An optimal linear approach to cruise controlling a fixed-wing aircraft in the presence of wind

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Linear Gaussian control (LQG) is a well suited control strategy when it comes to stabilize systems under a fixed set of operating conditions around an equilibrium point. It provides a great mechanism to ensure a stability margin for any real system with a high degree of non-linearity. It has been successfully used in conjugation with a system composed of states defined in different scales. This paper aims to develop a control strategy for a fixed wing aircraft which operates in windy conditions at a cruise altitude, focusing on the case of a small scale Unmanned Aerial vehicle (UAV) with no access to any wind sensors, thus relying on an estimator to identify the wind's intensity and direction. This approach seems really promising especially when paired with a gain-scheduling mechanism.

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I. INTRODUCTION

This research proposes a control strategy for a small aircraft flying in windy conditions, with a lack of weather sensors, based on a linear Gaussian control and using an estimator to make up for the lack of sensors. It will walk through all the stages of development for such a controller and present the reader will all the necessary information for conducting a similar research in order to compare results between approaches when different parameters are taken into consideration. It will try to prove the strategy by simulating the aircraft in windy conditions and giving it a path to follow.

While there has been a strong trend to automate flight in the recent years in both military and civil fields, as presented in [15], we can see that the market for low computational power solutions is still wide open. While studies like [14] and [13] do a great job for solving the problem of autonomous flight, using different control structures, they both rely heavily on a ground station with a high degree of computational power available. This is a very common, but this study aims to develop a low power controller, placed on board of the aircraft, yielding similar results.

This study is divided into 8 sections. The first section is the Introduction, while the second section focuses on the frames of reference used when developing the dynamic model of our system. The third and forth chapters focus on the mathematical model and incorporating the effects of wind into the differential equations. In the fifth section we will describe the implemented control structure that aims to keep the aircraft in the air and estimate the wind disturbances, while the sixth chapter will deal with ensuring a null steady state error when using a predefined path even when wind is present. In the last chapter the testing results will be listed and the conclusion of the study will be drawn.

II. SYSTEM OF REFERENCE

In order to apply Newton's equations of motion to the location and orientation of an aircraft when examining its dynamics, an appropriate inertial coordinate system must be used. Additionally, it is necessary to specify relative locations and velocities using multiple frames of reference, with the choice of which frame to use being a matter of convenience. A coordinate system fixed on the surface of the Earth is preferred to construct the velocity equations because sensors like the Global Positioning System (GPS) measure the aircraft's velocity in relation to the Earth. The information provided by the accelerometer and the gyroscope are specific to the system of reference fixed on the body of the vehicle. In light of this a second frame of reference shall be used, namely the body frame, which is fixed on the body of the aircraft itself.

A. Body-fixed and inertial frames of reference

Solving a dynamic problem requires an inertial reference frame, F^I, which is a system of reference fixed or in a uniform rectilinear translation relative to the distant stars. Meeting this requirement leads to the possibility of using the Newton's second law for the motion of a particle, which relates the external forces acting on the particle to its mass and acceleration relative to F^I. Generally, the rotation of the Earth is neglected in the analysis of the flight dynamics when flying happens on small distances. Therefore, any coordinate frame with the origin at a defined location on the Earth can be used as an inertial frame. Let F^E denote the Earth-fixed reference, frame having the origin close to the launching location, it's X axis indicating North, it's Y axis indicating East, and it's Z axis indicating vertically. This coordinate system will further be used in order to describe the aircraft's position and orientation relative to a fixed point on the earth's surface. In order to use ignore Coriolis effect we will work in a flat-earth environment. The origin of the body fixed frame, depicted as F^B, coincides with the vehicles center of gravity. The x-axis of this frame will always be pointing through the nose of the aircraft, the y-axis will be pointing through the right wind, and the z axis will be pointing downwards. The body-fixed reference frame, F^B, has an angular velocity of $\varPhi$ in relation to F^I, depicted as $\varPhi = [p, q, r]^T$.

The propulsive force acts along x-axis of the body-fixed system and the aerodynamic forces are described by a vector having its center in the aerodynamic center of the aircraft. We will assume that the aerodynamic center of the aircraft and the center of gravity overlap, to simplify the model. The interaction of forces will be analyzed in this body fixed system, because it makes it is the native system for most of the forces present during flight. Furthermore, some onboard sensors, specially the inertial measurement unit, collect data in this particular frame.

The Euler angles, used to describe the three sequential rotations around the z y x axes, that can be used to describe the orientation of F^B with relation to F^I, are noted as (ϕ, θ, ψ). Yaw(ϕ), pitch(θ), and roll(ψ) angles rotate the airplane's body around the x, y and z-axes respectively. The transformations that describe the rotations are:

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \tag{2.1}$$

$$R_2(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{2.2}$$

$$R_3(\psi) = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

This allows us to define the complete rotational matrix from F^I to F^B as:

$$R_{BI} = R_1(\phi) R_2(\theta) R_3(\psi) \tag{2.4}$$

B. Wind axes coordinate frame

The wind reference frame, F^W has the same point of origin like the body-fixed frame, namely the center of gravity of the aircraft. As depicted his x-axes is rotated around the body x body axis so that it aligns with the velocity vector of the aircraft. The y-axis also reflects that by being rotated around the z-axis of the aircraft with an angle of sideslip β. The z-axis still points down, but now it has also been rotated around the y-axis with the same α angle, formerly known as the angle of attack of the vehicle, just like the x-axis. This frame will later be used to describe the aerodynamic forces acting upon our aircraft. The wind vector can also be denoted in the inertial reference frame and his components would be $\omega_w = [p_w, q_w, r_w]T$. The rotation matrix from body to wind reference frames can be defined as follows:

$$R_{WB} = \begin{bmatrix} C(\alpha)C(\beta) & -C(\alpha)S(\beta) & -S(\alpha) \\ S(\beta) & C(\beta) & 0 \\ S(\alpha)C(\beta) & S(\alpha)S(\beta) & C(\alpha) \end{bmatrix} \tag{2.5}$$

where $S(\phi)$ denotes sin and $C(\phi)$ denotes the cosine of the angle ϕ.

