Multivariate Meta Analysis with Potentially Correlated Marketing Study Results

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Abstract: A univariate meta analysis is often used to summarize various study results on the same research hypothesis concerning an effect of interest. When several marketing studies produce sets of more than one effect, multivariate meta analysis can be conducted. Problems one might have with such a multivariate meta analysis are: (1) Several effects estimated in one model could be correlated to each other but their correlation is seldom published and (2) an estimated effect in one model could be correlated to the corresponding effect in the other model due to similar model specification or the data set partly shared, but their correlation is not known. Situations like (2) happen often in military recruiting studies. We employ a Monte-Carlo simulation to evaluate how neglecting such potential correlation affects the result of a multivariate meta analysis in terms of Type I, Type II errors, and MSE. Simulation results indicate that such effect is not significant. What matters is rather the size of the variance component due to random error in multivariate effects. © 2000 John Wiley & Sons, Inc. Naval Research Logistics 47: 500–510, 2000

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1. INTRODUCTION

Upon combining various study results on a univariate effect, one can use a meta analysis (Assmus, Farley, and Lehmann [1], Brown and Peterson [4], Chandrashenkaran and Walker [5], Churchill et al. [6], Fern and Monroe [10], Glass [11], Hedges and Olkin [13], Jones [14], Rust, Lehmann, and Farley [16], and Sohn [21]). Meta analysis is often used to investigate the sources of variation in study results (Szymansky and Busch [26], Sultan, Farley, and Lehmann [25], Sohn [21], and Sohn and Stepleman [23]). One of the basic assumptions used in such a meta analysis is that the study results are independent of each other. Although some studies are not conducted in a controlled manner, they would not be easily eliminated from a meta analysis when collected study results are not many (Begg and Louise [2]).

When using these uncontrolled study results in a meta analysis, Sohn [20] has investigated the effect of negligence of potential correlation of estimated elasticities of one factor over various studies. The results of the simulation study indicate that such effect is not significant, when a relatively small number of study results is combined.

Situation considered in this paper is an extension of Sohn [20] in the sense that we consider combining study results of more than one effect by adopting a multivariate meta model. Many published military recruiting studies use econometric models to analyze the effects of several recruiting efforts such as military advertising, recruiting forces and incentives on contract changes along with various socio-demographic and economic factors (Dertouzous and Polich [8], Goldberg [12], Kearl, Horne, and Gilroy [15], and Warner [28]). For each study, effects of various factors on the military marketing performance can be estimated. Not many military marketing studies are done in a controlled manner and retrospective field data are often utilized for the estimation of effects of multivariate factors in the form of the elasticities. Therefore, the estimated elasticities of marketing performance with respect to several related factors could be correlated to each other in some degree. In addition, when these sets of parameters are estimated by many researchers, estimated elasticities for the same factor over individual models often vary widely and yet could be potentially correlated. This can happen when the data sets are partially overlapped or the specification of the two models are similar. Many military recruiting studies conducted during the last decade show this phenomenon.

The main purpose of this paper is to evaluate the impact of negligence of potential correlation not only between the estimated effects of one factor over different studies but also that between the estimated effects of two factors in a study.

Such negligence may reduce the accuracy of the results of multivariate meta analysis and can mislead the marketing policy. We use a Monte Carlo simulation to generate multivariate meta models by varying levels of the two kinds of correlation.

In Section 2, a multivariate meta model is described and several estimators are derived. In Section 3, a Monte-Carlo simulation is employed to find the impact of using independence assumption in a multivariate meta analysis when study results are in fact correlated. In Section 4, the results are summarized.

2. MODEL

Consider a group of N studies in which the common topic is to investigate the effect of marketing resources on the sales performance. Structural models employed in many econometric analyses may not be identical; however, the reduced form of each study i would be close to the following:

$$\ln Y_{ij} = \alpha_i + \beta 1_i \ln Z1_{ij} + \cdots + \beta K_i \ln ZK_{ij} + f_{ij} + \epsilon_{ij}, \tag{1}$$

where j is the unit used in case study i $(i = 1, \ldots, N, j = 1, \ldots, n_i), Y_{ij}$ is the jth observation of the marketing performance (e.g., sales, contract) in study $i, Z1_{ij}, \ldots, ZK_{ij}$ are the jth observation of K marketing factors related to Y_{ij} and used commonly in all studies $i = 1, \ldots, N$ (e.g., advertising, sales force, price, incentives), α_i is the intercept coefficient in study $i, \beta k_i$ is the elasticity of the marketing performance with respect to marketing factor Zk $(k = 1, \ldots, K)$ considered in study i, f_{ij} is the jth observation of the linear combination of marketing factors which are used only for study i, and ϵ_{ij} is the random error with $E(\epsilon_{ij}) = 0$ and $V(\epsilon_{ij}) = \tau_i^2$.

