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# A Survey of Fractional Order Calculus Applications of Multiple-Input, Multiple-Output (MIMO) Process Control

**Alexandre Marques de Almeida [1], Marcelo Kaminski Lenzi [1,*] and Ervin Kaminski Lenzi [2]**

[1] Fractional Systems Engineering Lab, Department of Chemical Engineering, Federal University of Paraná, Curitiba 81531-980, Brazil; alex.almeid@gmail.com
[2] Department of Physics, State University of Ponta Grossa, Ponta Grossa 84030-900, Brazil; eklenzi@uepg.br
* Correspondence: lenzi@ufpr.br; Tel.: +55-41-3361-3577

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**Abstract:** Multiple-input multiple-output (MIMO) systems are usually present in process systems engineering. Due to the interaction among the variables and loops in the MIMO system, designing efficient control systems for both servo and regulatory scenarios remains a challenging task. The literature reports the use of several techniques mainly based on classical approaches, such as the proportional-integral-derivative (PID) controller, for single-input single-output (SISO) systems control. Furthermore, control system design approaches based on derivatives and integrals of non-integer order, also known as fractional control or fractional order (FO) control, are frequently used for SISO systems control. A natural consequence, already reported in the literature, is the application of these techniques to MIMO systems to address some inherent issues. Therefore, this work discusses the state-of-the-art of fractional control applied to MIMO systems. It outlines different types of applications, fractional controllers, controller tuning rules, experimental validation, software, and appropriate loop decoupling techniques, leading to literature gaps and research opportunities. The span of publications explored in this survey ranged from the years 1997 to 2019.

**Keywords:** fractional order calculus; multivariable control; survey

## 1. Introduction

The literature reports the application of fractional calculus, which consists of non-integer order derivatives and integrals, to a broad range of research fields [1,2]. Different approaches exist towards the appropriate derivative operator definition, for example, Riemann–Liouville, Grünwald–Letnikov, Caputo, Atangana–Baleanu [3]. As the operator usually contains a definite integral, memory effects are inherently present in the formulation [4].

A successful controller design demands a representative mathematical model of the system that will be under control [5]. The typical techniques employed for suitable mathematical modeling of experimental systems frequently use integer-order derivatives and consider either empirical equations or mass, energy, and momentum conservation equations [6]. However, these models may not be suitable to describe some systems [7]. Hence, due to memory effects, the use of fractional calculus theory frequently leads to mathematical models with a better experimental behavior description capability, allowing in a more reliable controller design [8].

The majority of applications of fractional order calculus in control system design concerns SISO systems, as shown by pioneering references [8–13] and also by recent works [14–18], among others. Manage [19] presented the first study, considering the issues on the stability of a SISO control loop with a non-integer parameter in the controller design equation.

Among the pioneering works in process control with the application of fractional calculus, Axtell and Bise [9] studied the control loop responses in the domain of complex numbers (Laplace domain) with derivatives of order $n = 0.5$. The authors demonstrated their results in a feedback control loop with reference $R(s)$ and output $C(s)$ through simulation studies, but did not compare with the integer order control. The authors only explained the potential of the application of fractional calculation in a simple SISO control loop, with different values of the fractional parameter.

Oustaloup [10], and Oustaloup and Melchior [10,11] developed a milestone technique aimed at using fractional calculus to develop a novel control strategy called CRONE (French abbreviation for robust control of non-integer order—commande robuste d'ordre non-entier). In their results, the authors presented the applications of the first, second, and third-generation CRONE control through the analysis of open-loop responses in the frequency domain. In the same decade, Podlubny [8,13] reported the pioneering concept of a $PI^{\lambda}D^{\mu}$ controller. The controller contained both non-integer integral and derivative parts in the classical PID controller. The order of process transfer function was also considered in the analysis of the SISO control loop performance, resulting in a better performance when both the process and the controller were of fractional type. Nevertheless, the study did not consider the model uncertainties and the measurement noise evaluation.

The application of fractional calculus to control MIMO systems is recently drawing attention of the scientific community, due to the MIMO system complexity and the loop interactions. Lanusse et al. [20] reported the first study of fractional control application to MIMO systems. It described the robust multi-scalar control of MIMO plants with uncertainties using the CRONE methodology. Furthermore, Lanusse et al. [21] studied multi-SISO and MIMO loop configurations using the CRONE methodology and linear time-invariant (LTI) systems with uncertainties. A complex non-integral parametrization of each element of a diagonal open-loop transfer matrix function was the basis of the design. In both papers, the authors implemented simulation studies in the time and frequency domain, without experimental validation. Since then, a large number of publications started for MIMO systems. For example, Liang et al. [22] reported an enhancement of the stability of multivariable systems in discrete-time of fractional order and experimental validation in a two-input two-output (TITO) electrical circuit. Silva et al. [23] implemented a fractional control to a MIMO robot with mobile joints to control their movements, with Nyquist stability studies. Gruel et al. [24] and Pommier-Budinger et al. [25] discussed the application of the CRONE methodology to robust control of lightly damped multivariable systems.

After reviewing and discussing the main contributions and results of both pioneering and recent publications, this survey work addresses the analysis of the state-of-the-art fractional order control applications to identification, modeling, simulation, and control of MIMO systems. This paper has the following organization. Section 2, addresses FO control concepts and applications for controlling MIMO systems, highlighting the types of systems studied and the structures of the used fractional controllers. Section 3 highlights publications that investigated simulated systems or presented a real-time validation by experimental modules on a pilot or laboratory scale, while Section 4 lists different schemes of the fractional-order controller applied to MIMO systems. Section 5 analyses different types of tuning methods and techniques for FO control used in MIMO systems, while Section 6 lists loop decoupling techniques and Section 7 presents the types of software used in the simulation studies. Finally, Section 8 shows a brief evolution of publications from 1997 to 2019, and the gaps still little explored in the literature, along with some conclusions. It is important to stress that the fundamental aspects of the fractional calculus theory are beyond the scope of this work and reported elsewhere [3,26,27].

## 2. FOC Applications in MIMO Control

The main feature of a MIMO system is the interaction between the controlled and manipulated variables. Unstable behavior may occur as a consequence of different SISO loops' interaction. This situation can be avoided with the aid of loop decouplers or with the application of advanced control strategies. This last procedure may increase the complexity of the control procedure or lead to prohibitive computer calculations for real-time applications. The use of fractional-order controllers may simplify the difficulties of MIMO systems control. As of the 2000s, there has been an increasing number of publications in the area of fractional order control dealing with the simulation of SISO systems control and a few studies with experimental validation. These primary publications reported a potential of the fractional control approach. On the other hand, they showed mainly academic studies.

As previously mentioned, the CRONE methodology is a milestone contribution to fractional control, with many publications of impact in diverse applications as in references [10–12,28–30]. In this context, the study of Oustaloup and Nouillant [28] reported the application of the first generation CRONE control to a MIMO system for robust control of a two degree of freedom (DOF) manipulator robot, thus obtaining promising results. When it comes to applying fractional order control to MIMO systems, Lanusse et al. [21] presented a pioneering study of the CRONE methodology for a multi-SISO control design in LTI MIMO systems, incorporating increased robustness of MIMO plant stability margins. The proposed controller was defined by a non-integer order transfer matrix, as follows

$$C\left(s\right) = G_0(s)^{-1}\beta_0\left(s\right) \tag{1}$$

where $G_0(s)^{-1}$ is the inverse matrix of the transfer function of the MIMO plant; $\beta_0\left(s\right)$ is the transfer function based on complex non-integer integration with limited frequency.

Gruel et al. [31] reported an extension of the idea concerning the robust control of unstable MIMO plants with dead time through the third generation of CRONE methodology. Simulation studies applied to a TITO distillation column model extracted from the literature, showed that the CRONE control approach successfully achieved robust closed-loop stability, with decoupling and robust disturbance rejection.

### *2.1. Distillation Column*

This part of the work highlights the application of fractional control to distillation columns, unit operation equipments widely present in chemical process industries, commonly employed in the separation of liquid-liquid mixtures based on the relative volatility of the mixture components. Regarding theoretical studies, the mathematical TITO model reported by Wood and Berry [32], consisting of a matrix of first-order plus time delay (FOPTD) transfer functions, has been widely used as a benchmark for the dynamic behavior of the distillation column. Many authors, see references [33–38], applied fractional control techniques to the Wood and Berry distillation model, showing that in all cases, the use of fractional control improved the performances when compared to integer-order (IO) control. On the other hand, in some studies, this difference in performance was not so evident, as observed in the work of Sivananaithaperumal and Baskar [35], which obtained a reduction of the integral of the absolute error (IAE) index from 10.4378 with IO-proportional integral (PI) controller to 10.2069 with controller FO-PI. Gruel et al. [31] reported the third generation CRONE control methodology for a TITO distillation column considering the dynamic TITO model identified by Wang et al. [39]. Furthermore, other works addressed distillation columns according to models previously reported in the literature, such as in references [35,40], which used the model proposed by Ogunnaike and Ray [41] of a multivariable distillation column with a size of 3 × 3 and FOPTD model.

Finally, in references [42–44], the dynamic behavior of the multivariable cryogenic separation columns of the $^{13}C$ isotope was described by a 3 × 3 matrix transfer function with the FOPTD model. The authors implemented a FO-PI controller and compared its performance with the IO-PI. The transfer function of the controller considered the following form.

$$C_{FO-PI}\left(s\right) = K_P\left(1 + \frac{K_I}{s^{\mu}}\right) \tag{2}$$

where $K_P$, $K_I$ and $\mu$ are the design parameter which have to be determined.

Dulf and Kovacs [45] reported the control of the $^{13}C$ cryogenic separation column cascade system, where the authors developed a new fractional control scheme based on a fractional observer. Although, differently from references [42–44], Dulf and Kovacs [45], considered a matrix transfer function model of dimension 9 × 9, obtained by a linearized model around the equilibrium point.

### 2.2. Coupled Tanks

Coupled tank systems are of great interest in studies of FO-MIMO control due to the high interaction characteristics among the controlled and manipulated variables and also due to the intrinsic non-linearity features. TITO coupled tanks were reported in references [46–64]. Some works also provided successful validation of the fractional control in experimental modules, as seen in [48,50,51,53,55]. Other works [46,47,59,63] reported theoretical results of fractional control based on dynamic models available in the literature.

The references [46,47] reported the first works on coupled tank control, proposing a fractional hybrid control scheme plus sliding mode control (SMC). The authors applied fractional control schemes such as $PD^{\alpha}$ and $PI^{\lambda}D^{\mu}$ with SMC control, as well as fuzzy control. Their results demonstrated the robustness of the proposed control compared to the classical control techniques.

