only product or process innovation as typically assumed in innovation papers. For instance, Mukoyama and Osotimehin (2019), who also consider a broad concept of innovation similar to mine, find an innovation to output ratio of 12%.

The default probability of innovation is 0.47, which is 9 p.p. lower than the average innovation probability in the baseline economy. Given the structure of the innovation problem, most innovation investments are devoted to the choice of the next period's productivity. This is because the cost of choosing a distribution π is incorporated in the value of innovating, lowering gains for innovation, as shown in equation (4).

18. According to the OCDE Spanish firms spend around 1% of turnover on innovation. This data, however, only includes technological innovation (supply-side innovation) while I use a much broader definition of innovation.

Figure 2: Productivity growth. Next period's productivity distribution



Notes: The x-axis refers to the difference in log productivity $\Delta \log d$. The dark line is the chosen next period's distribution π for a low and a high productivity firm. The light line represents the default distribution, η, given by equation (11), which is the same for low and high-productivity firms by assumption.

5 Results

Before analyzing the effects of firing cost, it is worth describing firms' innovation behavior in the baseline equilibrium, to illustrate how my approach to model firm innovation can generate realistic productivity dynamics.[19]

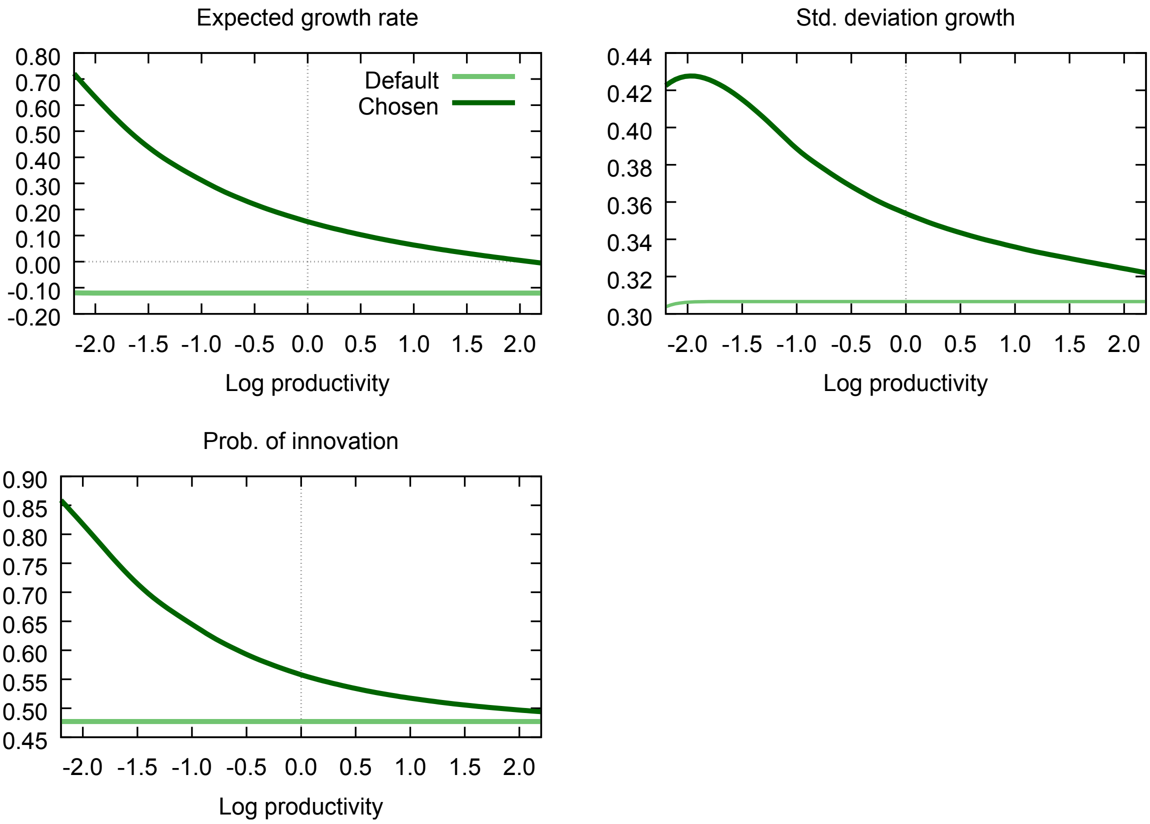
5.1 Endogenous productivity dynamics

Many papers in the literature of firm growth document the negative (unconditional) relationships between firm size and growth and between firm size and volatility of growth.[20] The model is consistent with these facts. Figure 2 presents the default and chosen distributions of productivity growth for a low- and high-productivity firm (in the 25th and 75th percentile of the productivity distribution respectively). The average productivity growth in case of innovation (thus, taking the chosen distribution π) is as high as 0.22 for low productivity firms and 0 for high productivity firms, who just offset the negative productivity trend. At the same time, the standard deviation of productivity growth is of 0.45 for low productivity firms, and of 0.35 for high productivity ones. Key for this result is the fact that the cost of innovation is assumed to be increasing in firm's productivity.

19. Figure B.3 presents the firm productivity distribution of firms and the firm size distribution in the baseline economy.

20. See figures B.1 and B.2 for the corresponding relationships in my data.

Figure 3: Innovation choices, by firm productivity



Notes: I compute the expected productivity growth rate and standard deviation of productivity growth for each point in the discretized state space using the corresponding distribution of next period's productivity, π (chosen) or η (default), and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d, where the default probability is $\bar{\lambda}$ and the chosen one is given by $\lambda(d, n)$. Figure B.5 replicates these graphs by number of employees.

This can be seen more generally in figure 3, where I plot the expected productivity growth rate, the standard deviation of firm productivity growth and the probability of innovation by firm productivity in the baseline economy in which $\kappa_F = 0.20$. Later we will discuss how these figures change when we increase/decrease the firing cost. Three main predictions arise from the model: (i) low productivity firms innovate more frequently, (ii) they undertake more aggressive innovations and (iii) their innovations are riskier, as measured by the expected productivity growth and the standard deviation of expected firm productivity growth, respectively. As a result, low productivity (small) firms in the model grow faster and face higher uncertainty. This, however, does not mean that small firms spend more on innovation. For instance, a firm with 10 workers spends 40% less in innovation than a firm with 30 workers.[21] This is because high-productivity firms prefer to lower the risk they face, which is also costly.