We assume that elasticities $(\beta k_i s)$ vary over different studies due mainly to some associated characteristics with each study $i, (x_i)$, and the random error (δ_i).

That is,

$$\beta = X\gamma + \delta, \tag{2}$$

where β is an $NK \times 1$ vector of βk_i's and X is a $NK \times mK$ block diagonal matrix. Each block consists of $N \times m$ factor matrix x_i related to the variation of βk_i. x_i is assumed to accommodate all factors related to βk_i for $k = 1, \ldots, K$. γ is an $mK \times 1$ vector of coefficients and δ is an $NK \times 1$ vector of random errors that take into account the remaining variation of βk_i's. We assume $\delta \sim N(0, D)$, where D is an $NK \times NK$ diagonal matrix consisting of N repeated diagonals of σ_k^2's for $k = 1, \ldots, K$. For more complex forms for D, see DerSimonian and Laird [7], Berkey et al. [3], and Stram [24].

When $\beta 1_i, \ldots, \beta K_i$'s in (1) are replaced with β in (2), model (1) is called a random effects model. Such a random effects model accommodates a fixed effects model which does not take into account the random error δ and has wide application areas (Sohn and Mazumdar [19], Berkey et al. [3], Smith, Spiegelhalter, and Thomas [18], Stram [24], and Sohn and Park [22]).

Since the true elasticity βk_i is unobservable, it is replaced with the estimated elasticity $\hat{\beta} k_i$ obtained from each study i. As a result, the estimation error ξ which is independent of δ is added to Eq. (2):

$$\hat{\beta} = X\gamma + \delta + \xi, \tag{3}$$

where an $NK \times 1$ mean matrix $E(\xi) = 0$ and an $NK \times NK$ variance–covariance matrix, $V(\xi) = V$, consists of $\widehat{\text{cov}}(\hat{\beta} k_i, \hat{\beta} k'_{i'}), (k = 1, \ldots, K, k' = 1, \ldots, K, i = 1, \ldots, N, i' = 1, \ldots, N)$. V becomes a diagonal matrix consisting of the estimated variance of $\hat{\beta} k_i$ $(\hat{v}(\hat{\beta} k_i))$, when $\hat{\beta} k_i$'s are not correlated to each other over i as well as over k. However, most of the econometric models utilize retrospective data rather than a controlled experiment. As a result, the estimated elasticity parameters are correlated in some degree. That is $\widehat{\text{cov}}(\hat{\beta} k_i, \hat{\beta} k'_{i'})$ may not be 0 often. Typically their correlation is not available either due to not publishing the results $(\widehat{\text{cov}}(\hat{\beta} k_i, \hat{\beta} k'_i))$ or due to not being able to estimate $(\widehat{\text{cov}}(\hat{\beta} k_i, \hat{\beta} k_{i'}))$. On the other hand, the estimated elasticity $(\hat{\beta} k_i)$ and standard error of $\hat{\beta} k_i (se(\hat{\beta} k_i) = \sqrt{\hat{v}(\hat{\beta} k_i))}$ are often obtained based on a large sample and are published.

Assuming $\delta + \xi$ follows $N(0, D + V)$ and ideally the variance due to estimation error V and the variance due to random error D are known to us, the generalized least square estimator for γ is obtained as follows:

$$\hat{\gamma} = (X'(D + V)^{-1} X)^{-1} (X'(D + V)^{-1} \hat{\beta}),$$

where the variance of $\hat{\gamma} = (X'(D + V)^{-1} X)^{-1}$.

When D is unknown, the maximum likelihood (ML) estimator of the diagonal element D, $\hat{\sigma}_k^2$'s, can be obtained by maximizing the following log likelihood function:

$$-0.5N \ln 2\pi - 0.5 \ln |D + V| - 0.5(\hat{\beta} - X\gamma)'(D + V)^{-1}(\hat{\beta} - X\gamma). \tag{4}$$

For other estimation methods for D, see Searle, Casella, and McCulloch [17] and Sohn and Mazumdar [19].

When the ML estimators $\hat{\sigma}_k^2$'s replace σ_k^2, the estimated generalized least square estimator for γ is obtained as

$$\hat{\gamma}^G = (X'(\hat{\Omega}^G)^{-1} X)^{-1} (X'(\hat{\Omega}^G)^{-1} \hat{\beta}), \tag{5}$$

where $\hat{\Omega}^G = \hat{D} + V$ and the estimated variance of $\hat{\gamma}^G$ is $(X'(\hat{\Omega}^G)^{-1} X)^{-1}$.

$\hat{\gamma}^G$ is an ideal estimator since we assume that V is known. Note that Van Houwelingen and Zwinderman [27] considered a bivariate meta analysis where they assume a known V. A realistic assumptions on V, however, is that only part of it ($\hat{v}(\hat{\beta}k_i) = \widehat{\text{cov}}(\hat{\beta}k_i, \hat{\beta}k_i)$) is known to us, because other parts of variance components are not available either due to not publishing the results ($\widehat{\text{cov}}(\hat{\beta}k_i, \hat{\beta}k_i')$) or due to not being able to estimate ($\widehat{\text{cov}}(\hat{\beta}k_i, \hat{\beta}k_{i'})$).