References [48,49] addressed the gain scheduling methodology applied to the design of FO-PID controllers for level control in a TITO system of coupled multi-tanks. Additionally, the authors implemented an extended Kalman filter to reduce the measurement noise propagation to the control law, improving pump performance. The main contribution of these studies was the proposal of a simple method of designing multivariable fractional controllers with only a static description of the FO-PID controllers. Therefore it can be used in automatic tuning for efficient control of non-linear systems over a wide operating range.

Several authors, such as [48,50,54–56,60–62,64], investigated the FO-PI controllers. The controller structure led to some divergences among the researchers. From these, only [48,50,60–62,64] worked with pure FO-PI. Roy and Roy [55,56] addressed the use of advanced fractional controllers based on PI control. Roy and Roy [55] investigated the proposed FO-PI control with feedforward and compared to PI, PID, 2DOF-PI, 3DOF-PI all with feedforward. This work showed the level control simulation and the experimental validation of the performance and the robustness of the proposed controller compared to conventional controls with feedforward. Roy and Roy [56] reported the dual mode adaptive fractional order PI controller (DMAFOPI) with feedforward with a comparison to several other structures found in the literature. The controllers were designed based on a new Variable Parameter Transfer Function model, showing the performance of the proposal by simulation and experimental studies. A comparison with previous similar works was presented and demonstrated the advantages of the novel proposed approach.

In the case of robust and advanced control, Muresan et al. [53] investigated a fractional-order internal model control (FO-IMC) type controller with Smith predictor applied to a TITO coupled tank system. The simulation results showed that the proposed FO-IMC controller ensured greater robustness to the modeling uncertainties. Experimental results validated the design of a multivariable FO-IMC controller with a Smith predictor for a quadruple tank system.

The work by Gurumurthy and Das [62], $FO-[PI]^{\lambda}$ investigated, and compared $FO-PI^{\lambda}$ controllers to PI and PID of integer order. The implementation of the proposed controller in a TITO system of coupled tanks of level control, proved the performance of the $FO-[PI]^{\lambda}$ controller as the best control strategy for the system. The authors also compared their results to the proposal by Roy and Roy [55] and showed a reduction of the IAE Index by 13.1481% for tank 1 and 22.269% for tank 2, from real-time experiments in coupled tank system (CTS). Lakshmanaprabu et al. Lakshmanaprabu et al. [63] compared the performance of the genetic algorithm (GA), the cuckoo search algorithm (CS) and the bat algorithm (BA) for parameters tuning of the multi-loop FO-PID controller to the IMC-PID. Yousfi et al. [64] implemented the design of independent PI and fractional PID controllers based on a bat colony algorithm (bat optimization) and validated the proposal by simulation and in real-time experiments. Both previous papers [63,64] applied the control proposal to a TITO experimental module of two interconnected conical tanks.

### *2.3. Robotics and Automotive*

Many authors, highlighting the references [23,28,65–71], reported applications in robotics. Silva et al. [23] applied fractional and integer-order control techniques in a flexible joint hexapod robot with multivariable characteristics to control its movements. The simulated results showed the robustness of the $PD^{\alpha}$ control when compared to the classic full-order PD approach with values of $0.8 \leq \alpha \leq 0.9$.

Other researchers adopted the selective compliance robot assembly (SCARA) robot model TITO for FO-MIMO control experiments [65,67–70]. The CRONE control methodology was applied together with the quantitative feedback theory (QFT) procedure by [65,67,69], achieving similar results. In these papers, the main contribution was the use of fractional pre-filters and the QFT approach in the design of the controller. The Davidson–Cole fractional pre-filter optimization determined the diagonal terms of the pre-filter.

Contrary to the previously mentioned studies, Rojas-Moreno [66] studied the control in experimental robotic devices, considering the MIMO and non-linear systems, which were given by a manipulator arm with two degrees of freedom, and a car with a translational manipulator with two degrees of freedom. The main objective was the experimental validation of the proposed FO-MIMO technique to control the position of the apparatus. The authors not only did not specify the control law, but also used a trial and error method to optimize the controller tuning parameters.

Damodaran et al. [71] presented a new two-stage method for designing a fractional order controller for linear SISO and MIMO speed control in a robot two-wheeled mobile. The main advantage of the proposed method was the simple selection of the reference model, using a linear quadratic regulator with integral action (LQRI). The ability to reject load disturbances and the robustness to variations in system parameters offered by the controller was evident in simulation results. Several authors, such as [72–75], reported promising results regarding engine control applications. The first references [72–74], investigated the design of a robust multivariable control system for controlling NOx emissions from the airpath of a diesel engine. On the other hand, the study by Lamara et al. [72] showed the implementation of the CRONE decentralized control methodology with a system dimension of $3 \times 2$ and an analysis of responses in the frequency domain. The results of system identification experiments showed a reduction of NOx emissions.

### *2.4. Miscellaneous and Generic Applications*

A substantial number of 55 publications published after the pioneering work of Lanusse et al. [20] regarding FO-MIMO control shows the research field is still growing. Lanusse et al. [20,21] implemented the CRONE methodology to LTI MIMO systems reporting only theoretical results. Liang et al. [22] carried out a study of stability properties of the zeros of discrete-time multivariable systems based on experiments in TITO electronic circuit with fractional adaptive control, demonstrating the performance of the proposal by improving the system stability. Many publications addressed theoretical models identified from the controlled system, mostly FOPDT SCARA models with delay time and no delay time as can be found in [31,49,71,76–85].

Liang et al. [22] investigated real systems for modeling and control with or without experimental validation, where a new adaptive fractional controller called an approximate fractional-order hold (AFROH) was developed with experimental validation in an electronic circuit. Tar and Bencsik [86] simulated a differential hydraulic cylinders system with a fractional adaptive controller for pressure control in the cylinder. Victor et al. [87,88] studied the temperature control problem at specific points on a long metal bar using third-generation CRONE control and comparison with integer-order PID. Pisano et al. [89], studied the same system, but with linear fractional modeling and sliding mode control (SMC). The third generation CRONE control methodology was also investigated by

Pommier-Budinger et al. [25] using the experimental apparatus of an aircraft wing model with a tank whose levels of filling may vary.

Furthermore, Jiacai et al. [90] studied a robust fractional-order sliding mode controller (FOSMC) for speed control of a permanent magnet synchronous motor (PMSM) non-linear MIMO system. Simulation studies evaluated the algorithm proposed by the authors, proving its performance by minimizing the integral of the squared error (ISE). Bucanovic et al. [91] developed an optimal FO-PID controller for the cryogenic process of air through simulation studies and application of multiobjective optimization using a genetic algorithm (GA). The results proved the robustness of the proposed controller considering the rejection of disturbances and improvement of the transient response compared to the IO-PID. Luo and Liu [92] investigated non-linear MIMO systems through the application of adaptive FO-FUZZY and IO-FUZZY controls. Moradi [93] implemented a new FO-PID controller based on genetic algorithm (GA) optimization for an ambient temperature control system applied to the pilot heating, ventilation, and air conditioning (HVAC) system with dimension $2 \times 2$, according to Chenikher et al. [78]. Das [94] presented the applicability of fractional calculus in LTI MIMO systems, where the state transition matrices, of the Gramian control, state trajectories, input control vector, and control energy for the fractional-order system was determined.

Finally, the references [95–105], investigated the FO-MIMO approach in actual experimental systems and prototypes. In this context, Vinagre et al. [95] explored the loop transfer recovery (LTR) concept with state estimation with a fractional Kalman filter to control two examples with state-space models: aircraft roll-dynamics and velocity-dynamics servomotor, with dimension $3 \times 3$. Aguila-Camacho et al. [96] designed fractional controllers of SISO in an ore milling plant, being a MIMO process. The authors used FO-PI controllers and a combination of adaptive controllers with a fractional-order reference model (FOMRAC). Their results indicated that fractional-order SISO controllers achieve similar or better results compared to the linear model predictive controller (LMPC) in the presence of parametric disturbances and process noise. Nasirpour and Balochian [97] reported a method was for optimum tuning of the FO-PID controller for the air conditioning variable air volume (VAV) system, using the particle swarm optimization algorithm (PSO). Feliu-Batlle et al. [98] proposed a robust FO-PI controller to control a reverse osmosis desalination plant with experimentally identified MIMO dynamics, demonstrated that the performance of the FO-PI controller surpasses the classic PI. Another important work was developed by Aguiar et al. [99] who applied an FO-PID controller to control the pH of a laboratory-scale process, proving the robustness of the FO-PID system compared to the conventional full-order PID.

Recent works, such as references [100–105], explored comparative studies of fractional and integer order controllers approaches. Yin et al. [100] proposed a new fractional-order multivariate approach to gradient search control for the optimization of MIMO systems. On the other hand, Kumar et al. [101] and Roy et al. [102] designed cascade FO-SMC and cascade FO-PID controllers, also comparing to their integer-order equivalents. Juchem et al. [103] applied FO control techniques to experimental devices, particularly to a MIMO $8 \times 8$ light system. Mondal and Dey [104] investigated a mass-spring immersed system in a Newtonian fluid, LTI unstable magnetic levitation system, SIMO inverted pendulum car system, and double rotor MIMO system. Finally, Quadros [105] studied FO control in an experimental thermal module composed of box, fan, lamp, and sensors for the TITO problem. Table 1 reports some of the most relevant publications with miscellaneous and generic applications reported.