III. MODELLING AN AIRCRAFT

This section derives the mathematical model of our aircraft. It will be considered as a rigid body with six degrees of freedom. The Earth is considered flat and still for the purpose of simplicity, making F^E a Newtonian frame of reference F^I. As long as the distances are close enough to the radius of the earth, we can assume the air density as constant, it should be emphasized that this approximation holds true for the majority of flight altitudes, that were taken into consideration for our small aircraft. The equations of motion are firstly derived under the presumption that the atmosphere is at rest with respect to the Earth, and the wind effect is then appropriately taken into account. The presentation in this section is mainly based on [5], [4], [2] textbooks.

A. State variables

The motion of an airplane in relation to the Earth can be represented by position, direction, velocity, and angular velocity. The position of the aircraft center of gravity in the inertial coordinate frame will be indicated by the vector P_E, whose elements are $P_E = [p_N, p_E, p_D]^T$. The real altitude of the vehicle will be equal to the inverse of the down position element $h = -p_D$ to account for the fact that the system's z-axis is pointing downward and to provide parallelism with the body-fixed reference frame when the latter is not rotated. As a result, the position of the airplane at any given time with respect to an inertial point of reference can be expressed as:

$$P = \begin{bmatrix} p_N \\ p_E \\ -p_D \end{bmatrix} \tag{3.1}$$

The Euler angles denote the angles needed in the equation 2.4. Thus, we can define the vector:

$$\varPhi = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{3.2}$$

where ϕ, θ, ψ represent the rotation angles between the body-fixed frame F^B and the inertial reference F^I around the x^B, y^B, z^B axis.

The inertial velocity vector V_E of an aircraft is widely represented in a variety of coordinate systems. Its constituents are provided by:

$$V_E = R_{EW} V_W = R_{EB} V_B \tag{3.3}$$

where

$$V_E = \begin{bmatrix} v_n \\ v_e \\ v_d \end{bmatrix} V_w = \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} V_B = \begin{bmatrix} v_u \\ v_v \\ v_w \end{bmatrix} \tag{3.4}$$

where v_n, v_e, v_d denotes the velocity of the aircraft in the earth-fixed frame on the north, east down axis respectively. V_a stands for the true airspeed in the wind frame and v_u, v_v, v_w denotes the velocity of the aircraft along the x, y, z-axis of the body frame respectively. The rotation matrices can be calculated starting from 2.4 and 2.5 as such:

$$R_{EW} = {R_{WE}}^T \tag{3.5}$$

$$R_{EB} = {R_{BE}}^T \tag{3.6}$$

$$R_{BW} = {R_{WB}}^T \tag{3.7}$$

$$R_{EW} = R_{EB} R_{BW} \tag{3.8}$$

The true airspeed V_a denotes the aircraft's velocity vector relative to the surrounding air, with the components $V_a = [u, v, w]^T$. If the atmosphere is at rest, a scenario that we will use for modelling without taking into consideration the effect of wind. The true airspeed will be equal to the aircraft's velocity relative to the fixed reference frame F^I. The equation that describes the true airspeed in the body reference frame is written below:

$${V_a}^B = V_b = R_{WB} V_W \tag{3.9}$$

this equation can be written more explicitly as:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = V_a \begin{bmatrix} \cos(\alpha)\sin(\beta) \\ \sin(\beta) \\ \sin(\alpha)\cos(\beta) \end{bmatrix} \tag{3.10}$$

Since we are just expressing the same value in a different reference frame its module should remain the same, this means:

$$|V_a| = \sqrt{v^2 + u^2 + w^2} \tag{3.11}$$

In order to rotate from the wind reference frame to the body reference frame, the angles of attack α and sideslip β have to be expressed relative to known states, thus:

$$\alpha = \arctan \frac{w}{u} \tag{3.12}$$

$$\beta = \arctan \frac{v}{V_a} \tag{3.13}$$

The angular velocity vector can be represented as $\omega = [p, q, r]$ in the body-fixed and wind-axes coordinate frames. Where p, roll rate, stands for the angular velocity around the x^b axis, q, pitch rate, stand for the angular velocity around the y^b axis and r, yaw rate, stands for the angular velocity around the z^b axis. This measure can also be defined in the wind reference frame using the rotation matrix R_{BW} as shown below:

$$\omega_w = R_{BW}\omega = \begin{bmatrix} p_w \\ q_w \\ r_w \end{bmatrix} \tag{3.14}$$

Thus the state vector can be described as a column vector formed from the vectors described before. Thus, the state vector:

$$X^T = \begin{bmatrix} V_B \\ \omega \\ \varPhi \\ p_E \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \\ \psi \\ p_n \\ p_e \\ p_h \end{bmatrix} \tag{3.15}$$

Then, in [6], the equations of motion for a rigid body aircraft when assuming a stationary flat-Earth and the atmosphere at rest are given, and they take the form:

$$\dot{V_B} = -\Omega_B V_B + R_{BE} \times g_0 + \frac{F_B}{m} \tag{3.16}$$

$$\dot{\varPhi} = \varsigma(\varPhi)\omega \tag{3.17}$$

$$\dot{\omega} = -J^{-1}\Omega_B J\omega + J^{-1} T_B \tag{3.18}$$

$$\dot{p_E} = R_{BE} V_A \tag{3.19}$$

Ω_B is the cross-product of the body angular rates, g_0 is the gravitational acceleration measured at sea level. F^B is the equivalent force acting on the aircraft's center of gravity, basically the sum of all forces considered in the model. m is the mass of the aircraft and J stands for the inertial matrix of our vehicle. The term T^B stands for the net torque acting on the vehicle. As will be shown later, the whole mathematical model presented in 3.19,3.17,3.16 and 3.18 can be divided into translational and rotational equations.