Therefore, some alternatives to $\hat{\gamma}^G$ are necessary. The first alternative is the weighted least square estimators for $\gamma, (\hat{\gamma}^W)$. The weighted least square estimator neglects the potential correlation between the estimated effects of one factor on two study results and that between the estimated effects of two factors in the same model—that is, $\widehat{\text{cov}}(\hat{\beta}k_i, \hat{\beta}k_{i'})$ and $\widehat{\text{cov}}(\hat{\beta}k_i, \hat{\beta}k_i')$ in V are assumed to be 0. As a result,

$$\hat{\gamma}^W = (X'(\hat{\Omega}^W)^{-1}X)^{-1}(X'(\hat{\Omega}^W)^{-1}\hat{\beta}), \tag{6}$$

where an $NK \times NK$ diagonal matrix $\hat{\Omega}^W$ consists of $\hat{\sigma}_k^{*2} + \hat{v}(\hat{\beta}k_i)$. Note that the ML estimator $\hat{\sigma}_k^{*2}$ is obtained in the same manner as $\hat{\sigma}_k^2$ by replacing V in (4) with a diagonal matrix of $\hat{v}(\hat{\beta}k_i)$'s.

Two other alternatives are the estimator ($\hat{\gamma}^H$) which reflects only the variance due to estimation error $\hat{v}(\hat{\beta}k_i)$ and the ordinary least square estimator ($\hat{\gamma}^O$) which ignores potential heterogeneity due to such variance component. They can be obtained in the following manner, respectively:

$$\hat{\gamma}^H = (X'(\hat{\Omega}^H)^{-1}X)^{-1}(X'(\hat{\Omega}^H)^{-1}\hat{\beta}), \tag{7}$$

$$\hat{\gamma}^O = (X'X)^{-1}(X'\hat{\beta}), \tag{8}$$

where an $NK \times NK$ diagonal matrix $\hat{\Omega}^H$ consists of only $\hat{v}(\hat{\beta}k_i)$'s and does not require one to estimate σ_k^2.

All these four ($\hat{\gamma}^G, \hat{\gamma}^W, \hat{\gamma}^H$, and $\hat{\gamma}^O$) estimators are unbiased when D is known. Typically, however, D is unknown and has to be estimated. In the following section, we conduct a Monte-Carlo simulation to compare the performances of the four estimators when they are used for a multivariate meta analysis. Particular situations considered in the simulation are (1) a meta analysis combines a relatively small number of study results over several effects; (2) some of the study results on an estimated elasticity are potentially correlated; but their correlation coefficients are not available; and (3) some of the estimated elasticities in one model could be correlated but their correlation is typically not published.

3. SIMULATION

We first construct model (3) based on the information obtained from a meta analysis conducted in Sohn [21]. The main interest of the study conducted in Sohn [21] was to examine why collected elasticities of several military recruiting analyses vary. Furthermore, it was of interest to model each elasticity as a function of study characteristics. In military recruiting, the number of contracts can be considered as the marketing performance where the elasticities of the military contract with respect to advertising, recruiter forces, and incentives are important parameters. These elasticities play significant roles when planning the military recruiting budget. Many researchers have made effort to estimate these elasticities but their values vary much. Sohn [21] collected the estimated contract elasticities with respect to advertising, recruiter forces, and incentive over 16 case studies. These 16 cases were obtained from four sources of literature (Dertouzos and Polich [8], Goldberg [12], Kearl, Horne, and Gilroy [15], and Warner [28]). Therefore, some cases are subset models of a larger model, and there could be some overlap in the data set used for the model estimation. It is suspected that there can be some correlation between two estimated elasticities for the same marketing factor, although they were obtained from different cases. However, this correlation is never considered nor published. In addition, data used in each case study are not carefully designed so that the estimated elasticities of several factors in one model also could be correlated. Analytically this correlation can be obtained but was not published except for the standard error of the estimated elasticity itself.

In Sohn's meta analysis [21], three separate models for advertising, recruiter forces, and incentives elasticities are formed. In order to explain the variation in each elasticity, three study characteristics are considered.