**Table 1.** Some papers on miscellaneous and generic applications.

| References | Applications | Controller | CV * | MV * | PS * |
|---|---|---|---|---|---|
| [21] | LTI square MIMO uncertain plants | Multi-SISO and MIMO CRONE | Not specified | Not specified | $J = [\bullet] \equiv \min.$ |
| [22] | Electronic circuit $2 \times 2$ | Adaptive control with AFROH * | Voltage (V) output 1 and 2 | Voltage (V) input 1 and 2 | Zeros of the discrete-time |
| [86] | Differential hydraulic cylinders $2 \times 2$ | FO Adaptive control | Pressure ($P_A$ and $P_B$) | Oil volume ($V_A$ and $V_B$) | Not specified |
| [24] | Active-suspension system $2 \times 2$ | CRONE 3*th* generation | $Y_1(s)$ $Y_2(s)$ | $U_1(s)$ $U_2(s)$ | $J = [\bullet] \equiv \min.$ |
| [25] | Aircraft wing model $2 \times 2$ | CRONE 3*th* generation | Wing vibrations | Vibration attenuators | Frequency domain and the Bode diagrams |
| [89] | Long aluminum rod FO model $3 \times 3$ | SMC * | Temperature | Thermal flux | - |
| [87,88] | Long aluminum rod $2 \times 2$ | CRONE 3*th* generation and PID | $T_1$ and $T_2$ two extreme points | Heat flow density | Nichols diagram and output error |
| [77] | FO Duffing–Holmes chaotic systems $2 \times 2$ | AITFSMC * | trajectories states $x$ and $y$ | Trajectory of the control effort $u(t)$ | Mean square errors |
| [106–108] | FO-MIMO systems $2 \times 2$ and $3 \times 2$ | CRONE and QFT * | Amplitude responses $y_1$ and $y_2$ | $u_1$ and $u_2$ | $J = [\bullet] \equiv \min.$ |
| [109,110] | FO-LTI MIMO systems and Hypersonic vehicle $m \times n$ and 6-DOF model | $PI^{\lambda}D^{\mu}$ nonlinear | State responses $x(t)$ and attack angles | Input $u(t)$ and input parameters | ITAE |
| [79,95,111] | NSS */ARSV */ FOPDT * $3 \times 3$ and $3 \times 2$ and $3 \times 3$ | FSMC */ LTR-FOKF */ FO-IMCSP * | State space $x_i$ and output $y_i$ | Input $u_i$ | ITAE and LQR * cost function |
| [96] | Single stage ore milling plant 5×3 | FOPI FOMRAC * | Particle size, ore load and mud volume | CFF *, OF *, CFW *, MFW *, MBF * | NRMSE *, NRMSI * |
| [81,82] | FO-MIMO with dead time and without dead time $3 \times 3$ and $2 \times 2$ | FFO-DDMC | Output $y_i$ | Input $u_i$ | Developed by authors and Nyquist and Bode diagram |
| [83–85] | FO-MIMO with dead time and without dead time $2 \times 2$ and $3 \times 2$ and $3 \times 3$ | PID, FO-IMC, FO-PID | Output $y_i$ | Input $u_i$ | Bode diagram, ITAE, settling time, overshoot, maximum sensitivity |

* CV: controlled variable; MV: manipulated variable; PS: performance index and/or stability analysis; AFROH: an approximate fractional order hold; SMC: sliding mode control; AITFSMC: Adaptive interval type-2 Fuzzy sliding mode control; QFT: quantitative feedback theory; NSS: nonlinear state space model; ARSV: aircraft roll-dynamics and servomotor velocity-dynamics; FOPDT: First order plus dead time model; FSMC: Fractional sliding-mode controllers; LTR-FOKF: Loop transfer recovery control with fractional Kalman filter; FO-IMCSP: IMC with Fractional order Smith predictor controller; LQR: linear quadratic regulator; FOMRAC: fractional order model reference adaptive controllers; NRMSE: normalized rootmean square error; NRMSI: normalized root meansquare input; CFF: Cyclone feed flow; OF: ore feed; CFW: collector feed water; MFW: mill feed water; MBF: mill ball feed water. [•] Cost function defined by authors.

## 3. Simulation and/or Experimental Validation

An experimental implementation is an important tool to validate the proposed approach. In this section, numerical simulations in the time domain and/or in the frequency domain and also experimental validations implemented by the authors in bench systems [100,102] or in experimental modules [60,62,105] contextualizes the FO-MIMO control applications.

### 3.1. Theoretical/Simulation Works

The literature reports several applications with only simulation studies of MIMO systems. The recent works by [36,38,40,68–70,101,112–114] are examples of theoretical studies on fractional control of MIMO systems with only simulation results considering the use of previously identified models. Table 2 shows the state-of-the-art of applications involving only simulated results of FO-MIMO control.

**Table 2.** Papers on simulation works.

| References | Applications | Controller | CV * | MV * | PS * |
|---|---|---|---|---|---|
| [112] | Refrigeration system TITO | CRONE decentralized, PID | $T_{out}$ * and $T_{do}$ * | $CE$ * and $EV$ * | Nichols analysis and objective function |
| [33,34,38] | Binary distillation column by [32] | Multi-loop FO-PID, FO-PI, IO-PI | $x_D$ and $x_B$ | $R$ and $S$ | ISE, IAE, ITSE, ITAE, overshoot, cost function defined by authors |
| [113] | Heavy oil fractionator by [115], distillation column by [32], flash distillation column by [116] | IMC FO-PI/PID and Smith predictor | $y_1$ and $y_2$ | $u_1$ and $u_2$ | IAE, ITAE and TV * |
| [36,117] | FO-TITO systems: binary distillation column by [32] and thermal system by [117] | FO-PI and IO-PI multiloop | $x_D$ and $x_B$, $Y_1$ and $Y_2$ | $R$ and $S$, $U_1$ and $U_2$ | ISE, ITAE, settling time, overshoot |
| [65,68–70] | SCARA * robot model TITO | FO-PD and $QFT$ *, CRONE and $QFT$, $PI^{\alpha}D$-$QFT$ | Amplitude | $u_1$ and $u_2$ | Rise time, settling time, cost function defined by authors |
| [31,35,40,114] | Distillation column by [32,39,41,118] | IMC-PID, FO-PI, optimal PI, FO-PID, PID, PI, CRONE 3th generation | Distillate and bottom composition | Reflux flow and reboiler steam flow | Overshoot, settling time, IAE, analysis of responses in frequency domain |
| [45] | Isotopic separation column cascade 9 × 9 | FO observer and FO-PI | Top and bottom pressure, liquid CO level | Waste flow, feed flow, electrical power supplied to the boiler | Gain crossover frequency, phase margin, isodamping property |
| [43,44] | $^{13}C$ isotope separation plant 3 × 3 | FO-PI with Smith's predictor, PI | $y_1$, $y_2$, $y_3$ | $u_1$, $u_2$, $u_3$ | Overshoot, settling time |
| [90,101] | Synchronous generator excitation system, permanent magnet synchronous motor (PMSM) | Cascaded IO-PID and FO-PID, FOSMC * | Excitation voltage, rotor speed | Synchronous voltage obtained by synchronous transformer, Current, torque | Routh's criterion, ISE, minimization of robustness stability scale function, control effort |
| [119] | Wind turbine with space state model 2 × 5 | MPC-LPV * and LFT * | RS *, AS *, TF *, GT *, BP * | GT reference, BP reference | Wind speed variation, power efficiency coefficien |
| [97] | Air-conditioning VAV * system 2 × 2 | PSO *-$PI^{\lambda}D^{\mu}$, PSO-PID, GA *-PID | Temperature supply air, temperature thermal space | Flow of cold water, flow air supply | Rise time, overshoot, settling time, ITSE |
| [110,120] | Inverted pendulum system, hypersonic vehicle 6-DOF model | FO-PID, IO-PID | Pendulum angle (rad), pendulum velocity (rad/s), state responses $x(t)$ and attack angles | Input $u(t)$, input parameters | Control error, ITAE |
| [121] | Twin rotor TITO system | FO-PID, IO-PID | Pitch (elevation) angle, yaw (azimuth) angle | Input voltage main rotor, input voltage tail rotor | Control effort, ISE |
| [91] | Expansion turbine in cryogenic air separation TITO process | FO-PID, IO-PID | Inlet temperature and air flow in the expansion turbine | Air outlet and inlet flow in the separator | Overshoot, settling time, IAE |
| [87–89] | Long aluminum rod 2 × 2, 3 × 3 | CRONE 3$th$ generation, PID, SMC * | Temperature $T_i$ | Heat flow density, thermal flux | Nichols diagram and output error |
| [46,47] | Coupled tanks 2 × 2, 2-DOF polar robot manipulator | PD-SMC, FO-PD, FO-PID, SMC, FO-PD, Fuzzy-SMC | Level $h_1$, $h_2$, tracking response of joint 1 and 2 | The inflow rate into Tank 1, control signal $u_1$ and $u_2$ | ISE, cost function defined by authors |

**Table 2.** *Cont.*

| References | Applications | Controller | CV * | MV * | PS * |
|---|---|---|---|---|---|
| [24] | Widely studied TITO active-suspension system | CRONE 3*th* generation | Output $Y_1$ and $Y_2$ | Intput $U_1$ and $U_2$ | Minimizes the robustness cost function |
| [86] | Differential hydraulic cylinders | FO adaptive control | Piston trajectory | Variation of the pressure | Not specified |

* CV: controlled variable; MV: manipulated variable; PS: performance index and/or stability analysis; $T_{out}$: Output temperature of the secondary evaporator flow, which represents the cooling demand (°C); $T_{do}$: the degree of overheating (°C); Output temperature of the secondary evaporator flow, which represents the cooling demand (°C) and the degree of overheating (°C); CE: Compressor speed (Hz); EV: expansion valve opening (%); TV: total variation of the manipulated variable; SCARA: Selective compliance robot assembly; QFT: quantitative feedback theory; RS: Rotor speed; AS: angular speed of the generator, TF: twist of the flexible drive train; GT: generator torque; BP: blade pitch angle; FOSMC: fractional order sliding mode controller; LPV: linear parameter-varying; LFT: linear fractional transformation; VAV: variable air volume; PSO: particle swarm optimization; GA: genetic algorithm; SMC: sliding mode control.

Table 3 reports publications that explored generic systems defined by the authors or other works, without specifying a physical system or process. These works had, in general, as main objective the demonstration of applications of fractional calculus techniques to MIMO systems. Furthermore, the studies concerned the analysis of performance, robustness, and rejection of disturbances of proposed methods compared to conventional controls of integer order with simulation tests and numerical analysis [81,122].

**Table 3.** Papers on simulation works in general systems.

| References | Applications | Controller | PS * |
|---|---|---|---|
| [80,82,85] | FOPDT TITO model | PID FO-EOTF *, optimal FO-PID | Nyquist and Bode diagram |
| [83,84] | FO-TITO system by [117,123] | FO-IMC and FO-PID | Maximum sensitivity, overshoot, settling time |
| [109,124,125] | LTI MIMO plant $3 \times 2$ and $2 \times 2$ | FO-MIMO, FO-PID and SOF * | $J = [\bullet] \equiv \min.$ |
| [79,81] | FO-MIMO FOPDT model $3 \times 3$ and $3 \times 2$ | FO-DDMC, DDMC *, IMC and FO-Smith predictor | $SD$ *, $RMSE$ *, ITAE, undamped oscillation frequency, overshoot, the peak time |
| [122,126,127] | TITO models of heterogeneous systems, Generic FO-TITO system | FO-PI, FO-IMC, MRAC *, CMRAC * | ISE, IAE, tracking error, control cost |
| [111] | State space model single input $1 \times 3$ and multiple inputs $3 \times 3$ | Sliding-mode-based fractional control and PI based | Not specified |
| [106–108] | MIMO uncertain system $3 \times 2$, $2 \times 2$ | CRONE 3*th* generation and QFT, Davidson–Cole prefilter MIMO QFT and CRONE | SSE *, overshoot, integral gap optimization |
| [92] | Non-linear MIMO systems: generic system $2 \times 2$ and a chaotic 3D saturated multiscroll system $3 \times 3$ | Adaptive FO-FUZZY and IO-FUZZY | Tracking errors, time response of Fuzzy system parameters |
| [93] | HVAC pilot system TITO model by [78] | $H_\infty$-FOPID, GA-FOPID | IAE, ISE, ITAE |
| [78] | TITO plant with time delay proposed by [128] | FO-PID, IO-PID | Stability robustness, tracking with disturbance and noise minimization |

* PS: performance index and/or stability analysis; PID FO-EOTF: PID with open-loop fractional transfer function method; [•] Performance index developed by the authors based on minimizing the vectors errors of the controlled and manipulated variables and through the analysis of perfect control energy and state vector energy; DDMC: distributed PID-type dynamic matrix control; SD: standard deviation; RMSE: root mean square; MRAC: model reference adaptive control; CMRAC: composite model reference adaptive control; SSE: minimize the sum of square error; SOF: static output feedback.