B. Force and moments equations

The forces and moments operating on the aircraft center of gravity drive the last two equations derived in 3.16.

They have both components as a result of a variety of causes, the most important of which are the propulsive and aerodynamic impacts.

The propulsive and aerodynamic components, F_p and F_a, can be used to describe the force vector from equation 3.16 like this:

$$F_b = F_p + F_a \tag{3.20}$$

The effects of the thrusters can be found in the translational equations in the form of F^B and in the rotational equations in the form of T_B. The engine thrust, represented in vector form as $\vec{T}$, represents the propulsive force generated by the propellers. Generally, the engines are positioned along the aircraft's longitudinal body axis so that the resultant force has only one not null component in the body-fixed reference frame pointing in the direction of the x-axis, i.e.

$$F_p = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \tag{3.21}$$

The aerodynamic force can be described in both body and wind axes. Regardless of whether frame is used, the components are connected by the rotation matrix 2.5. To make the equations look cleaner lift (L), drag (D), and side force (Y) are specified in the wind axes.

$$F_a{}^W = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} = \begin{bmatrix} \bar{q}SC_D \\ \bar{q}SC_Y \\ \bar{q}SC_L \end{bmatrix} \tag{3.22}$$

where S is the wing area, C_D, C_L and C_Y are physical coefficients that depend on the angle of attack, angle of sideslip and the practical design of the aircraft, which will be discussed later, and $\bar{q}$ is the free-stream dynamic pressure described by the formula:

$$\bar{q} = \frac{\rho(h){V_a}^2}{2} \tag{3.23}$$

where $\rho(h)$ is the dynamic static atmospheric pressure at the attitude h. They can be represented in terms of body-axes dimensionless aerodynamic coefficients C_x, C_y, C_z by denoting the components of the aerodynamic force in the body axes by (X_a, Y_a, Z_a):

$$F_a = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = \begin{bmatrix} \bar{q}SC_x \\ \bar{q}SC_y \\ \bar{q}SC_z \end{bmatrix} \tag{3.24}$$

or in terms of the aerodynamic force's wind axis components:

$$\begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = \begin{bmatrix} -DC(\alpha)C(\beta) - YC(\alpha)S(\beta) + LS(\alpha) \\ -DS(\alpha) + YC(\beta) \\ DS(\alpha)C(\beta) - YS(\alpha)S(\beta) - LC(\alpha) \end{bmatrix} \tag{3.25}$$

It is obtained by extending the force equation 3.16 and inserting the above notations:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rv - qw - g_0 \sin(\theta) + \frac{X_a+T}{m} \\ -rv + qw + g_0 \sin(\phi)\cos(\theta) + \frac{Y_a}{m} \\ qu - pv + g_0 \cos(\phi)\cos(\theta) + \frac{X_a+T}{m} \end{bmatrix} \tag{3.26}$$

The system of a rigid body aircraft flying through a still atmosphere can be expressed into a 12 state highly coupled differential model with a great degree of non-linearity. This expression can be derived from 3.16, 3.17, 3.18, and 3.19. Note that for both the navigation and force equations given in 3.24, two alternates have been presented 3.22. Although not directly visible in these equations, the control vector determines the thrust force and deflections of the moveable surfaces that manage the aerodynamic forces (D, L, Y) and moments $(\bar{L}, M, N)$.

The mathematical model created by putting these equations together is predicated on the following assumptions:

1. The aircraft is a rigid body with a plane of symmetry.
2. The Earth will be considered as a flat earth with no movement relative to far-off stars.
3. There is no wind in the surround atmosphere.

IV. FLYING IN A TURBULENT ENVIRONMENT

In this chapter we will revise the effects of wind on our aircraft, and an estimating strategy will be outlined in order to overcome our lack of pressure sensors in our scenario.

A. Wind description

We initially define wind as a component of the velocity field that the aircraft flies in order to understand and research how air motion impacts the system. The air mass is continually moving as a result of solar heating, Earth rotation, and other thermodynamic and electromagnetic processes. The velocity vector of the atmosphere is often variable in space and time and has a mean value and variations. Atmospheric turbulence or gust is the remaining fluctuating proportion, while mean wind is the steady-state velocity at a certain point.

The wind is used mostly for navigation and direction, whereas turbulence predominantly impacts airplane stability. $\vec{W}$ is the traditional notation for the air mass's velocity vector relative to the Earth. The overall velocity field within the atmosphere is defined as follows based on the above considerations:

$$\vec{W} = \vec{W_M} + \vec{W_F} \tag{4.1}$$

where $\vec{W_M}$ is the mean wind vector and $\vec{W_F}$ is the atmospheric turbulence.

The north, east, and down velocity components of local wind are expressed in the direction of the Earth-fixed reference frame by W_n, W_e, and W_h, respectively. The transformation matrices produced in the previous section can be reused to represent the air mass velocity vector in various coordinate systems for convenience.

Therefore:

$$\vec{W} = \begin{bmatrix} W_n \\ W_e \\ W_h \end{bmatrix} = \begin{bmatrix} W_{n_M} + W_{n_F} \\ W_{e_M} + W_{e_F} \\ W_{h_M} + W_{h_F} \end{bmatrix} = R_{WB} \begin{bmatrix} u_w \\ v_w \\ w_w \end{bmatrix} \tag{4.2}$$

where (u_w, v_w, w_w) represent the wind components in the body-fixed reference frame and R_{IB} is given by equation 3.8.