In simulating model (3), we consider x matrix and estimated elasticities $(\hat{\beta}k_i, k = 1, 2, 3; i = 1, \ldots, 16)$ collected in Sohn [21] as the true x and β. These values are listed in Table 1. The true values of γ are assumed as in Table 2. These values are based on the estimated γ in Sohn [21]. Insignificant γ are replaced with 0 and are used to evaluate the power of the significance test. Next, $\delta + \xi$ is generated from $N(0, \Omega^G)$, where Ω^G is a (48×48) matrix, $D + V$. That is,

$$\Omega^G = \begin{pmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{pmatrix},$$

where

$$Akk' = \begin{pmatrix} B1 & & & & & & & \\ & B2 & & & & & & \\ & & B3 & & & & & \\ & & & B4 & & & & \\ & & & & B5 & & & \\ & & & & & B6 & & \\ & & & & & & B7 & \\ & & & & & & & B8 \end{pmatrix}$$

and

$$B1 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_1, \hat{\beta}k'_1) & \mathrm{cov}(\hat{\beta}k_1, \hat{\beta}k'_2) \\ \mathrm{cov}(\hat{\beta}k'_1, \hat{\beta}k_2), & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_2, \hat{\beta}k'_2) \end{pmatrix},$$

$$B2 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_3, \hat{\beta}k'_3) & \mathrm{cov}(\hat{\beta}k_3, \hat{\beta}k'_4) \\ \mathrm{cov}(\hat{\beta}k'_3, \hat{\beta}k_4) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_4, \hat{\beta}k'_4) \end{pmatrix},$$

$$B3 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_5, \hat{\beta}k'_5) & \mathrm{cov}(\hat{\beta}k_5, \hat{\beta}k'_6) \\ \mathrm{cov}(\hat{\beta}k_6, \hat{\beta}k'_5) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_6, \hat{\beta}k'_6) \end{pmatrix},$$

$$B4 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_7, \hat{\beta}k'_7) & \mathrm{cov}(\hat{\beta}k_7, \hat{\beta}k'_8) & \mathrm{cov}(\hat{\beta}k_7, \hat{\beta}k'_9) \\ \mathrm{cov}(\hat{\beta}k_8, \hat{\beta}k'_7) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_8, \hat{\beta}k'_8) & \mathrm{cov}(\hat{\beta}k_8, \hat{\beta}k'_9) \\ \mathrm{cov}(\hat{\beta}k_9, \hat{\beta}k'_7) & \mathrm{cov}(\hat{\beta}k_9, \hat{\beta}k'_8) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_9, \hat{\beta}k'_9) \end{pmatrix},$$

$$B5 = \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{10}, \hat{\beta}k'_{10}),$$

$$B6 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{11}, \hat{\beta}k'_{11}) & \mathrm{cov}(\hat{\beta}k_{11}, \hat{\beta}k'_{12}) \\ \mathrm{cov}(\hat{\beta}k_{12}, \hat{\beta}k'_{11}) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{12}, \hat{\beta}k'_{12}) \end{pmatrix},$$

Table 1. Factors and estimated elasticities.

i	Study*	x_i: $x2_i$	$x3_i$	$x4_i$	$\hat{\beta}1_i$ / $\sqrt{\hat{v}(\hat{\beta}1_i)}$	$\hat{\beta}1_i$ / $\sqrt{\hat{v}(\hat{\beta}2_i)}$	$\hat{\beta}3_i$ / $\sqrt{\hat{v}(\hat{\beta}3_i)}$
1	A	0	1	1	0.015	0.412	0.477
					0.008	0.047	0.024
2	A	0	1	1	−0.001	0.0459	0.441
					0.0070	0.047	0.022
3	B	0	1	1	−0.034	−0.045	0.203
					0.0150	0.050	0.028
4	B	0	1	1	−0.038	−0.168	0.139
					0.0150	0.048	0.027
5	C	0	1	1	−0.017	0.487	0.483
					0.0050	0.076	0.030
6	C	0	1	1	0.001	0.957	0.402
					0.0200	0.066	0.031
7	D	1	0	0	0.720	1.150	0.650
					0.1080	0.095	0.071
8	D	1	0	0	0.580	0.680	0.650
					0.1060	0.138	0.069
9	D	1	0	1	0.430	0.480	0.570
					0.1070	0.133	0.064
10	E	1	0	1	0.023	0.542	0.512
					0.0044	0.064	0.197
11	F	1	0	1	0.340	0.160	0.430
					0.0436	0.080	0.040
12	F	1	0	1	0.010	0.290	0.720
					0.0076	0.216	0.075
13	G	1	0	1	0.050	0.150	0.590
					0.0058	0.031	0.018
14	G	1	0	1	0.020	0.350	0.760
					0.0119	0.330	0.126
15	H	1	1	1	0.103	0.371	0.554
					0.0400	0.074	0.026
16	H	1	1	1	0.198	0.482	0.451
					0.0410	0.076	0.025

$$B7 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{13}, \hat{\beta}k'_{13}) & \mathrm{cov}(\hat{\beta}k_{13}, \hat{\beta}k'_{14}) \\ \mathrm{cov}(\hat{\beta}k_{14}, \hat{\beta}k'_{13}) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{14}, \hat{\beta}k'_{14}) \end{pmatrix},$$

$$B8 = \begin{pmatrix} \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{15}, \hat{\beta}k'_{15}) & \mathrm{cov}(\hat{\beta}k_{15}, \hat{\beta}k_{16}) \\ \mathrm{cov}(\hat{\beta}k_{16}, \hat{\beta}k_{15}) & \sigma^2_{k,k'} + \mathrm{cov}(\hat{\beta}k_{16}, \hat{\beta}k'_{16}) \end{pmatrix}.$$