### *3.2. Theoretical/Simulation Works with Experimental Validation*

When dealing with innovative control system proposals, real-time validation is essential, as the simulated proposal is not always physically practicable due to a number of factors, such as the inadequate design of the decoupler in multi-loop systems, model uncertainties and mistakes, among others. Therefore, many investigations sought to highlight the advantages of the fractional MIMO control methodology through experimental modules and test benches of different types. As in references [51–57,60–64] reported the application of fractional order controller to an experimental multivariable coupled tank system. Other references like [72–75] developed experimental studies on diesel engines. On the other hand, references [23,66,71] reported applications to bench robots.

Investigations involving experimental studies in bench systems or experimental modules are still not easily found. Table 4 show the state-of-the-art of applications involving simulated results and with experimental validation for FO-MIMO control.

**Table 4.** Papers on experimental validation.

| References | Applications | Controller | CV * | MV * | PS * |
|---|---|---|---|---|---|
| [105] | EMBFL * | FO-PI, IO-PI | Temperature and luminescence | Air flow through the fan voltage, energy flow through the heating lamp voltage | IAE, ISE, ITAE, OS * |
| [104] | IPN *, LML *, CIP *, TRM * | 2-DOF FO-PID, IO-PID | Amplitude, pitch angle, yaw angle | Not specified | OS *, ST *, FLTF *, pole positions |
| [57,60,63,64] | TITO system of two coupled conical tanks, flash distillation column proposed by [116] | Multiloop FO-PID, IMC-PID, IO-PI, FO-PI, FO-PID, CFO-PID * and FO-PI, C/D-PI * | Level $h_1$, $h_2$ and $y_1$, $y_2$ | Pump rotation speed (flow into the tanks) | IAE, ISE, ITAE |
| [103] | Experimental test bench for an office lighting system $8 \times 8$ | FO-PI, IO-PI | Light intensity of the lamps | Lamps voltage | IAE, power consumption |
| [71] | FO/IO MIMO plants with time delays, wheeled mobile robot | FO-PID, IO-PID | Amplitude, velocity profiles right wheel and left wheel | Defined only as input | $J = [\bullet] \equiv \min.$ |
| [37] | TITO systems: binary distillation column by [32] and an experimental module with four coupled tanks | Auto-tuning FOPI, IO-PI | Distilled top and bottom composition, level tanks | Output flow tanks | IAE, ITAE, ISE, TV * |
| [61,62] | TITO system of coupled tanks | FO-$[PI]^{\lambda}$, FO-$PI^{\lambda}$, IO-PI, IO-PID, FO-PI | Level $h_1$, $h_2$ | Flow rates of inlet streams | ST *, OS *, IAE, NCI *, MAV *, Bode diagram |
| [58] | Laboratory scale pressure and level control modules | FO-PID Fuzzy, IO-PID | Level and pressure | Not specified | ISE, IAE, ITAE, ITSE, OS *, rise time, ST * |
| [51,52,55,56] | TITO system of coupled tanks | FF * FO-PI, FF * PI/PID/2DOF-PI/ 3DOF-PI, FF-DMAFOPI *, centralized FO-PID,FO-PID | Level $h_1$, $h_2$ | Flow rates of inlet streams | ISE, OS *, ST *, tracking error, rise time, IAE, ITAE |
| [53,54] | Experimental module with four coupled tanks for TITO problem | FO-IMC with Smith's predictor, IO-IMC, decoupled FO-PI, decentralized FO-PI, IMC | Level $L_1$, $L_2$ | Flow rates of inlet (voltage pumps) | Bode diagram, control effort, ISE |
| [50] | TSCT * | FO-PI, IO-PI | Level $h_1$, $h_2$ | Feed flow tanks | OS *, $t_s$ *, RT *, ISE, IAE |
| [48,49] | Several examples of FO-FOPDT * systems: TCT *, MTL *, MLS * | FO-PID, FO-PI, IO-PID, IO-PI | Output $y$, level tanks, ball position in the levitation module | Input $u$, feed flow tanks | ISE, IAE, ITSE, ITAE, stability analysis of time and frequency domain |

**Table 4.** *Cont.*

| References | Applications | Controller | CV * | MV * | PS * |
|---|---|---|---|---|---|
| [102] | Experimental TITO non-linear ball and plate system | Cascaded FO-SMC, SMC | Motion along the $x$ and $y$ axes | Applied current to the actuator | IAE, $t_s$ *, $t_{reach}$ *, $\bar{C}_h$ *, $\delta_{\sigma xy}$ *, $\delta_{\sigma\alpha\beta}$ * |
| [100] | MISO nonlinear system, light leds and sensor experimental model with Arduino 2560 board | FO-GESC *, IO-GESC, PID | Time responses of $y(t)$, indoor illumination | Energy consumption | Minimization of energy consumption |
| [99] | Laboratory scale pH neutralization TITO | FO-PID, IO-PID | pH and tank level | Speed Rotating pumps of $NaOH$ and $HNO_3$ | Control effort, ITAE |
| [98] | Reverse Osmosis Seawater Desalination Plant TITO | FO-PI, IO-PI | Permeate flow rate, permeate conductivity | Feed pressure, brine flow rate | OS *, $t_s$ *, frequency specifications |
| [75] | High dynamic engine testbeds TITO | CRONE MIMO | Engine speed (rpm) and torque (Nm) | Variation of current and throttle | Bode analysis and regression analysis for identification |
| [66] | RAM *, CTM * 2DOF | FO-PID MIMO and SISO | APM *, CAPTM * | Input $u$ not specified | Control force |
| [72,74] | Diesel engine 2 × 3 and 3 × 2 | Decentralized CRONE MIMO | Air-flow and boost pressure | EGR *, WG *, $th$ * | Bode plots and Nichols chart, robustness cost function |
| [73] | Engine–dynamometer test-bed: diesel engine 3 × 3 | CRONE MIMO based on 3$th$ generation | Mass air flow, boost pressure, NOx emissions | EGR *, geometry turbine, start of injection | Bode diagram of the real system |
| [25] | MIMO lightly damped plant: aircraft wing model 2 × 2 | CRONE MIMO based on 3$th$ generation | Wing vibrations | Vibration attenuators | Frequency domain and the Bode diagrams |
| [23] | Hexapod robot 2 × 2 | FO-PD, IO-PD | Motion trajectories of the multi-legged robot | Joint torques | Nyquist plots, indexes based on robot dynamics, indexes based on hip trajectory tracking errors |
| [22] | Electronic circuit 2 × 2 | Adaptive control with AFROH * | Voltage (V) output 1 and 2 | Voltage (V) input 1 and 2 | Zeros of the discrete-time |

* CV: controlled variable; MV: manipulated variable; PS: performance index and/or stability analysis; EMBFL: experimental module composed of box, fan, lamp and sensors for TITO problem; IPN: Immersed plate in a Newtonian fluid; LML: LTI unstable Magnetic Levitation system; CIP: cart-inverted pendulum SIMO system; TRM: twin rotor MIMO system; OS: overshoot, ST: settling time, FLTF: frequency response of loop transfer function; TV: total variation of manipulated variable; NCI: normalized control input; MAV: maximum absolute value of control input; FF: feedforward; FF-DMAFOPI: Dual-mode adaptive FO-PI with feedforward; CFO-PID: centralized FO-PID; C/D-PI: centralized and decentralized PI; [•]: objective function proposed by authors; TSCT: TITO process with two spherical coupled tanks and FOPDT model; OS: overshoot; RT: rise time; $t_s$: settling time; TCT: TITO coupled tanks; MTL: multi-tank laboratory system; MLS: magnetic levitation system; $t_{reach}$: reaching time (time within which output first reaches the reference); $\bar{C}_h$: average chattering magnitude; $\delta_{\sigma xy}$, $\delta_{\sigma\alpha\beta}$: average deviation of the sliding variables from zero in the steady state for the outer loop and for the inner loop; RAM: robot arm manipulator; CTM: car with translational manipulator; APM: angular positions of a manipulator; CAPTM: car and arm positions of a translational manipulator; FO-GESC: fractional-order gradient-based extremum seeking control; EGR: exhaust gas recirculation; WG: wastegate; $th$: throttle valve.

## 4. FO-MIMO Controller Scheme and Topology

The fractional or integer MIMO control can be classified in two basic ways.

- Multi-loop control: multiple simple loops in a single control strategy.
- Multivariable control: a complex loop control where each manipulated variable is adjusted based on the error of all controlled variables.

Fractional order SISO control is in a considerably developed stage. Mainly since the 1990s, several publications are available in the literature. References [8,13,129] reported interesting results, aimed at using fractional-order controllers of the type $PI^{\lambda}D^{\mu}$. Consequently, both the integral and derivative elements had an arbitrary (non-integer) order. The transfer function in the Laplace domain takes the following form.

$$G_C(s) = \frac{U(s)}{E(s)} = K_P + K_I s^{-\lambda} + K_D s^{\mu}, \quad (\lambda,\ \mu > 0) \tag{3}$$

where $K_P$, $K_I$, $K_D$ are the proportional, integrative and derivative tuning parameters for proper controller operation.

Some references, such as [83,130–132], reviewed relevant applications and frameworks of the FO-PID control. Shah and Agashe [130] discussed advances and computational tools developed for the design of FO-PID controllers. Tepljakov et al. [131] reported industrial applications and recent contributions to the FO-PID approach and analyzed the advantages of fractional controllers over classic integer-order controllers in real-time implementations. Chevalier et al. [133] presented a new method for tuning FO-PID controllers to make their use in real-time more attractive and convenient for industries, the proposed method of tuning reduces the number of parameters from five to just three to be calculated, which significantly reduces the complexity of the control system design. The authors tested 133 simulated SISO processes plus two experimental systems to prove the performance of the proposed method.

Recently, Birs et al. [132] analyzed the recent advances in structures and types of tuning methods for fractional-order control systems. The authors presented various existing tuning methods for FO-PID controllers and their extensions to fractional order and advanced control strategies.