Figure 3 highlights the importance of allowing firms to have (partial) control over the whole distribution of next period's productivity. Models based on Atkeson and Burstein (2010) allow firms to affect the probability of innovation while keeping fixed the "size" of the innovation. Alternatively, once could fix the probability of innovation and allow firms to invest in the average productivity growth. However, in both cases, the volatility of productivity growth is constant across firms, and unaffected by the distortion. In this model, firms endogenously face different degrees of uncertainty, which is key to account for the effects of firing costs (Bentolila and Bertola 1990).

5.2 Aggregate effects of firing costs

The main goal of this paper is to better understand the aggregate consequences of firing costs. To facilitate the exposition and the comparison with previous literature, I simulate the frictionless economy, in which $\kappa_F = 0$, and compare it with an economy with positive firing costs. In this exercise, the main object of interest is aggregate productivity. Following Da-Rocha et al. (2019), I define aggregate productivity as:

$$\text{aggregate productivity} = \left(\int_{x \in \mathcal{X}} d(x)^{1-\gamma} s(x) d\mu(x) \right)^{\frac{1}{1-\gamma}} \tag{12}$$

where $x = (d, n)$ is the firm's state vector, and $\mu(x)$ is the stationary mass of firms with state x, satisfying $\int_x d\mu(x) = 1$.[22]

Table 3 collects the results of this experiment. Table entries represent the percentage (negative) change in the corresponding variable relative to the frictionless economy. In the first column, I compare the frictionless economy with the one that arises from the calibration exercise presented in section 4, in which the firing cost parameter is $\kappa_F = 0.20$. The second column collects the results from simulating an economy in which I set the firing cost parameter to $\kappa_F = 0.40$, twice as large as the calibrated value. Finally, and to facilitate the comparison with the literature, I simulate an economy in which firing costs

21. Figure B.4 presents the relationship between innovation expenses and firm size.

22. As in Da-Rocha et al. (2019), this measure is a weighted average of firm-level productivity where weights are given by $s(x) = n^{\gamma}(x) / \int n(x)^{\gamma} d\mu(d, x)$.

Table 3: Aggregate effects of firing cost
(% fall relative to frictionless economy)

	$\kappa_F = 0.20$	$\kappa_F = 0.40$	$\kappa_F = 1.00$
Aggregate productivity	3.71	5.99	10.9
Output	1.91	3.33	6.57
Average productivity	1.41	2.25	4.47
Average firm size	1.84	3.17	6.10
Innovation expenses	2.68	4.23	8.04
Consumption	1.77	3.16	6.30
Job destruction rate	52.6	68.8	86.2
Job creation rate	29.1	39.8	50.8

are equivalent to one year's wage. All these results are general equilibrium outcomes.[23]

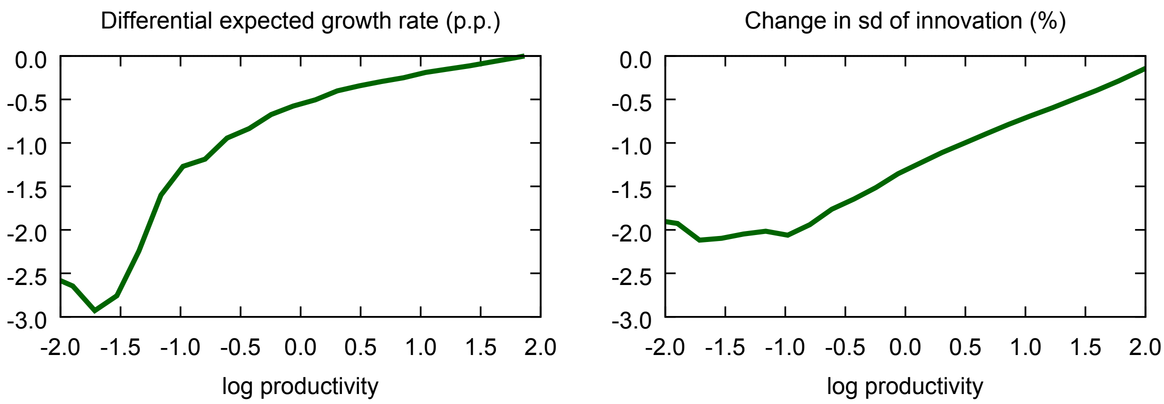
In line with the findings of previous literature, I find that firing costs damage aggregate productivity. In particular, a level of firing costs equivalent to 2.5 monthly wages generates a 3.7% fall in aggregate productivity relative to the frictionless economy. This is a large number compared to the literature. Setting firing costs to one year's wage Hopenhayn and Rogerson (1993) find a 2.1% decrease in productivity, and Da-Rocha et al. (2019) find a 4.2% fall. In my model, productivity falls by more than 10% with firing costs equivalent to one year's wage. These comparisons, however, must be taken with caution. Both Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2019) are calibrated to the US and use a different production function. Moreover, Hopenhayn and Rogerson (1993) consider a model with endogenous exit and mass of firms, while Da-Rocha et al. (2019) and I assume a constant mass of firms and exogenous exit. These differences make the comparison not perfect. However, it is still useful to compare my results to those found by these two papers to put the magnitude of my findings into some context. The conclusion that arises from this comparison is that the effects of firing costs on aggregate productivity are significantly larger than previously thought when the productivity distribution is endogenous.

The main additional channel compare to previous papers is that firing costs in my model affect the whole productivity distribution by changing firms' incentives to innovate.[24] In particular, firing costs make innovation riskier because the cost of a negative

23. Figure B.6 plots the percentage change in aggregate and average productivity for different values of κ_F, ranging from zero to 0.40, both in general and in partial equilibrium.