A mathematical model of such disturbances is required to investigate the wind vector effect on an aircraft's the flight qualities. Complete wind cannot be represented deterministically; in other words, it cannot be expressed using analytical formulations. The wind field, on the other hand, can be treated as a stochastic process with statistical features. The random-process theory is heavily used in the development of a wind gust model, and the reader is directed to [8] for a thorough discussion on the issue.

The Dryden wind turbulence model, often known as Dryden gusts, is a mathematical model of continuous gusts approved for use in certain aircraft design and simulation applications by the US Department of Defense. [2] The Dryden model handles continuous gusts linear and angular velocity components as spatially variable stochastic processes, with each component's power spectral density specified. Due to the rational power spectral densities of the Dryden wind turbulence model, precise filters that take white noise inputs and output stochastic processes with the Dryden gusts' power spectral densities can be developed. The Karman velocity spectra are used to filter a unit variance, band-limited white noise signal to generate the fluctuating wind vector with the necessary features. A Karman model's transfer functions are also listed from [7] as:

$$H_u(s) = \frac{\sigma_u \sqrt{\frac{2L_u}{\pi V}(1 + \frac{L_u}{4V} s)}}{1 + 1.357 \frac{L_u}{V} s + 0.1987 \frac{L_u}{V}^2 s^2} \tag{4.3}$$

$$H_v(s) = \frac{\sigma_v \sqrt{\frac{L_v}{\pi V}(1 + 2.7478 \frac{L_u}{V} s + 0.3398 \frac{L_v}{V}^2 s^2)}}{1 + 2.9958 \frac{L_v}{V} s + 1.9754 \frac{L_v}{V}^2 s^2 + 0.1539 \frac{L_v}{V}^3 s^3} \tag{4.4}$$

$$H_w(s) = \frac{\sigma_w \sqrt{\frac{L_w}{\pi V}(1 + 2.7478 \frac{L_w}{V} s + 0.3398 \frac{L_w}{V}^2 s^2)}}{1 + 2.9958 \frac{L_w}{V} s + 1.9754 \frac{L_w}{V}^2 s^2 + 0.1539 \frac{L_w}{V}^3 s^3} \tag{4.5}$$

where L_u, L_v, L_w denote the turbulence scale lengths. The turbulence intensities are represented by $\sigma_u, \sigma_v, \sigma_w$ and the speed of the vehicle is noted as V. In terms of implementing the Dryden model, there already exists a block given by Mathworks who takes into account all of those factors and returns the wind speed vector.

$$H_u(s) = \sigma_u \sqrt{\frac{2L_u}{\pi V}} \frac{1}{1 + \frac{L_u}{V} s} \tag{4.6}$$

$$H_v(s) = \sigma_u \sqrt{\frac{L_v}{\pi V}} \frac{1 + \frac{\sqrt{3} L_v}{V} s}{1 + \frac{L_v}{V} s^2} \tag{4.7}$$

$$H_w(s) = \sigma_u \sqrt{\frac{L_w}{\pi V}} \frac{1 + \frac{\sqrt{3} L_w}{V} s}{1 + \frac{L_w}{V} s^2} \tag{4.8}$$

The Dryden gust model, with mean values of $w_{n_M} =$ 4 m/s, $w_{e_M} = 1.7$ m/s, and $w_{h_M} = 0.4$ m/s, is shown in [13]. Under real-world conditions, the mean wind's properties change along the flight path as a result of how the air masses move in relation to one another. Wind shear, which is a significant shift in the wind's speed or direction over a brief distance, is extremely dangerous during takeoff and landing approaches[6]. In contrast to horizontal wind shear, which denotes the differences in the wind vector over horizontal distances, the vertical wind shear describes changes in the wind field through horizontal distances [9]. As described by [9], an aircraft's landing approach is impacted by horizontal wind shear. As shown by 4.9, the height above the ground influences the wind shear's intensity. Because the amount of air mass over the wing reduces as the wind velocity falls, the aircraft control system must take this into account as the plane descends. The wind speed at a reference height of 10 meters from [10] can be used to determine how exponentially the mean wind changes with altitude.

$$W(h) = W_{10} \frac{h}{h_{10}} a \tag{4.9}$$

where $W(h)$ states for the intensity of the wind at a certain height h. W_{10} stands for the intensity of the wind at a base altitude of height $h_{10} = 10$ m. The term a represents the Hellman exponent, which relates the wind to the surrounding terrain[13]. The shape of the terrain, as well as its aerodynamic 'roughness', influence wind shear. Because they have radically distinct roughness, the velocity profile of a wind shear over a major urban area will be very different from over a woodlands or flat grassland area. Furthermore, various wind profiles are produced at the bottom and top of a sloping land [12].

B. Effects of wind on an aircraft

In order to achieve flight an aircraft needs to have a lift force equal or greater than its gravitational pull towards the Earth. This lift force depends on a multitude of structural factors of the aircraft, on inputs for the control surface and is tightly coupled with the true airspeed. This quantity denotes the velocity of the aircraft relative to the surrounding masses of air. Aerodynamic forces and moments are, as seen in 3.26, affected by both horizontal and vertical components of the wind vector, thus giving birth to a complex system of highly nonlinear dynamics.

When an aircraft is moving through a windy atmosphere, it is subject to unknown external pressures that may endanger both the structural integrity of the aircraft and the internal stability of the flight controller. The performance of the vehicle as a whole might be harmed by additional factors, sometimes leading to aviation mishaps. When executing certain aircraft movements, such as a crosswind takeoff or landing, the control system must exert more control effort in order to prevent such undesirable conditions.