In the case of FO-MIMO control systems design, the first investigations addressed in the literature refer to applications of the CRONE methodology developed by Oustaloup [10]. In this context, as mentioned before, Lanusse et al. [20,21] applied to MIMO uncertain LTI systems for multi-scalar, multi-SISO and MIMO CRONE control design, but still using simulation studies. Some references, such as [24,25,31,73,88,134], used the third generation of the CRONE methodology in several MIMO systems. Lamara et al. [72,74] designed fractional controllers for MIMO systems using an experimental diesel engine. Many references, such as [65,67,69,106–108], reported simulation studies regarding the use of the CRONE plus QFT control approach to MIMO and SCARA robot systems. Finally, Table 5 presents the papers addressing the CRONE control methodology, originally developed and applied in references [10–12,28–30].

**Table 5.** Papers on CRONE control methodology.

| References | Applications | Controller | Experimental, Simulation or Both? |
|---|---|---|---|
| [112] | Refrigeration system TITO | CRONE decentralized, PID | Simulation |
| [65,67,69] | SCARA robot model TITO | CRONE and QFT MIMO using fractional prefilter of type Davidson Cole | Simulation |
| [75] | High dynamic engine testbeds TITO | FO-MIMO CRONE Control-System Design (CSD) | Experimental and Simulation |
| [72,74] | Diesel engine, $2 \times 3$ and $3 \times 2$ | Decentralized CRONE | Experimental and Simulation |
| [106–108] | MIMO systems, $3 \times 2$ and $2 \times 2$ | MIMO-QFT multi-SISO CRONE and Davidson–Cole prefilter | Simulation |
| [24,31,73,134] | Engine–dynamometer test-bed $3 \times 3$, distillation column by [39,118] $2 \times 2$, MIMO plants with time-delay, active-suspension system $2 \times 2$ | Third generation CRONE | Experimental and Simulation |
| [88] | Aluminum metal rod TITO system | Third generation CRONE, IO-PID | Simulation |
| [25] | MIMO lightly damped plant: aircraft wing model $2 \times 2$ | Third generation CRONE | Experimental and Simulation |
| [20,21] | MIMO uncertain LTI system | Multi-scalar, multi-SISO and MIMO CRONE | Simulation |

Several manuscripts explored control algorithms of the family $PI^{\lambda}D^{\mu}$, $PD^{\mu}$, $PI^{\lambda}$ in SISO and MIMO loops. Recent studies [36,37,40,51,52,57,60–64,103,105,122] of the FO-PI multi-loop control have received attention. Other authors have also investigated the FO-PI structure and comparison with IO-PI, such as Pradeepkannan and Sathiyamoorthy [50] and Feliu-Batlle et al. [98], who applied their fractional control proposals to a process with two spherical coupled tanks and a reverse osmosis seawater desalination plant, respectively, being both applications described by TITO models.

The references [33,34,54,96] explored the simulation and experimental studies regarding the design of FO-PI controllers compared to IO-PI. Muresan et al. [42–44] reported the incorporation of Smith's predictor into the fractional controller, where the authors studied the control of an isotope separation plant with dimension $3 \times 3$. Several researchers, as [35,38,48,49,51,52,57,58,60,63,64,66, 71,78,80,82,85,91,93,99,101,104,109,110,120,121], designed different FO-PID controllers and applied to MIMO systems with and without experimental validation. These works successfully show the superior performance of the fractional-based controllers. Tepljakov et al. [131] reported some applications on industrial-scale systems. Other references, such as [51,52,57,60,63,64,66,104,105], explored the use of real-time experimental modules, among others. Finally, it is important to mention different works [23, 46,47,68,70] that focused on the application of fractional controllers of the type $PD^{\alpha}$. Table 6 presents a survey and comparison of the publications reviewed including the application of FO-PID/PD/PI control structures.

The number of robust and advanced control systems incorporating fractional structures in the design control has been increasing [83,84,113,124]. Some references, such as [59,83,84,114,124], explored Internal Model-based FO control systems (FO-IMC) considering MIMO systems. Besides, references [53,79,81,113] reported the use of FO-IMC controllers with Smith's predictor. The works of [89,90,102,111] developed and applied FO sliding mode control (FO-SMC) techniques. In other publications, Li et al. [84] and Roy and Roy [55,56] adopted the FO feedforward hybrid control.

Some works, for example, references [22,79,81,86,92,126], studied and applied adaptive and advanced controllers in the design of FO-MIMO control systems. Finally, several references, such as [22,89,95,119,125,126,135], explored many novel approaches of FO robust and advanced control based on different structures. Table 7 lists a comparative survey of papers reported in the literature concerning FO-MIMO control with robust and advanced control approaches.

**Table 6.** Papers on FO-PI, FO-PD, FO-PID control.

| References | Applications | Controller | Experimental, Simulation or Both? |
|---|---|---|---|
| [105] | Experimental module composed of box, fan, lamp and sensors for TITO problem | FO-PI, IO-PI | Experimental and Simulation |
| [104] | Immersed plate in a Newtonian fluid, LTI unstable Magnetic Levitation system, cart-inverted pendulum SIMO system, twin rotor MIMO system | 2-DOF FO-PID, IO-PID | Experimental and Simulation |
| [80,82,85] | FO-FOPDT model TITO | PID with FO-EOTF *, optimal FO-PID | Simulation |
| [51,52,57,60,63,64] | Coupled conical tanks TITO, flash distillation column by [116], module with four coupled tanks for TITO problem | MCFOPID *, FO-PID, IMC, FO-PI, CFO-PI *, C-PI *, D-PI *, CFO-PID * | Experimental and Simulation |
| [103] | Experimental test bench for an office lighting system 8 × 8 | FO-PI, IO-PI | Experimental and Simulation |
| [38] | Binary distillation column by [32] | Multi-loop FO-PID | Simulation |
| [71] | FO/IO MIMO plants with time delays, wheeled mobile robot | FO-PID, IO-PID | Experimental and Simulation |
| [37] | TITO systems: binary distillation column by [32] and an experimental module with four coupled tanks | Auto-tuning FOPI | Experimental and Simulation |
| [61,62] | TITO system of coupled tanks | FO-$[PI]^{\lambda}$, FO-$PI^{\lambda}$, IO-PI, IO-PID, FO-PI | Experimental and Simulation |
| [36] | FO-TITO systems: binary distillation column by [32] and thermal system by [117] | FO-PI and IO-PI multiloop | Simulation |
| [68,70] | SCARA * robot model TITO | FO-PD and QFT, $PI^{\alpha}D$ and QFT | Simulation |
| [101] | Synchronous generator excitation system | Cascaded IO-PID, FO-PID | Simulation |
| [40] | Distillation column by [41] 3 × 3 | FO-PI, optimal PI | Simulation |
| [122] | Two models of heterogeneous TITO systems | FO-PI (cross-gain method), FO-PI (KC method), FO-IMC | Simulation |
| [99] | Laboratory scale pH neutralization TITO | FO-PID, IO-PID | Experimental and Simulation |
| [50,98] | Reverse Osmosis Seawater Desalination Plant TITO, process with two spherical coupled tanks TITO | FO-PI, IO-PI | Experimental and Simulation |
| [120] | Inverted Pendulum system | FO-PID multi-controller approach, IO-PID | Simulation |
| [58] | Laboratory scale pressure and level control modules | FO-PID Fuzzy, IO-PID | Experimental and Simulation |
| [96] | Single stage ore milling plant | FO-PI, FOMRAC * | Experimental and Simulation |
| [54] | Experimental module with four coupled tanks for TITO problem | Decoupled FO-PI, DFO-PI *, IMC | Experimental and Simulation |
| [42–44] | $^{13}C$ isotope separation plant 3 × 3 | FO-PI with Smith's predictor, FO-PI, IO-PI | Simulation |
| [66] | Robot armt manipulator 2DOF, car with translational manipulator 2DOF | FO-PID MIMO and SISO | Experimental and Simulation |
| [109,110] | FO-LTI MIMO systems and Hypersonic vehicle | $PI^{\lambda}D^{\mu}$ nonlinear, FO-PID and SOF * | Simulation |
| [48,49] | FO-FOPDT * systems: TITO coupled tanks, multi-tank laboratory system, magnetic levitation system | FO-PID, FO-PI, IO-PID, IO-PI | Experimental and Simulation |
| [35] | Binary distillation columns by [32] (2 × 2) and by [41] (3 × 3) | FO-PI, FO-PID, IO-PI, IO-PID | Simulation |
| [93] | VAC pilot system TITO model by [78] | $H_{\infty}$-FOPID, GA-FOPID | Simulation |
| [121] | Twin rotor TITO system | FO-PID, IO-PID | Simulation |

**Table 6.** *Cont.*

| References | Applications | Controller | Experimental, Simulation or Both? |
|---|---|---|---|
| [91] | Expansion turbine in the cryogenic air separation TITO process | FO-PID, IO-PID | Simulation |
| [78] | TITO plant with time delay proposed by [128] | FO-PID, IO-PID from literature | Simulation |
| [34] | Binary distillation column by [32] | FO-PI, IO-PI | Simulation |
| [46,47] | Coupled tanks 2 × 2, 2-DOF polar robot manipulator | PD-SMC, FO-PD, FO-PID, SMC, FO-PD, Fuzzy-SMC | Simulation |
| [136] | Two common systems in the petrochemical industry with multivariable parameter estimation | P, PI, PD and PID of fractional order and FO-IMC | Simulation |
| [33] | Binary distillation column by [32] | Decentralized and centralized FO-PI, IO-PI | Simulation |
| [23] | Hexapod robot 2 × 2 | FO-PD, IO-PD | Experimental and Simulation |

* FO-EOTF: open-loop fractional transfer function method; CFO-PI: centralized FO-PI; C-PI: centralized PI; D-PI: decentralized PI; CFO-PID: centralized FO-PID controller; MCFOPID: Multivariable Centralized Fractional Order PID; SCARA: selective compliance robot assembly; FOMRAC: fractional order model reference adaptive controllers; DFO-PI: decentralized FO-PI; SOF: static output feedback; FOMRAC: fractional order model reference adaptive controllers.