Note that these diagonal matrices represent a situation in which six sets of two study results (1, 2, 3, 6, 7, 8) and one set of three results (4) are correlated within the set while one study result (5) is independent of others. This pattern is again simulated based on Sohn [21]. We consider true values of σ^2_k's $= [0.0054, \ldots, 0.0054, 0.0681, \ldots, 0.0681, 0.0103, \ldots, 0.0103]$. These values are the ML estimates obtained from Sohn [20]. In order to analyze the impact of the relative size of the variance component due to the random error (σ^2_k's) and that due to the estimation error, $(v(\hat{\beta}k_i))$

Table 2. Values of γ used in simulation study.

FACTOR	$\gamma 1$	$\gamma 2$	$\gamma 3$
INTERCEPT	0.487	0.850	−0.505
$x2$	0.162	0.000	0.145
$x3$	0.000	0.000	−0.000
$x4$	−0.551	−0.606	−0.000

on the accuracy of the meta analysis, we consider one additional set of values of σ_k^2's. That is $[0.054, \ldots, 0.054, 0.681, \ldots, 0.681, 0.103, \ldots, 0.103]$.

The values of $\hat{v}(\hat{\beta}k_i)$ used in the simulation is listed in Table 1. This variance structure can accommodate many situations in which several sets of two or three study results are correlated within the set. Since information regarding cov$(\hat{\beta}k_i, \hat{\beta}k_i')$ is not given, we generate a within study correlation coefficient $\rho k_i k_i'$ from uniform distribution (i.e., $\rho k_i k_i' \sim U(a, b)$) so that cov$(\hat{\beta}k_i, \hat{\beta}k_i')$ can be obtained as $\rho k_i k_i' \sqrt{v(\hat{\beta}k_i) v(\hat{\beta}k_i')}$, where $k \neq k'$.

For the values of (a, b), the following two specifications are used: $|\rho k_i k_i'| \sim U(0, 0.3)$; and $|\rho k_i k_i'| \sim U(0.9, 1)$. This corresponds approximately to two different degrees of linear association such as $0 < (\rho k_i k_i')^2 < 0.1$ and $0.8 < (\rho k_i k_i')^2 < 1$. Moderate correlations $(0.1 < (\rho k_i k_i')^2 < 0.8)$ are not considered based on the lesson learned from Sohn [20] and the complexity that this additional level would cause in the simulation design. Following the same format as $\rho k_i k_i'$, we generate the between study correlation coefficient $\rho k_i k_{i'}$. So does cov$(\hat{\beta}k_i, \hat{\beta}k_{i'})$. However, we assume cov$(\hat{\beta}k_i, \hat{\beta}k_{i'}') = 0$.

In summary, as displayed in Table 3, eight combinations of $\rho k_i k_i'$, $\rho k_i k_{i'}$, and σ_k^2 are formed for simulation design. After model (3) is constructed based on this information, γ can be estimated using one of the following four estimators ($\hat{\gamma}^G$, $\hat{\gamma}^W$, $\hat{\gamma}^H$, and $\hat{\gamma}^O$), respectively. This procedure is repeated 1000 times. Performances of the four estimators are then compared in terms of the mean squared error of the $\hat{\hat{\beta}} = X\hat{\gamma}^*$ for $* = G, W, H, O$. Additional performance criteria used are type I errors of rejecting $H_0 : \gamma = 0$, where the true $\gamma = 0$ and the power of the test of rejecting $H_0 : \gamma = 0$ for γ's which are not 0. The nominal significance level used is 5%.

First, the results of the ANOVA for the mean squared error of the $\hat{\hat{\beta}} = X\hat{\gamma}^*$ indicate the following. The range of MSE covers. As shown in Table 4, there is a significant interaction effect between the variance component due to random error, σ_k^2, and the estimator on the size of MSE. According to Tukey's t-test, MSEs of the four estimator are not significantly different, when σ_k^2 is large. But $\hat{\gamma}^H$ results in the maximum MSE (0.0333) among the four estimators on average,

Table 3. Factorial design used in simulation.