To deal with steady crosswinds, you can use one of two methods. The first is known as 'crab' and it points the airplane's nose into the wind to balance the crosswind with engine thrust. The second is referred to as 'sideslip', and is achieved by keeping the aircraft's heading parallel with the inertial track while compensating for the drift generated by the wind with a modest bank angle. Both rudder and aileron are used during crosswind maneuvers. To accommodate for high crosswinds during the landing approach, some crucial flight situations may necessitate maximum control power. Structural modifications in the aircraft design phase may be considered to achieve enhanced rudder control. The 'Boeing B-52 Stratofortress' plane type, for example, has 'yaw-adjustable cross-wind landing gear' that points down the runway as the plane is yawed into the relative wind. Furthermore, turbulent winds can cause the plane to roll during takeoff or landing approaches, necessitating additional aileron control to counteract this motion. In high crosswinds, upper surface wing spoilers can be deployed to boost aileron efficacy and keep the upwind wing down [11].

The motion of the air masses relative to the earth's crust is effecting both the groundspeed and the airspeed thus influencing the inertial course with respect to the aircraft orientation. In extreme circumstances, turbulence may cause a vehicle to fly outside its safe operating range. To learn more about the effect of environmental wind on aircraft responsiveness we will examine the landing approach of an aircraft that is subject to intense wind shear caused by thunderstorms. As the aircraft approaches the wind shear, the outflow creates a headwind that causes the airplane to go faster. In order to keep the nominal airspeed, the engine power is consequently decreased. The airplane will pass through the outflow on the other side, flying into a tailwind, reducing lift and increasing sink rate since wind shear only affects short distances. Aggressive control input is needed since the aircraft is in a low-power, low-speed descent and might be close to the ground.

C. Incorporation into the equations of motion

The wind field has a substantial impact on tiny fixed-wing aircraft, like the ones we investigated, during all stages of flight because of their low operating speeds. According to the knowledge of aerodynamics gained in the previous part, the aerodynamic forces generated by their wings are very dependent on relative wind. The latter fluctuates in response to the shifting atmosphere, which causes a misalignment of the flying forces and, as a consequence, unpredictable aircraft behavior. Furthermore, stalling is conceivable if lift rapidly decreases due to unfavorable wind and drag soon replaces lift as the major component of aerodynamic force.

First, remember that wind affects both longitudinal and lateral aircraft dynamics. The ambient wind primarily alters lift force, but since it moves mostly horizontally with respect to the Earth, it may also create a lateral force that influences the directional stability of the aircraft. Therefore, it is necessary to consider the wind speeds along the three axes of the body-fixed reference frame.

By separating the vehicle's velocity in reference to the air from its velocity in relation to the ground, the mean component of the wind vector affects aviation navigation. The connection between the velocities is expressed as:

$$\vec{V_g} = \vec{V_a} + \vec{W} \tag{4.10}$$

where $\vec{V_g}$ is the aircraft velocity relative to the ground, $\vec{V_a}$ represents the air-relative aircraft velocity and $\vec{W}$ is the velocity of the wind with respect to the ground.

The mean of the wind velocity influences aircraft navigation by separating the vehicle's velocity in relation to the air from its velocity in relation to fixed point on the ground. This can be expressed as:

$$V_g{}^E = R_{EB} V_a{}^b + W^E \tag{4.11}$$

where the superscript ^{E} stands for representation in the inertial frame and ^{b} for representation in the body frame.

As a result, as the atmosphere moves relative to the Earth, it will directly and linearly impact the movement of the aircraft relative to the origin of the Earth-fixed frame, F_E. This phenomenon can be better illustrated by the equation:

$$\dot{p_E} = {V_g}^E = R_{EB}{V_a}^b + W^E \tag{4.12}$$

Where P_E is the position of the vehicle in the earth fixed reference frame.

D. Wind estimation

In a conventional large scale aircraft wind is directly measured using dynamic pressure sensors on the tip and on the wings of the aircraft. Thus, the controller has access to real-time data describing the surrounding air. In the case of small scale aircraft the complexity of these sensors make the virtually unfeasible for any project, thus an estimator must be designed in order to adapt to the surrounding wind conditions.

Since the aircraft dynamics are strictly reliant on the true airspeed, and since the true airspeed is calculated as the differential velocity between the aircraft and the surrounding masses of air, any change in the aircraft's velocity vector will be generated either by the inputs of the aircraft (known measures) or by a change in the surrounding atmospheric conditions.

Thus, the velocity of the wind, expressed in the body frame, can be used to extend the state vector when defining an estimator for the aircraft. In order to do that we will have to assume that the changes in wind speeds do not create any form of momentum on the aircraft, we assume perfectly symmetrical aircraft. The extended states vector can be described as follows:

$$X^T = \begin{bmatrix} V_B \\ \omega \\ \Phi \\ V_w \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \\ \psi \\ u_W \\ v_W \\ W_w \end{bmatrix} \tag{4.13}$$

It will use an extended linear model as explained in [1] we can use such a model in order to estimate the current unobservable perturbations as an unmeasured input. This approach is based on the mathematical model previously defined, linearized around a point of operation.

V. LINEAR GAUSSIAN CONTROL

A. State observer

As previously stated, in order to have continuous access to our states an observer will be used. The Lauenberg observer is a type of observer who uses the linear model of the system to estimate the expected outcome and also uses a gain matrix paired with the reading error in order to minimize position error. At first, we need to define our linear discrete system as:

$$X(k+1) = A_{E_d}X(k) + B_{E_d}U(K) \tag{5.1}$$
$$Y(k) = C_{E_d}X(k) \tag{5.2}$$

where $X(K)$ is the current state, $U(K)$ is the current input, $X(K+1)$ is the next state, $Y(K)$ is the current output and the matrices $(A_{E_d}, B_{E_d}, C_{E_d})$ are the matrices governing our discrete model. The Euler discretization method will be used. Now a definition of the discrete time estimator will be given:

$$\bar{X}(k+1) = A_{E_d}\bar{X}(k) + B_{E_d}U(K) + L(Y(K) - \bar{Y}(k)) \tag{5.3}$$
$$\bar{Y}(k) = C_{E_d}\bar{X}(k) \tag{5.4}$$