**Table 7.** Papers on robust and advanced fractional order controllers.

| References | Applications | Controller | Experimental, Simulation or Both? |
|---|---|---|---|
| [84] | FO-TITO system proposed by [117] | FFO-IMC, FO-PID | Simulation |
| [124] | LTI FO-MIMO plant 3 × 2 | GL-IMC | Simulation |
| [113] | Heavy oil fractionator by [115], distillation column by [32], flash distillation column by [116] | IMC FO-PI/PID and Smith predictor | Simulation |
| [83] | FO-TITO system by [117,123] | FO-IMC and FO-PID | Simulation |
| [45] | Isotopic separation column cascade 9 × 9 | FO observer and FO-PI | Simulation |
| [79,81] | FO-MIMO FOPDT model 3 × 3 and 3 × 2 | FO-DDMC, DDMC *, IMC and FO-Smith predictor | Simulation |
| [102] | Experimental TITO non-linear ball and plate system | Cascaded FO-SMC, SMC * | Experimental and Simulation |
| [114] | TITO system with transfer function matrix by [137], distillation column according by [41] | IMC-PID FO-Filter | Simulation |
| [135] | Chaotic system of $n$-dimensional fractional order with time delay model | MS-DE * | Simulation |
| [59] | Transfer function matrix of coupled conical tanks | IMC-PID, FO-IMC-$PID^2$, FO-IMC-$PID^5$ | Simulation |
| [126] | Generic FO-TITO system | MRAC *, CMRAC * | Simulation |
| [119] | Wind turbine with space state model 2 × 5 | MPC-LPV * and LFT * | Simulation |
| [125] | TITO LTI system in state space defined by authors | EMPC * | Simulation |
| [97] | Air-conditioning VAV * system 2 × 2 | PSO *-$PI^{\lambda}D^{\mu}$, PSO-PID, GA *-PID | Simulation |
| [55,56] | TITO system of coupled tanks | FF * FO-PI, FF * PI/PID/2DOF-PI/3DOF-PI, FF-DMAFOPI * | Simulation |
| [95] | Aircraft roll-dynamics, servomotor velocity-dynamics 3 × 3 | LTR-FOKF * | Simulation |

**Table 7.** *Cont.*

| References | Applications | Controller | Experimental, Simulation or Both? |
|---|---|---|---|
| [111] | State space model single input 1 × 3 and multiple inputs 3 × 3 | Sliding-mode-based fractional control and PI based | Simulation |
| [53] | Experimental module with four coupled tanks for TITO problem | FO-IMC with Smith's predictor, IO-IMC | Experimental and Simulation |
| [92] | Non-linear MIMO systems: generic system 2 × 2 and a chaotic 3D saturated multiscroll system 3 × 3 | Adaptive FO-FUZZY and IO-FUZZY | Simulation |
| [90] | TITO permanent magnet synchronous motor (PMSM) | FOSMC * | Simulation |
| [89] | Test bench involving long aluminum rod heated from one of its sides 3 × 3 | FSMC *, MSTSSMC * | Simulation |
| [86] | Differential hydraulic cylinders | FO adaptive control | Simulation |
| [22] | Electronic circuit 2 × 2 | Adaptive control and AFROH * | Experimental and Simulation |

* GL-IMC: Grünwald–Letnikov Inverse Model Control; FF: feedforward; FF-DMAFOPI: dual-mode adaptive FO-PI with feedforward; DDMC: DDMC: distributed PID-type dynamic matrix control; SMC: sliding mode control; MS-DE: multi-selection differential evolution method; MRAC: model reference adaptive control; CMRAC: composite model reference adaptive control; LPV: linear parameter-varying; LFT: linear fractional transformation; EMPC: energy-based minimization of the perfect control inputs; PSO: particle swarm optimization; GA: genetic algorithm; LTR-FOKF: loop transfer recovery control with fractional Kalman filter; FOSMC: fractional order sliding mode controller; FSMC: first-order sliding mode approach; MSTSSMC: modified version of the "super-twisting" second-order sliding mode control algorithm; AFROH: an approximate fractional order hold.

## 5. Tuning Methods and Techniques

When tuning the parameters of a controller, regardless of its integer or fractional order structure, the criterion of loop stability should be initially satisfied by having stable poles. Ideally, a closed-loop control system should meet the following performance criteria [6]:

- Stability of responses in closed-loop;
- Rejection to disturbances;
- Set-point tracking;
- Elimination of offset errors;
- Robustness, avoiding saturation in control actions.

Typically, the first PID controller tuning attempt regards the use of Ziegler and Nichols [138] rules. The literature [6] also reports other basic tuning rules, such as the direct synthesis (DS) method, internal model control (IMC) method, and controller tuning relations using frequency response techniques. According to the type of process transfer function model, such as first order plus time delay (FOPTD), second order plus time delay (SOPTD) models, and high order models, O'Dwyer [139] reported a list of several PI and PID tuning rules. Seborg et al. [6] also highlighted specific tuning rules for multi-loop PID-MIMO control systems, such as auto-tuning relay, detuning method [116], sequential loop tuning method [140]; independent loop method [141] and Skogestad and Morari [141,142]. Tuning methods based on closed-loop error minimization criteria are also popular in real-time applications, as found in references [6,139,143,144]. Expressions of the closed-loop error performance indexes take the following forms:

- Integral of the absolute value of the error (IAE):

$$IAE = \int_0^{\infty} |e\,(t)|dt \tag{4}$$

- Integral of the squared error (ISE):

$$ISE = \int_0^{\infty} e^2\,(t)dt \tag{5}$$

- Integral of the time-weighted absolute error (ITAE):

$$ITAE = \int_0^{\infty} t\,|e\,(t)|dt \tag{6}$$

Birs et al. [132] comprehensively reviewed the recent advances in FO-based control strategies for time-delay systems by evaluating and discussing suitable tuning techniques for FO-SISO controllers. The authors described the following methodologies for proper tuning of the FO-PID controllers with time delay:

- Frequency domain tuning method: determination the parameters of the controller by solving a system of nonlinear equations expressing specifications related to phase margin, gain crossover frequency, sensitivity functions and robustness to gain changes in a limited interval;
- Tuning methods based on time-domain cost functions and optimization routines: based on the minimization of the IAE, ISE and ITAE indexes.
- Fractional $M_s$ constrained integral optimization (F-MIGO) methods: fractional extension of the AMIGO (approximate $M_s$ constrained integral gain optimization) developed in references [145,146], where $M_s$ is sensitivity margin;
- Pontryagin and Hermite–Biehler theorems: theorem is described in references [147–149];
- Other tuning methods;
- Autotuning controllers.

Previously, Shah and Agashe [130] and Valério and da Costa [150], classified the methodologies for tuning fractional controllers into three different categories: (i) rule-based methods; (ii) analytical methods; and (iii) numerical methods, having characteristics similar to the descriptions of Birs et al. [132]. Table 8 deals with tuning methods and techniques based on heuristic and evolutionary approaches of multivariable optimization revised for FO-MIMO control.

For a better organization of the tuning techniques, Table 9 show the applications of several methods and techniques proposed in the revised papers that do not fully fit in the classification provided in Table 8.

**Table 8.** Papers on applications of tuning methods and techniques based on heuristic and evolutionary approaches.

| References | Applications | Tuning Methods | What Type FO-Controller? |
|---|---|---|---|
| [63,64] | Coupled conical tanks TITO | GA *, CS *, BA * | FO-PID multi-loop, FO-PI |
| [38] | Binary distillation column by [32] | DBA *, BA *, DiBA *, EBA *, PSO * | FO-PID multi-loop decentralized |
| [36] | FO-TITO systems: binary distillation column by [32] and thermal system by [117] | GA * | FO-PI |
| [71] | FO/IO MIMO plants with time delays, wheeled mobile robot | Minimum ITAE for equivalent transfer function and BA * | FO-PID |
| [45] | Isotopic separation column cascade $9 \times 9$ | PSO * | FO observer and FO-PI |

**Table 8.** *Cont.*

| References | Applications | Tuning Methods | What Type FO-Controller? |
|---|---|---|---|
| [101] | Synchronous generator excitation system | MEO * | Cascaded FO-PID |
| [57,59,60] | Coupled conical tanks | NBAO *, BA *, FGS *, HS * | FO-IMC-$PID^2$, FO-IMC-$PID^5$, centralized FO-PID, FO-PI, MCFOPID * |
| [58] | Laboratory scale pressure and level control modules | GA * | FO-PID Fuzzy |
| [96] | Single stage ore milling plant | PSO * | FO-PI, FOMRAC * |
| [79] | FO-MIMO FOPDT model $3 \times 2$ | DPP *, PSO * | IMC and FO-Smith predictor |
| [110] | Hypersonic vehicle 6-DOF model | NSPSO * | FO-PID |
| [51,52] | Module with four coupled tanks, two conical tank process | BA *, HS * | Adaptive multi-loop FO-PID, centralized FO-PID |
| [121] | Twin rotor TITO system | PSO * | FO-PID |
| [91] | Expansion turbine in the cryogenic air separation TITO process | GA * | FO-PID |
| [35] | Binary distillation columns by [32] and by [41] | Optimization with CMAES * and BLT * | FO-PI, FO-PID |
| [46] | 2-DOF polar robot manipulator, twin-tank model | GA * | FO-PD and SMC, FSMC * |
| [136] | Two common systems in the petrochemical industry with multivariable parameter estimation | MNDH * | FO-P/PI/PD/PID, FO-IMC |

* GA: genetic algorithm; CS: cuckoo search algorithm; BA: bat algorithm; DBA: dynamic BA; DiBA: directional BA; EBA: enhanced BA; PSO: particle swarm optimization algorithm; MEO: multiobjective evolutionary optimization; NBAO: novel bat optimization algorithm; HS: harmony search algorithm; DPP: dominant pole placement method; FGS: fuzzy gain schedulin method; NSPSO: based on a natural selection particle swarm algorithm; MNDH: multivariable nonlinear deterministic and heuristic optimization algorithms. MCFOPID: multivariable centralized fractional order PID; CMAES: covariance matrix adaptation evolution strategy; BLT: biggest log-modulus tuning algorithm; FOMRAC: fractional order model reference adaptive controllers; FSMC: fuzzy sliding mode controller.