24. The probability of innovation is almost unaffected by changes in κ_F. The reason is that both the value of innovating and the value of not innovating fall when firing costs increase and thus, gains from innovation are roughly equal to those in the frictionless economy. Despite being unaffected by changes in the firing cost, the innovation probability is still an important margin in the analysis. This is

Figure 4: Innovation choices. Experiment, $\kappa_F = 0.2$ *vs.* $\kappa_F = 0$



Notes: To compute the differences in expected productivity growth between non-innovative and innovative firms, I average expected growth over firm size for each productivity d using the corresponding distribution of next period's productivity (π for innovative firms and η for non-innovative firms) as in figure 3. I do the smae for computing the volatility of next period's distribution.

productivity shock increases. As a result, firms decide to decrease the dispersion in next period's productivity at the expense of lowering potential productivity growth, generating a decrease in the (unweighted) average productivity of 1.4%. To see how firing costs shift firms' innovation decisions, figure 4 presents the differential expected productivity growth rate (left panel) and the differential standard deviation of next period's productivity (right panel) between the frictionless economy and one in which firing costs are set to $\kappa_F = 0.2$.[25] The fall in expected productivity growth is of 3 p.p. for low productivity firms (and up to 10 p.p. if $\kappa_F = 1$) who lower the standard deviation of next period's productivity distribution decreases by 2% (and 6% if $\kappa_F = 1$).

Both the decrease in expected productivity growth and in the volatility of next period's productivity are smaller for high productivity firms, i.e. low productivity firms are the ones most affected by firing costs. This is because the probability of firing—i.e. the probability of getting a level of productivity next period that induces them to fire workers—is larger for low productivity firms. Thus, the expected (but not yet realized at the time of deciding on innovation) cost of firing is larger. This in turn increases their cost of failure more relative to high productivity firms, shaping more intensively their incentives to lower innovation volatility at the expense of productivity growth.[26] This

because the probability of innovation is (endogenously) different for low and high productivity firms.

25. Figures B.7 and B.8 plot the same results when the distorted economy has a level of firing costs of 0.4 and of one year's wage respectively.

26. Figure B.9 shows the ratio of the differential productivity growth and the percentage change in inno-

differential response of high and low productivity firms highlights again the importance of allowing firms to control both the outcome and the risk of innovation.

Since productivity and size are positively correlated, the fall in expected productivity growth is also higher for small firms compared to large ones. When the firing cost parameter is set to $\kappa_F = 0.2$ the growth rate of productivity for firms with less than 10 workers falls by 0.93 p.p. relative to the undistorted economy, while that of firms with more than 10 employees decreases by 0.14 p.p.. When the firing cost parameter is set to $\kappa_F = 0.4$ the growth rate of productivity falls by 1.1 p.p for firms with less than 10 workers and increases by 0.3 p.p. for firms with more than 10 workers, who benefit from lower wages.[27] These results show that small firms are particularly affected—consistent with larger effects among low productivity firms. The reason is that small firms face a higher probability of firing, and thus, face larger potential costs. Consequently, small firms endogenously choose to invest more on reducing the dispersion of next period's productivity at the expense of lowering potential productivity growth.

The distorted economy also exhibits lower job destruction and creation rates (defined as total firings/hirings over total employment). In particular, the share of newly hired workers in the economy falls by 30%, from 18.3% to 12.6%, while the share of fired workers drops by more than 53%, from 10.7% to 5.1%. Since firms find it costlier to fire workers with $\kappa_F > 0$, they decide to keep workers even if their size is larger than the optimal one. At the same time, firms below their optimal size decide not to hire due to precautionary motives. Since there is uncertainty about future productivity, the firms know that they may need to fire in the future, which prevents them from hiring in the first place. Note that the change in job creation is less pronounced than that in job destruction, as in Bentolila and Bertola (1990). These two distortions give rise to inefficiencies in the allocation of labor, which further damages aggregate productivity.

vation volatility. This ratio is decreasing in firm's productivity, meaning that the fall in productivity growth is larger than the fall in risk for low productivity firms.

27. Wages fall by 1.8%, 3.2% and 6.3% relative to the undistorted economy when firing costs are $\kappa_F = 0.2$, $\kappa_F = 0.4$ and $\kappa_F = 1$ respectively.

Table 4: Sensitivity analysis

(% fall in aggregate productivity relative to frictionless economy)

Parameter (benchmark % fall in aggregate productivity = 3.71)		Shock +10%	Shock −10%
σ_0	Standard deviation of initial productivity draw	3.68	3.73
μ	Depreciation of productivity (default distribution)	3.84	3.51
σ	Standard deviation of shocks (default distribution)	3.74	3.44
κ_0	Cost of innovation, level parameter	3.73	3.57
κ_1	Cost of innovation, shape parameter	3.79	3.32
$\bar{\lambda}$	Default probability of innovation	3.57	3.81

5.2.1 Sensitivity analysis

In this section, I check how robust the results presented in table 3 are to changes in the calibrated parameter values.[28] In particular, I compare the aggregate productivity losses from firing costs of 2.5 monthly wages shocking each calibrated parameter at a time, first increasing it by 10%, and then lowering it by 10%. To ensure comparability, I recompute the disutility of labor supply, θ, so that the equilibrium wage is equal to 1 for each alternative calibration.

The results, collected in table 4, suggest that effects of firing costs on aggregate productivity are very robust to changes in the calibrated parameters.[29] In general, changes in the parameter values that make innovation cheaper lower the overall impact of firing costs on aggregate productivity. This is the case for an increase in $\bar{\lambda}$ or a decrease in κ_0, κ_1, μ or σ. The reason is obvious: for a given value of firing cost, a lower cost of innovation allows firms to grow more, making the fall in aggregate productivity smaller. For instance, κ_1 controls how costly it is to innovate for high productivity firms. As it decreases, innovation becomes cheaper for those firms that are less affected by firing costs. Something similar happens when increasing the default probability of innovation: as $\bar{\lambda}$ increases, the cost of the extensive margin lowers, and firms use those resources to invest more in the intensive margin. In both cases, firms are able to reduce the probability of firing (by lowering the probability of a decrease in productivity) shrinking the overall effect of firing costs.

28. I do not include changes in the average productivity of entrants, μ_0, as it shifts the overall distribution of productivity and only has a level effect.

29. Table B.1 collects the results from this sensitivity analysis including all the relevant variables.

One important parameter in the model, not included in the previous table, is the degree of returns to scale, γ. In order to check the robustness of the results to changes in the value of γ, I set a $\gamma = 0.66$ (a 10% increase relative to its baseline value), recalibrate the rest of the parameters, and then compute the losses in aggregate productivity associated with firing costs. I find that aggregate productivity falls by 4.7%, 7.6% and 13.6% for a level of firing costs equivalent to 2.5 monthly wages ($\kappa_F = 0.2$), 5 monthly wages ($\kappa_F = 0.40$) and one year wages ($\kappa_F = 1$) respectively. These numbers are larger than the results presented in the first row of table 3, suggesting that my choice of γ is conservative.