Design	$\rho^2 k_i k_{i'}$	σ_k^2	$\rho^2 k_i k_i'$
1	Large 0.81 $\sim$ 0.99	Large	Large 0.81 $\sim$ 0.99
2	Large 0.81 $\sim$ 0.99	Large	Small 0.00 $\sim$ 0.10
3	Large 0.81 $\sim$ 0.99	Small	Large 0.81 $\sim$ 0.99
4	Large 0.81 $\sim$ 0.99	Small	Small 0.00 $\sim$ 0.01
5	Small 0.00 $\sim$ 0.10	Large	Large 0.81 $\sim$ 0.99
6	Small 0.00 $\sim$ 0.10	Large	Small 0.00 $\sim$ 0.10
7	Small 0.00 $\sim$ 0.10	Small	Large 0.81 $\sim$ 0.99
8	Small 0.00 $\sim$ 0.10	Small	Small 0.00 $\sim$ 0.10

Table 4. ANOVA: MSE.

Source	DF	Sum of squares	Mean square	F-Value	P-Value
Model	31	37.94109769	1.22390638	284.38	0.0001
Error	3168	13.63423858	0.00430374		
Corrected total	3199	51.57533627			
Source	**DF**	**Anova SS**	**Mean Square**	**F-Value**	**P-value**
$\rho k_i k_i'$	1	0.00007314	0.00007314	0.02	0.8963
$\rho k_i k_{i'}$	1	0.00016425	0.00016425	0.04	0.8451
$\rho k_i k_i' \times \rho k_i k_{i'}$	1	0.00000076	0.00000076	0.00	0.9894
EST	3	0.30400869	0.10133623	23.55	0.0001
$\rho k_i k_i' \times$ EST	3	0.00001308	0.00000436	0.00	1.0000
$\rho k_i k_{i'} \times$ EST	3	0.00000448	0.00000149	0.00	1.0000
$\rho k_i k_i' \times \rho k_i k_{i'} \times$ EST	3	0.00000071	0.00000024	0.00	1.0000
σ_k^2	1	37.44168256	37.44168256	8699.81	0.0001
$\rho k_i k_i' \times \sigma_k^2$	1	0.00000050	0.00000050	0.00	0.9914
$\rho k_i k_{i'} \times \sigma_k^2$	1	0.00000209	0.00000209	0.00	0.9824
$\rho k_i k_i' \times \rho k_i k_{i'} \times \sigma_k^2$	1	0.00000321	0.00000321	0.00	0.9782
EST $\times \sigma_k^2$	3	0.19514079	0.06504693	15.11	0.0001
$\rho k_i k_i' \times$ EST $\times \sigma_k^2$	3	0.00000141	0.00000047	0.00	1.0000
$\rho k_i k_{i'} \times$ EST $\times \sigma_k^2$	3	0.00000128	0.00000043	0.00	1.0000
$\rho k_i k_i' \times \rho k_i k_{i'} \times$ EST $\times \sigma_k^2$	3	0.00000072	0.00000024	0.00	1.0000

when σ_k^2 is relatively small. Performances of the remaining three estimators are not significantly different from each other [MSE($\hat{\gamma}^G$) $= 0.0290$, MSE($\hat{\gamma}^W$) $= 0.0289$, MSE($\hat{\gamma}^O$) $= 0.0286$]. Typically, σ_k^2 is small, when study characteristics (x) considered in the model explain most of the variation in the multivariate elasticities (β). So when one has a good idea about the influential study characteristics, $\hat{\gamma}^H$ is an estimator to be avoided for a multivariate meta analysis. It is interesting to note that there is no significant effect of $\rho k_i, k_i'$ or $\rho k_i, k_{i'}$ on the accuracy of the four estimators.

Next, ANOVAs are used to evaluate the effects of $\rho k_i k_i'$, $\rho k_i k_{i'}$, and σ_k^2 on the performances of the four estimators in terms of the Type I errors and the power of the test. As shown in Tables 5 and 6, different levels of $\rho k_i k_i'$, $\rho k_i k_{i'}$, and σ_k^2 do not affect the power of tests related to the four estimators while $\hat{\gamma}^H$ is associated with significantly higher type I error (0.4275) than the rest of the three estimators [Type I error ($\hat{\gamma}^G$) $= 0.3875$, Type I error ($\hat{\gamma}^W$) $= 0.3844$, Type I error ($\hat{\gamma}^O$) $= 0.3818$]. Note that the three estimators except for $\hat{\gamma}^G$ neglects both the within- and the between-study correlation while $\hat{\gamma}^H$ neglects additionally the variance component due to random error. Additional simulation by employing more levels of factors (e.g., smaller σ_k^2) confirm our study result.

4. DISCUSSION

In this paper, we consider a multivariate meta analysis in which a relatively small number of potentially correlated study results on several factors are combined. We compare the performances of the four estimators which can be applied to such a situation: $\hat{\gamma}^G$, $\hat{\gamma}^W$, $\hat{\gamma}^H$, and $\hat{\gamma}^O$. Our hypothesis was that the $\hat{\gamma}^G$ would perform best among the four estimators, especially when potential correlation between a set of two study results is relatively high or the correlation between the two estimated effects in one model is high. The results of a Monte-Carlo simulation, however, indicate that the performance of the estimators are not governed by these correlation.

Table 5. ANOVA: Type I error.