**Table 9.** Papers on applications of miscellaneous tuning methods and techniques.

| References | Applications | Tuning Methods | What Type FO-Controller? |
|---|---|---|---|
| [105] | EMBFL * | IMC method and pole allocation method | FO-PI |
| [44,85] | FO-FOPDT model TITO, ISP * | Graphical tuning method | PID and FO-EOTF *, FO-PI |
| [21,24,25,31,73,75, 83,88,112,120] | RS *, FOSLD *, IPS *, HDE *, EDT *, AMR *, DCW *, AWM *, ASS *, MULTI * | Based on CRONE methodology through Oustaloup's approximation method | CRONE decentralized, FO-IMC/FO-PID, FO-PID, CRONE, CRONE 3*th* generation |
| [37,72,104] | IPN *, LUML *, CIP *, BDW *, EMCT *, Diesel engine | Method based on frequency domain, Response analysis of controlled variables in closed loop | 2-DOF FO-PID, Auto-tuning FO-PI, decentralized CRONE |
| [84,89,92,111,126] | FO-TITO system by [117], FO-TITO system generic, State space model SISO and MIMO, Non-linear MIMO systems, long aluminum rod | Maximum sensitivity method, stability analysis by Lyapunov's method | FFO-IMC *, FO-PID, MRAC *, CMRAC *, FOSMC *, adaptive FO-Fuzzy, FSMC *, MSTSSMC * |
| [103] | Test bench for an office lighting | Based on Kissing Circle (KC) method | FO-PI |
| [124] | LTI FO-MIMO plant | Energy-based approach to perfect control robustness | GL-IMC * |
| [113] | HOF *, DCW *, FDL * | Tuning rules proposed by authors | IMC FO-PI/PID and Smith predictor |
| [62] | Coupled tanks | Specifications in the frequency domain: phase margin, crossover gain frequency and constant speed error | FO-$[PI]^{\lambda}$, FO-$PI^{\lambda}$ |

**Table 9.** *Cont.*

| References | Applications | Tuning Methods | What Type FO-Controller? |
|---|---|---|---|
| [69,70] | SCARA * robot | New tuning method and frequency responses, based on local optimization of the fractional pre-filter parameters | FO-PD and QFT, CRONE and QFT |
| [81] | FO-MIMO FOPDT model | Nash optimization and Monte Carlo method | FO-DDMC, DDMC * |
| [102] | Non-linear ball and plate system | Lyapunov's finite time stability criterion and Oustaloup's recursive approximation method | Cascaded FO-SMC * |
| [40] | Distillation column by [41] | Biggest Log-modulus Tuning (BLT) algorithm with IMC | FO-PI |
| [61] | TITO system of coupled tanks | Two methods proposed on literature | FO-PI |
| [68,107] | SCARA * robot model, MIMO uncertain system | Multiobjective optimization FO with anoptimized fractional prefilter of type FBLFD * | $PI^{\alpha}D$ and QFT, CRONE 3$th$ generation |
| [100,125] | Light leds and sensor experimental model, LTI MIMO plant | Arbitrated without specified rules | FO-GESC *, FO-perfect control |
| [114] | TITO system by [137], distillation column by [41] | Three steps described in paper using the Bode method for optimal closed-loop transfer function | IMC-PID and FO-Filter |
| [122] | Heterogeneous TITO systems | Self-tuning cross-gain method and KC method | FO-PI (cross-gain method), FO-PI (KC method), FO-IMC |
| [48,65,78,80,99, 108,109,117,119] | LpHN *, FO-FOPDT model, WT *, CMT *, FO-MIMO systems, TTM * and DCW *, SCARA robot, PTD *, FO-LTI MIMO systems | FOMCON, FOTF, tuning, optimization, CRONE, Ninteger by [151], LMI * - toolboxes of MATLAB | FO-PID, optimal FO-PID, MPC-LPV * and LFT *, FO-PI, CRONE and QFT, $PI^{\lambda}D^{\mu}$ nonlinear |
| [135] | CNFO * | MS-DE * and DE * | MS-DE * |
| [67] | SCARA robot model | Davidson–Cole fractional prefilter optimisation | CRONE and QFT |
| [55,56] | Coupled tanks | Tuning performed using a numerical solution and methods found on literature and parameter estimation algorithm | FF * FO-PI, FF *-PI/PID/2DOF-PI/ 3DOF-PI, FF-DMAFOPI * |
| [42,44,54] | Experimental module with four coupled tanks, ISP * | Tuning algorithm developed by the authors for 1st and 2nd order models with time delay and Oustaloup's recursive approximation method, set of equations according to [42,152] | FO-IMC and FO-PI with Smith's predictor, decoupled FO-PI, decentralized FO-PI |
| [95] | Aircraft roll-dynamics, servomotor velocity-dynamics | By minimizing a cost function defined by the authors | LTR-FOKF * |
| [66] | Robot arm manipulator, car with translational manipulator | Trial and error method | FO-PID |
| [50] | Two spherical coupled tanks | Optimization by minimum search algorithm for ISE and Ziegler–Nichols method | FO-PI |
| [49] | Coupled tanks, multi-tank laboratory system, magnetic levitation system | Optimization by Levenberg–Marquardt and Nelder–Mead simplex algorithm, Ziegler–Nichols, Cohen–Coon and AMIGO algorithm | FO-PI, FO-PID |
| [74] | Diesel engine | Based on ad-hoc trial and error methods | Decentralized CRONE MIMO |
| [106] | FO-MIMO systems | Based on direct optimization of the closed loop output | CRONE and QFT * |

**Table 9.** *Cont.*

| References | Applications | Tuning Methods | What Type FO-Controller? |
|---|---|---|---|
| [34] | Binary distillation column by [32] | Method proposed by the authors | FO-PI |
| [77] | FO Duffing–Holmes chaotic systems | Tuned on line by output feedback control law and adaptive law by using Lyapunov synthesis approach | AITFSMC * |
| [31] | Non-square multivariable plants with time-delay | Based on the use of the BRG * for the pairing of the manipulated inputs and controlled outputs | CRONE 3th generation |
| [33] | Binary distillation column by [32] | Inequality method | Decentralized and centralized FO-PI |
| [87] | Long aluminum rod | Flatness principle using polynomial matrices for linear fractional MIMO systems | CRONE 3th generation |
| [23] | Hexapod robot | Systematic method when establishing a compromise between the minimization indexes | FO-PD |
| [22] | Electronic circuit | Improvement of the stability properties of the zeros | AFROH * |

* FBLFD: frequency band limited fractional differentiation; FO-GESC: fractional-order gradient-based extremum seeking control; LPV: linear parameter-varying; LFT: linear fractional transformation; LMI: linear matrix inequality toolbox; LpHN: laboratory scale pH neutralization; WT: Wind turbine; CMT: coupled multi-tank system; TTM: thermoelectric temperature control test module; DCW: distillation column by [32]; PTD: plant with time delay; CNFO: chaotic system of $n$-dimensional fractional order with time delay model; MS-DE: multi-selection differential evolution method; MRAC: model reference adaptive control; DE: differential evolution method; FF: feedforward; FF-DMAFOPI: dual-mode adaptive FO-PI with feedforward; ISP: isotope separation plant; LTR-FOKF: loop transfer recovery control with fractional Kalman filter; QFT: quantitative feedback theory; AITFSMC: adaptive interval type-2 Fuzzy sliding mode control; BRG: block relative gain; AFROH: an approximate fractional order hold.

## 6. Decoupling Techniques

One of the fundamental characteristics of multivariable control is the effect of coupling and interaction between SISO loops. Therefore the elimination of such effects inherently depends on the adequate design of the control system. Towards this, the use of decoupling techniques represents an essential alternative. The classical relative gain matrix represents a powerful way of assessing the decoupling efficiency.

Practical applications of decoupling techniques may not be physically feasible because of the uncertainties inherent in the process models incorporated in the loops or due to the intrinsic process dynamics [36,117]. Some references, such as [6,115,153], reported and detailed typical decoupling strategies for MIMO systems. A typical decoupler is composed of two controlled variables and two manipulated variables (TITO) with $1-1/2-2$ pairing [6]. Therefore, in this system there are two conventional controllers ($G_{c1}$ and $G_{c2}$) and two decouplers ($D_{12}$ and $D_{21}$). The decouplers compensate for the unwanted interaction effects between the multiple variables of the system. Table 10 reports different studies using decoupling techniques.

**Table 10.** Papers on decoupling techniques.

| References | Applications | Has Decoupling? |
|---|---|---|
| [105] | EMBFL * | Yes, unspecified type |
| [85] | FO-FOPDT model TITO | FO-EOTF * method |
| [104] | IPN *, LTIML *, CIP *, TRMS * | $D_{12}(s) = -\frac{G_{21}(s)}{G_{22}(s)}$, $D_{21}(s) = -\frac{G_{12}(s)}{G_{11}(s)}$ |
| [84] | FO-TITO system proposed by [117] | $D_{12}(s) = -\frac{G_{p12}(s)}{G_{p11}(s)}$, $D_{21}(s) = -\frac{G_{p21}(s)}{G_{p22}(s)}$ |
| [59,60,64] | TCCT *, FDL * | EOTF * method and decoupler by [118] where the extra time delay is incorporated |

**Table 10.** *Cont.*

| References | Applications | Has Decoupling? |
|---|---|---|
| [103] | Experimental test bench for an office lighting system | Static decoupler: $D = I_8 Q_0^{-1}$ |
| [61,113] | HOF *, DCW *, FDL *, two coupled tanks | F-SDSP *: $D_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)}$ $D_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)}$ |
| [51,52,83] | FO-TITO system by [117,123], two interacting conical tank | Inverted decoupler $D_{12}(s)$ and $D_{21}(s)$ |
| [37] | DCW * and an experimental module with four coupled tanks | Ideal decoupler: $D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}$ |
| [62] | TITO system of coupled tanks | $D_1(s) = -\frac{G_{12}(s)}{G_{11}(s)} = -\frac{0.0462(s+0.0275)}{0.2535(s+0.0255)}$ $D_2(s) = -\frac{G_{21}(s)}{G_{22}(s)} = -\frac{0.0499(s+0.0372)}{0.2457(s+0.0202)}$ |
| [36] | FO-TITO systems: binary distillation column by [32] and thermal system by [117] | $T_{21}(s) = -\frac{G_{p21}(s)}{G_{p22}(s)}$ $T_{12}(s) = -\frac{G_{p12}(s)}{G_{p11}(s)}$ |
| [45] | Isotopic separation column cascade | Decoupled models |
| [98] | Reverse osmosis seawater desalination plant | $D(s) = \begin{bmatrix} \hat{g}_{22}(s) & -\hat{g}_{12}(s) \\ -\hat{g}_{21}(s) & \hat{g}_{11}(s) \end{bmatrix} \begin{bmatrix} e^{\kappa_1 s} & 0 \\ 0 & e^{\kappa_2 s} \end{bmatrix}$ |
| [80] | FO-FOPDT model TITO | Pseudodiagonalisation method |
| [97] | Air-conditioning VAV * system | Diagonal matrix method: $W_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)}$ $W_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)}$ |
| [79] | FO-MIMO FOPDT model | Cascade decoupler: $G(s) K_d(s) = \begin{bmatrix} p_{11} & \\ & p_{22} \end{bmatrix}$ |
| [53,54] | Experimental module with four coupled tanks | From the approximate decoupled process transfer function matrix |
| [50] | Process with two spherical coupled tanks | $D_{12}(s) = -\frac{0.0462}{1.8550s+0.0881}$, $D_{21}(s) = -\frac{0.0489}{1.919s+0.0612}$ |
| [42–44] | $^{13}C$ isotope separation plant | Through the decoupled process transfer function matrix and the inverse of the steady state gain matrix |
| [35] | Binary distillation columns by [32] and by [41] | Through the CMAES * algorithm |
| [117] | TITO system of a thermoelectric temperature control test module, distillation column by [32] | Simplified, ideal and inverted decoupler |
| [91] | Expansion turbine in the cryogenic air separation | Simplified decoupler |
| [72,74,75,112] | Diesel engine, refrigeration system, high dynamic engine testbeds | Through CRONE methodology, matrix: $\beta_0(s)$ |
| [65,106] | SCARA robot model, TITO systems | Through CRONE methodology, matrix: $\beta_0(s)$ |
| [89] | Long aluminum rod | Yes, unspecified type |
| [25,31] | Distillation column by [39,118], aircraft wing model | Through CRONE methodology, matrix: $\beta_0(s)$ |
| [21,24] | Active-suspension system, MIMO uncertain LTI system | Perfect decoupling through CRONE methodology, matrix: $\beta_0(s)$ |

* EMBFL: Experimental module composed of box, fan, lamp and sensors for TITO problem; FO-EOTF: open-loop fractional transfer function method; IPN: immersed plate in a Newtonian fluid; LTIML: LTI unstable Magnetic Levitation system; CIP: cart-inverted pendulum SIMO system; TRMS: twin rotor MIMO system; TCCT: two coupled conical tanks; FDL: flash distillation column proposed by [116]; EOTF: effective open loop transfer function; F-SDSP: simplified decoupling Smith predictor structure with approximated fractional order processes; HOF: heavy oil fractionator by [115]; DCW: distillation column by [32]; VAV: variable air volume; CMAES: covariance matrix adaptation evolution strategy.