5.3 What is the role of endogenous productivity dynamics?

In order to clearly identify the role of endogenous firm productivity in accounting for the fall in aggregate productivity, I repeat the experiments shown in section 5.2 fixing the innovation behavior from the frictionless economy. In short, I simulate a distorted economy in which I impose a law of motion for firm productivity given by

$$d' \sim \begin{cases} \pi(d'|d, n; \kappa_F = 0) & \text{w.p.} \quad \lambda(d, n|\kappa_F = 0) \\ \eta(d) & \text{w.p.} \quad 1 - \lambda(d, n|\kappa_F = 0) \end{cases}$$

where $\lambda(d, n|\kappa_F = 0)$ and $\pi(d'|d, n; \kappa_F = 0)$ are the resulting innovation probabilities and distributions from the frictionless economy in which firing costs are set to zero. To make the two economies comparable, I also keep fixed the cost of innovation which is now added as a fixed cost to the value of the firm. Results are collected in table 5. The first two columns collect the results from the exercise in section 5.2, in which innovation is endogenous, and thus, reacts to changes in firing costs. The two last columns collect the results from changing firing costs in an economy in which I fixed the innovation behavior that arises the frictionless economy.

In the model with exogenous innovation, a level of firing costs of $\kappa_F = 0.2$ implies a fall in aggregate productivity of 2.3% which is significantly lower than in a model with endogenous productivity dynamics. This fall in aggregate productivity represents a 65% of the estimated fall when innovation choices can react to changes in firing costs. This means that changes in firms' innovation choices account for 35% of the aggregate productivity losses associated to firing costs. Interestingly, productivity losses are still

Table 5: Aggregate effects of firing cost. Exogenous innovation
(% fall relative to frictionless economy)

	Endogenous Inn.			Exogenous Inn.		
Firing cost, κ_F	0.20	0.40	1.00	0.20	0.40	1.00
Aggregate productivity	3.71	5.99	10.9	2.38	3.86	7.10
Output	1.91	3.33	6.57	1.14	2.09	4.28
Average productivity	1.41	2.25	4.47	0.00	0.00	0.00
Average firm size	1.84	3.17	6.10	1.50	2.61	5.20
Innovation expenses	2.68	4.23	8.04	0.00	0.00	0.00

large when innovation is fixed compared to previous literature. There are two reasons for this. First, my model is calibrated to Spanish data, where small firms represent a bast majority of active firms in the economy. Since these are the most affected ones, the fall of productivity is expected to be larger than in models calibrated to the US. A second reason is that the dynamics of productivity when innovation is exogenous are the ones obtained in the frictionless economy. This means that the persistence and volatility of productivity are not constant across firms, as in models based on AR(1) productivity processes.[30] For instance, in the baseline economy with $\kappa_F = 0.2$ the persistence of productivity among the bottom 50% in the productivity distribution is 0.65 while that of the top 50% is 0.98.

Endogenous firm productivity dynamics are also important in accounting for the changes in aggregate output. In particular, the fall in aggregate output with exogenous innovation is equal to 1.1%, 2.1% and 4.3% when the firing cost parameter equals 0.2, 0.4 and 1 respectively. This represents a 40% to 35% of the overall fall in aggregate output. The effects of firing costs on the average firm size are also smaller with exogenous innovation. In particular, the effects of firing costs on innovation choices account for around 20% to 25% of the overall effect on average firm size.

6 Conclusions

This paper presented a firm dynamics model with endogenous productivity growth to analyze the aggregate effects of firing cost. Making the dynamics of productivity endogenous

30. This second channel is present in Da-Rocha et al. (2019) who also find larger effects of firing costs. In their paper, productivity dynamics are size dependent (so large and small firms face different laws of motion for productivity) but these differences do not change when firing costs increase.

allows the model to capture both the static effects of firing taxes—allocative efficiency—as well as the dynamic effects of such friction—changes in the distribution of firms' productivity. As opposed to existing literature, my model allows firms to control not only the probability of innovation but also the outcome. The model parameters are calibrated so as to match the firm size distribution and key features of the hiring and firing behavior of Spanish non-finalcial firms. I show that my flexible innovation technology is able to generate a distribution of firm size that is very close to that in the data, both in terms of size and in terms of dispersion.

I use the calibrated model to quantitatively asses the aggregate effects of firing cost. I show that firing costs equivalent to its calibrated value of 2.5 monthly wages generates a 3.7% drop in aggregate productivity relative to the frictionless economy. When firing costs are equivalent to one year's wage, the fall in productivity is of 10%. I show that the introduction of firing costs make firms to shift their innovation efforts towards decreasing the risk they face, at the expense of lowering the potential productivity growth. Moreover, low productivity firms—as well as small firms—are the ones most affected by the policy. The reason is that the probability of having to fire next period is larger for these firms, so that firing costs increase their cost of innovation failure relatively more.

I then decompose the fall in aggregate productivity between losses in allocative efficiency and changes in the distribution of firm productivity, by fixing the law of motion of firm-level productivity to the one that arises endogenously from the frictionless economy. I show that 65% of the aggregate productivity losses are explained a worse allocation of labor across firms, while the remaining 35% is accounted for changes in the distribution of productivities. This result suggests that researchers should take the effects of frictions on the dynamics of productivity into account when evaluating their aggregate effects. This paper applies this idea to firing cost, but it can be extended to any other frictions, such as distortionary corporate taxation or credit constraints.

My paper focuses on the effects of firing costs on firms. However, the literature has shown that firing costs may generate important welfare gains once we incorporate risk-averse workers into the model. An interesting avenue for future research would be to compute a welfare analysis of firing costs, incorporating heterogeneous risk-averse workers and hiring frictions into the model. It would also be interesting to see how employment protection can be redefined to overcome its negative impact of firms' incentives to grow.

An example would be making firing costs to depend on firm size, such that firing costs do not prevent small firms to invest in growth generating activities. I leave these questions for future research.