where $\bar{X}(k)$ denotes the estimated state at the point k and $\bar{Y}(k)$ denotes the estimates output at the point k. But before any observer can be developed, the observability of the system must be analyzed. Since we already consider 9 states as being accessible through our sensors the focus of this test is to determine the observability of the wind related states. This implies:

$$rank(\begin{bmatrix} C_{E_d} \\ A_{E_d}C_{E_d} \end{bmatrix}) = n \tag{5.5}$$

where n is the number of states of the model. In our case n=15, while our the rank of the matrix is also 15, so we are dealing with a completely observable extended model. We can describe the estimation error as the difference between the estimated and the true value of the states, thus:

$$e(k+1) = X(k) - \bar{X}(k+1) \tag{5.6}$$

Since our goal is to minimize error, a statement can be made that we need to find L such that:

$$\lim_{k\to\infty} e(k+1) -> 0 \tag{5.7}$$

After replacing 5.6, 5.2 and 5.4 into 5.7 we will get that:

$$\lim_{k\to\infty} e(k+1) = (A_{E_d} - LC_{E_d})e(k) + C_{E_d}y(k) \tag{5.8}$$

Since our goal is to ensure 5.7 to be true, which would imply that 5.8 must be null, the eigenvalues of the

$(A_{E_d} - LC_{E_d})$ matrix must be placed inside the unit circle. In order to achieve this, the pole-placement method can will be used to calculate a gain matrix such that the eigenvalues of $(A_{E_d} - LC_{E_d})$ are all inside the unit circle.

$$|eig((A_{E_d} - LC_{E_d}))| < 1 \tag{5.9}$$

An important aspect to keep in mind is the fact that the estimator also needs an offset to the point of equilibrium to which it was calculating around. As we can see

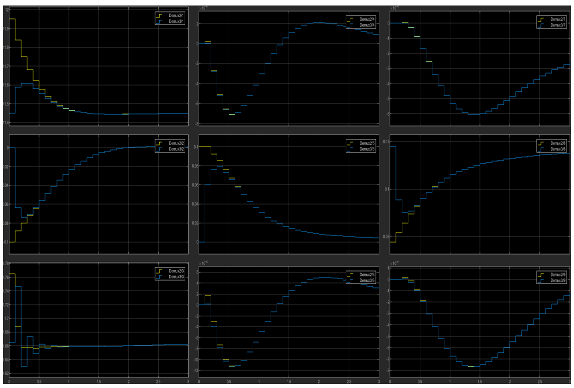

FIG. 1: Results of Lauenberg estimator with initial error

in 1 all the states and estimates converge over time even with initial error.

B. Noise filtering

Kalman filtering, also known as linear quadratic estimation (LQE) in statistics and control theory, is an algorithm that estimates a joint probability distribution over the variables for each time-frame using a series of measurements observed over time, including statistical noise and other inaccuracies, and produces estimates of unknown variables that are more accurate than those based on a single measurement alone.

Since our system will make use of sensor in order to access the available information about our aircraft, the effect of white noise might jeopardize our task, so in order to minimize its effect onto our control strategy we will adapt the estimator from an extended Lauenberg estimator to a Kalman filter with a high enough amount of trust into our linear model, that would allow us to minimize the effect of the white noise, while still achieving a null position error over time. First we need to add white noise to the system and the readings:

$$X(k+1) = A_{E_d}X(k) + B_{E_d}U(K) + w(k) \tag{5.10}$$

$$Y(k) = C_{E_d}X(k) + r(k) \tag{5.11}$$

where $w(k)$ represents the random noise at the moment k applied on the system and $r(k)$ represents the random noise on the sensors at the moment k. We will use the same definition as in 5.6 for the error of the system, and the goal still remains to have no position error at steady state. But a dynamic model of the estimator is needed:

$$\bar{X}(k) = X_{pred} + K_k(Y(k) - C_{E_d}x_{pred}) \tag{5.12}$$

where X_{pred} represent the predicted value and K_k represents the Kalman gain matrix at point k. All the other notations keep the same meaning as before. The main advantage of the Kalman filter is the continuous recalibration of the filter using the prediction error. The Kalman filter replaces the static matrix L from the Lauenberg estimator with a constantly changing K_k matrix governed by the equation:

$$K_k = P_{pred}{C_{E_d}}^T(C_{E_d}P_{pred}C_{E_d} + R_k)^{-1} \tag{5.13}$$

where R is a previously defined matrix that is constant throughout the whole operation, and it represents the 'thrust' we put in our linear model to predict the behavior of the real system. This recalibration happens through the matrix P_k which is calculated by the follow equation:

$$P_k = (I - K_kC_{E_d})P_{pred}(I - K_kC_{E_d})^T + K_kR{K_k}^T \tag{5.14}$$

The last element that needs to be discussed in the Kalman algorithm, is the predicted value X_{pred} which will be calculated at each step, and used to recalibrate the Kalman gain matrix. Together with the predicted state there will also be a prediction of the P matrix in P_{pred}:

$$X_{pred} = A_k\bar{X}(k-1) + B_ku(k-1) \tag{5.15}$$

$$P_{pred} = A_kP_{k-1}{A_k}^T + Q_{k-1} \tag{5.16}$$

since the Kalman filter recalibrates his Gain at each step, a big initial value of the P matrix is given in order to start from the reading and then start filtering out the noise.

$$P = 10I_{12\times12} \tag{5.17}$$

$$Q = I_{12\times12} \tag{5.18}$$

$$R = 3I_{9\times9} \tag{5.19}$$

Note that the size of the P and Q matrix have to match the size of the state vector, while the size of the R matrix has to match the size of the output vector. The values of R are greater than the values of Q in order to increase the trust we put in our model when clearing out noise.