Source	DF	Sum of Squares	Mean Square	F-Value	P-Value
Model	31	102.73968750	3.31418347	48.01	0.0001
Error	3168	218.69000000	0.06903093		
Corrected total	3199	321.42968750			
Source	**DF**	**Anova SS**	**Mean Square**	**F-Value**	**P-Value**
$\rho k_i k_i'$	1	0.01125000	0.01125000	0.16	0.6865
$\rho k_i k_{i'}$	1	0.00000000	0.00000000	0.00	1.0000
$\rho k_i k_i' \times \rho k_i k_{i'}$	1	0.00031250	0.0031250	0.00	0.9464
EST	3	1.11781250	0.37260417	5.40	0.0011
$\rho k_i k_i' \times$ EST	3	0.03437500	0.01145833	0.17	0.9193
$\rho k_i k_{i'} \times$ EST	3	0.00312500	0.00104167	0.02	0.9975
$\rho k_i k_i' \times \rho k_i k_{i'} \times$ EST	3	0.00281250	0.00093750	0.01	0.9978
σ_k^2	1	101.17531250	101.17531250	1465.65	0.0001
$\rho k_i k_i' \times \sigma_k^2$	1	0.36125000	0.36125000	5.23	0.0222
$\rho k_i k_{i'} \times \sigma_k^2$	1	0.00125000	0.00125000	0.02	0.8930
$\rho k_i k_i' \times \rho k_i k_{i'} \times \sigma_k^2$	1	0.00031250	0.00031250	0.00	0.9464
EST $\times \sigma_k^2$	3	0.02156250	0.00718750	0.10	0.9577
$\rho k_i k_i' \times$ EST $\times \sigma_k^2$	3	0.00062500	0.00270833	0.04	0.9896
$\rho k_i k_{i'} \times$ EST $\times \sigma_k^2$	3	0.00062500	0.00020833	0.00	0.9998
$\rho k_i k_i' \times \rho k_i k_{i'} \times$ EST $\times \sigma_k^2$	3	0.00156250	0.00052083	0.01	0.9991

In summary, except for $\hat{\gamma}^H$, overall performances measured in terms of MSE, Type I error, and Type II error of $\hat{\gamma}^G, \hat{\gamma}^W$, and $\hat{\gamma}^O$ are not significantly different. This recommends use of $\hat{\gamma}^W$ or $\hat{\gamma}^O$ in place of $\hat{\gamma}^G$, when some necessary correlation structure for a multivariate meta

Table 6. ANOVA: Type II error.

Source	DF	Sum of squares	Mean square	F-value	P-value
Model	31	0.89468750	0.02886089	0.38	0.9993
Error	3168	240.49000000	0.07591225		
Corrected total	3199	241.38468750			
Source	**DF**	**Anova SS**	**Mean square**	**F-value**	**P-value**
$\rho k_i k_i'$	1	0.03781250	0.03781250	0.50	0.4804
$\rho k_i k_{i'}$	1	0.00031250	0.00031250	0.00	0.9488
$\rho k_i k_i' \times \rho k_i k_{i'}$	1	0.00281250	0.00281250	0.04	0.8474
EST	3	0.37093750	0.12364583	1.63	0.1805
$\rho k_i k_i' \times$ EST	3	0.10843750	0.03614583	0.48	0.6989
$\rho k_i k_{i'} \times$ EST	3	0.00593750	0.00197917	0.03	0.9943
$\rho k_i k_i' \times \rho k_i k_{i'} \times$ EST	3	0.00343750	0.00114583	0.02	0.9975
σ_k^2	1	0.13781250	0.13781250	1.82	0.1780
$\rho k_i k_i' \times \sigma_k^2$	1	0.00031250	0.00031250	0.00	0.9488
$\rho k_i k_{i'} \times \sigma_k^2$	1	0.00031250	0.00031250	0.00	0.9488
$\rho k_i k_i' \times \rho k_i k_{i'} \times \sigma_k^2$	1	0.00281250	0.00281250	0.04	0.8474
EST $\times \sigma_k^2$	3	0.15843750	0.05281250	0.70	0.5546
$\rho k_i k_i' \times$ EST $\times \sigma_k^2$	3	0.05593750	0.01864583	0.25	0.8645
$\rho k_i k_{i'} \times$ EST $\times \sigma_k^2$	3	0.00593750	0.00197917	0.03	0.9943
$\rho k_i k_i' \times \rho k_i k_{i'} \times$ EST $\times \sigma_k^2$	3	0.00343750	0.00114583	0.02	0.9975

analysis is not available for $\hat{\gamma}^G$. Note that both $\hat{\gamma}^W$ and $\hat{\gamma}^O$ neglect potential correlation in the estimated study effects. But $\hat{\gamma}^W$ takes into account both variance component due to estimation error and that due to random error where $\hat{\gamma}^O$ neglects the variance component due to estimation error.