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Online appendix

A Computation

In this section, I briefly describe how to solve the model numerically. First, I discretize the state space is $\#_d \times \#_n$ points, where $\#_d = 60$ is the number of points in the grid for productivity and $\#_n = 50$ is the number of points in the grid for employment.[31]

Solving the value function

The problem in (2) is solved by value function iteration. For each point in the state space, (d, n), I find the optimal employment choice, n', using the Golden Search algorithm. This algorithm does not ensure finding a global maxima when the objective function is not well-behaved. To make sure I pick the optimal employment choice, I use the algorithm to solve for the optimal employment choice conditional on $n' > n$ and $n' < n$ separately, and then compare the two solutions with $n' = n$. Given the optimal choice of n', I compute the distribution of next period's productivity using equation (8). I repeat this algorithm until the value function converges.

Solving the innovation problem

The exponential term in equation (8) can easily go to infinity, depending on the maximum real number the computer can manage. To avoid this computational problem, one can redefine the value function and define equation (8) as:

$$\pi(d_i|d, n) = \frac{\eta(d_i|d) \exp\left(\tilde{V}(d_i, n)/\kappa_I\right)}{\sum_{j=1}^{D} \eta(d_j|d) \exp\left(\tilde{V}(d_j, n)/\kappa_I\right)} \tag{13}$$

where $\tilde{V}(d, n) = V(d, n) - \mathbb{C}$ and $\mathbb{C} = \max\{V(\cdot, n)\}$. Note that this normalization does not alter the value of $\pi(d'|d, n)$, but ensures that the exponential term is never larger than

31. The grid sizes are such that increasing them does not alter the results. The maximum number of workers allowed is 59, which corresponds to the 99th percentile in the firm size distribution in Spain.

one. Using this normalization, the cost of innovation becomes:

$$
\begin{aligned}
\mathcal{D}(\pi||\eta) &= \kappa_I \left[\sum_{i=1}^{D} \pi(d_i|d,n) \log \left(\frac{\pi(d_i|d,n)}{\eta(d_i|n)} \right) \right] = \\
&= \sum_{i=1}^{D} \pi(d_i|d,n) \tilde{V}(d_i,n) dx \;-\; \kappa_I \log \left[\sum_{i=1}^{D} \eta(d_i|d) \exp \left(\tilde{V}(d_i,n)/\kappa_I \right) \right] = \\
&= \sum_{i=1}^{D} \pi(d_i|d,n) V(d_i,n) dx \;-\; \mathbb{C} \;+\; \kappa_I \frac{1}{\kappa_I} \mathbb{C} \;-\; \kappa_I \log \left[\sum_{i=1}^{D} \eta(d_i|d) \exp \left(V(d_i,n)/\kappa_I \right) \right] = \\
&= \sum_{i=1}^{D} \pi(d_i|d,n) V(d_i,n) dx \;-\; \kappa_I \log \left[\sum_{i=1}^{D} \eta(d_i|d) \exp \left(V(d_i,n)/\kappa_I \right) \right]
\end{aligned}
$$

and the value function at the innovation stage:

$$
\mathcal{I}^I(d,n) \;=\; \sum_{i=1}^{D} \pi(d_i|d,n) V(d_i,n) - \mathcal{D}(\pi||\eta) \;=\; \kappa_I \log \left[\sum_{i=1}^{D} \eta(d_i|d) \exp \left(V(d_i,n)/\kappa_I \right) \right]
$$

which equals the expression derived in section 3.3.

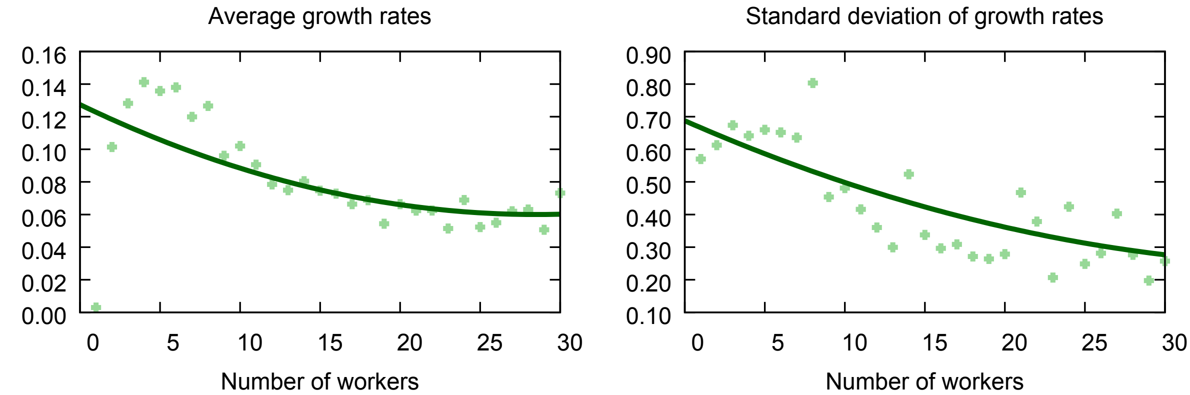
Note that this result makes the model particularly tractable. In fact, it allows to account for the effect of firing costs on the distribution of productivity without adding model complexity. Overall, the problem of a firm with state (d,n) is:

$$
\begin{aligned}
V(d,n) \;&=\; \max_{n'} \; \Pi(d,n,n') \;+\; \beta(1-\delta)\hat{V}(d,n') \;+\; \beta\delta V_E(n') \\
\text{s.t.} \quad & \hat{V}(d,n) \;=\; \lambda \mathcal{I}^I(d,n) + (1-\lambda)\mathcal{I}^N(d,n) \\
& \mathcal{I}^I(d,n) \;=\; \kappa_I \log \left[\sum_{i=1}^{D} \eta(d_i|d) \exp \left(V(d_i,n)/\kappa_I \right) \right] \\
& \mathcal{I}^N(d,n) \;=\; \sum_{i=1}^{D} \eta(d_i|d) V(d_i,n) \\
& \lambda \;=\; \frac{\bar{\lambda} \exp \left(\mathcal{I}^I(d,n)/\kappa_I \right)}{\bar{\lambda} \exp \left(\mathcal{I}^I(d,n)/\kappa_I \right) + (1-\bar{\lambda}) \exp \left(\mathcal{I}^N(d,n)/\kappa_I \right)}
\end{aligned}
$$

Note that the computational cost of solving this problem is similar to the one required to solve a standard firm dynamics model.