## 7. Software

Several reviewed papers used the MATLAB and Simulink software to perform the simulations and perform data acquisition from experimental modules. On the other hand, some references, such as [50,58,102], used the LabVIEW software together with MATLAB. Yin et al. [100] and Quadros [105] reported the use of data acquisition in an experimental module using Arduino.

*MATLAB*

Mondal and Dey [104] reported simulation studies and real-time experiments in a MATLAB-computational-based environment, aimed at implementing the FO-2DOF control structures in highly non-linear TRMS system and cart-inverted pendulum system. Lanusse and Tari [112] emphasized the advantages of applying the tool developed by Lanusse et al. [154] for a decentralized FO-MIMO CRONE control-system design (CSD), called CRONE CSD toolbox for MATLAB and made available for download since 2010. Li et al. [84] applied the MATLAB fsolve function to determine a correlation of the parameters of the fractional model in discrete time, it is not clear whether the authors used MATLAB for their simulations as well.

Lakshmanaprabu et al. [64] used MATLAB to implement the bat colony optimization algorithm (BA) for proper tuning the parameters of the FO-PI/PID controllers. Other works, such as [124,125,155], used the MATLAB/Simulink environment for simulation and optimization studies and the determination of the set of stable control poles based on the stable fractional-order perfect control law, confirming the potential of the proposed control approaches. Other simulations and experimental investigations used MATLAB/Simulink. Damodaran et al. [71] studied the control of a wheeled mobile robot. Baruah et. al [37] designed FO-PI/PID controller for a TITO system and also for a coupled tank system. Gurumurthy and Das [62] reported the control system of a four coupled tank system and also used MATLAB's systems identification tool to obtain the model of the TITO system of coupled tanks.

An investigation of FO-MIMO control using MATLAB, presented by Dulf and Kovacs [45], applied to the isotopic separation column cascade $9 \times 9$, where the LMI Toolbox in MATLAB was used to determine the linear matrix inequality (LMI) corresponding to the observer's gain. Other researchers used MATLAB's toolboxes, such as references [80,82], that applied fractional-order transfer function (FOTF) and multivariable frequency design (MFD) toolboxes to control FOPDT TITO systems. Chuong et al. [61] studied a coupled tank system. They used the toolboxes for experimental data acquisition. Aguiar et al. [99] implemented FO-PID control in a laboratory-scale pH neutralization TITO system. The FOMCON toolbox, proposed by Tepljakov et al. [156], tuned the controller. Tepljakov et al. [48] and Tepljakov [49] also reported the FOMCON toolbox. The authors explored different fractional controller designs and also used the toolbox to study coupled tanks and magnetic levitation systems.

Originally, Oustaloup and Melchior [11] and Oustaloup et al. [30] proposed the CRONE toolbox. Yousfi et al. [65,106,108] used the MATLAB optimization toolbox and CRONE toolbox, aimed at designing the third generation CRONE controllers to study to the control of SCARA robot and MIMO systems. For solving the fractional integration and differentiation, Li and Chen [117] used the aid of the MATLAB toolbox Ninteger developed by Valerio and da Costa [151] for TITO systems control studies. They proved the performance and applicability of fractional control with generalized decoupling from IO to FO. Chenikher et al. [78] used the Matlab2006b optimization toolbox for designing an FO-PID controller with robust stability and disturbance attenuation. Bucanovic et al. [91] reported the use of Simulink and MATLAB toolboxes to study fractional-order PID controllers. They studied an expansion turbine in the cryogenic air separation process.

Chekari et al. [83,114] implemented fractional IMC control algorithms based on PID and FO-PID in MATLAB environment, proving their proposals through numerical simulations to control TITO systems (see Tables 2, 3 and 7). In another study, Edet and Katebi [40] studied an FO-PI controller for a multivariable control problem applied to a distillation column with dimension $3 \times 3$. The authors used MATLAB to analyze the system stability and robustness. Other studies also explored the use of MATLAB/Simulink to implement their FO-MIMO control proposals, as reported in references [42,53,55,56,81,96]. Theses studies reported both simulation and experimental results.

Mori et al. [119] applied the MATLAB command pidTuner (Control System Toolbox) to design an integer order PI controller. The proposed fractional controller used the IO-PI as a comparative reference. Nasirpour and Balochian [97] used the particle swarm optimization (PSO) research toolbox in MATLAB to design the PSO-FOPID and compare it with PSO-PID and GA-PID. Furthermore, Song et al. [109] explored the application of the MATLAB linear matrix inequality (LMI) control toolbox for FO-PID and static output feedback (SOF) controllers design. Besides, Song et al. [110] presented the implementation of the natural selection particle swarm (NSPSO) algorithm for designing a non-linear FO-PID controller. The real-time software environment xPCTarget from MATLAB was applied by Nelson-Gruel et al. [73] for experimental investigations of FO-MIMO control in high dynamic engine-dynamometer test-bed. Finally, Jiacai et al. [90] used MATLAB/Simulink to perform their simulations of a permanent magnet synchronous motor (PMSM), aimed at exploring the fractional-order sliding mode controller (FOSMC).

## 8. Discussion and Conclusions

Based on the survey of the publications reviewed in this paper, it is possible to verify an increasing trend in the studies in the area of multivariable fractional control. Figure 1 exhibits the evolution of the publications reported between 1997 and 2019.

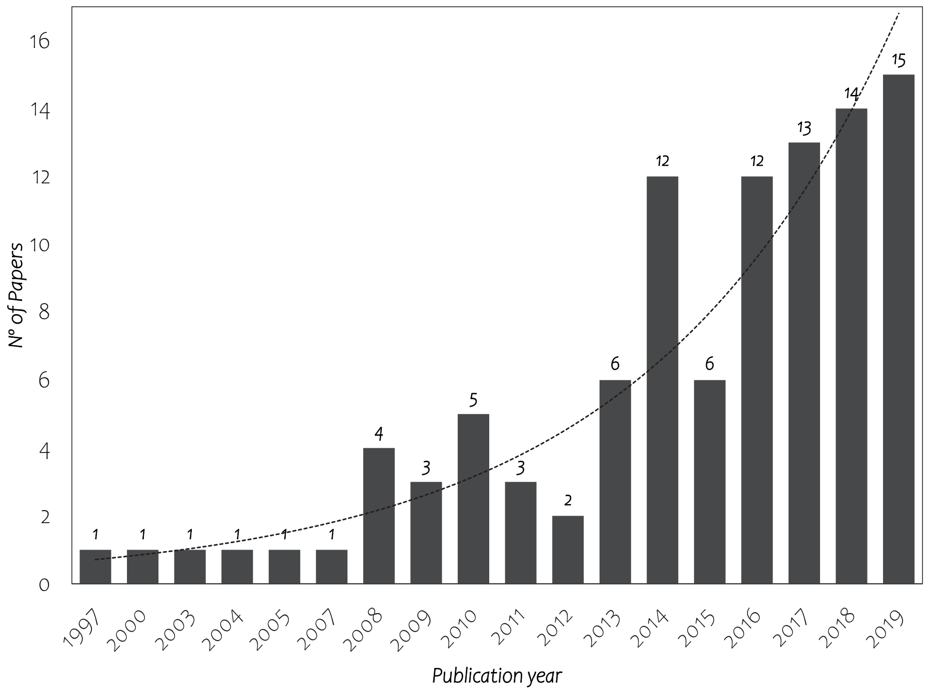


**Figure 1.** Number of publications on FO-MIMO per year.

This work reviewed the main advances in the application of fractional control techniques to MIMO systems. Recently, the works of Tepljakov et al. [131] and Chevalier et al. [133] approached some issues concerning the actual applications of fractional order controllers. In addition to the FO-MIMO control, Liu et al. [135] reported applications of multivariable identification techniques. However, investigations addressing fractional identification and incorporating fractional uncertainties and noises are still in a limited number.

Considering the works reviewed in this survey, 45.5% of the studies reported the use of MATLAB or Simulink software and toolboxes for MATLAB. On the other hand, 54.5% of the studies did not report the software used in simulations or data acquisition in experimental modules. Furthermore, by analyzing the results of Tables 2–4, 60.4% of the publications addressed studies only of simulations of fractional control proposals, without any experimental validation. Meanwhile, 39.6% investigated aspects in real-time and experimental studies to validate control proposals. Control studies FO-MIMO classified as advanced and robust, adding the CRONE methodology, comprise the majority of the researches, accounting for 56.4% of publications. Control structures based on FO-PID/PI/PD account for 43.6% of the papers published and reviewed here, as shown comparatively in Tables 5 and 6.

Based on the analysis of Tables 8 and 9, 20.8% of the reviewed papers applied evolutionary and heuristics techniques for tuning the FO-MIMO controllers. Other approaches, 79.2%, correspond to different methodologies and techniques developed by the authors [6,130,150,157]. Table 10 presents the results concerning the survey of different types of decouplers. Almost 40.6% of the works used and specified the decoupling system. However, 59.4% of the works did not mention or did not used decouplers.

Finally, real-time applications to industrial processes are still the main challenge fractional-order controllers need to overcome. Issues regarding stability and robustness demand further research, as well as stand-alone applications. The memory effects may be a limiting issue due to limited computational capabilities. There is still an open boundary regarding the use of analog fractional controllers. The tuning of the fractional-order controllers presents an evolution by incorporating heuristic and stochastic optimization algorithms, such as genetic algorithm (GA), cuckoo search algorithm (CS), bat algorithm (BA), and particle swarm optimization algorithm (PSO).

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