It would be ideal if one can compare many controlled study results which are not correlated. However, such comparable results may not be many due to expenses involved in controlled studies. Therefore, studies conducted based on historical data are utilized. Under such a situation, we expect that the simulation results give some insight into use of relatively simple approaches ($\hat{\gamma}^O$ or $\hat{\gamma}^W$) which provide insignificantly different degree of accuracy compared to the $\hat{\gamma}^G$. Although the simulation was conducted based on the military advertising studies, implication of our study results can be applied to many potential multivariate meta analysis of a small number of econometric study results.

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REFERENCES

[1] G. Assmus, J. Farley, and D.R. Lehmann, How advertising affects sales: Meta-analysis of econometric results, J Market Res 21 (1984), 65–74.

[2] C.B. Begg and P. Louise, A model for incorporating historical controls into a meta-analysis, Biometrics 47 (1991), 899–906.

[3] C.S. Berkey, D.C. Hoaglin, F. Mosteller, and G.A. Colditz, A random-effects regression model for meta-analysis, Stat Med 14 (1995), 395–411.

[4] S.P. Brown and R.A. Peterson, Antecedents and consequences of salesperson job satisfaction: Meta-analysis and assessment of casual effects, J Market Res 30 (1992), 63–77.

[5] M. Chandrashenkaran and B. Walker, Meta analysis with heteroscedastic effects, J Market Res 30 (1993), 246–255.

[6] G.A. Churchill, Jr., N.M. Ford, S.W. Hartley, and O.C. Walker, Jr., The determinants of salesperson performance: A meta analysis, J Market Res 22 (1985), 103–118.

[7] R. Der Simonian and N.M. Laird, Meta analysis in clinical trials, Control Clin Trials 7 (1986), 177–188.

[8] J. Dertouzos and M. Polich, Recruiting effects of Army advertising, RAND Report R-3577-FMP, RAND, Santa Monica, CA, 1989.

[9] J. Farley and D. Lehmann, Meta analysis in marketing: Generalization of response models, Lexington Books, Lexington, MA, 1985.

[10] E.F. Fern and K.B. Monroe, Effect-size estimates: Issues and problems in interpretation, J Consumer Res 23 (1996), 89–105.

[11] G.V. Glass, Primary, secondary, and meta analysis of research, Ed Res 5 (1976), 3–8.

[12] L. Goldberg, Recent estimates of enlistment supply models, Final Report, prepared for the Office of the Assistant Secretary of Defense, Reston, VA, Economic Research Laboratory, Inc., 1991.

[13] L. Hedges and I. Olkin, Statistical methods for meta-analysis, Academic, Orlando, FL, 1985.

[14] D.R. Jones, Meta-analysis: Weighting the evidence, Stat Med 14 (1985), 137–149.

[15] C. Kearl, D. Horne, and C. Gilroy, Army recruiting in a changing environment, Contemporary Policy Issues 8 (1990), 68–78.

[16] R.T. Rust, D.R. Lehmann, and J.U. Farley, Estimating publication bias in meta-analysis, J Market Res 27 (1990), 220–226.

[17] S. Searle, G. Casella, and C. McCulloch, Variance components, Wiley, New York, 1992.

[18] T.C. Smith, D.J. Spiegelhalter, and A. Thomas, Bayesian approaches to random-effects meta-analysis: A comparative study, Stat Med 14 (1995), 2685–2699.

[19] S.Y. Sohn and M. Mazumdar, A Monte Carlo study of variable selection and inferences in a two stage random coefficient linear regression model with unbalanced data, Commun Stat 20 (1991), 3761–3791.

[20] S.Y. Sohn, Meta analysis with potentially correlated study results, 1996 ASA Proceedings of the Section on the Quality and Productivity, American Statistical Association, Washington, DC, 1996.

[21] S.Y. Sohn, Random effects meta analysis of military recruiting, Omega 24 (1996), 141–151.

[22] S.Y. Sohn and C.J. Park, Random effects models for both process mean and variance, J Qual Technol 30 (1998), 33–39.

[23] S.Y. Sohn and R. Stepleman, Meta analysis on total braking time, Ergonomics (1998), to appear.

[24] D.O. Stram, Meta-analysis of published data using a linear mixed-effects model, Biometrics 52 (1996), 536–544.

[25] F. Sultan, J.U. Farley, and D.R. Lehmann, A meta-analysis of application of diffusion models, J Market Res 27 (1990), 70–76.

[26] D. Szymanski and P. Busch, Identifying the genetics-prone consumer: A meta-analysis, J Market Res 24 (1987), 425–431.

[27] H.C. Van Houwelingen and K.H. Zwinderman, A bivariate approach to meta-analysis, Stat Med 12 (1993), 2273–2284.

[28] J. Warner, Military recruiting programs during the 1980s: Their success and policy issues, Contemporary Policy Issues 8 (1990), 47–67.