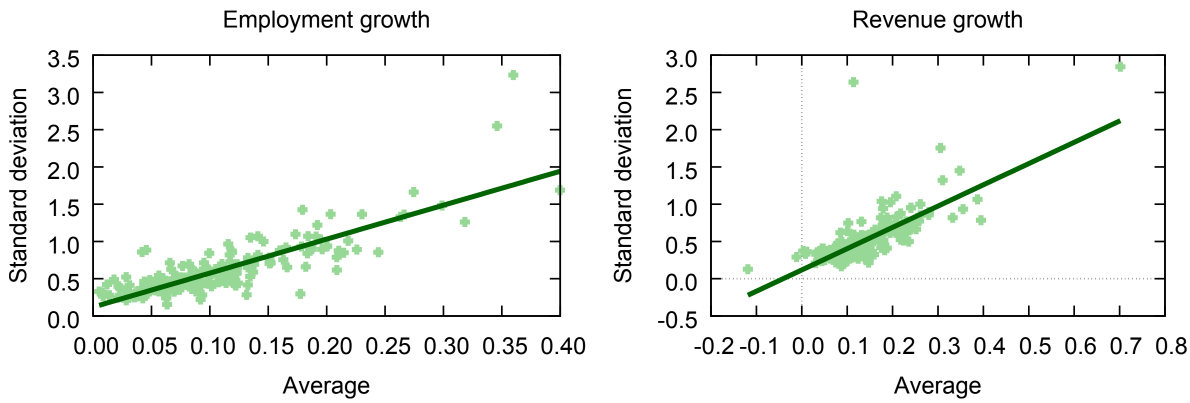
B Additional figures and tables

Figure B.1: Firm growth and growth volatility by firm size



Notes: Dots represent size-specific average and standard deviation of employment growth rates, and the dark line is a quadratic fit. *Source*: Central de Balances dataset, 2005-2007.

Figure B.2: Firm growth and growth volatility across sectors



Notes: Dots represent sector-specific average and standard deviation of employment and revenues growth rates, and the dark line is a linear fit. *Source*: Central de Balances dataset, 2005-2007.

Figure B.3: Distribution of productivity and employment. Baseline economy

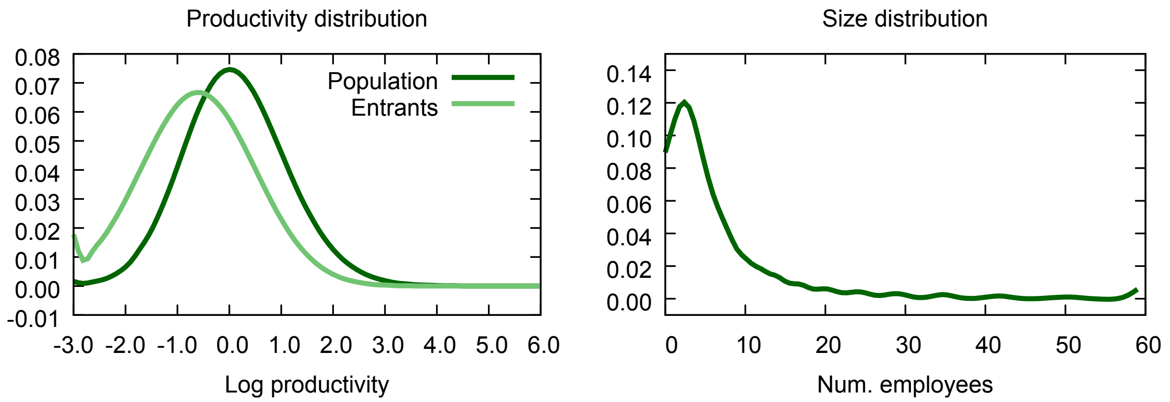
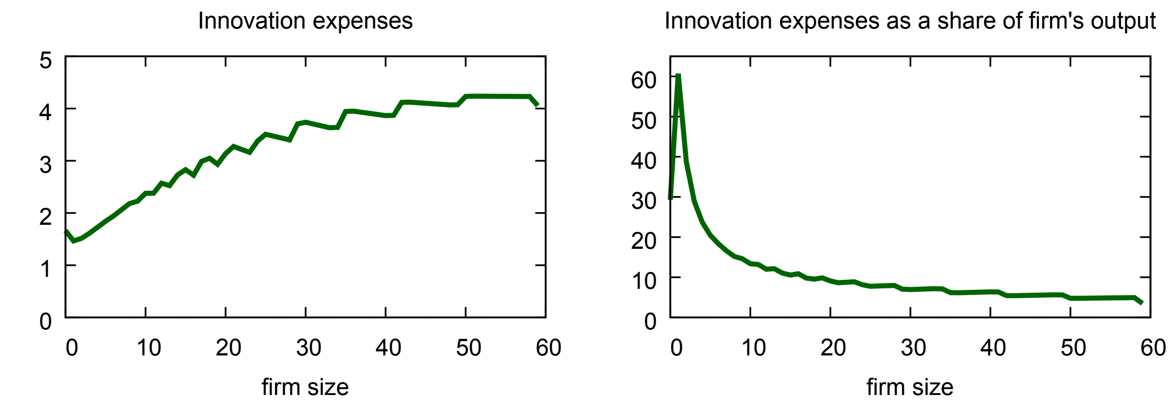
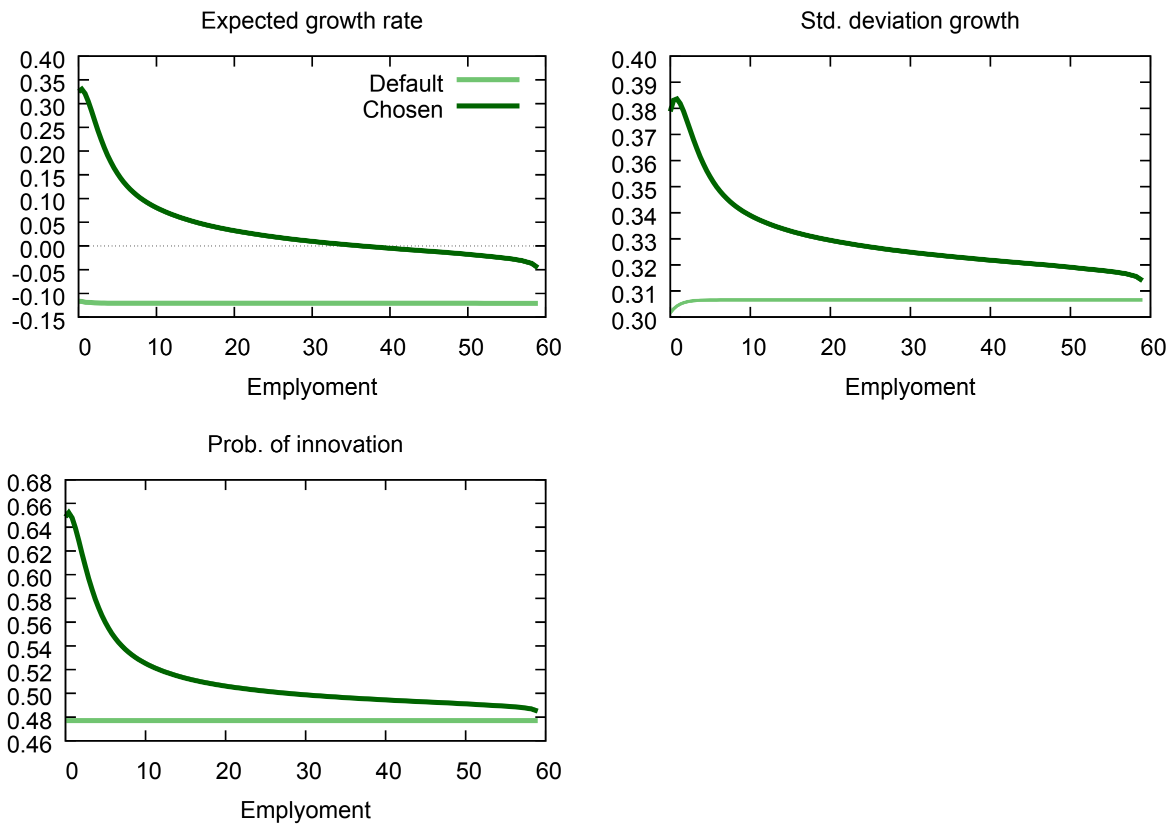