C. Linear Gaussian controller

The linear-quadratic-Gaussian (LQG) control problem is one of the fundamental optimum control issues in control theory. It concerns linear systems driven by additive white Gaussian noise. The objective is to identify an output feedback rule that minimizes the expected value of a quadratic cost criteria. The starting state is likewise regarded as a Gaussian random vector, and Gaussian noise is meant to distort output measurements.

This makes the LQG controller one of the best among the larger class of nonlinear controllers. That is given by the fact that applying a nonlinear control method won't make the anticipated value of the cost functional grow into account. This variation of the separation principle is a special case of the separation principle of stochastic control, which states that even when the process and output noise sources are potentially non-Gaussian, the optimal control separates into an optimal state estimator (which may no longer be a Kalman filter) and a LQR regulator as long as the system dynamics are linear.

Under these assumptions, an optimum control strategy may be created inside the class of linear control rules using the completion-of-squares argument. The linear-quadratic regulator (LQR) and linear-quadratic estimator (LQE) of the Kalman filter are combined to create the unique control rule known as the LQG controller (LQR). The state estimator and state feedback can be constructed independently, according to the separation idea. LQG control is a straightforward linear dynamic feedback control rule that can be computed and applied to both linear time-invariant and linear time-varying systems. The LQG controller is a dynamic system like the system it regulates. Both systems have the same state dimension.

Since a reliable optimal estimator has been developed we can now proceed to the next step, namely the construction of a controller. A linear quadratic controller is a full-state feedback controller designed with the purpose of minimize a quadratic cost function.

This function is known as the quadratic cost function:

$$J = x^T_{(t_1)} F_{(t_1)} x_{(t_1)} + \int_{t_0}^{t_1} (x^T Q_x + u^T R_u + 2x^T N_u)\, dx \tag{5.20}$$

Where x is the state vector of the system, u is the input vector of the system, F is the variable that will be used to minimize the cost, which in different notations is K, and R and Q are, just like in the Kalman filter, cost matrices.

A big disclaimer of this approach is the fact that this control method has been developed based on linear-time invariant system, and will require a level of linearization before it can be implemented onto highly non-linear systems. That is why this controller will be developed around the desired point of operation.

VI. COURSE CORRECTING

The linear quadratic controller will ensure the stability of the system in an inner loop, even in the presence of disturbances, but it will not be able to set a course for the aircraft. In order to do that there are two options:

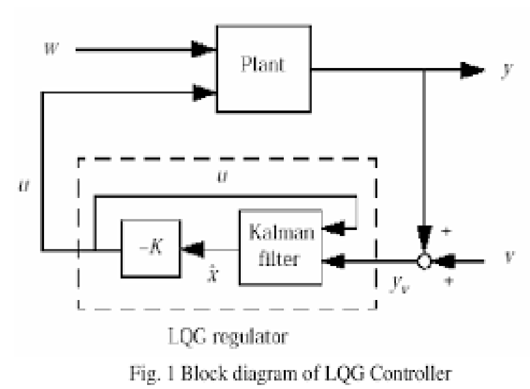

FIG. 2: Block diagram of a Gaussian control strategy

1. Adding a linear matrix, usually noted with N in the literature, that will linearly bind our 3×1 reference vector to our 5×1 input vector.
2. Using a decoupled approach on the stabilized system and adding 2 independent controllers with proportional and integrating effects to ensure no steady-state error.

The latter has been chosen for this study, in order to take complete advantage of the horizontal and vertical decoupling. A feedback-based control loop known as a proportional-integral-derivative controller (PID controller, also known as a three-term controller), is frequently used in industrial control systems and other applications that call for continuously modulated control. An error value $e(t)$ is calculated by a PID controller as the difference between a desired set point (SP) and a measured process variable (PV) on a continuous basis. A correction is then made using terms that are proportional, integral, and derivative (denoted P, I, and D, respectively) in nature.

In reality, PID automatically corrects a control function with an accurate and responsive adjustment. A car's cruise control, for instance, would slow down if constant engine power was used when climbing a slope. By progressively increasing the engine's power output, the PID algorithm in the controller brings the recorded speed back to the desired speed with the least amount of delay and overshoot.

Since we will do this on the stabilized system, that uses a discrete time estimator, discrete time transfer functions will be needed to define the PID controllers. We can check the validity of this approach by looking into the stability of the transfer function, which should reflect the stability of the linearly controlled system.

First in order to define the transfer functions we can start from our state space representation of the extended model with the linear quadratic controller. Thus:

$$\dot{X}(t) = AX(t) + BU(t) \tag{6.1}$$

$$Y(t) = CX(t) + DU(t) \tag{6.2}$$

Applying the Laplace transformation to the linear system yields:

$$sX(s) = AX(s) + BU(s) \tag{6.3}$$
$$Y(s) = CX(s) + DU(s) \tag{6.4}$$

In order to obtain the transfer function, a linear relation between $Y(s)$ and $U(s)$ needs to be defined.

$$sX(s) - AX(s) = BU(s) \tag{6.5}$$
$$(sI - A)X(s) = BU(s) \tag{6.6}$$
$$X(s) = (sI - A)^{-1}BU(s) \tag{6.7}$$

Replacing the new form of X in the output equation would yield a direct relationship between the input and output of the system. Thus:

$$Y(s) = C(sI - A)^{-1}BU(s) + DU(s) \tag{6.8}$$
$$Y(s) = (C(sI - A)^{-1}B + D)U(s) \tag{6.9}$$
$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{6.10}$$