Figure B.4: Expenses in innovation and firm size



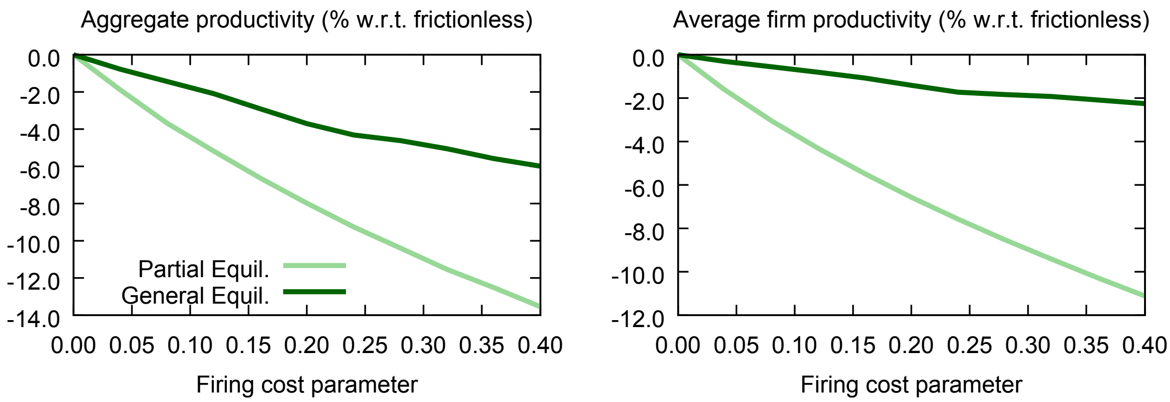
Notes: To obtain these numbers I compute the average innovation expenses across different productivity level for each given level of employment.

Figure B.5: Innovation choices, by firm size



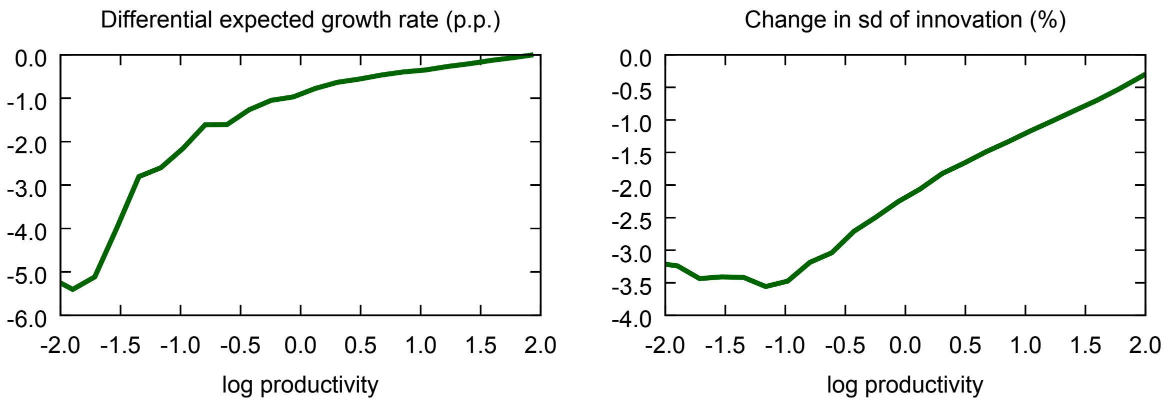
Notes: I compute the expected productivity growth rate and standard deviation of productivity growth for each point in the discretized state space using the corresponding distribution of next period's productivity, π or η, and then average across productivity for each value of n. The probability of innovation is also averaged across productivity for every value of n.

Figure B.6: Aggregate effects of firing costs. General *vs.* Partial equilibrium



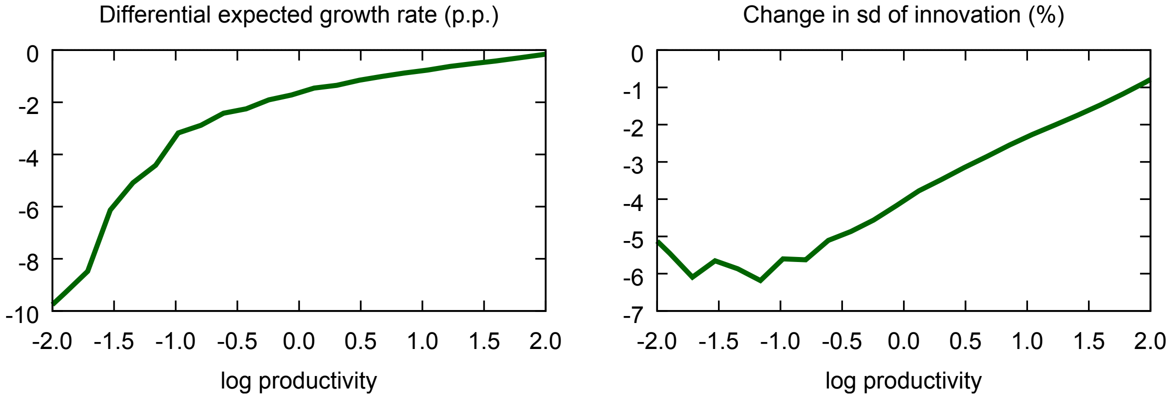
Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The light line represents the partial equilibrium results, where the wage rate is not adjusted. The dark line represents the general equilibrium results that emerge from adjusting the wage rate.

Figure B.7: Innovation choices. Experiment, $\kappa_F = 0.4$ *vs.* $\kappa_F = 0$



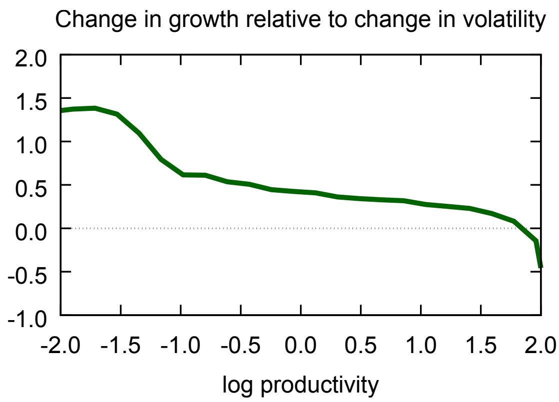
Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.8: Innovation choices. Experiment, $\kappa_F = 1$ *vs.* $\kappa_F = 0$



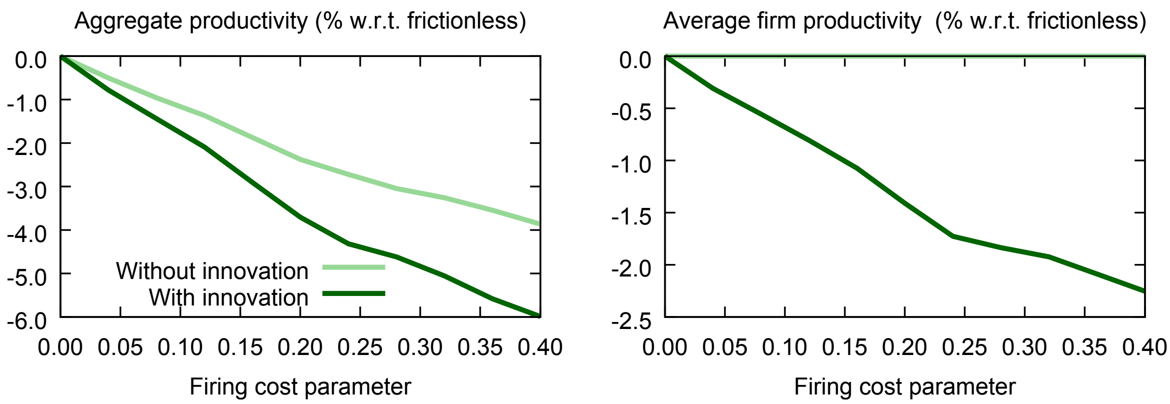
Notes: I compute the expected productivity growth rate for each point in the discretized state space using the chosen distribution of next period's productivity, π, and then average across firm size for each value of d. The probability of innovation is also averaged across size for every value of d.

Figure B.9: Innovation choices. Experiment, $\kappa_F = 0.2$ *vs.* $\kappa_F = 0$



Notes: This figure plots the ratio of the differential productivity growth rate and the percentage change in innovation volatility, both presented in figure 4.

Figure B.10: Aggregate effects of firing costs. Exogenous *vs.* Endogenous innovation



Notes: the y-axis refers to the percentage change of the relevant variable relative to the frictionless economy. The dark line represents the results when innovation is endogenous, and thus, firms' innovation choices react to changes in the firing cost. The light line represents the results when innovation is exogenous so that innovation choices are unaffected by changes in the firing cost.

Table B.1: Sensitivity Analysis – More results

(% fall relative to frictionless economy)

		Aggregate productivity	Average productivity	Innovation expenses	Aggregate output	Aggregate employment	Job Destruction	Job Creation
Benchmark		3.71	1.41	2.68	1.91	1.84	52.6	30.9
+10%	σ_z^0	3.68	1.42	2.70	1.92	1.84	52.6	30.8
	μ	3.84	1.36	2.70	1.94	1.90	51.8	31.0
	σ_z	3.70	1.62	2.71	2.00	1.84	51.4	30.3
	κ_0	3.73	1.27	2.65	1.88	1.88	51.8	30.9
	κ_1	3.79	1.25	2.65	1.89	1.89	52.4	31.3
	$\bar{\lambda}$	3.57	1.42	2.59	1.88	1.78	53.4	30.9
-10%	σ_z^0	3.73	1.40	2.66	1.91	1.84	52.7	29.5
	μ_z	3.51	1.44	2.63	1.87	1.76	53.5	28.8
	σ_z	3.44	0.99	2.33	1.69	1.75	53.8	32.0
	κ_0	3.57	1.50	2.62	1.90	1.77	53.6	28.2
	κ_1	3.32	1.56	2.58	1.88	1.71	53.5	28.1
	$\bar{\lambda}$	3.81	1.39	2.77	1.94	1.9	51.88	30.3