Since we are interested only in the position of the aircraft, and we have the Global Positioning System available to give us the current position, we can assume the only output of the system will be the position. Thus, The C matrix will be a 3×12 matrix. By using the 6.10 formula we can define the transfer function matrix $H(s)$ which is a 3-by-5 matrix containing the transfer function between the output i and the input j in the element $H_{(i,j)}(s)$. The standard form of our transfer function will be:

$$H(s) = \begin{bmatrix} H_{1,1}(s) & H_{1,2}(s) & H_{1,3}(s) & H_{1,4}(s) & H_{1,5}(s) \\ H_{1,1}(s) & H_{2,2}(s) & H_{2,3}(s) & H_{2,4}(s) & H_{2,5}(s) \\ H_{1,1}(s) & H_{3,2}(s) & H_{3,3}(s) & H_{3,4}(s) & H_{3,5}(s) \end{bmatrix} \tag{6.11}$$

Now in order to calculate the relative gain array(RGA), the steady state gain matrix has to be calculated. For that we only need to evaluate our system transfer function at steady state (s=0). Since our estimator and actuators are discrete we will first use the Euler discretization technique to rewrite our transfer function matrix in the discrete time space, and evaluate it at (z=1). So first our system will become:

$$H(z) = \begin{bmatrix} H_{1,1}(z) & H_{1,2}(z) & H_{1,3}(z) & H_{1,4}(z) & H_{1,5}(z) \\ H_{1,1}(z) & H_{2,2}(z) & H_{2,3}(z) & H_{2,4}(z) & H_{2,5}(z) \\ H_{1,1}(z) & H_{3,2}(z) & H_{3,3}(z) & H_{3,4}(z) & H_{3,5}(z) \end{bmatrix} \tag{6.12}$$

By evaluating the 6.12 at the point z=1 we will gain the relative gain matrix (RGA) as:

$$RGA = \begin{bmatrix} 0.8245 & 0.1508 & -0.0007 & 0.0343 & -0.0089 \\ -0.0004 & 0.0005 & 0.5646 & 0.2083 & 0.2269 \\ 0.0124 & 0.8370 & 0.0227 & 0.1101 & 0.0178 \end{bmatrix} \tag{6.13}$$

The relative gain matrix is a matrix of influence of the inputs to the outputs. In order to be able to safely decouple our system we are looking for the dominant members on each row. Because the terms $(1, 1)$,$(2, 3)$ and $(3, 2)$ are the dominant members on they're respective row, and since they are at least 5 times bigger than their counterparts, we can safely make the assumption that our system can be decoupled:

$$H(z) = \begin{bmatrix} H_{1,1}(z) & 0 & 0 & 0 & 0 \\ 0 & 0 & H_{2,3}(z) & 0 & 0 \\ 0 & H_{3,2}(z) & 0 & 0 & 0 \end{bmatrix} \tag{6.14}$$

Which also respects the intuitive answer that the forward velocity of the aircraft is directly controller by the thrust input, the lateral velocity of the aircraft is mainly influenced by the rudder and that the vertical velocity of the aircraft is mainly controller by the elevator. Since our aircraft is always expected to move forward we can even ignore the first row of our matrix and only develop 2 controllers for our lateral and vertical movements.

VII. CONCLUSION

After decoupling our system we can finally apply our complete control a structure on top of the non-linear model in Simulink to see the results. After making the connections from our control structure to our model we can simulate our system. It will be simulated on a longer time-frame with multiple disturbances all in the form of step inputs, sprinkled along the way.

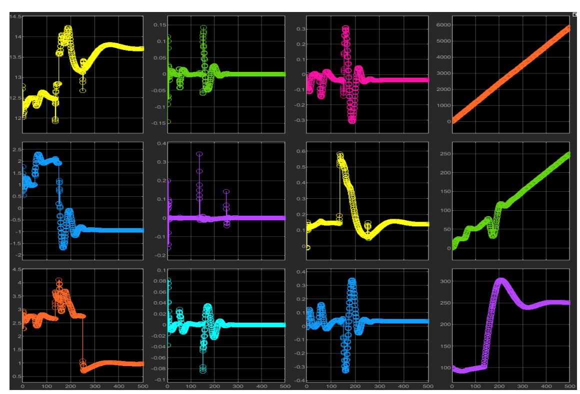

FIG. 3: System states on simulation

In the figure 3 all the states of the system as defined in previous sections except for the wind vector. In the figure 4 the reference trajectory is displayed in a 3 dimensional space together with the real trajectory followed by the aircraft. Both examples follow a trajectory mainly emphasizing lateral and vertical movement relative to the starting orientation. This has been chosen because of the limitations of our implementation.

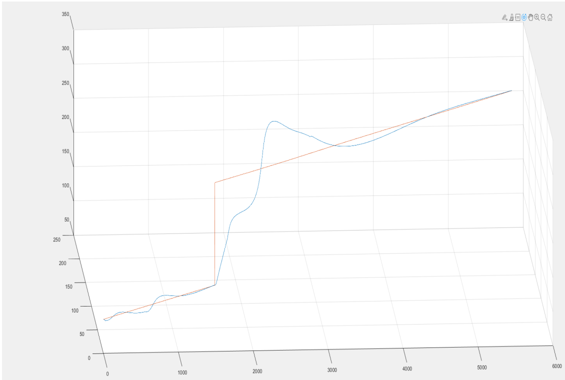

FIG. 4: Cruise flight mission simulation in 3 dimensional space

Achievements

In this paper the groundwork for the development of an aircraft controller has been developed using an optimal control approach and two proportional integrating controllers. This approach has been developed with the scope of a small-scale aircraft with limited computational resources which is expected to have an automated guidance system onboard. It takes into account the size limitations and proposes an efficient algorithm sufficient for basic cruise control.

Further development

In the future this approach could be extended to allow for fully autonomous aircraft with the ability to perform complex areal maneuvers and be able to land and take off safely. For this the before mentioned gain-scheduled architecture could be used, together with a performant mission planner and more efficient sensors for better data acquisition.

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